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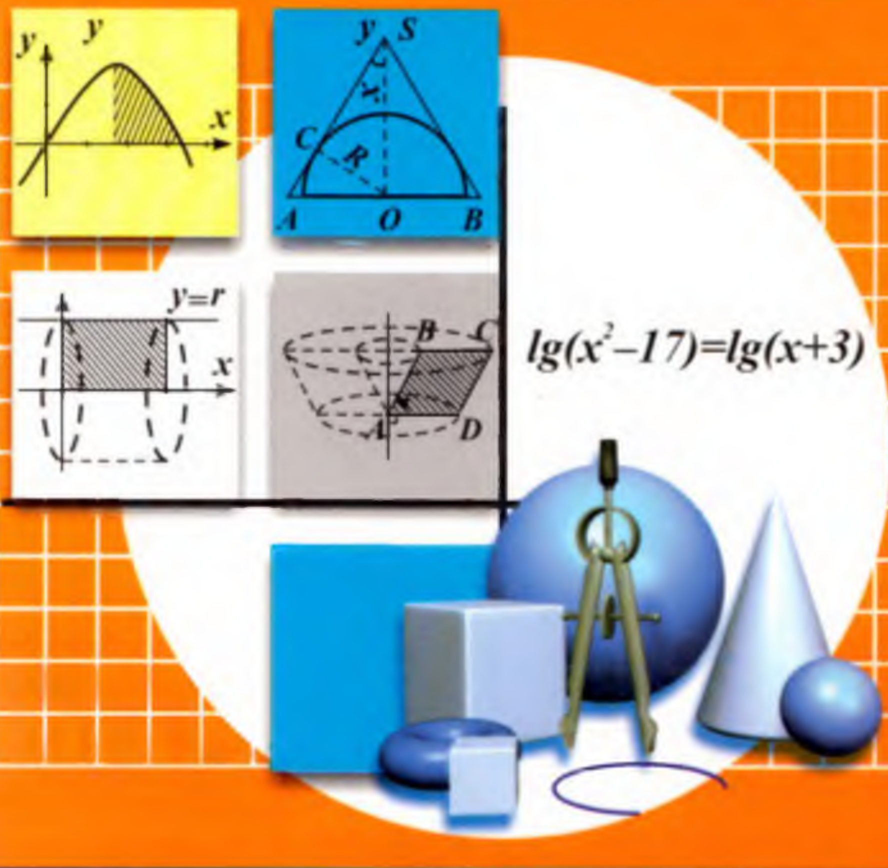
(qo'llanma)

II qism



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Taqrizchilar:

N. Qurbonov – Bo‘stonliq tumani XTMFMT va TEB mudiri,
K. Ashirqulova – Bo‘stonliq tumani XTMFMT va TEB metod markazi guruh rahbari,
M. Normatov – Bo‘stonliq tumani Matematiklarining uslubiyat seksiyasi rahbari.

Ortiqov M.

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Mazkur qo‘llanmadan, o‘qituvchilar, o‘rta umumta’lim maktab yuqori sinf o‘quvchilari, litsey va kollej talabalari va barcha qiziqqanlar foydalanishlari mumkin.

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SO'ZBOSHI

Bizlar hozirgi vaqtda axborot texnologiyalari asrida yashayapmiz. Bunda biz, albatta, zamonaviy taraqqiyotda fan va texnikaga suyanib olg'a boramiz. Bu taraqqiyotga ko'tarilishda dunyo olimlari va ajdodlarimiz yaratgan fan yangiliklari orqali muvaffaqiyatlarga erishib kelmoqdamiz. Bunday yangiliklarni yaratishda matematika fani asosiy o'rin tutadi.

O'rta bilim yurtlarini tamomlagan o'quvchilar zamonaviy texnologiyani o'zlashtirishda, ba'zi amaliy masalalarni yechishda va oliy o'quv yurtlariga kirishdagi sinov imtihonlari vaqtida algebradan olgan bilimlari yetarli emasligini his qilmoqdalar.

Bu qo'llanma o'quvchilarning yuqoridagi kamchiliklarga barham berish va mustahkam algebraik bilimga ega bo'lishini istaganlar uchun yaratildi.

Bu qo'llanmani o'rta maktabda ishlagan pedagogik davrida orttirgan pedagogik malakalarimiz va o'rta maktablar uchun chiqarilgan algebraik qo'llanmalar asosida yaratdik.

Mazkur qo'llanma II qism bo'lib, VI bob va 47-§ dan iborat. Bu qo'llanmada har bir mavzuga oid nazariy qism, masalalarning yechimlari, mavzuga oid savollar va mustaqil yechish uchun misol va masalalar berilgan.

Qo'llanmadan maktab o'quvchilari kollej, litsey o'quvchilari, oliy o'quv yurti qoshidagi tayyorlov kurslarining tinglovchilari to'liq foydalanishlari mumkin.

Muallif qo'llanmani yaratishda Toshkent viloyati, Bo'stonliq tumani matematiklarining uslubiyat seksiyasi rahbari, oliy toifali matematika

o‘qituvchisi M. Normetovga, Bo‘stonliq tumanidagi 20-umumta’lim maktabining 1-toifali matematika o‘qituvchisi A. Umirzoqovga, o‘zining qimmatli maslahatlarini bergan barcha o‘qituvchilarga minnatdorchilik bildiradi.

Hurmatli kitobxonlarning qo‘llanma haqidagi tanqidiy fikr va mulohazalarini chuqur mamnuniyat bilan qabul qilamiz.

Mualliflar

ALGEBRANI VUJUDGA KELISHIDAGI TARIXIY MA'LUMOTLAR

Insoniyat taraqqiyotining dastlabki bosqichlarida predmetlarni sanash arifmetikaning eng sodda tushunchalarini vujudga keltirdi. Og'zaki sanoq sistemasi asosida yozma sanoq sistemasi paydo bo'ldi va asta-sekin natural sonlar ustida to'rt arifmetik amalni bajarish takomillasha boshladi. Ayniqsa, qadimgi Misr va Bobilda arifmetika, geometriya fan sifatida shakllana boshlagan, savdo-sotiq va astronomiyaning rivojlanishi asosida algebra va trigonometriyaga oid ma'lumotlar to'plana boshlagan edi.

Qadimgi Misr va Bobilda matematik faktlar tarqoq holda bo'lgan bo'lsa, qadimgi Yunonistonda mantiqiy sistemaga solingan matematik bilimlar vujudga kela boshladi.

Qadimgi Yunonistonda matematikaning taraqqiyoti Fales va Pifagor nomi bilan bog'liq. Pifagor falsafiy maktabining vakillari matematik bilimlarni to'plab, sistemaga solishda, matematikaning ilk rivojlanishida katta o'rin tutdi.

Matematika qadimgi Yunon va ellinizm davri bilan bir vaqtda Xitoy va Hindistonda ham rivojlangan. Masalan, Chjan San va Szin Chou-channing «To'qqiz bobli arifmetika» asarida natural sonlardan kvadrat va kubildiz chiqarish qoidalari keltirilgan. Qadimgi Xitoy matematiklari asarlarida chiziqli tenglamalar sistemasini yechishda noma'lumni chiqarish usuli, Pifagor teoremasining arifmetik varianti, chegirmalarga oid masalalar uchraydi. Hind matematikasining rivojlanishi, asosan, V–XII asrlarda, yuqori bosqichga ko'tarilgan.

O'nli sanoq sistemasi, nol raqami, kvadrat irratsionalliklar va manfiy sonlar Hindistonda keng qo'llanilgan.

IX asrga kelib arab tilida ijod qilgan matematiklar matematik-astronomik jadvallar tuzishda, algebra, geometriya, trigonometriya sohasida qator yutuqlarga erishdilar.

IX asrda bizning vatandoshimiz, buyuk o‘zbek olimlaridan biri **Abu Abdulloh Muhammad ibn Muso Al-Xorazmiy** arab tilida algebra fanini mustaqil fan sifatida vujudga kelishi va rivojlanishida buyuk asos bo‘lgan «Al-kitob al-muxtasar fi hisob Al-jabr val-muqobala» **(«Al-jabr val-muqobala hisobi haqida qisqacha kitob»)** nomli risolasini yaratdi. Bu risola dunyoda birinchi marta algebrani sistemali ravishda bayon qilgan asar edi. Bu asar keyinchalik «Al-jabr val-muqobala» nomi bilan butun dunyoga mashhur bo‘ldi.

Kitob nomidagi «al-jabr» so‘zidan «algebra» so‘zi kelib chiqqan. «Al-jabr» so‘zi Yevropaga g‘arbiy arablar orqali o‘tgan. G‘arbiy arablar «j»ni «g» deb talaffuz qiladilar. Shu yo‘sinda «Al-jabr» Yevropada va keyinchalik, yer yuzining barcha joyida **«algebra»** deb yuritiladigan bo‘ldi.

Avvalo Muhammad Xorazmiy asaridagi «Al-jabr», «val-muqobala» terminlarining ma’nosini bilib olaylik. Bu arabcha terminlar algebradagi ikki amalni bildiradi:

«Al-jabr» – «to‘ldirish» – tenglamaning biror qismidagi ayriluvchi hadni uning ikkinchi tomoniga qo‘shiluvchi qilib o‘tkazish demakdir. Masalan, $3x^2-4x+2=2x^2+7$ tenglamaga «al-jabr» amali ishlatilsa, tenglama quyidagi ko‘rinishga keladi: $3x^2+2=2x^2+4x+7$.

«Al-muqobala» – «qarama-qarshi qo‘yish» – tenglamaning har ikkala qismidagi teng qo‘shiluvchilarni o‘zaro yeyishtirish (tashlab yuborish) demakdir. Oldingi tenglamaga «al-muqobala» amalini tatbiq etsak, u quyidagi ko‘rinishda bo‘ladi: $\mathbf{x^2=4x+5}$.

Bu asar juda ko‘p Yevropa tillariga tarjima qilingan. Ulardan eng qimmatlisi 1145-yili ingliz **Rober Chester** va 1160-yili italiyalik **Gerardo** qilgan lotincha tarjimalardir.

Bu asar bizgacha 1342-yili ko‘chirilgan arabcha nusxada yetib kelgan. U Oksford universiteti kutubxonasida saqlanadi. Muhammad Xorazmiyning «Al-jabr val-muqobala hisobidan qisqacha kitob» nomli asari rus tiliga matematika tarixchisi professor **B.A. Rozenfeld** tomonidan tarjima qilingan va 1964-yili O‘zSSR «Fan» nashriyotida chop etilgan.

Arifmetikada konkret sonlar ustida birinchi to'rt amal o'rganiladi. Algebrada esa bu amallarning har qanday son va son bo'lmagan boshqa matematik obyektlar uchun o'rinli umumiy xossalari tekshiriladi.

Bunda hosil qilinadigan natijalarning umumiy bo'lishiga erishish uchun miqdorlarning qiymatlari harflar bilan belgilanib, harfiy ifodalar ustida bajariladigan amallarning qoida va qonunlari ko'rsatiladi, ifodalar shaklini o'zgartirish va tenglamalarni yechish qoidalari o'rganiladi.

«Mashhur shoir va matematik **Umar Xayyom** algebrani tenglamalar yechish haqidagi fan deb ta'riflagan edi. Uning bu ta'rifi o'tgan asrgacha ham o'z kuchini yo'qotmay keldi.

1074-yilda Umar Xayyom «Al-jabr val-muqobala» masalalariga oid mashhur risolasini yozdi. Bu asar fransuz tiliga tarjima qilinishi bilan butun Yevropaga tarqaldi. Uning «Al-jabr» degan boshqa bir kitobida chiziqli va kvadrat tenglamalarni yechish, uchinchi darajali tenglamalarning ildizlarini geometrik usul bilan izlash va boshqa juda ko'p masalalarni yechish yo'llari ko'rsatilgan.

O'nli sanoq sistemasining Hindistondan butun dunyoga tarqalishi va takomillashuvida Xorazmiy, Beruniy, Koshiy va boshqalarning xizmati katta.

Tabobat ilmining buyuk arbobi **Abu Ali Ibn Sino** asarlarida ham o'sha zamon uchun alohida ahamiyatga ega bo'lgan arifmetika va algebra masalalarining yechimlari berilgan. Uning algebra va arifmetikaga oid ishlarida sonlarni kvadrat va kubga ko'tarish amallari tekshirilgan.

Shunisi diqqatga sazovorki, qadimgi dunyo tarixidan to Al-Xorazmiy davriga qadar matematika algebra va arifmetika kabi bo'limlarga ajralmagan edi. Faqat Al-Xorazmiy davridan boshlab, algebra matematikaning alohida bo'limi bo'lib ajraldi.

XV asr oxirida «+» (plyus) va «–» (minus) ishoralari algebraga kiritildi. Bundan keyingi davrda masalada qatnashadigan miqdorlar, shuningdek noma'lumlar harflar bilan belgilanadigan bo'ldi.

XVI asrda hozirgi zamon algebrasi uchun xarakterli bo'lgan harfiy formulalar birinchi marta paydo bo'ldi.

XVII asrda noma'lum sonlar uchun x, y, z, ... harflarni ishlatish **Dekartdan** boshlangan bo'lib, hozir ham shunday qilinmoqda.

aniqlanish sohasi [–1; 1] oraliqdan iborat bo‘ladi.

qiymatlari [0; π] oraliqdan iborat bo‘ladi.

$\cos\frac{\sqrt{2}}{2}=\frac{\pi}{4}$, chunki $\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$;

$=\frac{5\pi}{6}$, chunki $\cos\frac{5\pi}{6}=\frac{-\sqrt{3}}{2}$.

$8)=\pi-\arccos0,8608=180°-30°35'=149°25'$.

nksiya $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqda o‘sib, barcha qiymatlarni

gi) qabul qiladi. $y=\mathrm{tg}x$ funksiyaga $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ oraliqda

ya mavjud bo‘ladi.

siyani ***arktangens (arctg)*** deyiladi.

ing arktangensi deb $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ **oraliqqa tegishli iladiki, uning tangensi *a* ga teng bo‘lgan.**

aniqlanish sohasi (–∞; +∞) oraliqdan iborat.

qiymatlari $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqdan iborat.

$\mathrm{tg}\frac{\sqrt{3}}{3}=\frac{\pi}{6}$, chunki $\mathrm{tg}\frac{\pi}{6}=\frac{\sqrt{3}}{3}$;

$=-\frac{\pi}{3}$, chunki $\mathrm{tg}\left(-\frac{\pi}{3}\right)=-\sqrt{3}$.

$67°37'$, chunki $\mathrm{tg}67°37'=2,428$.

funksiya (0; π) oraliqda kamayib, barcha haqiqiy teng bo‘ladi. $y=\mathrm{ctg}x$ funksiyaga ham (0; π) oraliqda navjud bo‘ladi. Bu teskari funksiyani ***arkkotangens***

ning arkkotangensi deb (0; π) oraliqqa tegishli tiladiki, uning kotangensi *a* ga teng bo‘lgan.

Arkkotangensning aniqlanish sohasi barcha haqiqiy sonlardan iborat.

Arkkotangensning qiymatlari $(0; \pi)$ oraliqdan iborat.

Masalan: 1) $\mathrm{arcctg}\sqrt{3} = \frac{\pi}{6}$, chunki $\mathrm{ctg}\frac{\pi}{6} = \sqrt{3}$;

2) $\mathrm{arcctg}(-1) = \pi - \mathrm{arcctg}1 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ chunki $\mathrm{ctg}\frac{3\pi}{4} = ctg\left(\pi - \frac{\pi}{4}\right) = -\mathrm{ctg}\frac{\pi}{4} = -1$.

TAKRORLASH UCHUN SAVOLLAR

1. Qanday funksiyaga teskari funksiya mavjud?
2. $y=\sin x$ funksiyaga qaysi oraliqda teskari funksiya mavjud bo'ladi va u qanday yoziladi?
3. $y=\cos x$ funksiyaga qaysi oraliqda teskari funksiya mavjud bo'ladi va u qanday yoziladi?
4. $y=\mathrm{tg}x$ funksiyaga qaysi oraliqda teskari funksiya mavjud bo'ladi va u qanday yoziladi?
5. $y=\mathrm{ctg}x$ funksiyaga qaysi oraliqda teskari funksiya mavjud bo'ladi va u qanday yoziladi?

MASALALARNI YECHING

1. Tengliklar bajariladimi?

a) $\arcsin\frac{1}{2} = \frac{\pi}{6}$;

b) $\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$;

d) $\arcsin 1{,}2 = 0$.

e) $\mathrm{arctg}\sqrt{3} = -\frac{\pi}{3}$;

f) $\mathrm{arctg}(-1) = -\frac{\pi}{4}$

g) $\arccos\frac{1}{2} = -\frac{\pi}{3}$

2. Quyidagilarning qiymatini toping:

a) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$;

b) $\arcsin(-0{,}7801)$;

d) $\arccos\left(-\frac{1}{2}\right)$;

e) $\arccos 0{,}8033$;

f) $\arcsin\frac{1}{\sqrt{3}}$;

g) $\mathrm{arctg}(-2{,}7)$;

h) $\arccos\left(-\frac{\sqrt{3}}{3}\right)$;

i) $\mathrm{arcctg}(-3)$.

Arkkosinusning aniqlanish sohasi [–1; 1] oraliqdan iborat bo‘ladi.

Arkkosinusning qiymatlari $[0; \pi]$ oraliqdan iborat bo‘ladi.

Masalan: 1) $\arccos\frac{\sqrt{2}}{2}=\frac{\pi}{4}$, chunki $\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$;

2) $\arccos\left(-\frac{\sqrt{3}}{2}\right)=\frac{5\pi}{6}$, chunki $\cos\frac{5\pi}{6}=\frac{-\sqrt{3}}{2}$.

3) $\arccos(-0{,}8608)=\pi-\arccos 0{,}8608=180^\circ-30^\circ35'=149^\circ25'$.

III. Tangens funksiya $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqda o‘sib, barcha qiymatlarni ($(-\infty; +\infty)$ oraliqdagi) qabul qiladi. $y=\mathrm{tg}x$ funksiyaga $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ oraliqda ham teskari funksiya mavjud bo‘ladi.

Bu teskari funksiyani ***arktangens (arctg)*** deyiladi.

Ta'rif. ***a* sonning arktangensi deb** $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ **oraliqqa tegishli shunday sonni aytiladiki, uning tangensi *a* ga teng bo‘lgan.**

Arktangensning aniqlanish sohasi $(-\infty; +\infty)$ oraliqdan iborat.

Arktangensning qiymatlari $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqdan iborat.

Masalan: 1) $\mathrm{arc\,tg}\frac{\sqrt{3}}{3}=\frac{\pi}{6}$, chunki $\mathrm{tg}\frac{\pi}{6}=\frac{\sqrt{3}}{3}$;

2) $\mathrm{arctg}(-\sqrt{3})=-\frac{\pi}{3}$, chunki $\mathrm{tg}\left(-\frac{\pi}{3}\right)=-\sqrt{3}$.

3) $\mathrm{arctg}2{,}428\approx 67^\circ37'$, chunki $\mathrm{tg}67^\circ37'=2{,}428$.

IV. Kotangens funksiya $(0; \pi)$ oraliqda kamayib, barcha haqiqiy sonlar $(-\infty; +\infty)$ ga teng bo‘ladi. $y=\mathrm{ctg}x$ funksiyaga ham $(0; \pi)$ oraliqda teskari funksiya mavjud bo‘ladi. Bu teskari funksiyani ***arkkotangens (arcctg)*** deyiladi.

Ta'rif. ***a* sonning arkkotangensi deb $(0; \pi)$ oraliqqa tegishli shunday sonni aytiladiki, uning kotangensi *a* ga teng bo‘lgan.**

Arkkotangensning aniqlanish sohasi barcha haqiqiy sonlardan iborat.

Arkkotangensning qiymatlari $(0; \pi)$ oraliqdan iborat.

Masalan: 1) $\text{arcctg}\sqrt{3} = \frac{\pi}{6}$, chunki $\text{ctg}\frac{\pi}{6} = \sqrt{3}$;

2) $\text{arcctg}(-1) = \pi - \text{arcctg}1 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ chunki $\text{ctg}\frac{3\pi}{4} = ctg\left(\pi - \frac{\pi}{4}\right) =$ $= -\text{ctg}\frac{\pi}{4} = -1$.

TAKRORLASH UCHUN SAVOLLAR

1. Qanday funksiyaga teskari funksiya mavjud?
2. $y=\sin x$ funksiyaga qaysi oraliqda teskari funksiya mavjud bo'ladi va u qanday yoziladi?
3. $y=\cos x$ funksiyaga qaysi oraliqda teskari funksiya mavjud bo'ladi va u qanday yoziladi?
4. $y=\text{tg}x$ funksiyaga qaysi oraliqda teskari funksiya mavjud bo'ladi va u qanday yoziladi?
5. $y=\text{ctg}x$ funksiyaga qaysi oraliqda teskari funksiya mavjud bo'ladi va u qanday yoziladi?

MASALALARNI YECHING

1. Tengliklar bajariladimi?

a) $\arcsin \frac{1}{2} = \frac{\pi}{6}$;

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d) $\arcsin 1{,}2 = 0$.

e) $\text{arctg}\sqrt{3} = -\frac{\pi}{3}$;

f) $\text{arctg}(-1) = -\frac{\pi}{4}$

g) $\arccos \frac{1}{2} = -\frac{\pi}{3}$

2. Quyidagilarning qiymatini toping:

a) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$;

b) $\arcsin(-0{,}7801)$;

d) $\arccos\left(-\frac{1}{2}\right)$;

e) $\arccos 0{,}8033$;

f) $\arcsin\frac{1}{\sqrt{3}}$;

g) $\text{arctg}(-2{,}7)$;

h) $\arccos\left(-\frac{\sqrt{3}}{3}\right)$;

i) $\text{arcctg}(-3)$.

3 Yulduzchalar o‘rniga tenglik yoki tengsizlik belgisini qo‘ying, natijada to‘g‘ri mulohaza hosil bo‘lsin:

a) $\arccos\frac{1}{2} * \arcsin\frac{1}{2}$; d) $\arcsin\frac{1}{5} * \arcsin\frac{1}{6}$;

b) $\arcsin 0 * \arccos 0$; e) $\operatorname{arctg}2 * \operatorname{arctg}\sqrt{3}$;

f) $\arccos\frac{1}{2} * \arcsin\frac{\sqrt{2}}{2}$; g) $\operatorname{arctg}3 * \frac{\pi}{3}$.

4. Yig‘indining qiymatini toping:

a) $\arcsin\frac{1}{2} + \arccos\frac{1}{2}$;

b) $\arcsin(-1) + \arccos(-1)$;

d) $\arcsin\left(-\frac{\sqrt{2}}{2}\right) + \arccos\left(-\frac{\sqrt{3}}{2}\right)$;

e) $\arcsin 1 + \arccos 1 + \operatorname{arctg}1$;

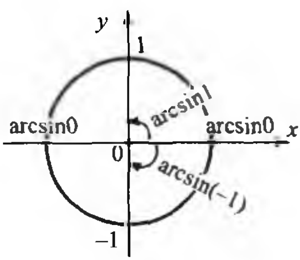
f) $\arcsin\left(-\frac{\sqrt{2}}{2}\right) + \arccos\left(-\frac{1}{2}\right) + \operatorname{arctg}\sqrt{3}$;

g) $\arcsin(-1) + \arccos(-1) + \operatorname{arctg}(-1) + \operatorname{arcctg}(-1)$.

2-§. Eng sodda trigonometrik tenglamalar

Eng sodda trigonometrik tenglamalar deb, $\sin x = a$; $\cos x = a$; $\operatorname{tg}x = a$; $\operatorname{ctg}x = a$ ko‘rinishdagi tenglamalarga aytiladi, bunda a – berilgan son.

Eng sodda trigonometrik tenglamalarni yechish, trigonometrik funksiyaning *a* qiymatiga ega bo‘luvchi hamma burchaklar to‘plamini topish demakdir.



1-chizma.

1. $\sin x = a$ tenglamani yechamiz.

Bu tenglama $a > 1$ va $a < -1$ da yechimga ega emas, chunki $-1 \leq \sin x \leq 1$.

$a = 1$ da $\sin x = 1$ bo‘lib, $x = \arcsin 1 = \frac{\pi}{2}$;

$a = -1$ da $\sin x = -1$ bo‘lib, $x = \arcsin(-1) = -\frac{\pi}{2}$;

$a = 0$ da $\sin x = 0$ bo‘lib, $x = \arcsin 0 = \pi$. va $x = \arcsin 0 = 0$. $x = 0$ va π.

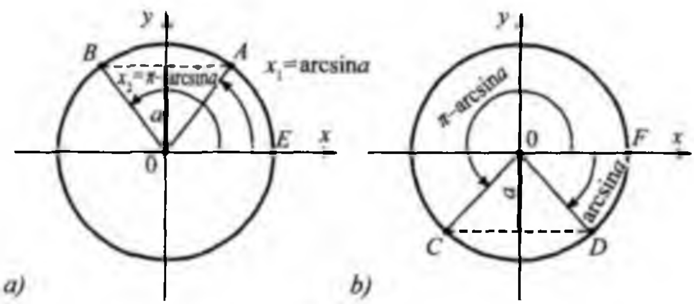
Bu ildizlar $\sin x=a$ tenglamaning boshlang‘ich yechimlari bo‘lib, umumiy yechimlari tenglamani qanoatlantiruvchi hamma burchaklar to‘plami esa, $\frac{\pi}{2}$ va $-\frac{\pi}{2}$ larga to‘la aylanishlarni (sinusning davrini) qo‘shish bilan hosil qilinadi, ya’ni **$\sin x=1$ da $x=\frac{\pi}{2}+2n\pi$; $\sin x=-1$ da $x=-\frac{\pi}{2}+2n\pi$; $\sin x=0$ da $x=n\pi$,** bunda n – butun son (1-chizma).

Agar $-1<a<1$ bo‘lsa, $\sin x=a$ tenglamani ikki holga ajratib yechamiz:

1-hol. $0<a<1$ da x burchak $0<x<\pi$ oraliqda topiladi.

Bu holda: $x_1=\arcsin a$ va $x_2=\pi-\arcsin a$ boshlang‘ich yechimlar topiladi. Umumiy yechimni topish uchun boshlang‘ich yechimlarga sinusning davri qo‘shib topiladi (2-a chizma), ya’ni $\left.\begin{matrix} x_1=\arcsin a+2n\pi \\ x_2=\pi-\arcsin a+2\pi \end{matrix}\right\}=$

$=\left.\begin{matrix} \arcsin a+2n\pi \\ -\arcsin a+(2\pi+1)\pi \end{matrix}\right\}=(-1)^k\arcsin a+k\pi$ kelib chiqadi.



2-chizma

Demak, **$\sin x=a$** ning ildizi **$x=(-1)^k\arcsin a+k\pi$,** bunda k – butun son.

2-hol. $-1<a<0$ da x burchak $-\pi<x<0$ oraliqda topiladi (2-b, chizma), ya’ni

$$\left.\begin{matrix} x_1=\arcsin a+2n\pi \\ x_2=\pi-\arcsin a+2n\pi=-\arcsin a+(2n+1)\pi \end{matrix}\right\}=\{(-1)^k\arcsin a+k\pi$$

kelib chiqadi.

Demak $\boldsymbol{x=(-1)^k \arcsin a+k\pi}$ (k – butun son).

1-misol. $\sin x = \frac{\sqrt{2}}{2}$ tenglamani yechamiz.

Umumiy yechim formulasiga asosan

$$x = (-1)^k \arcsin \frac{\sqrt{2}}{2} + k\pi = (-1)^k \cdot \frac{\pi}{4} + k\pi$$

Javob: $x = (-1)^k \frac{\pi}{4} + k\pi$ ($k \in z$, z – butun son).

2-misol. $\sin x = -\frac{\sqrt{3}}{2}$ tenglamani yechamiz.

$$x = (-1)^k \arcsin\left(-\frac{\sqrt{3}}{2}\right) + k\pi = (-1)^k \cdot \left(-\frac{\pi}{3}\right) + k\pi = (-1)^{k+1} \frac{\pi}{3} + k\pi (k \in z).$$

Javob: $x=(-1)^{k+1} \frac{\pi}{3}+k\pi(k \in z)$.

2. $\boldsymbol{\cos x=a}$ tenglamani yechamiz.

Bu tenglama $a>1$ va $a<-1$ da yechimga ega emas.

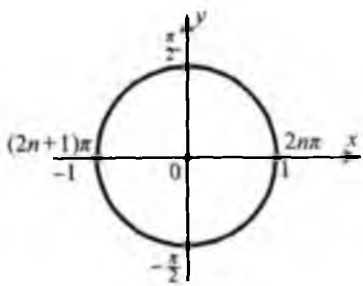
$a=1$ da $\cos x=1$ bo‘lib, $x=\arccos 1=2n\pi(n \in z)$

$a=-1$ da $\cos x=-1$ bo‘lib, $x=\arccos(-1)=\pi+2n\pi=(2n+1)\pi(n \in z)$.

$a=0$ da $\cos x=0$ bo‘lib, $x=\frac{\pi}{2}+n\pi(n \in z)$.

Agar $-1<a<1$ bo‘lsa, $\cos x=a$ tenglamani ikki holga ajratib yechamiz:

1-hol. $0<a<1$ da x burchak $-\frac{\pi}{2} < x < \frac{\pi}{2}$ oraliqda topiladi.



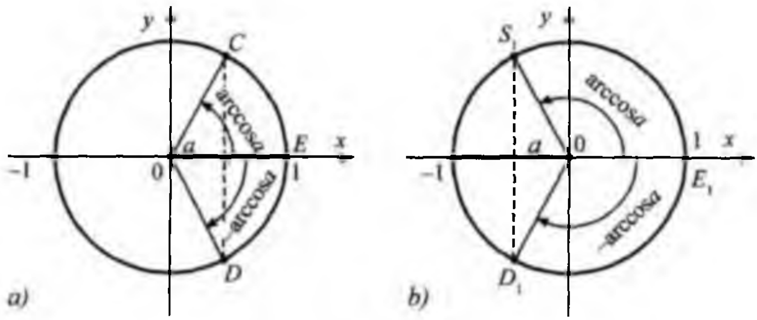
3-chizma

Kosinusi a ga teng bo‘lgan ikkita boshlang‘ich $x_1=\arccos a$ va $x_2=-\arccos a$ burchaklar mavjud.

Umumiy yechim esa, boshlang‘ich yechimga kosinusning davrini qo‘shib topiladi (4-*a*, chizma).

$$\left.\begin{array}{l} x_1 = \arccos a + 2n\pi \\ x_2 = -\arccos a + 2n\pi \end{array}\right\} \text{dan } x=\pm\arccos a+2k\pi.\ (k \in z).$$

Demak, $\cos x=a$ ning ildizi $x=\pm\arccos a+2k\pi(x \in z)$.



4-chizma.

2-hol. $-1<a<0$ da x burchak, x burchak $\frac{\pi}{2}<x<\frac{3\pi}{2}$ oraliqda topiladi (4-*b*, chizma).

Bu holda ham $\cos x=a$ tenglamaning ildizi $x=\pm\arccos a+2k\pi$ topiladi. Demak, $\boldsymbol{x=\pm\arccos a+2k\pi}$ **(*k*∈*z*).**

1-misol. $\cos x=\frac{1}{2}$ tenglamani yechamiz.

Topilgan formulaga asosan:

$x=\pm\arccos\frac{1}{2}+2k\pi=\pm\frac{\pi}{3}+2k\pi$ $(k\in z)$.

Javob: $x\pm\frac{\pi}{3}+2k\pi$ $(k\in z)$.

2-misol. $\cos 4x=0$ tenglamani yechamiz.

$a=0$ holda $4x=\frac{\pi}{2}+k\pi$ $(k\in z)$.

$x=\frac{\pi}{8}+\frac{k\pi}{4}$ $(k\in z)$. *Javob:* $\frac{\pi}{8}+\frac{k\pi}{4}$ $(k\in z)$.

3. **tg*x*=*a* tenglamani yechamiz.**

Tangens funksiya $\left]-\frac{\pi}{2};\frac{\pi}{2}\right[$ oraliqda o‘suvchi bo‘lgani uchun har bir a ning qiymatiga faqat bitta burchak mos kelgani uchun $a\geq 0$ bo‘lganda $0\leq \operatorname{arctg} a<\frac{\pi}{2}$ va $a\leq 0$ bo‘lganda $-\frac{\pi}{2}<\operatorname{arctg} a\leq 0$ oraliqda bo‘ladi.

Tangensni davri $k\pi$ bo'lgani uchun $\mathrm{tg}x=a$ tenglamaning ildizi

$x=\mathrm{artctg}\,a+k\pi$ $(k\in i)$ ga teng.

$a=0$ da $\mathrm{tg}x=0$ ildizi $x=k\pi$, $k\in z$.

Demak, **$\mathrm{tg}x=a$** ning ildizi **$x=\mathrm{arctg}a+k\pi$, $(k\in z)$**.

Masalan: 1) $\mathrm{tg}x=\sqrt{3}$ tenglamaning ildizini topamiz. Topilgan formulaga asosan:

$x=\mathrm{arctg}\sqrt{3}+k\pi=\frac{\pi}{3}+k\pi\cdot(k\in z)$.

Javob: $x=\frac{\pi}{3}+k\pi$ $(k\in z)$.

3) $\mathrm{tg}x=-1$ tenglamani yechamiz.

Formulaga asosan $x=\mathrm{arctg}(-1)+k\pi=-\frac{\pi}{4}+k\pi$ $(k\in z)$.

Javob: $x=-\frac{\pi}{4}+k\pi$ $(k\in z)$.

4. $\mathrm{ctg}x=a$ tenglamani yechamiz.

Kotangens funksiya $]0;\ \pi[$ oraliqda kamayuvchi bo'lgani uchun har bir a ning qiymatiga faqat bitta burchak mos keladi. $a\geq 0$ da $0<\mathrm{arcctg}\,a\leq\frac{\pi}{2}$, $a\leq 0$ da $\frac{\pi}{2}\leq\mathrm{arcctg}\,a<\pi$ oraliqda bo'ladi.

Kotangensning davri $k\pi$ bo'lgani uchun $\mathrm{ctg}x=a$ tenglamaning ildizi **$x=\mathrm{arcctg}\,a+k\pi$** $(k\in i)$ bo'ladi.

$a=0$ da $\mathrm{ctg}x=0$ ildizi $x=\frac{\pi}{2}+k\pi$, $k\in z$.

Masalan: 1) $\mathrm{ctg}x=\frac{\sqrt{3}}{3}$ tenglamani yechamiz.

Formulaga asosan: $x=\mathrm{arcctg}\frac{\sqrt{3}}{3}+k\pi=\frac{\pi}{3}+k\pi(k\in z)$;

Javob: $\frac{\pi}{3}+k\pi(k\in z)$.

2) $\mathrm{ctg}x=-\sqrt{3}$ tenglamani yechamiz.

Formulaga asosan

$x=\mathrm{arcctg}\,(-\sqrt{3})+k\pi=-\frac{\pi}{6}+k\pi(k\in z)$

yoki $\mathrm{ctg}\left(-\frac{\pi}{6}\right)=\mathrm{ctg}\left(\pi-\frac{\pi}{6}\right)=\mathrm{ctg}\frac{5\pi}{6}$, bo'yicha

$x=\frac{5\pi}{6}+k\pi \ (k\in z)$ ko'rinishda yozish mumkin.

Javob: $x=\pi-\frac{\pi}{6}+k\pi (k\in z)$, yoki $x=\frac{5\pi}{6}+k\pi (k\in z)$.

TAKRORLASH UCHUN SAVOLLAR

1. Qanday tenglamalarni eng sodda trigonometrik tenglamalar deyiladi?
2. Trigonometrik tenglamalarni yechish deganda biz nimani tushunamiz?
3. $\sin x=a$ tenglamaning ildizi formulasini yozing.
4. $\cos x=a$ tenglamaning ildizi formulasini yozing.
5. $\operatorname{tg} x=a$ tenglamaning ildizi formulasini yozing.
6. $\operatorname{ctg} x=a$ tenglamaning ildizi formulasini yozing.

MASALALARNI YECHING

5. Tenglamani yeching:

a) $\sin x=\frac{1}{2}$; f) $\cos x=\frac{\sqrt{3}}{2}$;

b) $\sin 2x=\frac{\sqrt{3}}{2}$; g) $\cos 2x=-\frac{1}{2}$;

d) $\sin\left(4x-\frac{\pi}{6}\right)=-0,5$; h) $\cos\frac{1}{3}x=\frac{\sqrt{2}}{2}$;

e) $\sin\left(2x-\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{2}$; i) $\cos(2x-1)=-\frac{1}{2}$.

6. Tenglamani yeching:

a) $\operatorname{tg} x=\frac{1}{\sqrt{3}}$; e) $\operatorname{ctg} x=\sqrt{3}$;

b) $\operatorname{tg} 2x=\sqrt{3}$; f) $\operatorname{ctg}\frac{1}{2}x=1$;

d) $\operatorname{tg} 3x=3,5$; g) $\operatorname{ctg}\frac{1}{5}x=0$.

7. Ayniyatni isbotlang:

a) $\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}$; b) $\frac{\sin\alpha+\operatorname{tg}\alpha}{1+\cos\alpha}=\operatorname{tg}\alpha$.

3-§. Trigonometrik tenglamalarni yechishga doir misollar

Eng sodda trigonometrik tenglamalardan boshqa birmuncha murakkabroq trigonometrik tenglamalar ham uchraydi.

Bunday tenglamalarni yechish uchun trigonometrik funksiyalarning holatlarini ifodalovchi formulalarni bilish talab qilinadi.

Ba'zi misollarni qarab chiqamiz:

1-misol. $\sqrt{3}\operatorname{ctg}\left(3x-\frac{\pi}{3}\right)+1=0$ tenglamani yechamiz.

Yechish. $\sqrt{3}\operatorname{ctg}\left(3x-\frac{\pi}{3}\right)=-1$ bundan $\operatorname{ctg}\left(3x-\frac{\pi}{3}\right)=-\frac{1}{\sqrt{3}}$;

$$\left(3x-\frac{\pi}{3}\right)=\operatorname{arcctg}\left(-\frac{1}{\sqrt{3}}\right)+k\pi;$$

$$3x-\frac{\pi}{3}=\pi-\operatorname{arcctg}\frac{1}{\sqrt{3}}+k\pi;$$

$$3x=\frac{\pi}{3}+\pi-\frac{\pi}{3}+k\pi;$$

$$3x=\pi+k\pi;$$

$$x=\frac{\pi}{3}+\frac{k\pi}{3}\ (k\in z).$$

2-misol. (kvadrat tenglamaga keltiriladigan).

$2\cos^2x+\cos x-1=0$ tenglamani yechamiz.

Bunda $\cos x=t$ deb belgilab olib, tenglamani $2t^2+t-1=0$ kvadrat tenglamaga keltiramiz.

$$t_{1/2}=\frac{-0,5\pm\sqrt{0,25+2}}{2}=\frac{-0,5\pm1,5}{2};\quad t_1=-1;\quad t_2=\frac{1}{2}.$$

1) $\cos x=t_1$; $\cos x=-1$; $x_1=\pi+2k\pi$. $(k\in z)$.

2) $\cos x=t_2$; $\cos x=\frac{1}{2}$; $x_2=\pm\arccos\frac{1}{2}+2k\pi=\pm\frac{\pi}{3}+2k\pi$. $(k\in z)$.

Javob: $\pi+2k\pi$ va $\pm\frac{\pi}{3}+2k\pi$ $(k\in z)$.

3-misol. (Bir ismli funksiyaga keltiriladigan).

$\sin^2x+\cos x+1=0$ tenglamani yechamiz.

$\sin^2=1-\cos^2x$ ni berilgan tenglamalar qo'yib, tenglamani $1-\cos^2x+\cos x+1=0$, $\cos^2x-\cos x-2=0$ ko'rinishga keltiramiz.

Bu tenglamani $\cos x$ ga nisbatan yechamiz ($\cos x=t$ deb belgilab yechish ham mumkin).

$$\cos x = 0,5 \pm \sqrt{0,25+2} = 0,5 \pm 1,5$$

$\cos x_1=-1$ va $\cos x_2=2$ bu tenglama ildizga ega emas.

$\cos x=-1$ dan

$x=\pi+2k\pi$ $(k\in z)$. *Javob:* $\pi+2k\pi$ $(k\in z)$.

4-misol. (Bir jinsli tenglamani yechish).

$\cos x-\sin x=0$ tenglamani yechamiz.

Tenglama $\sin x$ va $\cos x$ ga nisbatan hadlari bir xil darajali ko'phaddan iborat bo'lsa, y ni bir jinsli ko'p had deyiladi. Bu tenglamada $\sin x$ va $\cos x$ ning darajasi 1 ga teng.

Bu tenglamani yechish uchun $\cos x\neq 0$ deb $\cos x$ ga bo'lamiz. Natijada

$1-\mathrm{tg}x=0$ hosil bo'ladi.

$\mathrm{tg}x=1$; $x=\frac{\pi}{4}+k\pi$ $(k\in z)$, bunda $\cos\left(\frac{\pi}{4}+k\pi\right)\neq 0$.

Javob: $\frac{\pi}{4}+k\pi$ $(k\in z)$.

5-misol. $2\sin^2x-7\sin x\cdot\cos x+6\cos^2x=0$ tenglamani yechamiz. Bu hadlarining darajasi 2 ga teng bo'lgan bir jinsli tenglama.

Bu tenglamani $\cos x\neq 0$ deb $\cos^2x$ ga bo'lamiz:

Bunda $2\mathrm{tg}^2x-7\mathrm{tg}x+6=0$ hosil bo'ladi.

Bu tenglamani yechib, $\mathrm{tg}x_1=1,5$ va $\mathrm{tg}x_2=2$ larni topamiz.

$\mathrm{tg}x_1=1,5$ dan $x_1=\mathrm{arctg}1,5+k\pi$; $x_1=56°19'+180\,k$ $(k\in z)$.

$\mathrm{tg}x_2=2$ dan $x_2=\mathrm{arctg}2+k\pi$; $x_2=\mathrm{arctg}2+k\pi$ $(k\in z)$.

Javob: $x_1=\mathrm{arctg}1,5+k\pi$; $x_2=\mathrm{arctg}2+k\pi$ $(k\in z)$.

6-misol. $\cos 2x+\sin x=0$ tenglamani yechamiz.

Bu tenglamani bir xil ismga va bir xil o'zgaruvchiga keltirilib yechiladi. $\cos 2x$ ni $1-2\sin^2x$ ($2\sin^2x=1-\cos 2x$ formuladan foydalanib) ifodaga almashtirib, so'ngra $\sin x$ ni y bilan belgilab, kvadrat tenglama hosil qilamiz: $\sin x=y$ bo'lsin.

$1-2\sin^2x+\sin x=0$ dan $2y^2-y-1=0$ hosil bo'ladi. Bu tenglamani yechib, $y_1=-\frac{1}{2}$; $y_2=1$ topiladi.

1) $\sin x = y_1$; $\sin x = -\frac{1}{2}$; $x_1 = (-1)^k \arcsin\left(-\frac{1}{2}\right) + k\pi$.

$x_1 = (-1)^{k+1}\frac{\pi}{6} + k\pi$ $(k \in z)$.

2) $\sin x = y_2$; $\sin x = 1$; $x_2 = \frac{\pi}{2} + 2k\pi$ $(k \in z)$.

Javob: $(-1)^{k+1}\frac{\pi}{6} + k\pi$; $x_2 = \frac{\pi}{2} + 2k\pi$ $(k \in z)$.

7-misol. $4\sin x + 3\cos x = 2$ tenglamani yechamiz.

$a\sin x + b\cos x = c$ ni $\sqrt{a^2 + b^2}\sin(x + \varphi) = c$ ko'rinishga keltiramiz.

Bunda: $r = \sqrt{a^2 + b^2} = \sqrt{4^2 + 3^2} = 5$;

$\cos\varphi = \frac{a}{r} = \frac{4}{5} = 0{,}8$; $\sin\varphi = \frac{b}{r} = \frac{3}{5} = 0{,}6$; $\mathrm{tg}\varphi = \frac{b}{a} = \frac{3}{4} = 0{,}75$ φ burchak 1-chorakda joylashgan.

$\cos\varphi = 0{,}8$ dan. $\varphi \approx 36°52'$ yoki $\sin\varphi = 0{,}6$ dan $\varphi \approx 36°52'$.

Berilgan tenglama $4\sin x + 3\cos x = 2$ ni $5\sin(x + 36°52') = 2$ ko'rinishda yozib, uni yechamiz. $\sin(x + 36°52') = \frac{2}{5} = 0{,}4$. Bundan

$x + 36°52' = (-1)^k \arcsin 0{,}4 + 180°k$.

$x = -36°52' + (-1)^k 23°35' + 180°k$ $(k \in z)$.

Javob: $(-1)^k\, 23°35' - 36°52' + 180°k$ $(k \in z)$.

yoki $(-1)^k\, 0{,}4116 - 0{,}6437 + k\pi$ $(k \in z)$.

2-usul. $\sin x = \dfrac{2\mathrm{tg}\frac{x}{2}}{1 + \mathrm{tg}^2\frac{x}{2}}$ va $\cos x = \dfrac{1 - \mathrm{tg}^2\frac{x}{2}}{1 + \mathrm{tg}^2\frac{x}{2}}$ formuladagi $\mathrm{tg}\frac{x}{2}$ ni y deb belgilab, berilgan tenglamadan $4 \cdot \frac{2y}{1 + y^2} + 3 \cdot \frac{1 - y^2}{1 + y^2} = 2$ hosil qilamiz.

Bu tenglamani yechib: $\left(\mathrm{tg}\frac{x}{2}\right)_1 = -0{,}117$ va $\left(\mathrm{tg}\frac{x}{2}\right)_2 \approx 1{,}717$ topamiz.

1) $\mathrm{tg}\frac{x}{2} \approx -0{,}117$ dan $x_1 \approx -13°20' + 360°k$.

2) $\mathrm{tg}\frac{x}{2} \approx 1{,}117$ dan $x_2 \approx 119°34' + 360°k$ $(k \in z)$.

8-misol. (Ko'paytuvchilarga ajratish usuli).

$\sin x + \cos 6x = \sin 9x$ tenglamani yechamiz.

Yechish. $(\sin 3x - \sin 9x) + \cos 6x = 0$ ko'rinishda yozib, qavs ichidagi ayirmani ko'paytmaga keltirsak:

$-2\sin 3x \cdot \cos 6x + \cos 6x = 0$ yoki

$-\cos 6x(2\sin 3x - 1) = 0.$

Ko'paytuvchilarning har birini nolga tenglaymiz: $\cos 6x = 0$ va $2\sin 3x - 1 = 0$.

Bu tenglamalarni yechamiz:

1) $\cos 6x = 0$

$6x = \frac{\pi}{2} + k\pi$

$x_1 = \frac{\pi}{12} + \frac{\pi}{6}k = \frac{\pi}{12}(1 + 2k)$;

2) $2\sin 3x - 1 = 0$

$\sin 3x = \frac{1}{2}$

$3x = (-1)^k \frac{\pi}{6} + k\pi$, yoki

$x_2 = (-1)^k \frac{\pi}{18} + \frac{\pi}{3} k \ (k \in z)$.

Javob: $\frac{\pi}{12}(2k+1)$; $(-1)^k \frac{\pi}{18} + \frac{\pi}{3}k = \frac{\pi}{18}((-1)^k + 6k) \ (k \in z)$.

9-misol. $\sin 3x \cdot \sin 7x = \sin 4x \cdot \sin 6x$ tenglamani yechamiz.

Yechish. Tenglamaning har ikki qismidagi ko'paytmalarni yig'indi ko'rinishga keltiramiz:

$\sin 3x \cdot \sin 7x = \frac{1}{2}(\cos 4x - \cos 10x)$ va $\sin 4x \cdot \sin 6x = \frac{1}{2}(\cos 2x - \cos 10x)$ bularni tenglamaga qo'yib, $\cos 4x - \cos 10x = \cos 2x - \cos 10x$ topiladi.

Bundan $\cos 4x = \cos 2x$ yoki $\cos 4x - \cos 2x = 0$ ni ko'paytma ko'rinishga keltiramiz:

$-2\sin 3x \cdot \sin x = 0$. Bundan:

$\sin x = 0$ va $\sin 3x = 0$ tenglamalar hosil bo'ladi.

1) $\sin x = 0$

$x_1 = n\pi \ (n \in z)$

2) $\sin 3x = 0$

$3x = k\pi$

$x_2 = \frac{\pi}{3}k.$

$x_2 = \frac{\pi}{3}k$ dagi $k = 3n$ da x_1 ildizni x_2 ildiz o'z ichiga oladi.

Javob: $\frac{\pi}{3}k \ (n \in z)$.

8. Tenglamalarni yeching:

a) $\sin^2 x - \sin x = 0$;
d) $2\cos^2 x - 3\cos x + 1 = 0$;
b) $5\sin^2 x - 9\sin x + 4 = 0$;
e) $3\cos x - 2\sin 2x = 0$.

9. a) $\mathrm{tg}^2 x + 10 = 7\mathrm{tg}x$;
d) $8\mathrm{tg}x - 3\sec^2 x = 2$;
b) $\sqrt{3}\mathrm{tg}^2 x - 4\mathrm{tg}x + \sqrt{3} = 0$;
e) $\sin^2 x - 4\mathrm{tg}x + 2 = 0$.

10. a) $\sin^3 x + \sin x = 0$;
d) $(1 + \mathrm{tg}x) \cdot \cos x = 0$.
b) $\cos^3 x - \cos x = 0$;
e) $2\sin^5 x = 3\sin^3 x - \sin x$.

11. a) $1 + \cos x + \cos 2x = 0$;
d) $1 - \cos x = 2\sin \frac{x}{2}$;
b) $3 - \cos^2 x - 3\sin x = 0$;
e) $\cos 2x = 2\frac{1}{3}\sin x$.

12. a) $\cos\alpha \cdot \cos(\alpha + x) + \sin\alpha \cdot \sin(\alpha + x) = 0$;

b) $\sin(30° + x) + \sin(30° - x) = \frac{1}{2}$;

d) $\cos(x - 30°) - \cos(x + 30°) = 0$;

e) $\sin(x + 70°) - \sin(x - 70°) = 0$.

13. a) $\sin \frac{x}{2} \cdot \cos \frac{x}{2} = \frac{1}{4}$;

b) $4\sin x \cdot \cos x \cdot \cos 2x = 1$;

d) $\cos(45° + x) \cdot \cos(x - 45°) = -0{,}5$;

e) $\mathrm{ctg}x \cdot \sin 3x = 0$.

14. Tenglamani ko‘paytma holga keltirib yeching:

a) $\sqrt{3}\sin x - \cos x = 0$;

b) $\sqrt{3}\sin x + \cos x = \sqrt{2}$;

d) $5\cos x + 12\sin x = 13$;

e) $8\sin x + 6\cos x = -5$.

15. Bir jinsli tenglamalarni yeching:

a) $\sin x - \sqrt{3}\cos x = 0$;

b) $3\sin x - 5\cos x = 0$;

d) $\sin^2 x - 8\sin x \cdot \cos x + 7\cos^2 x = 0$;

e) $9\sin^2 x + 30\sin x \cdot \cos x + 25\cos^2 x = 25$;

f) $\cos^2 x - 3\sin x \cdot \cos x + 1 = 0$;

g) $2\sin^2(-x) + \cos^2(-x) - 3\sin x \cdot \cos(-x) = 0$.

16*. Tenglamani ko‘paytuvchilarga ajratib yeching:

a) $\sin x + \sin 5x - 2\cos^2 2x = 0$;

b) $\sin 6x = \sin 10x - 2\sin 2x$;

d) $\sin x + 2\sin 2x + \sin 3x = \cos x + 2\cos 2x + \cos 3x$;

e) $2\sin 3x \cdot \cos x - \sin 2x = \cos 2x$.

16*. ga ko‘rsatma.

d) $(\sin x + \sin 3x) + 2\sin x = (\cos 3x + \cos x) + 2\cos 2x$.

$2\sin 2x \cdot \cos x + 2\sin 2x = 2\cos 2x \cdot \cos x + 2\cos 2x$.

$\sin 2x(\cos x + 1) = \cos 2x(\cos x + 1)$

$(\cos x + 1)(\sin 2x - \cos 2x) = 0$. Bundan:

1) $\cos x + 1 = 0$; 2) $\sin 2x - \cos 2x = 0$ tenglamalar yechilib ildiz topiladi.

4-§. Eng sodda trigonometrik tengsizliklarni yechish

Eng sodda trigonometrik tengsizliklar:

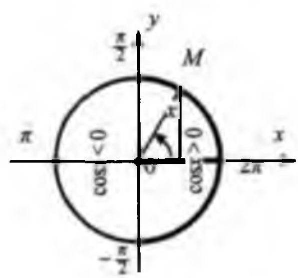
1) $\sin x > 0$, $\sin x < 0$; 2) $\cos x > 0$, $\cos x < 0$; 3) $\mathrm{tg}\, x > 0$; $\mathrm{tg}\, x < 0$;

4) $\mathrm{ctg}\, x > 0$; $\mathrm{ctg}\, x < 0$ ko‘rinishda bo‘ladi.

Bu tengsizliklar bilan biz qo‘llanmaning I qismidagi «Trigonometrik funksiyalarning ishoralari» (81-§) mavzusidan tanishganmiz.

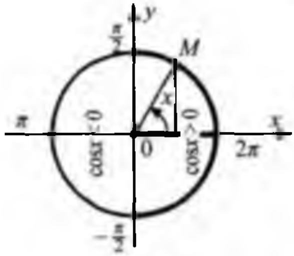
1. $\sin x > 0$ va $\sin x < 0$ larning yechimi:

$\sin x > 0$ tengsizlikning boshlang‘ich yechimi $0 < x < \pi$ bo‘lib, bunga sinusning davri qo‘shilib $2k\pi < x < (2k+1)$ umumiy yechim topiladi (yuqorigi yarim aylana yoyi) (5-chizma).



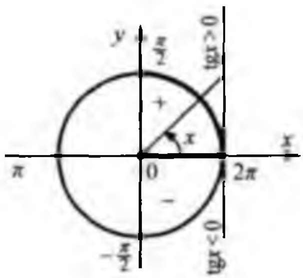
5-chizma.

2. $\cos x>0$ va $\cos x<0$ larning yechimi:



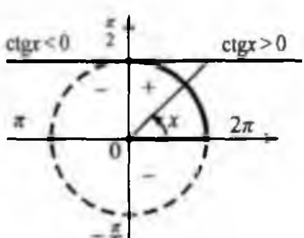
6-чизма.

$\cos x>0$ tengsizlikning boshlang'ich yechimi – $-\frac{\pi}{2}<x<\frac{\pi}{2}$ bo'lib, unga kosinusning davrini qo'shib $-\frac{\pi}{2}+2k\pi<x<\frac{\pi}{2}+2k\pi$ umumiy yechim topiladi (aylananing o'ng yarim yoyi) $\cos x<0$ tengsizlikning boshlang'ich yechimi $\frac{\pi}{2}<x<\frac{3\pi}{2}$ bo'lib, unga kosinusning davrini qo'shib $\frac{\pi}{2}+2k\pi<x<\frac{3\pi}{2}+2k\pi$ umumiy yechim topiladi (aylananing chap yarim yoyi) $k\in z$ (6-chizma).



7-chizma.

3. $\mathrm{tg}x>0$ va $\mathrm{tg}x<0$ larni yechamiz: $\mathrm{tg}x>0$ tengsizlikning boshlang'ich yechimi $0<x<\frac{\pi}{2}$ bo'lib, unga tangensning davrini qo'shib $k\pi<x<\frac{\pi}{2}+k\pi$ umumiy yechim topiladi (aylananing I choragini yoyi) (7-chizma). $\mathrm{tg}x<0$ tengsizlikning boshlang'ich yechimi $-\frac{\pi}{2}<x<0$ bo'lib, unga tangensning davrini qo'shib, $-\frac{\pi}{2}+k\pi<x<k\pi$ umumiy yechim topiladi (aylananing IV choragini yoyi) $k\in z$ (7-chizma).



8-chizma.

4. $\mathrm{ctg}x>0$ va $\mathrm{ctg}x<0$ larni yechamiz: $\mathrm{ctg}x>0$ tengsizlikning boshlang'ich yechimi $0<x<\frac{\pi}{2}$ bo'lib, unga kotangensning davrini qo'shib $k\pi<x<\frac{\pi}{2}+k\pi$ umumiy yechim topiladi (aylananing I choragini yoyi) (8-chizma).

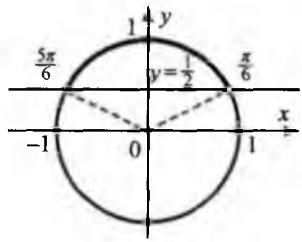
$\text{ctg}x<0$ tengsizlikning boshlang‘ich yechimi $\frac{\pi}{2}<x<\pi$ bo‘lib, unga kotangensning davrini qo‘shib $\frac{\pi}{2}+k\pi<x<(k+1)\pi$ umumiy yechim topiladi (aylananing II choragini yoyi) $k\in z$ (8-chizma).

5-§. Trigonometrik tengsizliklar

$\sin x>a$, $\sin x<a$, $\sin x\geq a$, $\cos x>a$, ... va hokazo ko‘rinishdagi tengsizliklarni yechishga keltiriladi.

Tengsizliklarni yechishga misollar keltiramiz:

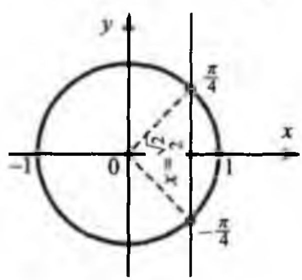
1-misol. $\sin x>\frac{1}{2}$ tengsizlikni yechamiz.



9-chizma.

Yechish. Tekislikdagi $y=\frac{1}{2}$ to‘g‘ri chiziq birlik aylana yoyini $\frac{\pi}{6}$ va $\pi-\frac{\pi}{6}=\frac{5\pi}{6}$ burchaklarda kesib o‘tadi. x o‘zgaruvchining tengsizlikni qanoatlantiruvchi qiymatlari oralig‘i $\left(\frac{\pi}{6};\frac{5\pi}{6}\right)$ dan iborat (9-chizmada qora qilib ko‘rsatilgan). Bu yoy oralig‘i $\sin x>\frac{1}{2}$ tengsizlikning boshlang‘ich yechimi $\frac{\pi}{6}<x<\frac{5\pi}{6}$ bo‘ladi. Tengsizlikning umumiy yechimi uning boshlang‘ich yechimiga sinusning umumiy davrini qo‘shib topiladi, ya’ni tengsizlikning yechimi $\frac{\pi}{6}+2k\pi<x<\frac{5\pi}{6}+$ $+2k\pi$ $(k\in z)$ ga teng.

2-misol. $\cos 3x<\frac{\sqrt{2}}{2}$ tengsizlikni yechamiz.

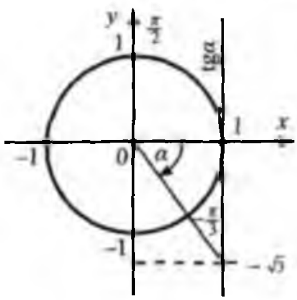


10-чизма.

Yechish. Tekislikdagi $x=\frac{\sqrt{2}}{2}$ to‘g‘ri chiziq birlik aylana yoyini $\frac{\pi}{4}$ va $2\pi-\frac{\pi}{4}=\frac{7\pi}{4}$ burchaklarda kesib o‘tadi. $3x$ o‘zgaruvchining tengsizlikni qanoatlantiruvchi qiymatlari oralig‘i $\frac{\pi}{4} < 3x < \frac{7\pi}{4}$ dan iborat bo‘lib, bunda tengsizlikning boshlang‘ich yechimi $\frac{\pi}{4} < 3x < \frac{7\pi}{8}$ oraliqda bo‘ladi (10-chizma). Tengsizlikning umumiy yechimi uning boshlang‘ich yechimiga kosinusning umumiy davrini qo‘shib topiladi, ya’ni tengsizlikning yechimi $\frac{\pi}{4} + 2k\pi < 3x < \frac{7\pi}{12} + 2k\pi$, $(k \in z)$ dan x ni topamiz:

$\frac{\pi}{12} + \frac{2}{3}k\pi < x < \frac{7\pi}{12} + \frac{2}{3}k\pi$, $(k \in z)$.

3-misol. $\mathrm{tg}\left(2x - \frac{\pi}{5}\right) > -\sqrt{3}$ tengsizlikni yechamiz.



11-chizma.

Yechish. Tekislikda birlik aylana olib, unda tangens chizig‘ini o‘tkazamiz.

Bunda $\alpha = 2x - \frac{\pi}{5}$ ga teng bo‘lsin (11-chizma). Chizmada tengsizlikni qanoatlantiruvchi $2x - \frac{\pi}{5}$ argumentning qiymatlari $\left(-\frac{\pi}{3}; \frac{\pi}{2}\right)$ oraliqdan iborat. Tengsizlikning boshlang‘ich yechimi $-\frac{\pi}{3} < 2x - \frac{\pi}{5} < \frac{\pi}{2}$ bo‘ladi.

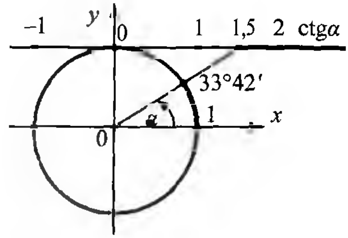
Bunga tangensning umumiy davrini qo‘shib tengsizlikning to‘la (umumiy) yechimi topiladi, ya’ni $-\frac{\pi}{3} + k\pi < 2x - \frac{\pi}{5} < \frac{\pi}{2} + k\pi$. Bundan x ni topamiz. Buning uchun tengsizlikka $\frac{\pi}{5}$ ni qo‘shib, so‘ngra 2 ga bo‘lamiz: $-\frac{\pi}{3} + \frac{\pi}{5} + k\pi < 2x < \frac{\pi}{2} + \frac{\pi}{5} + k\pi$; $-\frac{2\pi}{15} + k\pi < 2x < \frac{7\pi}{10} + k\pi$;

$-\frac{\pi}{15} + \frac{1}{2}k\pi < x < \frac{7\pi}{20} + \frac{1}{2}k\pi$.

Javob: $-\frac{\pi}{15} + \frac{1}{2}k\pi < x < \frac{7\pi}{20} + \frac{1}{2}k\pi$, $k \in z$.

4-misol. $\mathrm{ctg}\frac{1}{5}x > 1{,}5$ tengsizlikni yechamiz.

Yechish. Tekislikda birlik aylana olib, unda kotangens chizig'ini o'tkazamiz (12-chizma).



12-чизма.

Chizmada $\frac{1}{5}x$ argumentning qiymatlari (0°; arcctg1,5≈33°42′) oraliqdan iborat. Tengsizlikning boshlang'ich yechimi $0<\frac{1}{5}x<33°42'$ bo'ladi.

Bunga kotangensning umumiy davrini qo'shib tengsizlikning to'la (umumiy) yechimi topiladi, ya'ni $0°+180°k<\frac{1}{5}x<33°42'+180°k$.

Bu tengsizlikni 5 ga ko'paytirib, x ni topamiz.

$$0°+900°<x<168°30'+900k \text{ yoki}$$
$$0+5k\pi<x<0{,}936\pi+5k\pi,\ k\in z.$$

Javob: $0+5k\pi<x<0{,}936\pi+5k\pi,\ k\in z.$

5-misol. $2\sin^2\left(x+\frac{\pi}{4}\right)+2<-5\cos\left(x+\frac{\pi}{4}\right)$ tengsizlikni yechamiz.

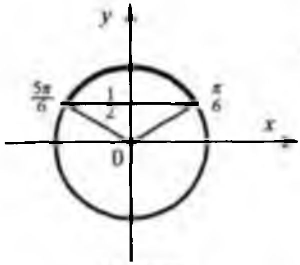
Yechish. $-5\cos\left(x+\frac{\pi}{4}\right)=-5\sin\left(\frac{\pi}{2}-\left(x+\frac{\pi}{4}\right)\right)=-5\sin\left(\frac{\pi}{4}-x\right)=5\sin\left(x-\frac{\pi}{4}\right)$ni berilgan tengsizlikka qo'yamiz. $2\sin^2\left(x-\frac{\pi}{4}\right)-5\sin\left(x-\frac{\pi}{4}\right)+2<0$. Bunda $\sin\left(x-\frac{\pi}{4}\right)=y$ bo'lsin. $2y^2-5y+2<0$ tengsizlikni yechamiz. $2y^2-5y+2=0$ ning ildizi: $y_1=\frac{1}{2}$ va $y_2=2$.

Tengsizlik $\frac{1}{2}<y<2$ yechimga ega. $y=\sin\left(x-\frac{\pi}{4}\right)$ ni tengsizlikka qo'yib, $\frac{1}{2}<\sin\left(x-\frac{\pi}{4}\right)<2$ ni hosil qilamiz.

Bu tengsizlikni ajratib:

a) $\sin\left(x-\frac{\pi}{4}\right)>\frac{1}{2}$ va b) $\sin\left(x-\frac{\pi}{4}\right)<2$ tengsizliklarni hosil qilamiz.

$\sin\left(x-\frac{\pi}{4}\right)<2$ tengsizlikning yechimi yo'q.



13-chizma.

$\sin\left(x-\frac{\pi}{4}\right)>\frac{1}{2}$ ni yechamiz: Bu tengsizlikning boshlang'ich yechimi (13-chizma).

$\frac{\pi}{6}<x-\frac{\pi}{4}<\frac{5\pi}{6}$ dan iborat.

Tengsizlikning to'liq yechimi $\frac{\pi}{6}+2k\pi<x-\frac{\pi}{4}<\frac{5\pi}{6}+2k\pi$ dan iborat bo'lib, bundan x ni topamiz.

$$\frac{\pi}{6}+\frac{\pi}{4}+2k\pi<x<\frac{5\pi}{6}+\frac{\pi}{4}+2k\pi;$$

$$\frac{5\pi}{12}+2k\pi<x<\frac{13\pi}{12}+2k\pi$$

Javob: $\frac{\pi}{12}(24k+5)<x<\frac{\pi}{12}(24k+13)$; $k\in z$. (z butun son).

TAKRORLASH UCHUN SAVOLLAR

1. Eng sodda trigonometrik tengsizliklar qanday ko'rinishda bo'ladi?
2. $\sin x>0$ va $\sin x<0$ tengsizliklarning yechimi qanday bo'ladi?
3. $\cos x>0$ va $\cos x<0$ tengsizliklarning yechimi qanday bo'ladi?
4. $\mathrm{tg}x>0$ va $\mathrm{tg}x<0$ tengsizliklarning yechimi qanday bo'ladi?
5. $\mathrm{ctg}x>0$ va $\mathrm{ctg}x<0$ tengsizliklarning yechimi qanday bo'ladi?
6. Trigonometrik tengsizliklarning boshlang'ich yechimidan uning to'liq (umumiy) yechimi qanday topiladi?

MASALA (TENGSIZLIK)LARNI YECHING

17. Trigonometrik tengsizliklarni yeching:

a) $\sin x>\frac{\sqrt{2}}{2}$; d) $\mathrm{tg}x<\sqrt{3}$;

b) $\cos x>-\frac{\sqrt{3}}{2}$; e) $\mathrm{ctg}x>-1$.

18. a) $\sin 2x\le\frac{\sqrt{2}}{2}$; d) $\mathrm{tg}\left(x-\frac{\pi}{4}\right)<1$;

b) $2\cos>-1$; e) $\mathrm{ctg}x<-\sqrt{3}$.

19. a) $2\sin x \cdot \cos x \geq \frac{\sqrt{2}}{2}$;

d) $\cos\frac{\pi}{6} \cdot \cos x + \frac{1}{2}\sin x \geq \frac{\sqrt{3}}{2}$;

b) $\sin\left(\frac{3\pi}{2} - x\right) > \frac{\sqrt{3}}{2}$;

e) $\mathrm{ctg}(\pi - x) < -1$;

20*. a) $\mathrm{ctg}\left(\frac{\pi}{2} - x\right) > \frac{1}{\sqrt{3}}$;

d) $\mathrm{tg}^2 x + 0{,}75\mathrm{tg}x > \frac{1}{4}$;

b) $(1 + \mathrm{ctg}x)\mathrm{tg}x < 0$;

e) $\cos^2 x + \cos 2x > \frac{5}{4}$.

Ko'rsatma. e) $\cos^2 x = \frac{1 + \cos 2x}{2}$ deb, $\cos 2x > \frac{1}{2}$ yechiladi.

6-§. Ko'rsatkichli funksiya

Musbat a asosli ko'rsatkichli funksiya haqida sizga ma'lum bo'lgan ma'lumotlarni eslatamiz. Bu funksiyaning qiymatlari oldin butun x lar uchun, so'ngra kasr $x=\frac{p}{q}$ lar uchun (bunda – p butun son, q – natural son) ushbu $a^x=a^{\frac{p}{q}}=\sqrt[q]{a^p}$ formula bo'yicha hisoblanadi.

Ratsional sonlar sohasida aniqlangan bu funksiya $a>1$ da o'sadi, $0<a<1$ da esa kamayadi.

Bu **ko'rsatkichli funksiya** deb ataladi va $y=a^x$ ko'rinishda yoziladi ($a\neq 1$ musbat son, x – o'zgaruvchi).

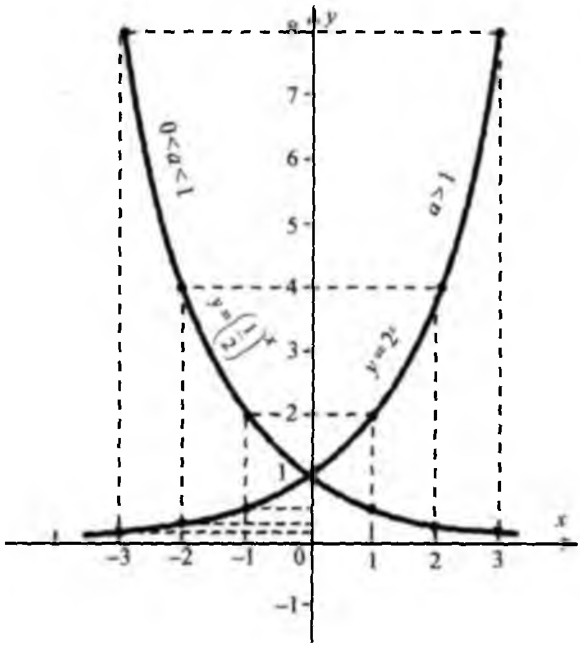
Koordinatalar tekisligida koordinatalari ratsional x larda yetarli miqdorda nuqtalar $(x;\ a^x)$ nuqtalar yasab (14-chizma), bu nuqtalarni silliq egri chiziq bilan birlashtirish mumkinligini ko'ramiz, bu egri chiziqni sonlar to'g'ri chizig'ining hamma yerlarida aniqlangan, ratsional $x=\frac{p}{q}$ larda $a^{\frac{p}{q}}$ qiymatlarni qabul qiluvchi va $a>1$ da o'suvchi yoki $0<a<1$ da kamayuvchi biror funksiyaning grafigi deb olinadi.

Masalan, quyidagi ikki funksiyaning grafigini o'zgaruvchi x songa bir qancha butun qiymatlar berib yasaymiz.

1) $a>1$ hol uchun $y=2^x$; 2) $0<a<1$ hol uchun $y=\left(\frac{1}{2}\right)^x$. $x=-3$ da: $y=2^{-3}=\frac{1}{2^3}=\frac{1}{8}$; $y=\left(\frac{1}{2}\right)^{-3}=1:\left(\frac{1}{2}\right)^3=1:\frac{1}{8}=8$ va hokazo qiymatlar quyidagi jadvalda topilgan.

Bu qiymatlarni koordinata tekisligida tasvirlab, hosil bo'lgan nuqtalarni uzluksiz egri chiziqlar bilan biriktirib (14-chizma), olingan funksiyalarning ikkita grafigini yasaymiz.

x	–3	–2	–1	0	1	2	3
$y=2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$y=\left(\frac{1}{2}\right)^x$	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$



14-chizma.

Ko'rsatkichli funksiyalarning grafiklarini tekshirib, biz ularda quyidagi xossalarning tasdig'ini ochiq-oydin ko'ramiz:

1. a^x funksiyaning aniqlanish sohasi – haqiqiy sonlar to'plami.

2. a^x funksiyaning qiymatlari sohasi ($a \neq 1$ da) – barcha musbat sonlar to'plami.

3. $a>1$ da a^x funksiya sonlar to'g'ri chizig'ining hamma yerida o'sadi; $0<a<1$ da a^x funksiya kamayadi.

4. x va y ning barcha qiymatlarida $a^x \cdot a^y = a^{x+y}$; $(a^x)^y = a^{xy}$.

3-xossaning isbotini ko'rib chiqamiz:

a) Agar x_1 va x_2 ikkita musbat son bo‘lib, $x_2>x_1$ bo‘lsa, ma’lumki $a>1$ bo‘lganda darajada ko‘rsatkichi katta darajada katta bo‘ladi, ya’ni $a^{x_2}>a^{x_1}$.

b) Agar x_1 va x_2 musbat kasrlar, masalan $x_1=\frac{m}{n}$ va $x_2=\frac{p}{q}$ bo‘lsin. Bunda $x_2>x_1$ deylik. Bu holda $\frac{p}{q}>\frac{m}{n}$. Kasrlarni bir xil maxrajga keltirib $\frac{pn}{qn}>\frac{mq}{nq}$ tengsizlikni hosil qilamiz.

Bu tengsizlikdan $pn>mq$ (maxrajlari teng ekanligidan). pn va mq butun sonlar bo‘lgani uchun yuqoridagi tasdiqqa asosan $a^{pn}>a^{mq}$ o‘rinli bo‘ladi. Bu darajalardan qn darajali ildizlarni chiqaramiz.

Demak, $\sqrt[nq]{a^{pn}}>\sqrt[nq]{a^{mq}}$ yoki $a^{\frac{pn}{nq}}>a^{\frac{mq}{nq}}$ bundagi ko‘rsatkichlarni qisqartirib, $a^{\frac{p}{q}}>a^{\frac{m}{n}}$ ya’ni $a^{x_2}>a^{x_1}$ ni hosil qilamiz.

Demak, $y=a^x$ funksiya $a>1$ da o‘suvchi bo‘ladi. $0<a<1$ da kamayuvchi bo‘ladi.

1-misol. $y=3^{\sqrt{x^2-2x}}$ funksiyaning aniqlanish sohasini topamiz.

Yechish. Bu funksiyada $x^2-2x\geq 0$ bo‘lishi kerak.

$x^2-2x=0$; $x(x-2)=0$. $x_1=0$; $x-2=0$; $x_2=2$.

$x^2-2x\geq 0$ bo‘lishi uchun $x\leq 0$ va $x\geq 2$ bo‘lishi kerak. Funksiyani aniqlanish sohasi $x\leq 0$ va $x\geq 2$.

2-misol. $y=(5)^{-x+2}$ funksiyani o‘suvchi yoki kamayuvchi ekanligini tekshiring.

Yechish. $y=5^{-x+2}=5^{-(x-2)}=\frac{1}{5^{x-2}}=\frac{1}{5^x\cdot 5^{-2}}=\frac{1}{5x\cdot\frac{1}{25}}=\frac{25}{5^x}=25\cdot\frac{1}{5^x}=25\cdot\left(\frac{1}{5}\right)^x.$

Demak, $y=25\cdot\left(\frac{1}{5}\right)^x$ funksiya hosil bo‘ldi. $\left(\frac{1}{5}\right)^x$ darajada $0<\frac{1}{5}<1$ bo‘lgani uchun $\left(\frac{1}{5}\right)^x$ funksiya kamayuvchi bo‘ldi.

$25\cdot\left(\frac{1}{5}\right)^x$ funksiya ham kamayuvchi bo‘ladi.

Javob: $y=5^{-x+2}$ funksiya kamayuvchi.

TAKRORLASH UCHUN SAVOLLAR

1. Qanday funksiyani ko‘rsatkichli funksiya deyiladi? Bundagi a va x qanday qiymatlarga teng?
2. $y=ax$ ning grafigi qanday chiziladi, ular qaysi holda o‘suvchi yoki kamayuvchi bo‘ladi?
3. Ko‘rsatkichli funksiyaning xossalarini ayting.

MASALALARNI YECHING

21. Daraja ko‘rsatkichiga -1; 0; $\frac{1}{2}$; 1; $\frac{3}{2}$; 2; $2\frac{1}{2}$; 3 qiymatlarni berib, funksiya $y=3^x$ ning $x=-1$ bilan 3 oralig‘idagi grafigini yasang.

22. Funksiyaning grafigini yasang:

a) $y=4^x$; b) $y=0{,}2^x$; d) $y=\pi^x$.

23. Funksiyaning aniqlanish sohasini toping:

a) $y=2^{\frac{1}{x+1}}$; b) $y=0{,}5^{\sqrt{x}}$; d) $y=5^{\sqrt{-2x+1}}$; e) $y=3^{\sqrt{x^2-x}}$.

24. Funksiyaning o‘sishi yoki kamayishini aniqlang:

a) $y=3^{x^2}$; b) $y=5^{x}+20$; d) $y=0{,}5^{-2x}$.

7-§. Ko‘rsatkichli tenglamalar

Ta’rif. **Noma’lum son daraja ko‘rsatkichida qatnashgan tenglamani ko‘rsatkichli tenglama deyiladi.**

Masalan: 1) $3^x=\frac{1}{81}$; 2) $\left(\frac{2}{3}\right)^{2x}=\frac{729}{64}$; 3) $8^{\sqrt{x+1}}=64\cdot 2^{\sqrt{x+1}}$.

Ko‘rsatkichli tenglamalarni yechishning umumiy usuli mavjud emas. Ammo ularning asosiy turlarini yechish yo‘llarini ko‘rsatish mumkin.

Ko‘rsatkichli tenglamani yechish natijasida ba’zan chet ildiz hosil bo‘lishi mumkin. Shu sababdan topilgan ildizni lozim bo‘lgan hollarda tekshirib ko‘rish kerak.

I. Asoslarini tenglash bilan yechiladigan ko'rsatkichli tenglamalar

1-misol. $3^x = \frac{1}{81}$ tenglamani yechamiz.

Yechish. $\frac{1}{81}$ kasrni 3 asosli daraja ko'rinishida yozamiz. Natijada $3^x=3^{-4}$ tenglama hosil bo'ladi. Bir xil asosli darajalar, faqat, daraja ko'rsatkichlari teng bo'lganda to'g'ri bo'ladi.

Demak, $x=-4$ – tenglamaning ildizi.

2-misol. $\left(\frac{2}{3}\right)^{2x}=\frac{729}{64}$ tenglamani yechamiz.

Yechish. $\frac{729}{64}$ kasrni $\frac{2}{3}$ asosli darajaga keltiramiz: $\frac{729}{64}=\frac{3^6}{2^6}=$ $=\left(\frac{3}{2}\right)^6=\left(\frac{2}{3}\right)^{-6}$ ko'rinishda yozib $\left(\frac{2}{3}\right)^{2x}=\left(\frac{2}{3}\right)^{-6}$ tenglamani hosil qilamiz.

Bundan $2x=-6$ tenglamani yechib, $x=-3$ ildiz topiladi.

Javob: $x=-3$.

3-misol. $0{,}0625\cdot 4^{\frac{x+2}{x-4}}=8\cdot(0{,}5)^{\frac{x+5}{x-5}}$ tenglamani yechamiz.

Yechish. Bu tenglamaning asosini 2 ga keltiramiz. $0{,}0625=(0{,}5)^4=\left(\frac{1}{2}\right)^4=2^{-4}$; $0{,}5^{\frac{x+5}{x-5}}=2^{-\frac{x+5}{x-5}}$ ko'rinishda yozib, tenglamani $2^{-4}\cdot 2^{\frac{2(x+2)}{x-4}}=2^3\cdot 2^{-\frac{x+5}{x-5}}$ hosil qilamiz.

$2^{\frac{2(x+2)}{x-4}-4}=2^{3-\frac{x+5}{x-5}}$ bundan $\frac{2x+4}{x-4}-4=3-\frac{x+5}{x-5}$ tenglamani yechib, x ni topamiz:

$$\frac{2x+4}{x-4}-4=3-\frac{x+5}{x-5};\quad \frac{^{x-5/}2x+4}{x-4}+\frac{^{x-4/}x+5}{x-5}=\frac{(x-4)(x-5)}{7}.$$

$2x^2+4x-10x-20+x^2+5x-4x-20=7(x^2-5x-4x+20)$ bundan $4x^2-58x+$ $+180=0$ tenglamani hosil qilamiz.

Bu tenglamani yechib $x_1=4{,}5$ va $x_2=10$ ildizlar topiladi.

Tekshirish: $x_1=4{,}5$ da, $0{,}0625\cdot 4^{\frac{6,5}{0,5}}=8\cdot(0{,}5)^{\frac{9,5}{-0,5}}$; $2^{-4}\cdot 2^{\frac{13}{0,5}}=2^3\cdot 2^{19}$; $2^{22}=2^3\cdot 2^{19}$; $2^{22}=2^{22}$; $x_2=10$ da $1=1$ hosil bo'ladi.

Javob: 4,5 va 10.

II. Algebraik tenglamaga keltirib yechiladigan ko'rsatkich tenglamalar

4-misol. $8^{\sqrt{x+1}} = 64 \cdot 2^{\sqrt{x+1}}$ tenglamani yechamiz.

Yechish. $8^{\sqrt{x+1}} = (2^3)^{\sqrt{x+1}} = (2^{\sqrt{x+1}})^3$ ko'rinishda yozib, $2^{\sqrt{x+1}} = y$ kabi belgilab, $y^3 = 64y$ tenglamani hosil qilamiz. Bu tenglamani yechamiz: $y^3 - 64y = 0$; $y(y^2 - 64) = 0$.

Bundan 1) $y_1 = 0$; 2) $y^2 - 64 = 0$; $y_{2/3} = \pm 8$.

1) $2^{\sqrt{x+1}} = y_1$ dan $2^{\sqrt{x+1}} = 0$ tenglama yechimga ega emas.

2) $2^{\sqrt{x+1}} = y_2$; $2^{\sqrt{x+1}} = -8$ bu tenglama ham yechimga ega emas, chunki $2^{\sqrt{x+1}} > 0$ bo'lgani uchun;

3) $2^{\sqrt{x+1}} = y_3$; $2^{\sqrt{x+1}} = 8$; $2^{\sqrt{x+1}} = 2^3$; $\sqrt{x+1} = 3$; $x+1 = 9$; $x = 8$.

Tekshirish: $x = 8$ da $8^{\sqrt{8+1}} = 64 \cdot 2^{\sqrt{8+1}}$; $8^3 = 8^2 \cdot 2^3$; $8^3 = 8^2 \cdot 8^1$; $8^3 = 8^3$.

Javob: 8.

5-misol. $12 \cdot 11^x - 121^x - 11 = 0$ tenglamani yechamiz.

Yechish. $121^x = (11^2)^x = (11^x)^2$ kabi yozib, $11^x = y$ deb belgilaymiz.

Berilgan tenglama $12y - y^2 - 11 = 0$ ko'rinishda yoziladi. $y^2 - 12y + 11 = 0$ tenglamani yechib, $y_1 = 1$ va $y_2 = 11$ lar topiladi.

1) $11^x = y_1$; $11^x = 1$; $11^x = 11^0$; $x_1 = 0$.

2) $11^x = y_2$; $11^x = 11$; $x_2 = 1$.

Tekshirish. $x_1 = 0$ da $12 \cdot 11^0 - 121^0 - 11 = 0$.

$$12 - 1 - 11 = 0;\ 0 = 0.$$

$x_2 = 1$ da $12 \cdot 11 - 121^1 - 11 = 0$;

$$132 - 132 = 0;\ 0 = 0.$$

Javob: 0 va 1.

III. Logarifmlab yechiladigan ko'rsatkichli tenglamalar

Bunday ko'rsatkichli tenglamalarni yechishni logarifmlar mavzusidan keyin o'rganiladi.

TAKRORLASH UCHUN SAVOLLAR

1. Qanday tenglamalarni ko'rsatkichli tenglamalar deyiladi?
2. Ko'rsatkichli tenglamalarni yechishning qanday usullarini bilasiz?
3. Tenglamalarni og'zaki yeching:

a) $3^x = 27$; b) $2^x = \frac{1}{8}$; d) $4^x = 8$; e) $16^x = \frac{1}{4}$.

MASALALARNI YECHING

25. Tenglamalarni yeching:

a) $5^x=125$; b) $3^x=\frac{1}{243}$; d) $9^x=\sqrt{27}$; e) $\sqrt{3^x}=\sqrt[3]{9}$.

26. a) $\left(\frac{2}{5}\right)^x=\left(\frac{5}{2}\right)^4$; b) $\sqrt{2^x}\cdot\sqrt{3^x}=36$; d) $\left(\frac{2}{3}\right)^x\cdot\left(\frac{9}{8}\right)^x=\frac{27}{64}$.

27. a) $7^{(x+1)(x-2)}=1$; b) $2^{x^2+x-0,5}=4\sqrt{2}$; d) $3^{x^2-x-2}=81$.

28. a) $4^{x+1}+4^x=320$; b) $2\cdot3^{x+1}-4\cdot3^{x-2}=150$;

d) $7^{x+2}+4\cdot7^{x-1}=347$.

29. a) $10\cdot2^x-4^x=16$; b) $4^{\sqrt{x-2}}+16=10\cdot2^{\sqrt{x-2}}$.

30. a) $7^x-7^{3-x}=42$; b) $4^{2x+0,5}-17\cdot4^x+4^{1,5}=0$.

31. a) $2^{3x}\cdot3^x-2^{3x-1}\cdot3^{x+1}=-288$.

Ko'rsatma $2^{3x}\cdot3^x=(8\cdot3)^x$ va $2^{3x-1}\cdot3^{x+1}=\frac{3}{2}(8\cdot3)^x$ kabi yozib yechiladi.

b) $10\cdot4^x-29\cdot10^x+10\cdot25^x=0$. **Ko'rsatma.** Tenglamaning har bir hadini 25^x ga bo'lib, tenglamani yechamiz.

32. a) $2^2\cdot4^2\cdot8^2\cdot16^2\cdot...2^{2x}=(0,25)^{-28}$;

Ko'rsatma. $2^{2+4+6+8+...+2x}=2^{56}$.

$2+4+6+8+...+2x=\frac{2+2x}{2}\cdot x=2^{56}$; $2^{x(x+1)}=2^{56}$ dan x topiladi.

b) $3^{2+4+6+8+...+2x}=\left(\frac{1}{9}\right)^{-6}$.

8-§. Ko'rsatkichli tengsizliklar

I. Bir xil asosli darajalarga keltirib ko'rsatkichlarni solishtirish bilan yechiladigan tengsizliklar

$y=a^x$ funksiya $a>1$ bo'lganda o'suvchi, $0<a<1$ bo'lganda kamayuvchi bo'lgani uchun $a^{f(x)}>a^{\varphi(x)}$ $(a\neq1)$ tengsizlikning yechimi $a>1$ bo'lganda $f(x)>\varphi(x)$ tengsizlikni yechish bilan, $0<a<1$ bo'lganda esa $f(x)<\varphi(x)$ tengsizlikni yechish bilan aniqlanadi.

$y=a^x$ funksiya x ning har qanday qiymatida ham musbat bo‘lgani uchun $a^{f(x)}>0$ tengsizlikning yechimi x ning $f(x)$ mavjud bo‘lgan barcha qiymatlaridan iborat bo‘ladi, $a^{\varphi(x)}<0$ tengsizlikning yechimi mavjud emas.

1-misol. $0{,}2^x>\frac{1}{25}$ tengsizlikni yechamiz.

Yechish. Avval asoslarini bir xilga keltirib olamiz: $0{,}2^x=\left(\frac{1}{5}\right)^x=5^{-x}$; $\frac{1}{25}=5^{-2}$.

Berilgan tengsizlik $5^{-x}>5^{-2}$ ko‘rinishga keladi. Bundagi asos $5>1$ bo‘lganidan

$-x>-2$ tengsizlikning yozamiz, ya’ni

$x<2$. *Javob:* $x<2$.

2-misol. $2^{\frac{2x+7}{x+3}}>0$ tengsizlikni yechamiz.

Yechish. $2^{\frac{2x+7}{x+3}}$ daraja faqat musbat bo‘ladi, ammo $\frac{2x+7}{x+3}$ kasr $x\neq-3$ da ma’noga ega bo‘lgani uchun tengsizlikning yechimi $x\neq-3$ barcha sonlardan iborat.

3-misol. $2^{3x^2-x+1}>0{,}5^{3-x-3x^2}$ tengsizlikni yechamiz.

Yechish. $0{,}5^{3-x-3x^2}=(2^{-1})^{3-x-3x^2}=2^{3x^2+x-3}$ ko‘rinishda yozib, $2^{3x^2-x+1}>2^{3x^2+x-3}$ da $2>1$ bo‘lgani uchun $3x^2-x+1>3x^2+x-3$ tengsizlikni yechamiz.

$2x<4$ tengsizlikdan $x<2$ ni hosil qilamiz.

Javob: $x<2$.

4-misol. $\left(\frac{1}{4}\right)^{x-2}+2^{2-2x}<10$ tengsizlikni yechamiz.

Yechish. $\left(\frac{1}{4}\right)^{x-2}$ ni 2 asosli daraja ko‘rinishida yozamiz. $\left(\frac{1}{4}\right)^{x-2}=\left(\frac{1}{2}\right)^{2x-4}=$

$=2^{4-2x}=2^2\cdot2^{2-2x}=4\cdot2^{2-2x}$.

Natijada berilgan tengsizlik $4\cdot2^{2-2x}+2^{2-2x}<10$ yoki $5\cdot2^{2-2x}<10$ bu tengsizlikni 5 ga bo‘lib $2^{2-2x}<2$ ni hosil qilamiz.

Bundan $2-2x<1$; $-2x<-1$ yoki $x>\frac{1}{2}$ topiladi.

Javob: $x>\frac{1}{2}$.

II. Yordamchi noma'lum kiritib kvadrat tengsizlikka keltirib yechiladigan ko'rsatkichli tengsizliklar

5-misol. $10\cdot\left(\frac{4}{25}\right)^x - 29\cdot\left(\frac{5}{2}\right)^{-x} + 10 < 0$ tengsizlikni yechamiz.

Yechish. Darajaning asoslarini bir xilga keltiramiz: $\left(\frac{4}{25}\right)^x = \left(\frac{2}{5}\right)^{2x}$ va $\left(\frac{5}{2}\right)^{-x} = \left(\frac{2}{5}\right)^{-(-x)} = \left(\frac{2}{5}\right)^x$ ko'rinishda yozib, $10\left(\frac{2}{5}\right)^{2x} - 29\left(\frac{2}{5}\right)^x + 10 < 0$ tengsizlikni hosil qilamiz.

$\left(\frac{2}{5}\right)^x = y$ deb belgilab, $10y^2 - 29y + 10 < 0$ tengsizlikni hosil qilamiz. Bu tengsizlikni yechib, $\frac{2}{5} < y < \frac{5}{2}$ tengsizlik topiladi. Bunga $y = \left(\frac{2}{5}\right)^x$ ni qo'yib $\frac{2}{5} < \left(\frac{2}{5}\right)^x < \frac{5}{2}$ qo'sh tengsizlik yechiladi:

1) $\frac{2}{5} < \left(\frac{2}{5}\right)^x$ dan $1 > x$ topiladi;

2) $\left(\frac{2}{5}\right)^x < \frac{5}{2}$ yoki $\left(\frac{2}{5}\right)^x < \left(\frac{2}{5}\right)^{-1}$ dan $x > -1$ topiladi.

Bu ikkala yechimdan $-1 < x < 1$ ni yozamiz.

Javob: $-1 < x < 1$.

TAKRORLASH UCHUN SAVOLLAR

1. $a^{f(x)} > 0$ tengsizlikning yechimi nimaga teng?
2. $a^{f(x)} < 0$ tengsizlikning yechimi nimaga teng?
3. $7^x > 7^{-2}$ tengsizlikning yechimi nimaga teng?
4. $0{,}8^{2x} < 0{,}8^4$ tengsizlikning yechimi nimaga teng?

MASALALARNI YECHING

33. Tengsizliklarni yeching:

a) $2^x > 16$; b) $0{,}4^x > 0{,}064$; d) $\left(\frac{3}{7}\right)^x \le 1$; e) $\frac{1}{3x} \ge 27$.

34. a) $(0{,}3)^{2x} < 0{,}09$; b) $(0{,}5)^x < 4$; d) $5^{2x-4} < 1$; e) $2^{2-3x} > 0{,}25$.

35. a) $35^{x+10}>0$; b) $\left(\frac{1}{3}\right)^{1-5x}<0$; d) $5^{\frac{2x+3}{x+1}}>0$; e) $(0,5)^{x^2-6x+8}<1$.

36. a) $2^{3x^2-x+1}>(0,5)^{3-x-3x^2}$; b) $\sqrt[x]{16^{2x-2}}>\sqrt[3]{(0,25)^{1-3x}}\,(x>0)$.

37. a) $27\cdot 81^x-28\cdot 9^x+1<0$; b) $2\cdot 64^{x-\frac{1}{3}}-9\cdot 8^{x-\frac{2}{3}}+1>0$.

Ba'zida asosida va ko'rsatkichida noma'lum bo'lgan ko'rsatkichli tengsizliklarni yechishga duch kelamiz.

Namuna. $(x+1)^{4x-3}>(x+1)^{x+6}$ tengsizlikni yechamiz.

Yechish. 1) Asos $x+1>1$ bo'lsin. U holda ko'rsatkichlari $4x-3>x+6$ bo'ladi.

Yechim $\begin{cases} x+1>1 \\ 4x-3>x+6 \end{cases}$; $\begin{cases} x>0 \\ x>3 \end{cases}$; $x>3$;

2) Asos $0<x+1<1$ da $4x-3<x+6$ bo'lib.

Bu tengsizliklarni yechimi $\begin{cases} 0<x+1<1 \\ 4x-3<x+6 \end{cases}$ tengsizliklar sistemasining yechimi bo'ladi.

$\begin{cases} -1<x<0 \\ x<3 \end{cases}$ sistemaning yechimi $-1<x<0$.

Javob: $-1<x<0$ va $x>3$.

38. Tengsizliklarni yeching.

a) $x^{2x}<x^{3x-2}$; b) $x^{3x-4}>x^5$.

39. a) $(2x-3)^{x^2-2}>(2x-3)^{2-x+x^2}$; b) $(2x+3)^{x+4}<1$.

40*. a) $(0,5x+2)^{x^2+5x+4}<1$; b) $(x^2-7x+11)^{x^2-6x+5}<1$.

b) ga ko'rsatma. 1) $\begin{cases} x^2-7x+11>1 \\ x^2-6x+5<0 \end{cases}$; 2) $\begin{cases} 0<x^2-7x+11<1 \\ x^2-6x+5>0 \end{cases}$ sistemalar yechilib, javob topiladi.

9-§. Logarifmning ta'rifi

Masalan, 16 ni hosil qilish uchun 4 sonini qanday darajaga ko'tarish talab qilinadi?

Buning uchun 4 ni ko'tarish kerak bo'lgan daraja ko'rsatkichni x bilan belgilab, $4^x=16$ tenglamani hosil qilamiz.

Berilgan daraja va berilgan asosga ko'ra daraja ko'rsatkichni topish amalini **logarifmlash** deymiz. Bizning misolda: $x=2$, chunki $4^2=16$. Logarifmlash amali darajaga ko'tarishga teskari amal hisoblanadi, chunki darajaga ko'tarishda asos va daraja ko'rsatkichlar orqali daraja topiladi.

Ta'rif. **Berilgan sonning berilgan asosga ko'ra logarifmi deb, berilgan sonni hosil qilish uchun shu asosni ko'tarish kerak bo'lgan daraja ko'rsatkichiga aytiladi.**

«4 asosga ko'ra 16 ning logarifmi» jumlani qisqacha $\log_4 16$ ko'rinishda yoziladi.

4 asos, log belgisining pastrog'iga yoziladi. Yuqoridagi misol yechimini qisqacha $\log_4 16=2$ ko'rinishda yozamiz.

Masalan, asosi 2 bo'lgan 2; 4; 16; 1; $\frac{1}{2}$; sonlarning logarifmlarini topamiz:

$\log_2 2=1$, chunki $2^1=2$; $\log_2 4=2$, chunki $2^2=4$;

$\log_2 16=4$, chunki $2^4=16$; $\log_2 1=0$, chunki $2^0=1$;

$\log_2 \frac{1}{2}=-1$, chunki $2^{-1}=\frac{1}{2}$; $\log_2\sqrt{8}=\frac{3}{2}$, chunki $2^{\frac{3}{2}}=\sqrt{2^3}=\sqrt{8}$.

Quyidagi logarifmlarni topamiz:

$\log_3 9=2$; $\log_3 81=4$; $\log_3 \frac{1}{9}=\log_3 3^{-2}=-2$;

$\log_5 625=\log_5 5^4=4$; $\log_5 \frac{1}{125}=\log_5 5^{-3}=-3$. $\log_{\frac{1}{2}} \frac{1}{16}=4$; $\log_{\frac{1}{3}} 27=-3$.

Umumiy holda M sonining a asosga ko'ra logarifmi $\log_a M$ ko'rinishda yoziladi.

M sonining a asosga ko'ra logarifmi n ga teng bo'lsa, uni $\boldsymbol{\log_a M=n}$ ko'rinishda yoziladi.

Bunda: $a \neq 1$ **– musbat son,** $M > 0$, n **– ratsional son.**

$\log_a a = 1$, $\log_a 0$ – mavjud emas, chunki $a^n \neq 0$.

$\log_a M$ da $M \leq 0$ bo‘lsa, $\log_a M$ – mavjud emas.

Masalan, $\log_2(-8)$ – mavjud emas.

Agar asos $a = 10$ bo‘lsa, bunday logarifmni o‘nli logarifm deyiladi. Uni $\mathbf{\log_{10} a = \lg a}$ ko‘rinishda yoziladi.

Masalan, $\lg 10 = 1$; $\lg 1000 = 3$; $\lg 0{,}1 = -1$; $\lg 0{,}0001 = -4$.

TAKRORLASH UCHUN SAVOLLAR

1. Qanday amalni logarifmlash deyiladi?
2. Logarifmning ta’rifini ayting.
3. Asosi 2; 5; 10 bo‘lgan logarifmga misollar keltiring.
4. $\log_a M = n$ dagi a; M va n lar qanday sonlarga teng bo‘lishi mumkin?
5. Qanday logarifmni o‘nli logarifm deyiladi va u qanday yoziladi?

MASALALARNI YECHING

41. Quyidagi tengliklarni log belgisi bilan yozing:

$2^3 = 8$; $7^2 = 49$; $5^4 = 625$; $10^{-2} = 0{,}01$ $2^{-5} = \frac{1}{32}$;

$3^{\frac{1}{2}} = \sqrt{3}$; $3^{\frac{3}{4}} = \sqrt[3]{81}$; $b^x = N$; $a^{\sqrt{x}} = M$.

42. Tenglikni log belgisisiz yozing:

$\log_5 125 = 3$; $\log_3 81 = 4$; $\log_2 \frac{1}{32} = -5$; $\log 1000 = 3$; $\log 0{,}01 = -2$.

43. Asos 16 bo‘lganda, quyidagi sonlarning logarifmlari qanday bo‘ladi?

16; 256; $\frac{1}{16}$; $\frac{1}{256}$; 4; $\frac{1}{4}$; 2; $\frac{1}{2}$.

44. Quyidagilarni toping:

$\log_2 16$; $\log_3 9$; $\log_3 729$; $\log_3 1$; $\log_3 \frac{1}{3}$; $\log_3 \frac{1}{\sqrt{3}}$.

45. Agar $a \neq 1$ musbat son bo‘lsa, quyidagi ifodalar nimaga teng?

$\log_a a^2$; $\log_a a^n$; $\log_a \frac{1}{a}$; $\log_a \sqrt{a}$; $\log_a \frac{1}{\sqrt{a}}$.

46. x qanday songa teng?

1) $\log_2 x=3$; 2) $\log_5 x=2$; 3) $\log_4 x=-5$; 4) $\log_x 4=2$.

10-§. Logarifmik funksiya va uning grafigi

$y=a^x$($a\neq 1$ musbat) funksiya $a>1$ bo‘lganda o‘suvchi, $0<a<1$ bo‘lganda esa kamayuvchi bo‘lgani, uchun unga teskari funksiya mavjud bo‘ladi.

Teskari funksiyada y argument, x esa y ning biror funksiyadagi argumenti, x esa y ning biror funksiyasi bo‘ladi. Bu teskari funksiya $x=\log_a y$, ya’ni x miqdor a asosli y sonining logarifmi (funksiyasi) demakdir. Bu ifodada odatdagidek argumentni x bilan, funksiyani esa y bilan belgilasak, y funksiya quyidagi ko‘rinishga keladi.

$$y=\log_a x.$$

$y=a^x$ funksiyaga teskari bo‘lgan $y=\log_a x$ funksiyani logarifmik funksiya deb ataladi.

$y=a^x$ va $y=\log_a x$ funksiyalarning grafiklarini I va III koordinat burchaklarining bissektrisasiga nisbatan simmetrik bo‘lishini ko‘rsatamiz.

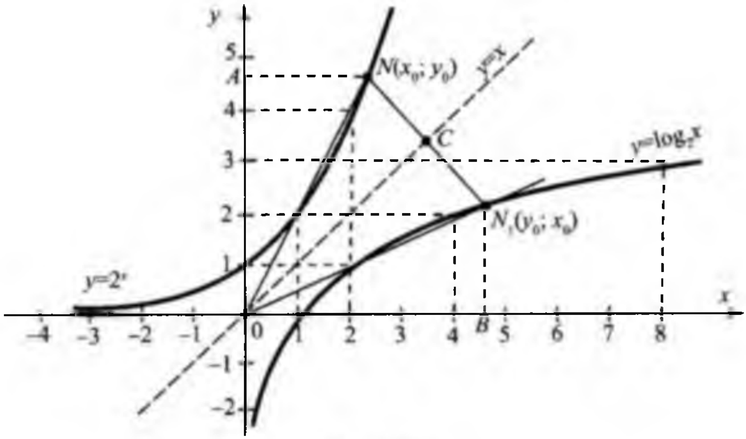
O‘zaro teskari $y=2^x$ va $y=\log_2 x$ funksiyalarning grafiklarini I va III koordinat burchaklarining bissektrisasiga nisbatan simmetrik joylashganligini ko‘rib chiqamiz. $y=2^x$ va $y=\log_2 x$ funksiyalarning grafiklarini quyidagi jadvallar yordamida chizamiz.

x	...	-3	-2	-1	0	1	2	3	...
$y=2^x$	...	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	...

Bu jadvaldagi x bilan y ning qiymatlarini almashtirib qo‘ysak, $y=\log_2 x$ funksiyaning quyidagi jadvali hosil bo‘ladi.

x	...	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	...
$y=\log_2 x$	...	-3	-2	-1	0	1	2	3	...

$y=2^x$ funksiya grafigida biror $N(x_0;\ y_0)$ va $y=\log_2 x$ funksiya grafigida $N_1(y_0;\ x_0)$ nuqtalarni olamiz. ON va ON_1 larni o'tkazib, $\Delta AON=\Delta BON$ (katetlar tengligidan). Bundan $\angle AON=\angle BON$ va $ON=ON_1$ kelib chiqadi. ΔNON_1 teng yonli bo'lgani uchun $\angle NON_1$ ning bissektrisasi NN_1 tomonga o'tkazilgan mediana va balandlik bo'ladi, ya'ni $NC=CN_1$ va $NN_1 \perp OC$ (15-chizma).



15-chizma.

Demak, N va N_1 nuqtalar OC bissektrisaga nisbatan o'zaro simmetrik bo'ladi.

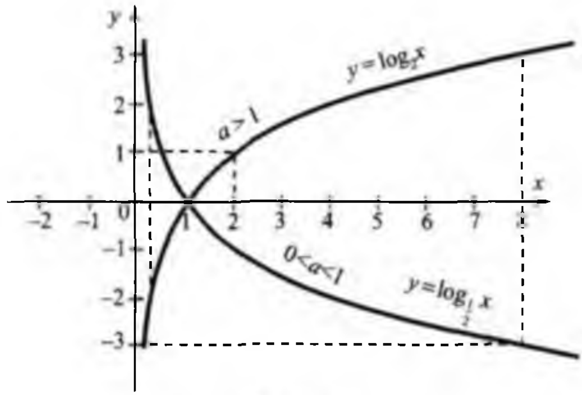
Shunday qilib, $y=2^x$ funksiya grafigida olingan ixtiyoriy $N(x_0;\ y_0)$ nuqta unga teskari bo'lgan $y=\log_2 x$ funksiya grafigi ustida olingan $N_1(y_0;\ x_0)$ nuqta I va III koordinat burchagining bissektrisasiga nisbatan simmetrik ekan.

Demak, o'zaro teskari $y=a^x$ va $y=\log_a x$ funksiyalarning grafiklari I va III koordinat burchagining bissektrisasiga nisbatan simmetrik joylashadi ($y=x$ ga nisbatan simmetrik bo'ladi).

$y=\log_{\frac{1}{2}} x$ funksiyaning grafigini jadval asosida yasaymiz.

x	...	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	...
$y=\log_{\frac{1}{2}} x$	...	3	2	1	0	–1	–2	–3	...

Bu grafikka 15-chizmadagi $y=\log_2 x$ funksiyaning grafigini chizamiz (16-chizma).



16-chizma

Bu chizmada $y=\log_a x$ funksiyaning $0<a<1$ $\left(0<\frac{1}{2}<1\right)$ va $a>1(2>1)$ bo'lgandagi grafiklari chizilgan. Bu chizma (16-chizma)dan foydalanib $y=\log_a x$ funksiyaning xossalarini sanab o'tamiz:

1. $y=\log_a x$ funksiyaning grafigi OY o'qining o'ng tomonida joylashgani uchun uning aniqlanish sohasi barcha musbat sonlardan iborat. (Manfiy sonlar va nol sonining logarifmi mavjud emas).

2. $y=\log_a x$ funksiyaning o'zgarish sohasi (qiymatlari) barcha haqiqiy sonlardan iborat.

3. $y=a^x(a>1$ da) o'suvchi bo'lgani uchun unga teskari $y=\log_a x$ funksiya ham o'suvchi bo'ladi.

4. $y=a^x(0<a<1$ da) kamayuvchi bo'lgani uchun unga teskari $y=\log_a x$ funksiya ham kamayuvchi bo'ladi.

5. $a>1$ da: $x>1$ bo'lsa, $\log_a x>0$;

$0<x<1$ bo'lsa, $\log_a x<0$ bo'ladi.

6. $0<a<1$ da: $x>1$ bo'lsa, $\log_a x<0$;

$0<x<1$ bo'lsa, $\log_a x>0$ bo'ladi.

1-misol. a) $y=\log_a(-x)$; b) $y=\log_a(7-2x)$ funksiyalarning aniqlanish sohasini topamiz.

Yechish. a) $y=\log_a(-x)$ funksiya aniqlangan bo‘lishi uchun $-x>0$ yoki $x<0$ bo‘lishi kerak. Aniqlanish sohasi: $x<0$ yoki $(-\infty;\ 0)$.

b) $y=\log_a(7-2x)$ funksiyaning aniqlangan bo‘lishi uchun $7-2x>0$ bo‘lishi kerak. Buning uchun shu tengsizlikni yechamiz.

$-2x>-7$; $x<3,5$. Aniqlanish sohasi: $x<3,5$.

2-misol. Quyidagi sonlardan qaysi biri katta. Javobingizni asoslab bering:

1) $\log_{10}6$ va $\log_{10}7$. Bunda $\log_{10}6<\log_{10}7$, chunki $10>1$ da $6<7$. Katta sonning logarifmi katta bo‘ladi.

2) $\log_{\frac{1}{2}}6$ va $\log_{\frac{1}{2}}7$. Bunda $\log_{\frac{1}{2}}6>\log_{\frac{1}{2}}7$, chunki $\frac{1}{2}<1$ da kichik sonning logarifmi katta bo‘ladi.

3-misol. Quyidagi logarifmlarni kichigidan boshlab tartib bilan yozing:

$\log_7 3$; $\log_7 7$; $\log_7 \frac{1}{5}$; $\log_7 12$; $\log_7 49$.

Yechish. Bunda asos $7>1$ bo‘lgani uchun katta sonning logarifmi katta bo‘ladi. *Javob:* $\log_7 \frac{1}{5}$; $\log_7 3$; $\log_7 7$; $\log_7 12$; $\log_7 49$.

TAKRORLASH UCHUN SAVOLLAR

1. Qanday funksiyani logarifmik funksiya deyiladi?
2. Logarifmik funksiya asos a qanday bo‘lganda o‘suvchi, a qanday bo‘lganda kamayuvchi bo‘ladi?
3. $y=a^x$ va $y=\log_a x$ funksiyalarning grafiklari o‘zaro qanday joylashadi?
4. $y=\log_a x$ funksiyaning aniqlanish sohasini ayting.
5. $y=\log_a x$ funksiyaning o‘zgarish sohasini ayting?
6. a) $\log_{0,2}5$ va $\log_{0,2}7$; b) $\log_{12}8$ va $\log_{12}11$ larni taqqoslang.

MASALALARNI YECHING

47. Funksiyalarning aniqlanish sohasini toping:

a) $y=\log_3 7x$; b) $\log_5(-3x)$; d) $y=\log_7(3x-12)$; e) $y=\log_4 \frac{12}{2x-8}$.

48. Quyidagi tengsizliklar m va n ning qanday qiymatlarida to‘g‘ri bo‘ladi?

a) $\log_{10}m<\log_{10}n$; b) $\log_{0,1}m>\log_{0,1}n$;

d) $\log_5 m<\log_5 n$; e) $\log_{\frac{1}{5}} m<\log_{\frac{1}{5}} n$.

49. Quyidagi tenglik va tengsizliklar m ning qanday qiymatlarida to‘g‘ri bo‘ladi?

a) $\log_{10}m=-2$; b) $\log_{10}m>1$; d) $\log_{0,1}m>-1$; e) $\log_{0,01}m<-2$.

50. Jadval yordamida $y=\log_4 x$ funksiyaning grafigini chizing, grafikdan foydalanib $\log_4 5$; $\log_4 8$; $\log_4 10$ va $\log_4 0,5$ larni toping.

11-§. Ko‘paytma, bo‘linma, daraja va ildizlarning logarifmlari

Logarifm ta’rifiga asosan $\log_2 8=3$ dan $2^3=8$ tenglikni yozamiz. Bundagi 3 ni $\log_2 8$ bilan almashtirib $2^{\log_2 8}=8$ ayniyatni hosil qilamiz. Bu ayniyat $a\neq 1$ musbat sonlarda to‘g‘ri bo‘ladi.

Umumiy holda $a^{\log_a M}=M$ (1) ko‘rinishda bo‘ladi.

1-teorema. **Ikki musbat son ko‘paytmasining logarifmi ko‘paytuvchilar logarifmlarining yig‘indisiga teng.**

Bunda: asos $a\neq 1$ – musbat son, $x>0$; $y>0$ bo‘lsin.

Isbot qilish kerak $\log_a(xy)=\log_a x+\log_a y$.

Isbot. Logarifmning asosiy ayniyatiga asosan

$a^{\log_a(xy)}=xy$.

$a^{\log_a x+\log_a y}=a^{\log_a x}\cdot a^{\log_a y}=xy$.

Bu tengliklarning o‘ng tomonlarini tengligidan $a^{\log_a(xy)}=a^{\log_a x+\log_a y}$ tenglik kelib chiqadi. Bir xil asosli ikki darajaning tengligidan darajalarning ko‘rsatkichlarini tengligi kelib chiqadi, ya’ni

$$\log_a(xy)=\log_a x+\log_a y \qquad (2)$$

Bu teorema ko‘paytuvchilar ikkitadan ortiq musbat sonlar uchun ham to‘g‘ri bo‘ladi, ya’ni $\log_a(xyz)=\log_a x+\log_a y+\log_a z$.

1-misol. a) $\log_3 30=\log_3(3\cdot 10)=\log_3 3+\log_3 10=1+\log_3 10.$

b) $\log_6 4+\log_6 9=\log_6(4\cdot 9)=\log_6 36=2.$

2-teorema. **Ikki musbat son bo‘linmasining logarifmi bo‘linuvchining logarifmidan bo‘luvchining logarifmini ayrilganiga teng.**

Bunda: $a\neq 1$ – musbat son, $x>0$; $y>0$ bo‘lsin.

Isbot qilish kerak: $\log_a \frac{x}{y}=\log_a x-\log_a y$.

Isbot. $\frac{x}{y}\cdot y=x$ tenglikni olib, y ni a asosga ko‘ra logarifmlaymiz:

$\log\left(\frac{x}{y}\cdot y\right)=\log_a x$ yoki $\log_a \frac{x}{y}+\log_a y=\log_a x$ bundan

$$\log_a \frac{x}{y}=\log_a x-\log_a y \qquad (3)$$

2-misol. a) $\log_3 \frac{27}{25}=\log_3 27-\log_3 25=3-\log_3 25.$

b) $\log_2 1000-\log_2 125=\log_2 \frac{1000}{125}=\log_2 8=3.$

3-teorema. **Musbat son darajasining logarifmi daraja ko‘rsatkichini asos logarifmiga ko‘paytmalariga teng.**

Bunda: $a\neq 1$ musbat son, $x>0$; k – haqiqiy son.

Isbot qilish kerak: $\log_a x^k=k\cdot\log_a x.$

Isbot. $a^{\log_a x^k}=x^k$ (ayniyatga ko‘ra).

$$a^{k\log_a x}=\left(a^{\log_a x}\right)^k=x^k.$$

Bu tengliklarning o‘ng tomonlarini tengligidan $\log_a x^k=k\cdot\log_a x$ tenglik kelib chiqadi. Bir xil asosli ikki darajaning tengligidan, ya’ni

$$\log_a x^k=k\cdot\log_a x \qquad (4)$$

3-misol. a) $\log_3 243=\log_3 3^5=5\cdot\log_3 3=5\cdot 1=5.$

b) $\log_2\cdot 64^{\sqrt{3}}=\log_2(2^6)^{\sqrt{3}}=\log_2 2^{6\sqrt{3}}=6\sqrt{3}\cdot\log_2 2=6\sqrt{3}\cdot 1=6\sqrt{3}.$

4-teorema. **Musbat son ildizining logarifmi ildiz ostidagi son logarifmining ildiz ko'rsatkichiga bo'linganiga teng.**

Bunda: $a \neq 1$ musbat son, $x > 0$ va n – natural son.

Isbot qilish kerak: $\log_a \sqrt[n]{x} = \frac{1}{n}\log_a x$.

Isbot. $\sqrt[n]{x} = x^{\frac{1}{n}}$ ko'rinishda yozib, darajani logarifmlash teoremasiga asosan logarifmlaymiz, ya'ni

$$\log_a \sqrt[n]{x} = \log_a x^{\frac{1}{n}} = \frac{1}{n}\log_a x,$$ ya'ni

$$\mathbf{\log_a \sqrt[n]{x} = \frac{1}{n}\log_a x} \qquad (5)$$

4-misol. a) $\log_2 \sqrt[5]{32} = \frac{1}{5}\log_2 32 = \frac{1}{5}\cdot 5 = 1.$

b) $\log_3 \sqrt{81} = \frac{1}{2}\log_3 81 = \frac{1}{2}\cdot 4 = 2.$

Ba'zida bir xil asosli logarifmdan boshqa asosli logarifmga o'tish ishni osonlashtiradi. Shuning uchun $\log_a b$ ni biror c_1 – musbat asosli logarifmga o'tadigan formulani chiqaramiz.

Buning uchun $a^{\log_a b} = b$ ayniyatni c asosli logarifmlaymiz:

$\log_c a^{\log_a b} = \log_c b$ olib,

$\log_a b \cdot \log_c a = \log_c b$ ko'rinishda yozib, bundan

$$\mathbf{\log_a b = \frac{\log_c b}{\log_c a}} \qquad (6)$$

formulani topamiz.

5-misol. $\log_8 32$ ni hisoblang.

Bu logarifmni 2 asosli logarifmlaymiz:

$$\log_8 32 = \frac{\log_2 32}{\log_2 8} = \frac{5}{3} = 1\frac{2}{3}$$

Ba'zan $\log_b a$ da a bilan b ni almashtirish kerak bo'lsa, uni quyidagi formula bo'yicha almashtiriladi:

$\log_b a$ ni a asosga almashtiradigan (6) formuladan foydalanamiz, ya'ni

$$\log_b a = \frac{\log_a a}{\log_a b} = \frac{1}{\log_a b};$$

Demak, $$\mathbf{\log_b a = \frac{1}{\log_a b}} \qquad (7)$$

Masalan, $\log_{25} 5 = \frac{1}{\log_5 25} = \frac{1}{2}$.

Agar logarifm asosida daraja ko‘rsatkich bo‘lsa, uni quyidagi formula bo‘yicha topiladi.

$$\log_{a^n} b = \frac{\log_a b}{\log_a a^n} = \frac{\log_a b}{n \cdot \log_a a} = \frac{\log_a b}{n \cdot 1} = \frac{1}{n} \log_a b$$

$$\log_a \frac{1}{6} = \log_a b^{-1} = -\log_a b.$$

Demak, $$\mathbf{\log_{a^n} b = \frac{1}{n} \log_a b} \qquad (8)$$

$$\mathbf{\log_a \frac{1}{b} = -\log_a b} \qquad (9)$$

Masalan, $\log_{32}16 = \log_{32}16 = \frac{1}{5} \log_2 16 = \frac{1}{5} \cdot 4 = 0{,}8$.

6-misol. $\log_2 5 + \log_5 2 > 2$ ekanligini isbotlaymiz.

Isbot. $\log_2 5 > \log_2 4 = 2$; $\log_5 2 = \frac{1}{\log_2 5} > 0$.

$\log_2 5 + \log_5 2 > 2 + \frac{1}{\log_2 5} > 2$. Demak, $\log_2 5 + \log_5 2 > 2$.

Eslatma. Bizda yig‘indi va ayirmali ifodalarni logarifmlash formulasi mavjud emas.

TAKRORLASH UCHUN SAVOLLAR

1. Logarifmning asosiy ayniyatini yozing.
2. Musbat sonlar ko‘paytmasining logarifmi nimaga teng?
3. Musbat sonlar bo‘linmasining logarifmi nimaga teng?
4. Musbat son darajasining logarifmi nimaga teng?

5. Musbat son ildizining logarifmni nimaga teng?
6. Logarifmi berilgan asosdan boshqa asosga o'tish formulasini yozing?
7. Sonning logarifmi bilan asosini almashtiruvchi formulani yozing?

MASALALARNI YECHING

51. Logarifmlarni hisoblang:

a) $\log_6 3+\log_6 2$; e) $\log_5 100-\log_5 4$;

b) $\log_{10} 40+\log_{10} 25$; f) $\log_3 7-\log_3 \frac{7}{27}$;

d) $\log_{12} 4+\log_{12} 36$; g) $\log_{0,1} 50-\log_{0,1} 0,5$.

52. Quyidagi ifodalarni hisoblang:

a) $\log_2 128$; e) $\log_2 \sqrt[3]{16}$;

b) $\log_5 \frac{1}{625}$; f) $\log_2 \sqrt{0,5^3}$;

d) $\log_{\frac{1}{3}} 81$; g) $\log_{\frac{1}{5}} \sqrt[3]{25}$.

53. Logarifmlarni hisoblang:

a) $\log_4 32$; d) $\log_{625} \frac{1}{25}$;

b) $\log_{\frac{1}{9}} 243$; e) $\log_{\frac{1}{343}} \frac{1}{2401}$.

54. $\log_{a^n} b^m = \frac{m}{n}$ ekanligini isbotlang.

55. $\log_{\sqrt[m]{b}} \sqrt[m]{a} = \log_b a$ ekanligini isbotlang.

56. $\log_{bn} an = \frac{\log_b a+\log_b n}{1+\log_b n}$ ekanligini isbotlang.

16-§. Algebraik ifodalarni logarifmlash

Algebraik ifodalarni logarifmlash bu ifodani tashkil qiluvchi ayrim sonlarning logarifmlari vositasi bilan uning logarifmini ifoda qilish demakdir. O'tgan 15-§. formulalari yordamida berilgan ifodalar logarifmlanadi.

Masalan: 1) $\log_3 \frac{13\cdot 9^7}{0.72} = \log_3(13\cdot 9^7) - \log_3 0,72 = \log_3 13 + \log_3 9^7 - \log_3 0,72 = \log_3 13 + 7\cdot\log_3 9 - \log_3 0,72 = \log_3 13 + 7\cdot 2 - \log_3 0,72 = \log_3 13 + 14 - \log_3 0,72.$

2) $\log_a \frac{9a^4b^3}{\sqrt[3]{b^2}} = \log_a(9a^4b^3) - \log_a \sqrt[3]{b^2} = \log_a 9 + \log_a a^4 + \log_a b^3 - \frac{\log_a b^2}{3} = \log_a 9 + 4 + 3\log_a b - \frac{2}{3}\log_a b = \log_a 9 + 4 + 2\frac{1}{3}\log_a b.$

3) $x = \frac{30}{\sqrt[4]{a^3}}\sin^{-5}\varphi$ ifodani a asosga ko'rgan logarifmlaymiz.

$$\log_a x = \log_a\left(\frac{30}{\sqrt[4]{a^3}}\sin^{-5}\varphi\right) = \log_a \frac{30}{\sqrt[4]{a^3}} + \log_a \sin^{-5}\varphi = \log_a 30 - \log \sqrt[4]{a^3} -$$

$$-5\log_a \sin\varphi = \log_a 30 - \frac{1}{4}\log_a a^3 - 5\log_a \sin\varphi = \log_a 30 - \frac{3}{4} - 5\log_a \sin\varphi.$$

Demak, $\log_a x = \log_a 30 - \frac{3}{4} - 5\log_a \sin\varphi$.

MASALALARNI YECHING

57. Quyidagi ifodalarni 10 asosga ko'ra logarifmlang ($\log_{10}x = \lg x$ ko'rinishda yozing):

a) $x = 3a^7$; e) $x = a\sqrt[4]{ab^5}$;

b) $x = 15\sqrt[3]{b^2}$; f) $x = a\sqrt[4]{a^3b^2}$;

d) $x = 0,35a^3b^3\sqrt{c}$; g) $x = \sqrt[3]{\frac{a^3}{b^2}}$.

58. a) $x=\frac{a+b}{a-b}\sqrt[3]{(a+b)^2}$; d) $x=\frac{a\sin\alpha}{2b^3}\sqrt{b^2\operatorname{tg}\alpha}$;

b) $x=\left(\sqrt[5]{\frac{a}{c^2}}\right)^3$; e) $x=\left(\sqrt[7]{\frac{\sin\varphi}{\cos^2\varphi}\operatorname{ctg}\varphi}\right)^3$.

59. a) $x=\sqrt[3]{a^2\sqrt{b}}$; d) $x=\left(\frac{\sqrt[3]{ab^2}}{3ab}\right)^4$;

b) $x=\sqrt[4]{\frac{7}{b^2}}\cdot\sqrt[3]{\frac{a^4}{5}}$; e) $x=0{,}8\sqrt[3]{a^2\sqrt[4]{a}}$.

60. a) $x=\frac{\sqrt{a\sqrt{a\sqrt{a}}}}{\sqrt[3]{a\sqrt[3]{a}}}$; b) $x=\frac{a\sqrt{b\sqrt{a\sqrt{b}}}}{b\sqrt{a\sqrt{b\sqrt{a}}}}$.

13-§. Popensirlash

Logarifmlangan ifodadan logarifmlashdan oldingi ifodani topish (logarifmlashga teskari) amali ***popensirlash*** deyiladi. Masalan:

1-misol. $\log_a x=\log_a 2+\log_a 3$ dan x ni topamiz.

Bunda $\log_a 2+\log_a 3=\log_a(2\cdot 3)=\log_a 6$.

$\log_a x=\log_a 6$ tenglikdan $x=6$.

2-misol. $\log_a x=2\log_a 8-4\log_a 2$ dan x ni topamiz.

Bunda $2\log_a 8-4\log_a 2=\log_a 8^2-\log_a 2^4=\log_a\frac{8^2}{2^4}=\log_a 4$.

$\log_a x=\log_a 4$ tenglikdan $x=4$.

3-misol. $\log_2 x=\log_2 a+n\log_2(a+b)-\frac{1}{n}\log_2(a-b)$ dan x ni topamiz.

Bunda $\log_2 a+n\log_2(a+b)-\frac{1}{n}\log_2(a-b)=\log_2 a+\log_2(a+b)^n-\log_2\sqrt[n]{a-b}=$

$=\log_2\left(a\cdot(a+b)^n\right)-\log_2\sqrt[n]{a-b}=\log_2\frac{a(a+b)^n}{\sqrt[n]{a-b}}$; $\log_2 x=\log_2\frac{a(a+b)^n}{\sqrt[n]{a-b}}$ tenglikdan $x=\frac{a(a+b)^n}{\sqrt[n]{a-b}}$.

MASALALARNI YECHING

Quyidagi logarifmlardan x ni toping (popensirlang):

61. a) $\log_a x = \log_a 2 + \log_a 10$; d) $\log_a x = 2\log_a 4 + 3\log_a 2$;

b) $\log_a x = \log_a 3 - \log_a 27$; e) $\log_a x = 3\log_a 5 - 2\log_a 10$.

62. a) $\log_3 x = \frac{1}{3}\log_3 8$; d) $\log_5 x = \frac{2\log_5 3}{5} - \frac{\log_5 7}{10}$;

b) $\log_3 x = \frac{2}{3}\log_3 8 + \frac{1}{2}\log_3 16$;

e) $\log_2 x = \log_2 b + 3\log_2(a+b) - 2\log_2(a-b)$.

63. a) $\log_2 x = -\frac{1}{2}\log_2 a + \frac{1}{4}\left[\log_2 b - \frac{2}{3}\log_2 a + \frac{2}{3}\log_2(a-b) - \frac{1}{2}\log_2(a+b)\right]$;

b) $\log_5 x = -\log_5(a+b) + \frac{2}{5}\left[2\log_5 a + \frac{1}{2}\log_5 b - \frac{1}{3}(\log_5 a - \log_5 b) - \log_5 a\right]$;

d) $\log_a x = \frac{2}{3}\log_a \sin\varphi + \frac{1}{3}\log_a tg\varphi - \frac{2}{3}\log_a \cos\varphi$.

14-§. O'nli logarifmlarning xossalari

1-xossa. Ma'lumki, $10 = 10^1$; $100 = 10^2$; $1000 = 10^3$ va hokazo bo'lgani uchun lg10 = 1; log100 = 2; log1000 = 3 va hokazo.

Demak, **bir va nollar bilan tasvirlanadigan butun sonning logarifmi sondagi nollar sonicha birlardan iborat bo'lgan butun musbat sondir.**

Masalan: lg1000 = 5; lg10000000 = 7 va hokazo.

2-xossa. $0{,}1 = \frac{1}{10} = 10^{-1}$; $0{,}01 = \frac{1}{100} = 10^{-2}$; $0{,}001 = \frac{1}{1000} = 10^{-3}$ va hokazo. lg0,1 = 1; lg0,01 = –2; lg0,001 = –3 va hokazo.

Demak, **bir va uning oldida nollar bilan tasvirlanadigan o'nli kasrning logarifmi, kasrning nol butun bilan birgalikda hammasida qancha nol bo'lsa, shuncha manfiy birlardan iborat bo'lgan butun manfiy sondir.**

Masalan: lg0,0001 = –4; lg0,00000001 = –8 va hokazo.

3-xossa. 100 bilan 1000 orasida yotgan, masalan, 843 ni olamiz. lg843 soni $\lg 100=2$ va $\lg 1000=3$ lar orasida bo'ladi, ya'ni $\lg 100<\lg 843<\lg 1000$ yoki $2<\lg 843<3$, chunki asos $10>1$ da katta sonning logarifmi katta bo'ladi.

Demak, $\lg 843=2,\overline{\alpha\beta\gamma}\ldots$ ($\alpha, \beta, \gamma, \ldots$ – kasr qismining raqamlari).

Bundagi: 2 soni logarifmning butun qismi – **xarakteristika,** $0,\overline{\alpha\beta\gamma}$ soni logarifmning kasr qismi – **mantissa** deyiladi.

Son logarifmining xarakteristikasini hamma vaqt sonning ko'rinishidan topish mumkin. Buning uchun berilgan butun sonda qancha raqam yoki berilgan aralash sonning butun qismida qancha raqam borligini sanab, undan bitta kam son logarifmning xarakteristikasi bo'ladi.

Masalan: 1) 63,2 soni $10<63,2<100$ oraliqda

$\lg 10<\lg 63,2<\lg 100$

$1<\lg 63,2<2$

$\lg 63,2=1,\ldots;$

2) 9348,7 soni $1000<9348,7<10000$;

$\lg 1000<\lg 9348,7<\lg 10000$

$\lg 9348,7=3,\ldots$.

Demak, **butun yoki aralash son logarifmining xarakteristikasi sonning butun qismidagi raqamlar sonidan bitta kam bo'lgan musbat birlikka teng.**

Masalan: 1) $\lg 7,205=0,\ldots$; 2) $\lg 830,47=2,\ldots$ va hokazo.

4-xossa. 1 dan kichik (musbat) bo'lgan bir necha o'nli kasr olaylik: 0,35; 0,07; 0,0056; 0,00084 va hokazo.

Ma'lumki:	Demak:
$0,1<0,35<1$	$-1<\lg 0,35<0;$
$0,01<0,07<0,1$	$-2<\lg 0,07<-1;$
$0,001<0,0056<0,01$	$-3<\lg 0,0056<-2;$
$0,0001<0,00084<0,001$	$-4<\lg 0,00084<-3.$

Bu logarifmlarning har biri, bir-biridan bitta birlik miqdorda farq qilgan ikki manfiy son orasida turadi; shu sababli u logarifmlarning har biri shu manfiy sonlarning kichigini birdan kichik biror musbat son miqdorida orttirishdan hosil bo‘lgan songa teng bo‘ladi. Masalan, $\lg 0,0056=-3+0$, ... Ikkinchi qo‘shiluvchi 0,7482 bo‘ladi, deb faraz qilaylik. U holda:

$\lg 0,0056=-3+0,7482=-2,2518$

$-3+0,7482$ ga o‘xshash yig‘indilarni logarifmik hisoblarda qisqacha bunday yozish shart qilingan: $-3+0b7482=\bar{3},7482$ («minus 3 butun, o‘n mingdan 7482») kabi yoziladi, ya’ni $\lg 0,0056=\bar{3},7482$. Bunday belgilanishda manfiy ishora faqat xarakteristikaga tegishli bo‘lib, mantissa musbat ekanligini anglatadi.

Shunday qilib, yuqoridagi misollarni quyidagicha yozamiz:

$\lg 0,35=\bar{1},\ldots$; $\lg 0,07=\bar{2},\ldots$; $\lg 0,00084=\bar{4},\ldots$; va hokazo.

Umumiy holda: $\mathbf{\lg 0,\overbrace{000\ldots0}^{m}\alpha\beta = -m + 0,\ldots = \bar{m},\ldots}$.

Demak, **1 dan kichik (musbat) bo‘lgan o‘nli kasr logarifmining xarakteristikasi berilgan o‘nli kasrning birinchi qiymatli raqamigacha (nol butun bilan birga) qancha nol bo‘lsa, shuncha manfiy birliklardan iborat bo‘ladi, bunday logarifmning mantissasi musbat.**

Masalan: $\lg 0,875=\bar{2},\ldots$; $\lg 0,0000803=\bar{5},\ldots$.

5-xossa. Qandaydir biror *N* sonni (xoh butun, xoh kasr bo‘lsin) 10 ga, 100 ga; ... , umuman bir bilan nollardan iborat bo‘lgan butun songa ko‘paytiramiz. Buning natijasida lgN ning qanday o‘zgarishini qaraymiz. Ko‘paytmaning logarifmi ko‘paytuvchilar logarifmlarining yig‘indisiga teng bo‘lganlikdan:

$\lg(N\cdot 10)=\lg N+\lg 10=\lg N+1$;

$\lg(N\cdot 100)=\lg N+\lg 100=\lg N+2$;

$\lg(N\cdot 1000)=\lg N+\lg 1000=\lg N+3$; va hokazo.

lgN ga birorta butun son qo‘shsak, bu son faqat xarakteristikani orttiradi, mantissa esa o‘zgarmaydi. Ya’ni $\lg N=2,7804$ bo‘lsa, u holda $2,7804+1=3,7804$; $2,7804+3=5,7804$ va hokazo.

Shunga o‘xshash; agar $\lg N=\bar{3},5649$ bo‘lsa, u holda $\bar{3},5649+1=$ $=\bar{2},5649$; $\bar{3},5649+4=1,5649$ va hokazo.

Demak, sonni 10 ga, 100 ga, 1000 ga, ... umuman bir bilan nollardan iborat bo'lgan butun sonlarga ko'paytirishda logarifmning mantissasi o'zgarmaydi, xarakteristika esa ko'paytuvchida qancha nol bo'lsa, shuncha birlik qadar ortadi.

Masalan: $\lg N = \bar{3},8026$ bo'lsa:

a) $\lg(N \cdot 100) = \lg N + \lg 100 = \bar{3},8026 + 2 = \bar{1},8026;$

b) $\lg(N \cdot 10000) = \lg N + \lg 10000 = \bar{3},8026 + 4 = 1,8026.$

6-xossa. Ma'lumki, bo'linmaning logarifmi bo'linuvchining logarifmidan bo'luvchi logarifmini ayrilganiga teng ekanini e'tiborga olib, quyidagilarni yozamiz:

$$\lg\frac{N}{10} = \lg N - \lg 10 = \lg N - 1;$$

$$\lg\frac{N}{100} = \lg N - \lg 100 = \lg N - 2;$$

$$\lg\frac{N}{1000} = \lg N - \lg 1000 = \lg N - 3;$$ va hokazo.

Agar logarifmdan butun sonni ayirishda shu butun sonni har vaqt xarakteristikadan ayirib, mantissa o'zgartirmasdan yoziladi.

Demak, sonni bir bilan nollardan iborat bo'lgan butun songa bo'lishda logarifmning mantissasi o'zgarmaydi, xarakteristika esa bo'luvchida qancha nol bo'lsa, shuncha birlik kamayadi.

Masalan: $\lg N = \bar{1},4771$ bo'lsa:

a) $\lg\frac{N}{10} = \lg N - \lg 10 = \bar{1},4771 - 1 = \bar{2},4771;$

b) $\lg\frac{N}{1000} = \lg N - \lg 1000 = \bar{1},4771 - 3 = \bar{4},4771;$ va hokazo.

Natijalar. a) O'nli kasrdagi vergulni surish bilan shu o'nli kasr logarifmining mantissasi o'zgarmaydi (5 va 6-xossalar).

Shunday qilib, 0,00514; 0,0514; 0,514; 5,14 lar logarifmlarining mantissalari bir xil bo'lib, ular bir-biridan xarakteristikasi bilan farq qiladi.

b) Qiymatli qismlari bir xil bo'lgan sonlarning mantissalari bir xil bo'ladi.

Haqiqatan, 23; 230; 23000; 0,023 sonlarning logarifmlari faqat xarakteristikalari bilan farq qiladi, ya'ni lg23=1,3617; lg230=2,3617; lg0,023=$\bar{2}$,3617 va hokazo.

Har qanday berilgan sonning logarifmi xarakteristikasini jadval yordamisiz og'zaki topa olamiz (3 va 4-xossaga asosan). Shuning uchun logarifm jadvallariga faqat mantissalargina qo'yilgan.

TAKRORLASH UCHUN SAVOLLAR

1. Bir va nollardan tuzilgan butun sonning logarifmi nimaga teng?
2. Bir va uning oldida nollar bilan tasvirlanadigan o'nli kasrning logarifmi nimaga teng?
3. Son logarifmining xarakteristikasi va mantissasi deb nimaga aytiladi?
4. Butun yoki aralash son logarifmining xarakteristikasi nimaga teng?
5. Birdan kichik bo'lgan musbat son logarifmining xarakteristikasi nimaga teng?
6. Sonni 10 ga, 100 ga, 1000 ga. ..., umuman bir va nollardan tuzilgan butun sonlarga ko'paytirishda sonning logarifmi qanday o'zgaradi?
7. Sonni 10 ga, 100 ga, 1000 ga, ..., umuman bir va nollardan tuzilgan butun sonlarga bo'lishda sonning logarifmi qanday o'zgaradi?
8. O'nli kasrdagi vergulni surish bilan shu o'nli kasr logarifmining xarakteristika va mantissasi qanday o'zgaradi?

MASALALARNI YECHING

64. Quyidagi sonlarning o'nli logarifmlarini toping:

a) 1; 10; 100; 1000; 10 000;

b) 0,1; 0,01; 0,001; 0,0001.

65. Quyidagi sonlarning o'nli logarifmi xarakteristikalarini toping:

a) 2,00; 57,38; 632,84; 24956,01;

b) 0,17; 0,99; 0,0834; 0,001007.

66. lg2=0,301; lg3=0,477; lg7=0,845 lardan foydalanib, dastlabki 10 ta natural sonning o'nli logarifmlarini hisoblang.

67. lg2=0,30103 va lg3=0,47712 lar yordamida

a) lg0,0015; b) lg750 larni, hamda; d) $\lg 2^{100}$ ni hisoblang.

15-§. O'nli logarifmlarning mantissalari va antilogarifmlari jadvali

Amaliy masalalarning ko'plarini yechish uchun to'rt xonali jadvallar mutlaq kifoya qiladi. Jadvalning kichkina bir qismi shu betda berilgan.

Logarifmlar mantissalari

№	0	1	2	3	4	5	6	7	8	9	123	456	789
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	123	345	678
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	123	345	678
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	122	345	677
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	122	345	667

Jadvalda birdan 9999 gacha barcha butun sonlar logarifmlarining mantissalari bo'lib, to'rt o'nli xona bilan hisoblab qo'yilgan.

Butun son yoki o'nli kasr logarifmining xarakteristikasini o'nli logarifmlarning xossalariga asosan bevosita qo'ya olganimiz uchun jadvallarga faqat mantissalar joylashtirilgan.

Shuning uchun berilgan songa ko'ra mantissani topishda shu sondagi vergulni tashlab, bundan so'ng hosil bo'lgan butun son logarifmining mantissasini topamiz.

Bunda quyidagi hollar bo'lishi mumkin:

1. Butun son uch raqamdan iborat.

Masalan, 526 son logarifmining mantissasini topamiz. Bu sonning oldingi ikki raqamini, ya'ni 52 ni jadvalda chapdan birinchi vertikal ustunchadan topamiz (shu betdagi jadvalchadan) 52 ni topib, undan gorizontal satr bo'ylab, o'ngga qarab, jadvalning ustiga (yoki ostiga) qo'yilgan 0, 1, 2, 3, ..., 9 raqamlaridan, berilgan sonning uchinchi raqami (bizning misolda 6) turgan ustuncha bilan kesishguncha boramiz. Ularning kesishgan joyida 526 ning logarifmiga tegishli mantissa 7210 (ya'ni 0,7210) ni olamiz. Shunga o'xshash 518 ning logarifmi uchun mantissa 0,7143 ni topamiz.

2. Butun son ikki yoki bir raqamdan iborat.

Bu holda son yoniga fikrimizda bir yoki ikki nol qo‘yamiz va shu yo‘sinda tuzilgan uch xonali son uchun mantissa topamiz. Masalan, 51 ning yoniga bir nol qo‘yib, bundan 510 ning mantissasi 0,7076 ni topamiz. Agar sonimiz 5 bo‘lsa, uning yoniga ikkita nol qo‘yib, mantissa 0,6990 ni topamiz va hokazo.

3. Butun son to‘rt raqam bilan ifodalangan hol.

Masalan, lg5267 ning mantissasini topish kerak. Bu holda berilgan sonning oldingi uch raqami bilan tasvirlangan sonning, ya’ni 526 ning logarifmi uchun mantissa topamiz (bu 0,7210). Undan keyin topilgan mantissadan gorizontal satr bo‘ylab o‘ngga (vertikal qo‘sh chiziqdan keyin) qarab, jadvallarning shu qismi ustida yoki ostida) turgan raqamlar: 1, 2, 3, ..., 9 dan berilgan sonning to‘rtinchi raqami (misolda 7) turgan vertikal ustuncha bilan kesishgancha boramiz. Kesishgan joyda to‘ldiruvchi tuzatmani (6 ni) topamiz, buni dilda 7210 mantissaga qo‘shish kerak. Shunday qilib, 0,7216 mantissani topamiz.

4. Butun son besh yoki undan ortiq raqamlar bilan ifodalangan hol.

Bu holda oldingi to‘rt raqamdan boshqa raqamlarni tashlab, to‘rt xonali taqribiy son olamiz, agar tashlanadigan beshinchi raqam 5 ga teng yoki 5 dan ortiq bo‘lsa, olingan taqribiy sonning oxirgi raqamiga 1 qo‘shiladi. Haqiqatan, 57842 o‘rniga 5784 olamiz, 30257 o‘rniga 3026 olamiz. Bunday yaxlitlangan to‘rt xonali son mantissasi yuqorida topilgandek topiladi.

Masalan: $\lg 52{,}638 \approx \lg 52{,}64 = 1{,}7213;\quad \lg 0{,}0523 = \bar{2}{,}7185;$

$\lg 5156{,}3 \approx \lg 5156{,}0 = 3{,}7123;\quad \lg 528694 \approx \lg 528700 = 5{,}7232.$

Manfiy logarifmning shaklini o‘zgartirish:

1 dan kichik sonlarning logarifmlari manfiy ekanligini bilamiz (o‘nli logarifmda). Masalan, bizda sonning logarifmi –2,0873 bo‘lsa, uni quyidagicha o‘zgartirish mumkin.

$$-2{,}0873 = -2 - 0{,}0873 + 1 - 1 = (-2-1) + 1 - 0{,}0873 = -3 + 0{,}9127 = \bar{3}{,}9127.$$

Bunda biz manfiy logarifmning mantissasi musbat, xarakteristikasi manfiy qilib shakl almashtirdik.

Buning uchun mantissaga musbat 1, xarakteristikaga –1 qo‘shish kifoya bo‘ladi.

Yuqoridagi shakl almashtirishni qisqacha quyidagicha yoziladi:

$$-2,0873 = -\overset{-1}{2},0\overset{+1}{8}73 = \bar{3},9127; \qquad -\overset{-1}{0},3\overset{+1}{8}47 = \bar{1},6153.$$

Aksincha, manfiy xarakteristikali va musbat mantissali har qanday logarifmni manfiy o‘nli kasrga aylantirish mumkin.

Buning uchun musbat mantissaga –1, manfiy xarakteristikaga esa musbat 1 qo‘shish kifoya.

Masalan. $\bar{7},8345 = \overset{\pm1}{\bar{7}},8\overset{-1}{3}45 = -6,1655;$

$$\bar{4},0037 = \overset{\pm1}{\bar{4}},0\overset{-1}{0}37 = -3,9963.$$

Manfiy xarakteristikali logarifmlar ustida amallar bajaramiz:

1) $\begin{array}{r} +\ \bar{2},9734 \\ 1,8036 \\ \hline 0,7770 \end{array} \qquad \begin{array}{r} +\ \bar{3},8334 \\ \bar{5},8804 \\ \hline \bar{7},7138 \end{array} \qquad \begin{array}{r} -\ \overset{-1}{\bar{1}},0\overset{+1}{3}84 \\ 5,9630 \\ \hline \bar{7},0754 \end{array} \qquad \begin{array}{r} -\ \overset{-1}{0},0\overset{+1}{0}52 \\ \bar{4},5736 \\ \hline 3,4316 \end{array}$

2) Logarifmlarni musbat songa ko‘paytirish:

$\begin{array}{r} \times\ \bar{3},5837 \\ 9 \\ \hline \overline{22},2533 \end{array} \qquad \begin{array}{r} \bar{2},4735 \\ \times\ 14 \\ \hline 18940 \\ +\ 4735 \\ \hline 6,6290 \\ +\ \bar{28} \\ \hline \overline{22},6290 \end{array}$

$0,4735 \cdot 14 = 6,629$ bajarilab, so‘ngra $\bar{2} \cdot 14 = \overline{28}$ topilib, $6,629 + \overline{28} = \overline{22},629$ topiladi.

3) Logarifmni manfiy songa ko‘paytirish va bo‘lish:

$$\overset{\pm1}{\bar{2}},51\overset{-1}{3}4 \cdot (-5) = -1,4896 \cdot (-5) = 7,448.$$

$$\overset{\pm1}{\bar{3}},7\overset{-1}{6}08 : 8 = -2,2392 : 8 = -\overset{-1}{0},27\overset{+1}{9}9 = \bar{1},7201.$$

Antilogarifm jadvallari

Berilgan logarifmga ko‘ra sonni topish uchun, berilgan sonlar logarifmlarining mantissalarini topish jadvallari xizmat qila oladi, lekin **antilogarifmlar,** ya’ni berilgan mantissaga mos keladigan sonlar joylashtirilgan boshqa jadvallardan foydalanish qulayroq (V.M. Bradis «To‘rt xonali matematik jadvallar», XIV jadval, 68-bet). To‘rt xonali mantissasi 2863 berilgan va unga mos keladigan butun sonni topish talab qilingan bo‘lsin. Bu sonni topishda antilogarifmlar jadvallaridan yuqorida mantissani topishdek ish bajariladi. Birinchi ustundan mantissaning ikki raqamini topamiz (raqamlar oldidagi nuqta logarifmining butun sonini mantissadan ajratuvchi vergul o‘rniga qo‘yilgan), ya’ni 28 ko‘rinishda. So‘ngra va raqamlardan gorizontal satr bo‘ylab o‘ng tomonga qarab mantissaning uchinchi raqami turgan vertikal ustun bilan kesishguncha boramiz. Kesishgan joyda 286 mantissaga mos kelgan to‘rt xonali 1932 sonni topamiz. Undan keyin bu sondan gorizontal satr bo‘ylab o‘ngga qarab mantissaning to‘rtinchi raqami turgan vertikal ustun bilan kesishguncha boramiz. Kesishgan joydagi 1 tuzatmani oldin topilgan 1932 ga qo‘shib, 2863 mantissaga mos kelgan 1933 sonni topamiz.

Bundan keyin logarifm xarakteristikasiga e’tibor berib, 1933 sonning tegishli xonasiga vergul qo‘yamiz. Agar berilgan mantissada xarakteristika 1 bo‘lsa, ya’ni 1,2863 bo‘lsa, izlangan son 19,33 bo‘ladi; agar berilgan mantissada xarakteristika –2 bo‘lsa, ya’ni $\bar{2},2863$ bo‘lsa, izlangan son 0,01932 bo‘ladi.

Masalan: 1) agar $\lg x=2,2863$ bo‘lsa, $x=193,3$;

2) $\lg x=0,2863$ bo‘lsa, $x=1,933$;

3) $\lg x=3,5029$ bo‘lsa, $x=3184$;

4) $\lg x=\bar{3},0436$ bo‘lsa, $x=0,001106$;

5) $\lg x=-1,2365$ bo‘lsa, $\overset{-1}{1},\overset{+1}{2}365=\bar{2},7635$ ko‘rinishda yozilib, $x=0,05801$ topiladi.

TAKRORLASH UCHUN SAVOLLAR

1. Uchta raqamli son logarifmining mantissasi jadvaldan qanday topiladi?
2. Ikki va bir raqamli son logarifmining mantissasi jadvaldan qanday topiladi?
3. To‘rt raqamli son logarifmining mantissasi jadvaldan qanday topiladi?
4. Besh yoki undan ortiq raqamli son logarifmining mantissasi qanday topiladi?
5. Manfiy logarifmning mantissasini qanday qilib musbatga almashtiradi (Masalan, –2,4837 ni)?

MASALALARNI YECHING

68. V.M. Bradisning jadvalidan foydalanib, quyidagi sonlarning o‘nli logarifmini toping:

a) 257; 301; 25; 2; 7; 3799; 10325;

b) 3,84; 263,56; 0,02803; 0,000082.

69. Jadvaldan foydalanib, quyidagilarni hisoblang:

a) $\log_3 5$; b) $\log_{0,5} 17$; d) $\log_9 0{,}004$.

70. Logarifmlar ustida amallar bajaring:

a) $\bar{1},4792+\bar{2},5706+4,0056$;

b) $0,9329+\bar{5},0060-\bar{1},2605$;

d) $\bar{1},2396-0,5974-\bar{2},9328$.

Ko‘rsatma. $\bar{1},8432-\bar{2},4031-0,6831$ ni hisoblash uchun mantissalarni musbatga aylantiramiz:

$$-\overset{-1}{\bar{2}},\overset{+1}{4}031=\bar{1},5969; \quad -\overset{-1}{0},\overset{+1}{6}831=\bar{1},3169$$

$$\bar{1},8432-\bar{2},4031-0,6831=\bar{1},8432+\bar{1},5969+\bar{1},3169=\bar{1},7570.$$

71. Quyidagilarni 0,001 aniqlikda hisoblang:

a) $\bar{3},1728\cdot 5$; b) $\bar{2},0296\cdot 0,36$; d) $\bar{2},6302:7$; e) $\bar{3},0280\cdot\frac{2}{5}$.

72. Quyidagilarni 0,001 aniqlikda hisoblang:

a) $\bar{4},4437\cdot(-0,2)$; b) $\bar{5},2709:\left(-\frac{3}{4}\right)$.

73. Antilogarifmlar jadvalidan foydalanib, tenglamani yeching:

a) $\lg x = 0{,}3625$; d) $\lg x = \bar{1}{,}6072$;

b) $\lg x = 4{,}0002$; e) $\lg x = -3{,}0257$.

16-§ Logarifmlar jadvali yordamida hisoblash

1-misol. $x = 12{,}48^5 \cdot \sqrt[7]{5{,}76^3}$ ni hisoblaymiz.

Hisoblash: O‘nli logarifmlaymiz:

$$\lg x = \lg\left(12{,}48^5 \cdot \sqrt[7]{5{,}76^3}\right) = \lg 12{,}48^5 + \lg \sqrt[7]{5{,}76^3} = 5\lg 12{,}48 + \frac{3}{7}\lg 5{,}76 =$$

$$= 5 \cdot 1{,}0962 + \frac{3}{7} \cdot 0{,}7604 = 5{,}4810 + 0{,}3259 = 5{,}8069;\ \lg x = 5{,}8069.$$

Antilogarifmdan 5,8069 ni topamiz. $x = 641000$ (jadvaldan 6410 topilib, xarakteristika 5 bo‘lgani uchun butun olti xona bo‘ladi).

Javob: $x \approx 641000$.

2-misol. $x = \dfrac{0{,}475^2 \cdot \sqrt[3]{\sin 238^\circ 22'}}{\mathrm{tg} 51^\circ 13'}$ ni hisoblaymiz.

O‘nli logarifmlaymiz:

$$\lg x = \lg\left(0{,}475^2 \cdot \sqrt[3]{\sin^2 38^\circ 22'}\right) - \lg tg 51^\circ 13' = 2\lg 0{,}475 +$$

$$+ \frac{2}{3}\lg \sin 38^\circ 22' - 0{,}0950 = 2 \cdot \bar{1}{,}6767 + \frac{2}{3} \cdot \bar{1}{,}7929 - 0{,}095 = \bar{2} + 1{,}3534 +$$

$$+ \frac{2 \cdot \bar{1}{,}7929}{3} - 0{,}095 = \bar{1}{,}3534 + \frac{\bar{1}{,}5858}{3} - 0{,}095 = \bar{1}{,}3534 + \bar{1}{,}8619 - \overset{-1}{\bar{0}}{,}09\overset{+1}{5} =$$

$$= \bar{1}{,}2153 + \bar{1}{,}905 = \bar{1}{,}1203; \quad \lg x \approx \bar{1}{,}1203.$$

Antilogarifmdan x ni topamiz, ya’ni $x = 0{,}1319$ (jadvaldan 1203 topilib, xarakteristika -1 bo‘lgani uchun $x = 0{,}1319$ bo‘ladi).

Javob: 0,1319.

3-misol. $x = \sqrt[6]{\sqrt[3]{35} - \sqrt[4]{30}}$ ni hisoblaymiz.

Hisoblash: Bu misolda 6-darajali ildiz ostidagi ifoda ayirma bo‘lgani uchun uni to‘liq logarifmlab bo‘lmaydi. Buning uchun $\sqrt[3]{35}$ va

$\sqrt[4]{30}$ larni alohida hisoblab topiladi. Natijada, ayirmadan 6-darajali ildiz chiqariladi:

1) $\lg\sqrt[3]{35}=\frac{1}{3}\lg 35=\frac{1}{3}\cdot 1,5441=0,5147;$

$\sqrt[3]{35}\approx 3,271.$

2) $\lg\sqrt[4]{30}=\frac{1}{4}\lg 30=\frac{1}{4}\cdot 1,4771=0,3693;$

$\sqrt[4]{30}\approx 2,341.$

$\sqrt[3]{35}-\sqrt[4]{30}=3,271-2,341=0,93.$

$x=\sqrt[6]{0,93};\ \lg x=\frac{1}{6}\lg 0,93=\frac{1}{6}\cdot\bar{1},9685=\bar{1},9948.$

$\lg x=\bar{1},9948$ bo'lsa, $x\approx 0,9881.$

Javob: 0,9881.

MASALALARNI YECHING

74. Quyidagi ifodalarni o'nli logarifm yordamida hisoblang:

a) $x=13^{13}$; b) $x=0,74^{11}$; d) $x=0,032^{8}$; e) $\sqrt[8]{111}$.

75. a) $\sqrt[6]{421,3}$; b) $\sqrt[5]{0,03715}$; d) $\sqrt[5]{\frac{13}{17}}$; e) $\left(8\frac{1}{5}\right)^{10}$.

76. a) $\frac{1,84^6\cdot\sqrt[5]{11}}{1,17^4}$; b) $\sqrt[8]{\frac{716,5^3}{\sqrt[4]{7}}}$; d) $\frac{3,89^{-2}\sqrt[3]{0,1536}}{0,924^2}$.

77. a) $\sqrt[3]{5\sqrt[5]{2}+\sqrt[5]{36}}$; b) $\frac{\sqrt[3]{\sin 32^\circ 14'\cdot 26,73^2}}{\sqrt{13,65\cdot\operatorname{ctg}29^\circ 18'}}$; d) $\sqrt[3]{5\sqrt[5]{2}+\sqrt[5]{3}-2\sqrt[5]{5}}$.

17-§. Logarifmik tenglamalar

Ta'rif. **Noma'lum son logarifm belgisi ostida qatnashgan tenglamalarni logarifmik tenglamalar deyiladi.**

Masalan: $\log_2 x=5$; $\log_a(x-3)=0$ va hokazo logarifmik tenglamalardir.

Bunday tenglamalar «sonlar teng bo‘lsa, ularning logarifmlari teng va aksincha logarifmlar teng bo‘lsa, ularga mos kelgan sonlar ham teng» degan asosga suyanib yechiladi.

Sodda logarifmik tenglama umumiy ko‘rinishda $\log_a x=b$ bo‘lib, bunda: $a\neq 1$ – musbat son, $x>0$ noma’lum son, b – haqiqiy son. Logarifm ta’rifiga asosan $x=a^b$ bo‘ladi.

Masalan, $\log_3 x=4$ bo‘lsa, bundan $x=3^4=81$.

Tenglamaning ildizi: $x=81$.

1-misol. $\log_x(x^2-3x+6)=2$ tenglamani yechamiz.

Yechish. Bunda $x^2-3x+6>0$ va $x>0$ bo‘lishi kerak. Logarifmning ta’rifiga asosan tenglamani $x^2-3x+6=x^2$ ko‘rinishda yozib, uni yechamiz: $-3x+6=0$; $3x=6$ $x=2$. Tekshirish: $x=2$ da, $\log_x(x^2-3x+6)=$ $=\log_2(4-6+6)=\log_2 4=2$. Demak, $x=2$ tenglamaning ildizi bo‘ladi.

Javob: $x=2$.

2-misol. $\lg(x^2-17)=\lg(x+3)$ tenglamani yechamiz.

Yechish. Bunda $x^2-17>0$ va $x+3>0$ bo‘lishi kerak. «Ikki sonning bir xil asosli logarifmlari teng bo‘lsa, u sonlar ham teng bo‘ladi» deyilgan xossaga asosan tenglamani $x^2-17=x+3$ ko‘rinishda yozib, uni yechamiz: $x^2-x-20=0$. tenglamani yechib, $x_1=-4$ va $x_2=5$ ildizlarni topamiz.

Tekshirish: a) $x=-4$ da $\lg(x^2-17)=\lg(16-17)=\lg(-1)$.

$\lg(-1)$ – mavjud emas, $\lg(x+3)=\lg(-1)$ – mavjud emas.

Demak, $x=-4$ ildiz emas.

b) $x=5$ da $\lg(x^2-17)=\lg(x+3)$.

$\lg(25-17)=\lg(5+3)$.

$\lg 8=\lg 8$. $x=5$ tenglamaning ildizi.

Javob: $x=5$.

3-misol. $2\lg(x-1)=\frac{1}{2}\lg x^5-\lg\sqrt{x}$ tenglamani yechamiz.

Yechish. Tenglamani quyidagicha shakl almashtiramiz.

$2\lg(x-1)=\lg(x-1)^2$.

$$\frac{1}{2}\lg x^5-\lg\sqrt{x}=\lg x^{\frac{5}{2}}-\lg x^{\frac{1}{2}}=\lg\frac{x^{\frac{5}{2}}}{x^{\frac{1}{2}}}=\lg x^2.$$

Berilgan tenglama $\lg(x-1)^2=\lg x^2$ ko'rinishga keldi va uni $(x-1)^2=x^2$ kabi yozamiz.

Bu tenglamani yechib, $x=\frac{1}{2}$ topiladi.

Tekshirish: $x=\frac{1}{2}$ da $2\lg\left(\frac{1}{2}-1\right)=\frac{1}{2}\lg\left(\frac{1}{2}\right)^4-\lg\sqrt{\frac{1}{2}}$.

Bundagi $\lg\left(\frac{1}{2}-1\right)=\lg\left(-\frac{1}{2}\right)$ bo'lib, logarifm mavjud emas, demak, $x=\frac{1}{2}$ – ildiz emas.

Javob: tenglamaning ildizi yo'q.

4-misol. $\log_3^2 x-3\log_3 x-10=0$ tenglamani yechamiz.

Yechish. Bunday tenglamani $\log_3 x=y$ deb belgilab, kvadrat tenglamaga keltirib yechiladi. $y^2-3y-10=0$. Bu tenglamani yechib, $y_1=-2$ va $y_2=5$ larni topamiz.

1) $\log_3 x=-2$ dan $x_1=3^{-2}=\frac{1}{9}$;

2) $\log_3 x=5$ dan $x_2=3^5=243$ topiladi.

Tekshirish: $x_1=\frac{1}{9}$ da $\left(\log_3\frac{1}{9}\right)^2-3\log_3\frac{1}{9}-10=4+6-10=0$.

$x_2=243$ da $(\log_3 243)^2-3\log_3 243-10=25-15-10=0$.

Ildizlar tenglamani qanoatlantiradi.

Javob: $\frac{1}{9}$ va 243.

5-misol. $x^{\lg x-1}=100$ tenglamani yechamiz.

Yechish. Bu tenglamani hadlab logarifmlaymiz:

$\lg(x^{\lg x-1})=\lg 100$.

$(\lg x-1)\cdot\lg x=2$, bunda $\lg x=y$ belgilab,

$(y-1)y=2$ tenglama tuzib, uni yechamiz.

$y^2-y-2=0$. Bu tenglamani yechib,

$y_1=-1$ va $y_2=2$ topiladi.

a) $\lg x=-1$ dan $x=10^{-1}=0{,}1$; $x_1=0{,}1$.

b) $\lg x=2$ dan $x=10^2=100$; $x_2=100$.

Tekshirish:

$x_1=0{,}1$ da $0{,}1^{\lg 0{,}1-1}=0{,}1^{-1-1}=0{,}1^{-2}=\frac{1}{0{,}1^2}=100.$

$x_2=100$ da $100^{\lg 100-1}=100^{2-1}=100^1=100.$

Demak, $x_1=0{,}1$ va $x_2=100$ tenglamani ildizlari.

Javob: 0,1 va 100.

6-misol. $\log_2 x+\log_3 x=1$ tenglamani yechamiz.

Yechish. Bunda logarifmning asoslari har xil bo‘lgani uchun, ularni bir xil asosga keltiramiz:

$$\frac{\lg x}{\lg 2}+\frac{\lg x}{\lg 3}=1 \text{ bundan } \lg x\cdot\left(\frac{1}{\lg 2}+\frac{1}{\lg 3}\right)=1;$$

$$\lg x=1:\frac{\lg 2+\lg 3}{\lg 2\cdot\lg 3}=\frac{\lg 2\cdot\lg 3}{\lg 2+\lg 3};\ \lg x=\frac{\lg 2\cdot\lg 3}{\lg 2+\lg 3}=\frac{\lg 2\cdot\lg 3}{\lg 6}.$$

$$x=10^{\frac{\lg 2\cdot\lg 3}{\lg 6}}=(10^{\lg 2})^{\frac{\lg 3}{\lg 6}}=2^{\frac{\lg 3}{\lg 6}};\ x=2^{\frac{\lg 3}{\lg 6}}.$$

Tekshirish: $x=2^{\frac{\lg 3}{\lg 6}}$ da.

$$\log_2 x+\log_3 x=\frac{\lg x}{\lg 2}+\frac{\lg x}{\lg 3}=\frac{\lg x\cdot(\lg 2+\lg 3)}{\lg 2\cdot\lg 3}=\frac{\lg 2^{\frac{\lg 3}{\lg 6}}\cdot\lg 6}{\lg 2\cdot\lg 3}=-\frac{\lg 3\cdot\lg 2\cdot\lg 6}{\lg 2\cdot\lg 3\cdot\lg 6}=1$$

Javob: $2^{\frac{\lg 3}{\lg 6}}$.

7-misol. $\sqrt[3]{x^{\lg x-1}}=10000$ tenglamani yechamiz.

Yechish. Tenglamani $x^{\frac{\lg x-1}{3}}=10^4$ ko‘rinishda yozib, 10 asosga ko‘ra logarifmlaymiz: $\frac{\lg x-1}{3}\lg x=4\cdot\lg 10$ yoki $(\lg x-1)\lg x=12$.

$\lg^2 x-\lg x-12=0$. Bunda $\lg x=y$ bo‘lsin, u holda $y^2-y-12=0$. Bu tenglamani yechib, $y_1=-3$ va $y_2=4$.

a) $y=-3$ da $\lg x=-3$; $x=10^{-3}=0{,}001$; $x_1=0{,}001$.

b) $y=4$ da $\lg x=4$; $x=10^4=10000$; $x_2=10000$.

Tenglamani tekshirish uchun tenglamaning aniqlanish sohasini topib, tenglamaning topilgan ildizi aniqlanish sohaga tegishli ekanligini aniqlash kifoya.

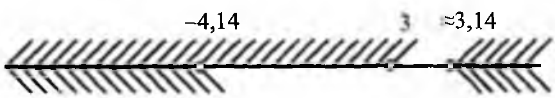
Bu tenglamaning aniqlanish sohasi $x>0$ dan iborat. Demak, $0{,}001>0$ va $10000>0$.

Javob: 0,001 va 10000.

8-m i s o l. $\frac{1}{2}\lg(9-6x+x^2)+\lg(x^2+x-13)=\log_{\sqrt{10}}(x-3)+2\lg\sqrt{7}$ tenglamani yechamiz.

Yechish. Bu tenglamani aniqlanish sohasini topamiz, ya'ni:

$$\begin{cases} 9-6x+x^2>0 \\ x^2+x-13>0 \\ x-3>0 \end{cases} \quad \begin{cases} x\neq 3 \\ x<\approx(-4{,}14) \text{ va } \ x>\approx 3{,}14 \\ x>3 \end{cases} \quad \begin{cases} x>\approx 3{,}14. \end{cases}$$



Bu tenglamani yechish uchun avval uni potensirlaymiz.

$\lg\sqrt{(x-3)^2}+\lg(x^2+x-13)=2\lg(x-3)+\lg 7$;

$\lg(x-3)(x^2+x-13)=\lg(7\cdot(x-3)^2)$;

$(x-3)(x^2+x-13)=7(x-3)^2$;

$(x-3)(x^2+x-13-7(x-3))=0$.

a) $x-3=0$; $x_1=3$.

b) $x^2+x-13-7x+21-0$; $x^2-6x+8=0$; $x_2=2$; $x_3=4$.

Bu $x_2=2$ va $x_1=3$ ildizlar tenglamani aniqlanish sohasiga tegishli emas.

Javob: 4.

TAKRORLASH UCHUN SAVOLLAR

1. Qanday tenglamani logarifmik tenglama deyiladi? Misollar keltiring.
2. Logarifm ta'rifidan foydalanib yechiladigan tenglamalarga misol keltiring?

3. Logarifmlari teng bo'lgan tenglamalar qanday yechiladi?
4. Qo'shimcha noma'lum kiritib yechiladigan tenglamalarga misol keltiring.
5. Popensirlab yechiladigan tenglamaga misol keltiring.

MASALALARNI YECHING

78. Quyidagi logarifmik tenglamalarni yeching:

a) $\log_3 x = -2$; d) $4 + \log_3 x = 6$;

b) $2\log_5 x = 6$; e) $\log_4 x^3 = 3$.

79. a) $\log_2(\log_3 x) = -1$; d) $\log_2(\log_3(\log_4 x)) = 0$;

b) $\log_3(\log_2(x-3)) = 1$; e) $\log_2(\log_3(\log_4(2x-4))) - 0$.

80. a) $\lg x = -\lg 2$; d) $\log_2(x-1) = \log_2(x^2 - x - 16)$;

b) $\lg x = 3 - \lg 5$; e) $2\lg\sqrt{x} = \lg(15 - 2x)$.

81. a) $\lg^2 x = 4 - 3\lg x$; d) $4 + \lg^2 x + 4\lg x - \lg 10 = 0$;

b) $\lg^2 x + \lg 100 = 3\lg x$; e) $\lg^2 x - \lg x^2 = \lg\sqrt[10]{x} + 0{,}2\lg 0{,}1$.

82. a) $\log_2 x \cdot \log_3 x = \log_2 3$; b) $\log_3 x + \log_5 x = \log_3 15$;

d) $\log_9 x + \log_{x^2} 3 = 1$; e) $\log_{3x} 3 = (\log_3 3x)^2$.

83. a) $x^{\lg x + 1} = 1\,000\,000$; d) $x^{\log_3 3x} = 9$;

b) $\sqrt{(1000x)^{\lg x}} = 0{,}1$; e) $x^{\log_2 8x} = 16$.

84. **Ko'rsatma.** 83 masaladagi tenglamalar logarifmlab yechiladi.

a) $\log_3(2x-1)^2 + \log_3(2x-1) = 6$;

b) $\lg\sqrt{2x-3} - 0{,}5\lg(9 - 12x + 4x^2) = \log_3\sqrt{3}$;

d) $\lg 20 - \lg(y^2 - 4y + 5) = 2\left(1 - \lg\sqrt{y^2 + 4y + 5}\right)$;

e) $2\lg 2 + \left(1 + \frac{1}{2x}\right)\lg 3 - \lg\left(\sqrt[x]{3} + 27\right) = 0$.

Ko'rsatma.

$$\lg 4+\lg_3^{1+\frac{1}{2x}}=\lg\left(3^{\frac{1}{x}}+27\right);\ \lg 12\cdot 3^{\frac{1}{2x}}=\lg\left(3^{\frac{1}{x}}+27\right);$$

$12\cdot 3^{\frac{1}{2x}}=3^{\left(\frac{1}{2x}\right)^2}+27$; $3^{\frac{1}{2x}}=y$ bo'lsin. $12y=y^2+27$ ni echib, $y_1=3$, $y_2=9$ larda x_1 va x_2 lar topiladi.

85. a) $\log_3 x+\log_{27} x+\log_{81} x=\frac{19}{12}$;

b) $\log_a x+\log_{a^2} x+\log_{a^4} x=3,5$.

86*. $3\log_3 x\log_x 3\cdot\log_{\frac{x}{9}} 3+\log_3 9x=0$;

Ko'rsatma. $\log_3 x\cdot\log_x 3=1$ dan foydalaniladi.

87*. $\log_2 9^{x-3,5}\cdot\log_3\sqrt[3]{8^{2x-3}}=2x^2-7x+1$;

Ko'rsatma. $\log_2 3^{2x-7}\cdot\log_3 2^{2x-3}=2x^2-7x+1$;

$(2x-7)(2x-3)\cdot\log_2 3\cdot\log_3 2=2x^2-7x+1$ yoki

$(2x-7)(2x-3)\cdot 1=2x^2-7x+1$ tenglama yechiladi.

88*. $\lg x+2\lg x+3\lg x+...+x\lg x=2x^2+2x$.

Ko'rsatma. $(1+2+3+...+x)\lg x=2x^2+2x$;

$\frac{1+x}{2}\cdot x\cdot\lg x=2x(x+1)$; $x(x+1)(\lg x-4)=0$ tenglama yechiladi.

89*. $\log_{16} x+\log_{16}^2 x+\log_{16}^3 x+...=\frac{1}{3}(1<x<16)$.

Ko'rsatma. $a_1=\log_{16} x$ va $q=\log_{16} x$ bo'lgan geometrik progressiyadan iborat. $0<\log_{16} x<1$ bo'lganidan,

$S=\frac{a_1}{1-q}=\frac{1}{3}$; $\frac{\log_{16} x}{1-\log_{16} x}=\frac{1}{3}$ tenglama yechiladi.

18-§. Logarifmik tenglamalar sistemasi

I. Tenglamalar sistemasiga keltirish bilan yechiladigan logarifmik tenglamalar sistemasi

1-misol. $\begin{cases} \lg x + \lg y = -4 \\ \lg x - \lg y = 2 \end{cases}$ sistemani yechamiz.

Yechish.

1-usul. $+\begin{array}{l} \lg x + \lg y = -4 \\ \lg x - \lg y = 2 \\ \hline 2\lg x = -2; \end{array}$ bu sistemada $x>0$ va $y>0$. $\lg x=-1$; $x=10^{-1}=0,1$.

$\lg x=-1$; da $-1+\lg y=-4$; $\lg y=-3$; $y=10^{-3}=0,001$.
$x=0,1$; $y=0,001$.

2-usul. $\lg x=u$, $\lg y=\upsilon$ bo'lsin deb, $\begin{cases} u+\upsilon=-4 \\ u-\upsilon=2 \end{cases}$ sistemani yechamiz:

$+\begin{cases} u+\upsilon=-4 \\ u-\upsilon=2 \end{cases}$ $\quad$ $-\begin{cases} u+\upsilon=-4 \\ u-\upsilon=2 \end{cases}$

$2u=-2$; $\quad$ $2\upsilon=-6$;
$u=-1$. $\quad$ $\upsilon=-3$;
$\lg x=-1$ $\quad$ $\lg y=-3$
$x=0,1$. $\quad$ $y=0,001$. $\quad$ *Javob:* $x=0,1$; $y=0,001$.

2-misol. $\begin{cases} \log_x 8 + \log_y 9 = 5 \\ \log_x 4 + \log_y 27 = 5 \end{cases}$ sistemani yechamiz.

Yechish. Bundagi: $x\neq 1$ va $y\neq 1$ – musbat sonlar. $\log_x 8=\log_x 2^3=3\log_x 2$; $\log_y 9=2\log_y 3$; $\log_x 4=2\log_x 2$; $\log_y 27=3\log_y 3$ bo'lgani uchun sistema

$\begin{cases} 3\log_x 2 + 2\log_y 3 = 5 \\ 2\log_x 2 + 3\log_y 3 = 5 \end{cases}$ hosil bo'ladi.

$\log_x 2=u$, $\log_y 3=\upsilon$ bo'lsin deb, $\begin{cases} 3u+2\upsilon=5 \\ 2u+3\upsilon=5 \end{cases}$ sistemani yechamiz:

Bu sistemani yechib u=1 va $\upsilon=1$ lar topiladi.
$\log_x 2=1$ dan, $x=2$ va $\log_y 3=1$ dan, $y=3$.
Javob: $x=2$ va $y=3$.

II. Potensirlash bilan yechiladigan logarifmik tenglamalar sistemasi

3-misol. $\begin{cases} \lg(x^2+y^2+50)-\lg 7=2 \\ \lg(x+y)+\lg 6=\lg(x-y)+\lg 17 \end{cases}$ sistemani yechamiz.

Yechish. Sistemada $x+y>0$ va $x-y>0$ bo'lish kerak.

$\begin{cases} \lg(x^2+y^2+50)=\lg 700 \\ \lg(6x+6y)=\lg(17x-17y) \end{cases}$ bundan:

$$\begin{cases} x^2+y^2+50=700 \\ 6x+6y=17x-17y \end{cases}; \quad \begin{cases} x^2+y^2=650 \\ 11x-23y=0 \end{cases}; \quad \begin{cases} \left(\frac{23y}{11}\right)^2+y^2=650 \\ x=\frac{23y}{11} \end{cases};$$

$\frac{529y^2}{121}+y^2=650$; $529y^2+121y^2=78650$; $650y^2=78650$.

$y^2=121$; $y=\pm 11$; $x=\frac{23(\pm 11)}{11}=\pm 23$. Bu topilgan ildizlarning $x+y>0$ va $x-y>0$ shartni qanoatlantiradiganlari: $x=23$ va $y=\pm 11$.

III. Logarifmlash bilan yechiladigan logarifmik tenglamalar sistemasi

4-misol. $\begin{cases} \lg x+\lg y=5 \\ x^{\lg y}=10000 \end{cases}$ sistemani yechamiz.

Yechish. Sistemada $x>0$ va $y>0$ bo'lishi kerak.

$x^{\lg y}=10000$ ni 10 li logarifmlaymiz:

$\lg x^{\lg y}=\lg 10000$; $\lg x\cdot\lg y=4$. Sistema $\begin{cases} \lg x+\lg y=5 \\ \lg x\cdot\lg y=4 \end{cases}$ ko'rinishga keladi. Bunda $\lg x=u$ $\lg y=\upsilon$ bo'lsin deb, $\begin{cases} u+\upsilon=5 \\ u\cdot\upsilon=4 \end{cases}$ ni hosil qilamiz.

Bu sistemani yechib, $u_1=1$; $u_2=4$ va $\upsilon_1=4$; $\upsilon_2=1$ larni topamiz.

$\lg x_1=1$ dan, $x_1=10$; $\lg x_2=4$ dan, $x_2=10000$;

$\lg y_1=4$ dan, $y_1=10000$; $\lg y_2=1$ dan, $y_2=10$.

Javob: $x_1=10$; $y_1=10000$ va $x_2=10000$ va $y_2=10$.

IV. Bir asosdan ikkinchi asosga o'tish formulalaridan foydalanib yechiladigan logarifmik tenglamalar sistemasi

5-misol. $\begin{cases} \log_x 8+\log_y 4=3 \\ \log_8 x+\log_4 y=\frac{3}{2} \end{cases}$ sistemani yechamiz.

Yechish. $x\neq 1$ musbat; $y\neq 1$ musbat sonlar.

$\begin{cases} \log_x 8+\log_y 4=3 \\ \log_8 x+\log_4 y=\frac{3}{2} \end{cases}$; $\begin{cases} 3\log_x 2+2\log_y 2=3 \\ \frac{1}{3\log_x 2}+\frac{1}{2\log_y 2}=\frac{3}{2} \end{cases}$ bunda $\log_x 2=u$ va

$\log_y 2=\upsilon$ bo'lsin deb, $\begin{cases} 3u+2\upsilon=3 \\ \frac{1}{3u}+\frac{1}{2\upsilon}=\frac{3}{2} \end{cases}$ sistemani tuzamiz.

Bundan: $\begin{cases} 3u+2\upsilon=3 \\ 3u+2\upsilon=9u\upsilon \end{cases} \Rightarrow 9u\upsilon=3;\ \upsilon=\frac{1}{3u}$ buni

$3u+2\upsilon=3$ ga qo'yib, $3u+2\cdot\frac{1}{3u}=3$ dan

$9u^2-9u+2=0$ topib, bu tenglamani yechamiz:

$u_1=\frac{1}{3}$ va $u_2=\frac{2}{3}$; $\upsilon_1=\frac{1}{3u_1}=\frac{1}{3\cdot\frac{1}{3}}=1$ va $\upsilon_2=\frac{1}{3u_2}=\frac{1}{3\cdot\frac{2}{3}}=\frac{1}{2}$.

$\log_x 2=u_1$; $\log_x 2=\frac{1}{3}$; $x^{\frac{1}{3}}=2$; $x_1=8$.

$\log_x 2=u_2$; $\log_x 2=\frac{2}{3}$; $x^{\frac{2}{3}}=2$; $x_2=2^{\frac{3}{2}}=2\sqrt{2}$.

$\log_y 2=\upsilon_1$; $\log_y 2=1$; $y_1=2$.

$\log_y 2=\upsilon_2$; $\log_y 2=\frac{1}{2}$; $y^{\frac{1}{2}}=2$; $y_2=4$.

Javob: $x_1=8$; $y_1=2$ va $x_2=2\sqrt{2}$; $y_2=4$.

MASALALARNI YECHING

90. Quyidagi tenglamalar sistemasini yeching:

a) $\begin{cases} \lg x+\lg y=4 \\ \lg x-\lg y^2=1 \end{cases}$; b) $\begin{cases} 3\log_5 x+\log_3 y=5 \\ \log_5 x+4\log_3 y=6 \end{cases}$; d) $\begin{cases} \lg^3 x-\lg^3 y=7 \\ \lg x-\lg y=1 \end{cases}$

91. a) $\begin{cases} \log_2(x+y)+\log_2(x-y)=\log_2 12 \\ \log_2 x=3-\log_2 y \end{cases}$;

b) $\begin{cases} \log_5 \log_3 \log_2(x+y)=0 \\ xy=7 \end{cases}$;

d) $\begin{cases} \log_5 x+3^{\log_3 y}=7 \\ x^y=5^{12} \end{cases}$

92. a) $\begin{cases} \log_y x+\log_x y=-2,5 \\ xy=9 \end{cases}$;

b) $\begin{cases} \log_x y+\log_y x=2,5 \\ x+y=6 \end{cases}$;

d) $\begin{cases} \frac{1}{4}\log_y x-\log_{\frac{1}{\sqrt{x}}} y=1,5 \\ xy=32 \end{cases}$

93*. a) $\begin{cases} \log_4 x\cdot\log_3 y=4 \\ xy=144 \end{cases}$

Ko'rsatma. $\log_4 x$ ni $\frac{\log_3 x}{\log_3 4}$ kabi yozib, $\log_3(xy)=\log_3 144$ logarifmlaymiz.

$\log_3 x+\log_3 y=2+2\log_3 4$ lardan

$\begin{cases} \log_3 x\cdot\log_3 y=4\log_3 4 \\ \log_3 x+\log_3 y=2(1+\log_3 4) \end{cases}$ sistemani tuzamiz.

Bunda $\log_3 x=u$, $\log_3 y=\upsilon$ belgilab,

$$\begin{cases} u+\upsilon=4\log_3^4 \\ u\cdot\upsilon=2(1+\log_3^4) \end{cases}$$ sistemani yechib ildizlar topiladi.

b) $$\begin{cases} \log_3 x+\log_2 y=3 \\ xy=12 \end{cases}$$

19-§. Logarifmik tengsizliklar

Logarifmik tengsizliklar ham ko'rsatkichli tengsizliklar kabi $y=\log_a x$ funksiya asos $a>1$ da monoton o'suvchi va asos $0<a<1$ da monoton kamayuvchi xossalaridan foydalanib yechiladi.

1-misol. $\log_5(2x-1)<2$ tengsizlikni yechamiz.

Yechish. Tengsizlikdagi 2 ning o'rniga $\log_5 25$ ni qo'yamiz: $\log_5(2x-1)<\log_5 25$, bunda asos $5>1$ ekanligidan $2x-1<25$ yozamiz. Logarifmda $2x-1>0$ bo'lgan uchun, bu tengsizliklarni qanoatlantiruvchi x ning qiymatini topamiz.

$$\begin{cases} 2x-1<25 \\ 2x-1>0 \end{cases} \quad \begin{cases} x<13 \\ x>0,5 \end{cases}$$ bularning yechimi $0,5<x<13$.

Javob: $0,5<x<13$.

2-misol. $\log_{0,2}(x^2-5x+6)>-1$ tengsizlikni yechamiz.

Yechish. 1) Logarifmda $x^2-5x+6>0$ bo'lishi kerak.

2) -1 soni $\log_{\frac{1}{5}} 5=\log_{0,2} 5$ ga teng. Buni berilgan tengsizlikka qo'yib,

$\log_{0,2}(x^2-5x+6)>\log_{0,2} 5$ tengsizlikni hosil qilamiz.

Buna asos $0,2<1$ bo'lgani uchun $x^2-5x+6<5$ tengsizlikni yozamiz. Berilgan tengsizlikni yechimi

$$\begin{cases} x_2-5x+6>0 \\ x_2-5x+6<5 \end{cases}$$ sistemaning yechimidan iborat bo'ladi.

$x^2-5x+6>0$ ning yechimi $x<2$ va $x>3$.

$x^2-5x+6<5$; $x^2-5x+1<0$ ning yechimi

$$2{,}5-\sqrt{5{,}25}<x<2{,}5+\sqrt{5{,}25}.$$

Tengsizlikning yechimi $\begin{cases} x<2 \text{ va } x>3 \\ 2{,}5-\sqrt{5{,}25}<x<2{,}5+\sqrt{5{,}25} \end{cases}$ yoki

Javob: $2{,}5-\sqrt{5{,}25}<x<2$ va $3<x<2{,}5+\sqrt{5{,}25}$.

3-misol. $\frac{\lg(2x-3)}{x^2-5x+6}<0$ tengsizlikni yechamiz.

Yechish. Bu tengsizlikni yechish uchun ikki hol qarab chiqiladi.

I. $\begin{cases} \lg(2x-3)>0 \\ x^2-5x+6<0 \\ 2x-3>0 \end{cases}$ $\begin{cases} 2x-3>1 \\ 2<x<3 \\ x>1{,}5 \end{cases}$ $\begin{cases} x>2 \\ 2<x<3; \\ x>1{,}5 \end{cases} \Rightarrow 2<x<3$

II. $\begin{cases} \lg(2x-3)<0 \\ x^2-5x+6>0; \\ 2x-3>0 \end{cases}$ $\begin{cases} 2x-3<1 \\ x<2 \text{ va } x>3; \\ x>1{,}5 \end{cases}$ $\begin{cases} x<2 \\ x<2 \text{ va } x>3; \\ x>1{,}5 \end{cases} \Rightarrow 1{,}5<x<2$

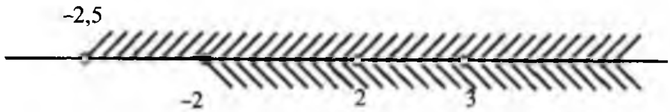
Javob: $1{,}5<x<2$ va $2<x<3$.

4-misol. $y=\sqrt{\lg(2x+5)}-\frac{4x+5}{x^2-5x+6}$ funksiyaning aniqlanish sohasini toping.

Yechish. Funksiya aniqlangan bo‘lishi uchun $\lg(2x+5)\geq 0$; $2x+5>0$ va $x^2-5x+6\neq 0$ bo‘lishi kerak. Bu tengsizliklarning umumiy yechimi

$\begin{cases} \lg(2x+5)\geq 0 \\ 2x+5>0 \\ x^2-5x+6\neq 0 \end{cases}$ sistemaning yechimi bo‘ladi.

$$\begin{cases} \lg(2x+5)\geq \lg 1 \\ 2x>-5 \\ x\neq 2 \text{ va } x\neq 3 \end{cases}; \quad \begin{cases} 2x+5\geq 1 \\ x>-2{,}5 \\ x\neq 2 \text{ va } x\neq 3 \end{cases}; \quad \begin{cases} x\geq -2 \\ x>-2{,}5 \\ x\neq 2 \text{ va } x\neq 3 \end{cases}.$$



Javob: $-2\le x<2$; $2<x<3$ va $x>3$.

5-misol. $\log_{2x-1}(4x+5)<0$ tengsizlikni yechamiz.

Yechish. Asos $2x-1>1$ va $0<2x-1<1$ bo'lgan ikki holni ko'rib chiqamiz:

$$\text{I.}\ \begin{cases} 2x-1>1 \\ 4x+5>0 \\ \log_{2x-1}(4x+5)<\log_{2x-1}1 \end{cases};\quad \begin{cases} x>1 \\ x>-1,25; \\ 4x+5<1 \end{cases}\quad \begin{cases} x>1 \\ x>-1,25 \Rightarrow \\ x<-1 \end{cases} \text{bu holda yechim yo'q.}$$

$$\text{II.}\ \begin{cases} 0<2x-1<1 \\ 4x+5>0 \\ 4x+5>1 \end{cases};\quad \begin{cases} 1<2x<2 \\ x>-1,25; \\ x>-1 \end{cases}\quad \begin{cases} 0,5<x<1 \\ x>-1,25; \Rightarrow 0,5<x<1 \\ x>-1 \end{cases}$$

Javob: $0,5<x<1$.

MASALALARNI YECHING

94. Quyidagi tengsizliklarni yeching:

a) $\log_5 x>3$; d) $\log_{0,2} x<-2$;

b) $\log_7 x<0,1$; e) $\lg(2x-7)\le 1$.

95. a) $\lg(x+1)>\lg(5-x)$; d) $\log_{\frac{1}{7}}(2x-6)<\log_{\frac{1}{7}}x$;

b) $\log_{\frac{1}{2}}(x-7)>4$; e) $\lg(x^2-3)>\lg(x+3)$.

96. a) $\log_2(x^2-x-4)<3$; d) $\lg(x-1)+\lg x<\lg 1$;

b) $\log_3(12-2x-x^2)>2$; e) $\lg(x^2-x+8)\ge 10$.

97. a) $\lg^2x+2\lg x>3$; d) $8\lg^2x-2\lg x>\lg 10$;

b) $\lg^2x-2\lg x-8\leq 0$; e) $\log_2(x^2-3x-5)<\log_2 x$.

98*. a) $(x^2+2x+2)\log_{0,3}(2x-1)>0$;

b) $(4x-x^2-5)\log_7(1+3x)>0$.

99*. Tengsizlikni yeching:

$$\sqrt{\log_3\frac{2x-1}{3-x}}<2.$$

Ko'rsatma. $\frac{2x-1}{3-x}\geq 1$ da $\log_3\frac{2x-1}{3-x}\geq 0$ bo'lib, kvadrat ildiz mavjud bo'ladi.

$\sqrt{\log_3\frac{2x-1}{3-x}}<2$ ning ikkala tomoni musbat bo'lgani uchun uni kvadratga ko'tarib, $\log_3\frac{2x-1}{3-x}<4$ ni hosil qilamiz. Bundan

$\log_3\frac{2x-1}{3-x}<\log_3 81$ yoki $\frac{2x-1}{3-x}<81$ kelib chiqadi.

Natijada $\begin{cases}\frac{2x-1}{3-x}\geq 1\\ \frac{2x-1}{3-x}<81\end{cases}$ bu sistemani yechib tengsizlik yechimi topiladi.

100*. $\frac{\lg^2 x-3}{\lg x}<-2$ tengsizlikni yeching:

Ko'rsatma. $\frac{\lg^2 x-3}{\lg x}+2<0$ yoki $\frac{\lg^2 x+2\lg x-3}{\lg x}<0$.

Bunda: $x>0$; $x\neq 1$; $\lg x=y$ bilan belgilab, $\frac{y^2+2y-3}{y}<0$ ni yechib, tengsizlikning yechimi topiladi.

101*. Quyidagi funksiyalarning aniqlanish sohasini toping:

$$y=\sqrt{\log_2(3x^2-x-2)}$$

102*. Funksiyaning aniqlanish sohasini toping: $y = \sqrt{\lg(2x+5)} + \sqrt{\lg(5-2x)}$

Ko'rsatma. Bu funksiyaning aniqlanish sohasi

$$\begin{cases} 2x+5>0 \\ \lg(2x+5)\geq 0 \\ 5-2x>0 \\ \lg(5-2x)\geq 0 \end{cases}$$ sistemani yechib topiladi.

20-§. Funksiyaning nuqtadagi limitining ta'rifi

Fizika kursidagi yerkin tushishni ko'rib chiqamiz. Erkin tushish S yo'l t vaqtning funksiyasi sifatida $S=\frac{gt^2}{2}$ formula bilan beriladi.

Vaqtning biror t_0 paytini olib va t_0 dan $t=t_0+\Delta t$ paytgacha bo'lgan vaqt oralig'ini qaraymiz. Bu vaqt oralig'ida jism $\Delta S=S(t+\Delta t)-S(t_0)=\frac{g(t_0+\Delta t)^2}{2}-\frac{g}{2}t_0^2=gt_0\Delta t+\frac{g\Delta t^2}{2}$ yo'lni o'tadi.

Fizikada $\frac{\Delta S}{\Delta t}=gt_0+\frac{g\Delta t}{2}$ munosabatni $(t_0;\ t_0+\Delta t)$ vaqt oralig'ida jismning o'rtacha tezligi deyiladi. t_0 o'zgarmas bo'lganda bu o'rtacha tezlik t ning funksiyasi bo'ladi, ya'ni $\upsilon_{\text{o'rta}}(\Delta t)=gt_0+\frac{g\Delta t}{2}$.

Bu formuladan ma'lumki, kichik Δt larda o'rtacha tezlik gt_0 dan juda kam farq qiladi. Masalan, $t_0=2$ da Δt ning 0 ga yaqinlashuvchi ba'zi qiymatlarida $\upsilon_{\text{o'rta}}(\Delta t)=2g+\frac{g}{2}\Delta t$ ning qiymatlari jadval bilan beriladi (soddalik uchun $g=9{,}8$ deb olamiz).

Δt	1	0,1	0,01	0,001
$\upsilon_{\text{o'rta}}$	24,5	20,09	19,649	19,6049

Bu jadvalda, agar Δt borgan sari 0 ga yaqin qiymatlarni qabul qilsa, u holda $\upsilon_{\text{o'rta}}$ ning tegishli qiymatlari $\upsilon_0=gt_0=19{,}6000$ qiymatga yaqinlashadi, ya'ni Δt nolga intilganda $\upsilon_{\text{o'rta}}$ o'rtacha tezlik υ_0 limitga intiladi. Bu limit t_0 paytdagi «haqiqiy» (yoki «oniy») tezlik deb hisoblaniladi.

Oniy tezik $\upsilon_0=\lim\limits_{\Delta t\to 0}\upsilon_{\text{o'rta}}(\Delta t)$ ko'rinishda yoziladi.

1-misol. Tomonining uzunligi ε aniqlikda berilgan (ε – musbat son) a ga teng kvadrat shaklidagi plastinkada o'lchashlar bajarilib, u ning P perimetrini topish talab qilinsin.

Buning uchun plastinka tomonining uzunligini $\frac{\varepsilon}{4}$ gacha aniqlikda o'lchash yetarli. Haqiqatan, agar x biz bajargan ishlar natijasi bo'lib, $|x-a|<\frac{\varepsilon}{4}$ bo'lsa, u holda $|4x-4a|=\varepsilon$ bo'ladi. $P=4a$ bo'lgani uchun, $|4x-4a|<\varepsilon$ tengsizlik ε gacha aniqlikda $4x=P$ ekanligini ko'rsatadi.

Qo'yilgan masalani yechishda biz har qanday $\varepsilon>0$ uchun a ga yetarlicha yaqin barcha x larda $f(x)=4x$ funksiyaning qiymatlari $P=4a$ sonidan ε ga qaraganda kichikroq farq qilishini ko'rdik. Demak, $f(x)=4x$ funksiyaning x ni a ga intilgandagi limiti $4a$ ga teng ya'ni $\lim_{x\to a} 4x=4a$.

Bu yozuv: «$f(x)=4x$ funksiyaning a nuqtadagi limiti $4a$ ga teng» kabi o'qiladi.

Yuqorida ko'rib chiqilgan misollardan funksiyaning nuqtadagi limitining quyidagi ta'rifini beramiz:

Ta'rif. **Agar ixtiyoriy $\varepsilon>0$ son uchun x ning a ga yetarlicha yaqin barcha qiymatlarida ($x\neq a$) $|f(x)-b|<\varepsilon$ tengsizlik bajarilsa, b soni $f(x)$ funksiyaning a nuqtadagi limiti deyiladi.**

Bu limit $\lim_{x\to a} f(x)=b$ ko'rinishda yoziladi.

2-misol. $f(x)=2x+3$ funksiyaning 1 nuqtadagi limiti 5 ga teng ekanini isbotlaymiz, ya'ni $\lim_{x\to 1}(2x+3)=5$.

Buning uchun $f(x)-5=2x+3-5=2x-2$ ayirmani qaraymiz. $f(x)-5$ ayirmaning $2|x-1|$ moduli har qanday x da $f(x)$ soni bilan 5 soni orasidagi masofadir. Agar $|x-1|<0{,}005$ bo'lsa, ya'ni x nuqtadan 1 gacha bo'lgan masofa 0,005 dan kichik bo'lsa, $2|x-1|$ masofa 0,01 dan kichik bo'ladi yoki $2|x-1|<0{,}01$.

Agar $|f(x)-5|<0{,}001$ tengsizlikning bajarilishini xohlasak, x soni $|x-1|<0{,}0005$ tengsizlikni qanoatlantiradi ($2|x-1|<0{,}001$ dan $|x-1|<0{,}0005$).

Umuman, har qanday $\varepsilon>0$ sonni olganimizda ham $f(x)-5$ ayirmaning moduli, agar $|x-1|<\frac{\varepsilon}{2}$ bo‘lsagina, ya’ni x nuqtadan 1 nuqtagacha bo‘lgan masofa dan kichik bo‘lsagina, ε dan kichik bo‘ladi, ya’ni $|f(x)-5|<\varepsilon$.

3-misol. O‘zgarmas funksiyaning limiti istalgan a nuqtada shu o‘zgarmasning o‘ziga teng ekanini isbotlaymiz.

Haqiqatan, agar a nuqtani o‘z ichiga oluvchi biror intervaldan bo‘lgan barcha x lar uchun $f(x)=k$ bo‘lsa, u holda bunday x lar uchun $|f(x)-k|=|k-k|=0<\varepsilon$ bajariladi.

Shunday qilib, $\lim\limits_{x\to a} k=k$.

TAKRORLASH UCHUN SAVOLLAR

1. Jismlarning erkin tushish masofasi qanday formula bilan topiladi?
2. Δt vaqt oralig‘idagi o‘rtacha tezlik qanday formula bilan topiladi?
3. Qanday tezlikni oniy tezlik deyiladi?
4. $f(x)$ funksiyaning a nuqtadagi limiti deb nimaga aytiladi?
5. O‘zgarmas funksiyaning limiti nimaga teng?

MASALALARNI YECHING

103. Tomonlari 15 va a ga teng bo‘lgan to‘g‘ri to‘rtburchak shaklida yer uchastkasi bor. To‘g‘ri to‘rtburchakning: a) perimetrini; b) yuzini 10^{-2} gacha aniqlikda hisoblash uchun uning a tomonini qanday aniqlikda o‘lchash kerak?

Ko‘rsatma. To‘g‘ri to‘rtburchakning perimetrini $P(x)=30+2x$; yuzini $S(x)=15x$ ko‘rinishdagi funksiya deb oling.

104. Agar a) $f(x)=3x+2$; b) $f(x)=\frac{x^2-4}{x+2}$ bo‘lsa, $\varepsilon=0,1; 0,01; 0,001$ uchun x o‘zgaruvchini qanday aniqlikda olish kerak, agar $|f(x)-(-4)|<\varepsilon$ bajarilishi uchun.

105. Funksiyaning nuqtadagi limiti ta’rifidan foydalanib isbotlang:

a) $\lim\limits_{x\to 2}(2x+3)=7$; b) $\lim\limits_{x\to -2}(5x-2)=-12$.

Ko‘rsatma: a) $|x-2|<\frac{\varepsilon}{2}$ va b) $|x+2|<\frac{\varepsilon}{5}$ kabi olinadi.

21-§. Funksiyalarning limitlari haqidagi asosiy teoremalar

Funksiyaning nuqtadagi limitini faqat limitning ta’rifidan foydalanib topish ancha murakkab. Quyidagi teoremalar bu ishni ancha osonlashtiradi (bu teoremalarning isboti bu kurs programmasiga kirmaydi).

1-teorema. **O‘zgarmas sonning limiti shu sonning o‘ziga teng:**

$$\lim_{x\to a} C = C \text{ (3-misolda isbotlangan).}$$

2-teorema. **O‘zgarmas ko‘paytuvchini limit ishorasidagi tashqariga chiqarish mumkin.**

$$\lim_{x\to a}(k\cdot f(x)) = k\cdot \lim_{x\to a} f(x).$$

3-teorema. **Funksiyalar yig‘indisining (ayirmasining) limiti shu funksiyalar limitlarining yig‘indisiga (ayirmasiga) teng:**

$$\lim_{x\to a}(f(x)\pm g(x)) = \lim_{x\to a}(f(x) \pm \lim_{x\to a} g(x).$$

4-teorema. **Funksiyalar ko‘paytmasining limiti shu funksiyalar limitlarining ko‘paytmasiga teng:**

$$\lim_{x\to a}(f(x)\cdot g(x)) = \lim_{x\to a}(f(x)\cdot \lim_{x\to a} g(x).$$

5-teorema. **Agar bo‘luvchining limiti nolga teng bo‘lmasa, ikki funksiya nisbatining limiti shu funksiyalar limitlarining nisbatiga teng:**

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)} \left(\lim_{x\to a} g(x)\neq 0\right)$$

Funksiyalarning limitlarini topishga doir bir nechta misollar ko‘rib chiqamiz:

1-misol. $\lim\limits_{x\to 3} x^2$ va $\lim\limits_{x\to 2}(2x+7)$ limitlarni hisoblaymiz.

$\lim\limits_{x\to 3} x^2 = \lim\limits_{x\to 3}(x\cdot x)$ bu limitni ko‘paytmaning limitini topishga ko‘ra:

$\lim\limits_{x\to 3}(x\cdot x) = \log\limits_{x\to 3} x \cdot \log\limits_{x\to 3} x = 3\cdot 3 = 9;$

$\lim\limits_{x\to 2}(2x+7)$ ni yig‘indining limitini topishga ko‘ra:

$\lim\limits_{x\to 2}(2x+7) = \lim\limits_{x\to 2}(2\cdot x) + \lim\limits_{x\to 2} 7 = 2\cdot \lim\limits_{x\to 2} x + 7 = 2\cdot 2 + 7 = 11.$

2-misol. $\lim\limits_{x\to 2}\frac{3x^2+2}{4x-5}$ limitni hisoblaymiz. Yig‘indi, ko‘paytma, bo‘linmalarning limitini topish teoremalariga asosan topiladi.

$\lim\limits_{x\to 3}\frac{3x^2+2}{4x-5} = \frac{\lim\limits_{x\to 3}(3x^2+2)}{\lim\limits_{x\to 3}(4x-5)}$ bundan:

1) $\lim\limits_{x\to 3}(3x^2+2) = \lim\limits_{x\to 3} 3x^2 + \lim\limits_{x\to 3} 2 = 3\cdot \lim\limits_{x\to 3} x^2 + 2 = 3\cdot \lim\limits_{x\to 3} x \times$

$\times \lim\limits_{x\to 3} x + 2 = 3\cdot 3\cdot 3 + 2 = 29.$

2) $\lim\limits_{x\to 3}(4x-5) = \lim\limits_{x\to 3} 4x - \lim\limits_{x\to 3} 5 = 4\lim\limits_{x\to 3} x - 5 = 4\cdot 3 - 5 = 7.$

3) $\frac{\lim\limits_{x\to 3}(3x^2+2)}{\lim\limits_{x\to 3}(4x-5)} = \frac{29}{7} = 4\frac{1}{7}$.

3-misol. $\lim\limits_{x\to 3}\frac{x^2-2x-3}{x^2-5x+6}$ ni hisoblaymiz.

Bu limitning surat va maxraji $x\to 3$ da nolga intiladi. Bu yerda bo‘linmaning limiti haqidagi teoremani bevosita tatbiq etib bo‘lmaydi. Ammo berilgan kasrni qisqartirib, uni boshqa ko‘rinishga keltiramiz: $\frac{x^2-2x-3}{x^2-5x+6} = \frac{(x-3)(x+1)}{(x-3)(x-2)} = \frac{x+1}{x-2}$.

Endi limit osongina topiladi.

$$\lim\limits_{x\to 3}\frac{x^2-2x-3}{x^2-5x+6} = \lim\limits_{x\to 3}\frac{x+1}{x-2} = \frac{3+1}{3-2} = 4$$

(Bundan buyon limitlardagi hisoblashlar og‘zaki bajariladi).

4-misol. $\lim\limits_{x\to 0}\frac{x+\sqrt{x}}{x-\sqrt{x}}$ ni hisoblaymiz.

Bu kasrning maxraji $x\to 0$ da nol bo'lgani uchun kasrni $\sqrt{x}$ ga qisqartiramiz:

$$\lim_{x\to 0}\frac{x+\sqrt{x}}{x-\sqrt{x}}=\lim_{x\to 0}\frac{\sqrt{x}(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)}=\lim_{x\to 0}\frac{\sqrt{x}+1}{\sqrt{x}-1}=\frac{0+1}{0-1}=-1$$

5-misol. $\lim\limits_{x\to 0}\frac{x}{\sqrt{1+x}-1}$ ni hisoblaymiz.

Bu kasrning surat va maxraji $x\to 0$ da nolga aylanadi. Shu sababli bo'linmaning limiti haqidagi teoremani tatbiq etib bo'lmaydi, kasrni qisqartirib ham bo'lmaydi. Bu holda kasrning surat va maxrajini $\sqrt{1+x}+1$ ga ko'paytirish lozim.

Natijada ushbu

$$\frac{x}{\sqrt{1+x}-1}=\frac{x(\sqrt{1+x}+1)}{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}=\frac{x(\sqrt{x+1}+1)}{1+x-1}=\sqrt{x+1}+1$$ hosil bo'ladi.

Bundan limit olamiz: $\lim\limits_{x>0}(\sqrt{x+1}+1)=1+1=2$

6-misol. $\lim\limits_{x\to 4}\frac{\sqrt{x}-2}{x-4}$ ni hisoblaymiz.

$x\to 4$ da kasrning surat va maxraji 0 ga intiladi. Buning uchun kasrning maxrajini ko'paytma ko'rinishida tasvirlab, uning ko'rinishini o'zgartiramiz: $\frac{\sqrt{x}-2}{x-4}=\frac{\sqrt{x}-2}{(\sqrt{x}-2)(\sqrt{x}+2)}=\frac{1}{\sqrt{x}+2}$.

Endi berilgan limitni topamiz:

$$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}=\lim_{x\to 4}\frac{1}{\sqrt{x}+2}=\frac{1}{2+2}=\frac{1}{4}.$$

LIMITLARNI HISOBLANG

Quyidagi limitlarni hisoblang:

106. $\lim\limits_{x\to 1}(2x^2+3x-5)$;

107. $\lim\limits_{x\to 5}(3x-x^2)$;

108. $\lim\limits_{x\to 0}\frac{x^2-1}{2x^2-x-1}$;

109. $\lim\limits_{x\to 1}\frac{x^2-3x+2}{x^2-4x+3}$;

110. $\lim\limits_{x\to 2}\dfrac{3x^2+5x-2}{5x^2+12x+4}$;

113. $\lim\limits_{x\to 0}\dfrac{2\sqrt{x}-3x}{3\sqrt{x}-2x}$;

111. $\lim\limits_{x\to 1}\dfrac{x-1}{1-\sqrt{x}}$;

114. $\lim\limits_{x\to 0}\dfrac{x^2}{\sqrt{x^2+4}-2}$;

112. $\lim\limits_{x\to 16}\dfrac{\sqrt[4]{x}-2}{\sqrt{x}-4}$;

115. $\lim\limits_{x\to 0}\dfrac{x}{\sqrt[3]{x+1}-1}$.

7-misol. $\lim\limits_{x\to\infty}\left(\dfrac{3x-5}{4x+1}-0,7^x\right)$ limitni hisoblaymiz.

Yechish. Bu funksiyada $x\to\infty$ bo'lgani uchun nisbat $\frac{\infty}{\infty}$ ko'rinishli kasr ma'noga ega emas. Bu holda funksiyaning surati va maxrajini x ga bo'lib kasr shaklini o'zgartiramiz:

$$\lim_{x\to\infty}\left(\frac{3x-5}{4x+1}-0,7^x\right)=\lim_{x\to\infty}\frac{3x-5}{4x+1}-\lim_{x\to\infty}0,7^x=\lim_{x\to\infty}\frac{3-\frac{5}{x}}{4+\frac{1}{x}}-0=\frac{3-0}{4+0}=0,75.$$

Quyidagi limitlarni hisoblang:

116. $\lim\limits_{x\to\infty}\left(5+\dfrac{7}{x}-\dfrac{1}{x^2}\right)$;

118. $\lim\limits_{x\to\infty}\dfrac{4-x^2}{1+2x^2}$;

117. $\lim\limits_{x\to\infty}\dfrac{2x}{x-1}$;

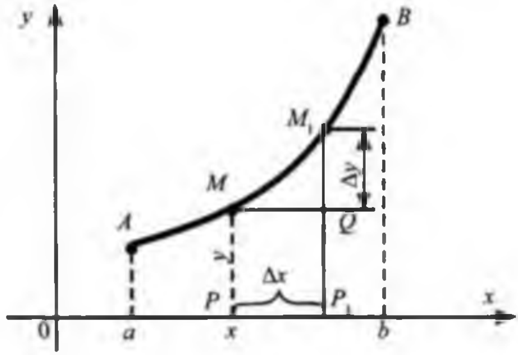
119. $\lim\limits_{x\to 0}\dfrac{2x^2+3x}{8x^3-9}$.

22-§. Funksiyaning uziuksizligi va uning tatbiqlari

AB yoy $y=f(x)$ funksiyaning grafigi bo'lsin (17-chizma). Bu yoyda ixtiyoriy $M(x;\,y)$ nuqtani olamiz va x ga $PP_1=\Delta t$ orttirma beramiz, unda ***u*** $QM_1=\Delta y$ orttirma oladi.

$\Delta x\to 0$ deb faraz qilsak, u holda $\Delta y\to 0$ bo'ladi, ya'ni $\lim\limits_{\Delta x\to 0}\Delta y=0$.

Bu $\Delta x\to 0$ da P_1M_1 ordinata PM ga cheksiz yaqinlashib kelib, M_1 nuqta M nuqtaga yaqinlashadi. Bunda, *AB* yoyda *M* nuqtaga juda yaqin bo'lgan nuqta topiladi. Bu holda x ning berilgan qiymatida $y=f(x)$ funksiya uzluksiz funksiya deyiladi.



17-chizma.

Ta'rif. **Agar x ning cheksiz kichik orttirmasiga y ning cheksiz kichik orttirmasi to'g'ri kelsa, ya'ni $\lim\limits_{\Delta x \to 0} \Delta y = 0$ bo'lsa, $y=f(x)$ funksiya x ning ushbu berilgan qiymatida uzluksiz funksiya deyiladi.**

Bu shart x ning $x=a$ dan $x=b$ gacha oraliqdagi har qanday qiymatida bajarilsa, funksiya bu oraliqda uzluksiz deyiladi.

Lekin hamma funksiyalar ham x ning har qanday qiymatlarida uzluksiz bo'lavermaydi. Masalan, $y=\frac{1}{x}$ funksiyaning $x=0$ nuqtada qiymati mavjud emas, shuning uchun berilgan funksiya $x=0$ nuqtada **uziladi** deb aytiladi (grafigi *OY* o'qini kesmaydi).

Bunday uzilishlar kasrli funksiyalarning maxraji nolga aylanadigan argumentning qiymatlarida mavjud bo'ladi.

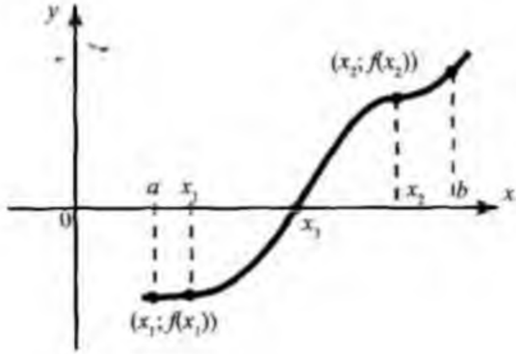
Masalan, $y=\frac{4x}{x-5}$ funksiya $x=5$ bo'lganda; $y=\frac{10}{x^2-9}$ funksiya esa $x=\pm 3$ bo'lganda uziladi.

Uzluksiz funksiyaning asosiy xossasini ko'rib chiqamiz.

Agar $f(x)$ funksiya (a; b) oraliqda uzluksiz va nolga aylanmasa, u holda bu funksiya shu oraliqda bir xil ishorali bo'ladi.

Isbot. Haqiqatan, (a; b) oraliqning shunday x_1 va x_2 nuqtalari topilib, ular $f(x_1)<0, f(x_2)>0$ bo'lsin deb faraz qilamiz (18-*a*, chizma).

$f(x_1)<0$ va $f(x_2)>0$ bo'lgani uchun ularga mos $(x_1; f(x_1))$ va $(x_2; f(x_2))$ nuqtalar *OX* o'qining turli tarafida yotadi. $f(x)$ funksiya uzluksiz bo'lgani uchun uning grafigi *OX* o'qini biror $(x_3; 0)$ nuqtada kesib o'tadi.



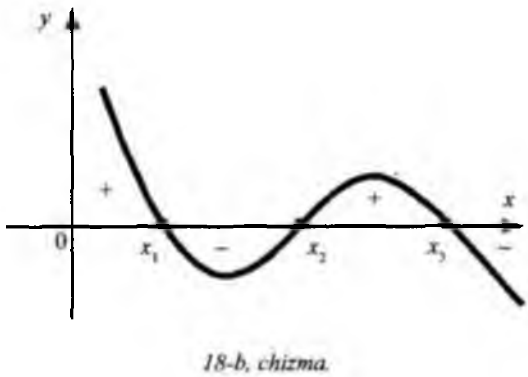
18-a, chizma.

Bu kesishish nuqtasida $f(x_3)=0$ bo'ladi.

Bu $f(x)$ funksiya (a; b) oraliqda nolga teng emas deyilganga zid bo'ladi.

Demak, uzluksiz $f(x)$ funksiya (a; b) oraliqda bir xil ishorali va nolga teng emas.

Uzluksiz funksiya o'z ishorasini grafigi faqat absissa o'qini kesib o'tgandagina o'zgartirishi mumkin (18-*a*, chizma).



18-b, chizma.

Masalan, 18-*b*, chizmada $f(x)$ funksiya (–; x_1) va (x_2; x_3) oraliqlarda musbat ($f(x)>0$), (x_1; x_2) va (x_3; –) oraliqlarda manfiy ($f(x)<0$).

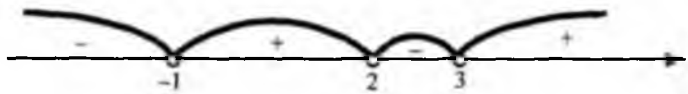
Bir o'zgaruvchili tengsizliklarni yechish metodi shu faktga asoslangan bo'lib, **intervallar** metodi deb ataladi.

1-misol. $(x+1)(x-2)(x-3)<0$ tengsizlikni oraliqlar (intervallar) metodi, bilan yechamiz.

Yechish. Avval *OX* o‘qini kesib o‘tadigan nuqtalarini topamiz:

$x+1=0 \quad x-2=0 \quad x-3=0$

$x_1=-1 \quad x_2=2 \quad x_3=3$. Bu nuqtalarni son o‘qida belgilab, oraliqlarni ajratamiz.



19-chizma.

Natijada (–∞; –1); (–1; 2); (2; 3) va (3; +∞) oraliqlarda berilgan tengsizlik ishorasini aniqlaymiz.

(–∞; –1) oraliqda –2 ni olib, berilgan tengsizlik ishorasini topamiz.

1) $x=-2$ da: $\left.\begin{array}{l} x+1=-2+1=-1<0. \\ x-2=-2-2=-4<0. \\ x-3=-2-3=-5<0. \end{array}\right| (x+1)(x-2)(x-3)<0$ (–∞; –1) oraliqda tengsizlik manfiy.

2) $x=0$ da: $\left.\begin{array}{l} x+1=0+1>0 \\ x-2=0-2<0 \\ x-3=0-3<0 \end{array}\right| (x+1)(x-2)(x-3)>0$ (–1; 2) oraliqda tengsizlik musbat.

3) $x=2{,}5$ da: $\left.\begin{array}{l} x+1=2{,}5+1>0 \\ x-2=2{,}5-2>0 \\ x-3=2{,}5-3<0 \end{array}\right| (x+1)(x-2)(x-3)<0$; (2; 3) oraliqda tengsizlik manfiy.

4) $x=4$ da: $\left.\begin{array}{l} x+1=4+1>0 \\ x-2=4-2>0 \\ x-3=4-3>0 \end{array}\right| (x+1)(x-2)(x-3)>0$; (3; ∞) oraliqda tengsizlik musbat.

19-chizma yordamida tengsizlikning yechimi (–∞; –1) va (2; 3) topiladi.

Javob: (–∞; –1) va (2; 3).

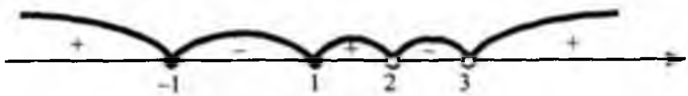
2-misol. $\frac{x^2-1}{x^2-5x+6} \geq 0$ tengsizlikni yechamiz.

Yechish. Ushbu $f(x)\frac{x^2-1}{x^2-5x+6}$ funksiya aniqlanish sohasining har bir nuqtasida uzluksiz.

Bu kasr $x^2-1=0$ da $x=\pm 1$ bo‘lib, –1 va 1 da nolga aylanadi.

Bu kasr $x^2-5x+6=0$ da aniqlanmagan.

$x_1=2$ va $x_2=3$ nuqtalarda funksiya uziladi. –1; 1; 2 va 3 nuqtalar funksiyaning aniqlanish sohasini 5 ta oraliqqa bo‘ladi (20-chizma).



20-chizma.

Bu oraliqlar: (–∞; –1]; [1; 1]; [1; 2); (2; 3) va (3; +∞) lardan iborat.

$x=-2$ da $\frac{(-2)^2-1}{(-2)^2-5\cdot(-2)+6}=\frac{3}{4+16}>0;$

$x=0$ da $\frac{0-1}{0-0+6}<0;$

$x=1,5$ da $\frac{1,5^2-1}{1,5^2-5\cdot 1,5+6}=\frac{2,25-1}{2,25-7,5+6}>0;$

$x=2,5$ da $\frac{2,5^2-1}{2,5^2-5\cdot 2,5+6}<0:$

$x=4$ da $\frac{16-1}{16-20+6}>0.$

20-chizma bo‘yicha tengsizlikning yechimi: (–∞; –1]; [1; 2) va (3; +∞).

TAKRORLASH UCHUN SAVOLLAR

1. Qanday funksiyani uzluksiz funksiya deyiladi (ta’rifi)?
2. Qanday funksiya uzluksiz bo‘lmaydi (uziladi)?
3. Uzluksizning qanday xossasi bor?
4. Uzluksiz funksiya o‘z ishorasini qachon (qaysi vaqtda) o‘zgartiradi?
5. Bir o‘zgaruvchili tengsizliklar intervallar metodi bilan qanday yechiladi?

120. Quyidagi tengsizliklarni yeching (intervallar metodi bilan).

a) $(x+2)(x-3)(x+0,5)>0$;

b) $3x^2+11x-4\leq 0$.

121. $\frac{3x^2-9x}{x^2-x-12}\geq 0$;

122. $\frac{(x-2)(x-4)}{(x+3)(x-1)}>0$;

123. $\sqrt{x^2-4}(x-1)<0$;

124. $x^4-5x^2+4\geq 0$;

125. $(x^2-1)(x^3-1)(x^4-1)\geq 0$;

126. $\frac{1}{x-1}\geq \frac{1}{x}+\frac{1}{x+1}$;

127. $\frac{(x^2+1)(x+2)^2\sqrt{16-x}}{x-3}\geq 0.$

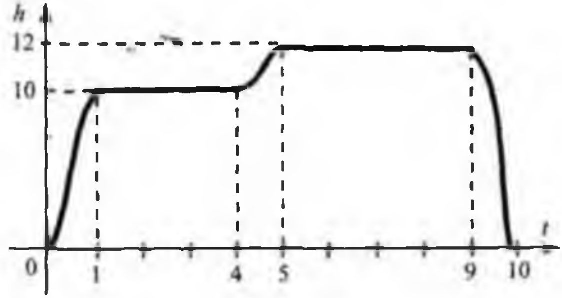
23-§. Funksiyaning o'sishi va kamayishi

Samolyotning 10 soat uchish davomidagi balandligining o'zgarish grafigi 21-chizmada tasvirlangan (vaqt soat bilan, balandlik kilometr bilan o'lchanadi). Birinchi soat davomida samolyot balandlikka ko'tarila boradi. Shundan keyin 3 soat davomida bir xil balandlikda uchadi, so'ngra bir soat davomida yana balandlikka ko'tariladi, yangi balandlikda to'rt soat uchadi, oxirgi soat davomida esa pastga tushadi.

Bu misolda h balandlik t vaqtning [0; 10] oraliqda aniqlangan funksiyasidan iborat.

Bu funksiya [0; 1] va [4; 5] t ning ortishi bilan o'sadi, [1; 4] va [5; 9] oraliqlarda o'zgarmaydi [9; 10] oraliqda nolgacha kamayadi.

Avvalgi boblarda o'suvchi (kamayuvchi) funksiyaga berilgan ta'rifni eslatamiz.



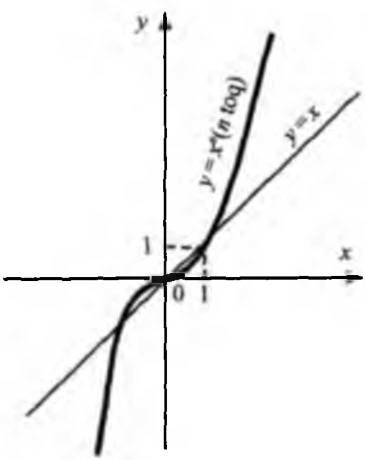
21-chizma.

Ta'rif. **1. Agar [*a*; *b*] oraliqqa tegishli har qanday x_1 va x_2 sonlar uchun $x_1<x_2$ tengsizlikdan $f(x_1)<f(x_2)$ tengsizlik kelib chiqsa, $f(x)$ funksiya [*a*; *b*] oraliqda o'suvchi deyiladi.**

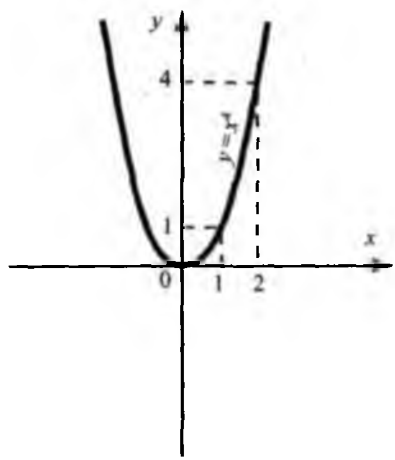
2. Agar [*a*; *b*] oraliqqa tegishli har qanday x_1 va x_2 sonlar uchun $x_1<x_2$ tengsizlikdan $f(x_1)>f(x_2)$ tengsizlik kelib chiqsa, $f(x)$ funksiya [*a*; *b*] oraliqda kamayuvchi deyiladi.

1-misol. $f_1(x)=x$, $f_2(x)=x^3$ va umuman $f(x)=x^n$ (n – har qanday natural toq son) funksiyalar sonlar to'g'ri chizig'ining hammasida o'sadi (22-chizma).

2-misol. $f(x)=x^2$, $f(x)=x^4$ va umuman $y=x^n$ funksiyalar har qanday juft n da [0; +∞) oraliqda o'sadi va (–∞; 0] da kamayadi (23-chizma).



22-chizma.



23-chizma.

3-misol. $f(x)=-3x+2$ funksiyaning o'sish va kamayish oraliqlarini topamiz:

Yechish. $f(x)=-3x+2$ funksiya grafigi I; II va IV choraklardan o'tuvchi kamayuvchi funksiya bo'ladi. Javob: $(-\infty;\ +\infty)$ oraliqda kamayadi.

4-misol. $f(x)=-2x^2+6x-7$ funksiyaning o'sish va kamayish oraliqlarini topamiz:

Yechish. Bu funksiyaning grafigi tarmoqlari pastga qaragan parabola bo'ladi. Parabolaning uchining koordinatalarini topamiz:

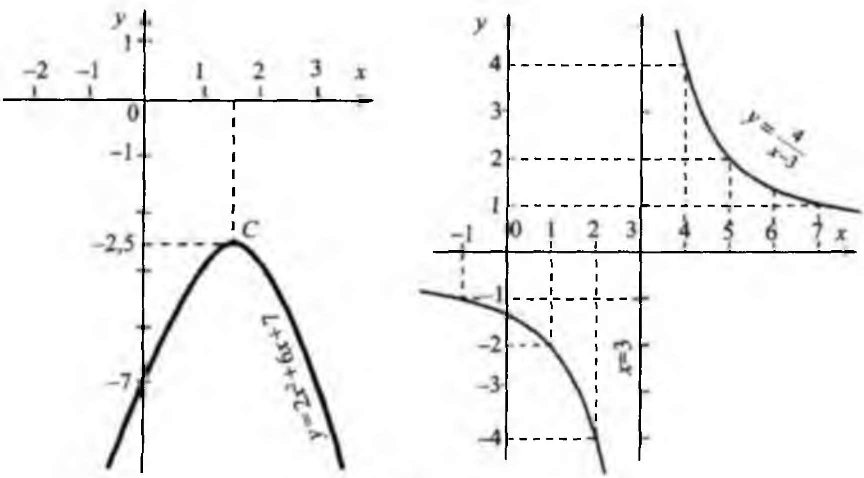
$$x_0=-\frac{b}{2a}=-\frac{6}{2\cdot(-2)}=1,5;\ y_0=-2\cdot 1,5^2+6\cdot 1,5-7=-4,5+9-7=-2,5.$$

Parabolaning uchi $C(1,5;\ -2,5)$.

Bu funksiyaning grafigini sxematik chizamiz. 24-chizmadan ma'lumki, bu funksiya $(-\infty;\ 1,5]$ oraliqda o'sadi va $[1,5;\ +\infty)$ oraliqda kamayadi.

5-misol. $f(x)=\frac{4}{x-3}$ funksiyaning o'sish va kamayish oraliqlarini topamiz.

Yechish. Bu funksiyaning grafigi $y=\frac{4}{x}$ funksiya grafigini 3 birlik o'ngga surish bilan hosil qilinadi.



24-chizma

$y=\frac{4}{x-3}$ funksiya $x=3$ nuqtada uziladi (ma'noga ega emas). Shuning uchun uning grafigi $x=3$ to'g'ri chiziqqa intilib boruvchi giperbola bo'ladi. Bu funksiya $(-\infty; 3)$ va $(3; +\infty)$ oraliqlarda kamayuvchi bo'ladi.

Yoki 1) $(-\infty; 3)$ da $x_1=-2$ va $x_2=1(x_1<x_2)$ da $f_1(-2)=-\frac{4}{5}, f_2(1)=-2.$

$f_1(-2)>f_2(1)$ bajariladi.

2) $(3; +\infty)$ oraliqda:

$x_1=4; x_2=5(x_1<x_2)$ da $f_1(4)=4; f_2(5)=2.$

$f_1(4)>f_2(5)$ bajariladi.

TAKRORLASH UCHUN SAVOLLAR

1. Samolyotning balandligi qaysi vaqt oralig'ida o'sadi?
2. Samolyotning balandligi qaysi vaqt oralig'ida o'zgarmaydi?
3. Samolyotning balandligi qaysi vaqt oralig'ida kamayadi?
4. Qanday funksiyani $[a; b]$ oraliqda o'suvchi deyiladi?
5. Qanday funksiyani $[a; b]$ oraliqda kamayuvchi deyiladi?
6. $f(x)=x^n$ (n – toq natural son) funksiyaning o'sish yoki kamayish oraliqlari qanday?
7. $f(x)=x^n$ (n – juft natural son) funksiyalarning o'sish va kamayish oraliqlari qanday?
8. $y=2x$ va $y=-3x$ funksiyalarning o'sish yoki kamayish oraliqlarini toping.

MASALALARNI YECHING

128. a) $f(x)=4x-3$; b) $f(x)=-3x+2$ funksiyalarning o'sish yoki kamayish oraliqlarini toping.

Quyidagi funksiyalarning o'sish va kamayish oraliqlarini toping:

129. a) $y=-2x^2$; b) $y=2x^2+4$.

130. a) $y=(x-2)^2$; b) $y=-(x-3)^2$.

131. a) $y=x^2+3x-108$; b) $y=-x^2+3x+4$.

132. a) $y=3x^2+8x-3$; b) $y=-x^2-1,8x+3,6$.

133. a) $y=\frac{2}{x-3}$; b) $y=-\frac{3}{x+2}$.

134. a) $y=\lg(x+2)$; b) $y=\log_{0,5}(x-1)$.

135. a) $y=\left(\frac{1}{3}\right)^{2x-5}$; b) $y=2^{3-6x}$.

24-§. Funksiyaning orttirmasi

x va x_0 – erkli o‘zgaruvchining $f(x)$ funksiyaning aniqlanish sohasidan olingan ikkita qiymati bo‘lsin. $x-x_0$ ayirma erkli o‘zgaruvchining x_0 nuqtadagi orttirmasi deyiladi, u Δx («delta iks») bilan belgilanadi. Ya’ni $\Delta x=x-x_0$ bundan $x=x_0+\Delta x$ topiladi.

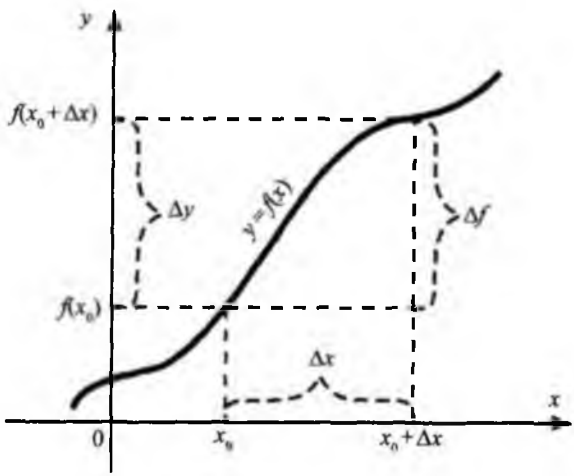
x_0 va Δx orttirmadan keyin funksiyaning qiymati $f(x)-f(x_0)=$ $=f(x_0+\Delta x)-f(x_0)$ miqdor qadar o‘zgaradi.

Funksiyaning $f(x_0+\Delta x)$ yangi qiymati bilan uning boshlang‘ich $f(x_0)$ qiymati orasidagi bu ayirma $f(x)$ funksiyaning x_0 nuqtadagi orttirmasi deyiladi, $\Delta f(x_0)$ («delta ef x_0») deb o‘qiladi, ya’ni

$\Delta f(x_0)=f(x_0+\Delta x)-f(x_0)$ yoki

$\Delta f(x_0)=f(x)-f(x_0)$ bunda $x=x_0+\Delta x$.

$\Delta f(x_0)$ erksiz o‘zgaruvchi y ning orttirmasi ham deyiladi, Δf yoki Δy kabi belgilanadi (25-chizma).



25-chizma.

$\Delta f(x)$ ning qiymati x_0 va Δx larga bog‘liq bo‘lgani uchun Δf orttirma Δx ning funksiyasi bo‘ladi.

1-misol. Agar $x_0=3$; $x=2{,}5$ va $y=x^2$ bo‘lsa, x_0 nuqtada Δx va Δy larni topamiz.

Yechish. Orttirmaning ta’rifiga ko‘ra:

$$\Delta x=x-x_0=2{,}5-3=-0{,}5; \quad x_0+\Delta x=3+(-0{,}5)=2{,}5$$

$$\Delta f=\Delta y=f(x_0+\Delta x)-f(x_0)=f(2{,}5)-f(3)=2{,}5^2-3^2=-2{,}75.$$

2-misol. $f(x)=2x-x^2$ funksiyaning x_0 *nuqtadagi* orttirmasini x_0 va Δx lar bilan ifodalang.

Yechish. $\Delta f(x_0)=f(x_0+\Delta x)-f(x_0)=2(x_0+\Delta x)-(x_0+\Delta x)^2-(2x_0-x_0^2)=$

$$=\cancel{2x_0}+2\Delta x-\cancel{x_0^2}-2x_0\cdot\Delta x-\Delta x^2-\cancel{2x_0}+\cancel{x_0^2}=2\Delta x-2x_0\Delta x-\Delta x^2.$$

$$\Delta f(x_0)=2\Delta x-2x_0\Delta x-\Delta x^2.$$

TAKRORLASH UCHUN SAVOLLAR

1. Erkli o‘zgaruvchining (argumentning) orttirmasi nimaga teng?
2. Funksiyaning x_0 nuqtadagi orttirmasi nimaga teng?
3. $y=x$ funksiyaning Δx va Δy orttirmalarini toping.

MASALALARNI YECHING

136. $y=2x+3$ funksiya uchun:

a) agar $x_0=3$ va $\Delta x=0{,}2$ bo‘lsa, x va Δy ni toping;

b) agar $x_0=4$ va $\Delta x=0{,}06$ bo‘lsa, x va Δy ni toping;

d) agar $x_0=7$ va $\Delta x=0{,}01$ bo‘lsa, Δy ni toping;

e) agar $x_0=-5$ va $\Delta x=0{,}001$ bo‘lsa, Δy ni toping.

137. Agar a) $x=2{,}5$ va $x_0=2$; b) $x=3{,}9$ va $x_0=3{,}75$ bo‘lsa, $y=x^2$ funksiya uchun Δx orttirmani va unga mos Δy orttirmani toping.

138. Agar a) $y=3x^2$; b) $y=2\sqrt{x}$ bo‘lsa, $f(x_0+\Delta x)$, $f(x_0+\Delta x)-f(x_0)$ va $\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$ larni toping (x_0 va Δx bilan ifodalang).

25-§. Hosilaning ta'rifi

Siz hosilalar tushunchasi bilan fizika kursida tanishgansiz. Unda vaqtning t_0 momentida oniy tezlik o'rtacha tezlikning $[t_0; t_0+\Delta t]$ vaqt oralig'ida t_0 nuqtadagi limiti sifatida ta'riflangan, ya'ni $\upsilon(t_0)=\lim\limits_{\Delta t\to 0}\frac{\Delta S}{\Delta t}$.

Bunda ΔS – bosib o'tilgan masofa orttirmasi, Δt – vaqt orttirmasi harakatlanayotgan jismning o'rtacha va oniy tezliklariga o'xshash funksiyaning $[x_0; x_0+\Delta x]$ oraliqdagi o'rtacha o'zgarish tezligi deb, funksiya orttirmasining erkli o'zgaruvchi orttirmasiga nisbati aytiladi, ya'ni $\frac{\Delta t(x_0)}{\Delta x}=\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$.

Erkli o'zgaruvchi orttirmasi Δx ning 0(nol)ğa intilgandagi o'rtacha o'zgarish tezlikning limiti, ya'ni $\lim\limits_{\Delta x\to 0}\frac{\Delta f(x_0)}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$ **funksiyaning x_0 nuqtadagi o'zgarish tezligi yoki hosila** deyiladi.

Ta'rif. $f(x)$ funksiyaning x_0 nuqtadagi hosilasi deb $\lim\limits_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$ ga aytiladi. $f(x)$ funksiyaning x_0 nuqtadagi hosilasi $f'(x_0)$ bilan belgilanadi («iks nol nuqtada ef shtrix»). Demak, $\lim\limits_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}=f'(x_0)$ yoki $y'(x_0)$.

Biror nuqtada hosilaga ega bo'lgan funksiya shu nuqtada differensiallanuvchi deyiladi. Berilgan funksiyaning hosilasini topish differensiallash deyiladi.

1-misol. $f(x)=x^2$ funksiyaning hosilasini topamiz.

Yechish. Hosila ta'rifiga asosan: $f'(x_0)=\lim\limits_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}=$

$$=\lim_{\Delta x\to 0}\frac{(x_0+\Delta x)^2-x_0^2}{\Delta x}=\lim_{\Delta x\to 0}\frac{2x_0\Delta x+(\Delta x)^2}{\Delta x}=\lim_{\Delta x\to 0}(2x_0+\Delta x)=2x_0.$$

Bu natijani bunday yozish qabul qilingan:

$$f'(x)=(x^2)'=2x.$$

2-misol. $f(x)=\frac{1}{x}$ funksiyaning hosilasini topamiz.

Yechish. $f(x)=\frac{1}{x}$ funksiya uchun $x_0 \neq 0$ bo‘lishi kerak.

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{\frac{1}{x_0+\Delta x}-\frac{1}{x_0}}{\Delta x}=\lim_{\Delta x\to 0}\frac{x_0-(x_0+\Delta x)}{\Delta x\cdot x_0(x_0+\Delta x)}=\lim_{\Delta x\to 0}\frac{-\Delta x}{\Delta x\cdot x_0(x_0+\Delta x)}=$$

$$=\lim_{\Delta x\to 0}\frac{-\Delta x}{\Delta x\cdot x_0(x_0+\Delta x)}=-\frac{1}{x_0^2}.$$

Demak, $f'(x)=\left(\frac{1}{x}\right)'=-\frac{1}{x^2}$.

3-misol. $f(x)=7x-4$ funksiyaning hosilasini topamiz.

Yechish. $f'(x_0)=\lim_{\Delta x\to 0}\frac{7(x_0+\Delta x)-4-(7x_0-4)}{\Delta x}=\lim_{\Delta x\to 0}\frac{7x_0+7\cdot\Delta x-4-7x_0+4}{\Delta x}=\lim_{\Delta x\to 0}\frac{7\cdot\Delta x}{\Delta x}=7.$

Demak, $f'(x)=(7x-4)'=7$.

4-misol. O‘zgarmas son C ning hosilasini topamiz.

Yechish. Haqiqatan, $C'=\lim_{\Delta x\to 0}\frac{C-C}{\Delta x}=\lim_{\Delta x\to 0}\frac{0}{\Delta x}=\lim_{\Delta x\to 0}0=0.$

Demak, $\mathbf{C'=0}$.

5-misol. $f(x)=\sqrt{x}$ funksiyaning hosilasini topib, $f'(3)$ va $f'\left(\frac{9}{16}\right)$ larni toping.

Yechish. $f'(x_0)=\lim_{\Delta x\to 0}\frac{\sqrt{x_0+\Delta x}-\sqrt{x_0}}{\Delta x}=\lim_{\Delta x\to 0}\frac{\left(\sqrt{x_0+\Delta x}\right)^2-\left(\sqrt{x_0}\right)^2}{\Delta x\cdot\left(\sqrt{x_0+\Delta x}+\sqrt{x_0}\right)}=$

$$=\lim_{\Delta x\to 0}\frac{\Delta x}{\Delta x\left(\sqrt{x_0+\Delta x}+\sqrt{x_0}\right)}=\lim_{\Delta x\to 0}\frac{1}{\sqrt{x_0+\Delta x}+\sqrt{x_0}}=\frac{1}{2\sqrt{x_0}}.$$

Demak, $f'(x_0)=\left(\sqrt{x}\right)'=\frac{1}{2\sqrt{x}}$.

$$f'(3)=\frac{1}{2\sqrt{3}}=\frac{\sqrt{3}}{6};\qquad f'\left(\frac{9}{16}\right)=\frac{1}{2\sqrt{\frac{9}{16}}}=\frac{4}{3\cdot 2}=\frac{2}{3}.$$

TAKRORLASH UCHUN SAVOLLAR

1. Fizika kursida vaqtning t_0 nuqtasidagi oniy tezlik qanday topiladi?
2. Funksiyaning $[x_0;\ x_0+x]$ oraliqdagi o‘rtacha o‘zgarish tezligi nimaga teng?
3. Funksiyaning x_0 nuqtadagi hosilasi deb nimaga aytiladi?
4. Qanday funksiyaning differensiallanuvchi deyiladi?
5. $y=2x$ ning hosilasi nimaga teng?

FUNKSIYANING HOSILASINI TOPING

139. Ta’rifdan foydalanib funksiyaning hosilasini toping:

a) $5-3x$; b) $\frac{2}{3}x+0{,}4$; d) $3x^2+1$; e) $5x^2-x$.

140. Quyidagilarni isbotlang:

a) $(x^3)'=3x^2$; b) $\left(4\sqrt{x}\right)'=\frac{2}{\sqrt{x}}$; d) $\left(\sqrt[3]{x}\right)'=\frac{1}{3\sqrt[3]{x^2}}$.

141. $f(x)=\sqrt[3]{x^2}$ funksiyadan a) $f'(1)$; b) $f'(8)$; d) $f'\left(\frac{1}{8}\right)$.

26-§. Hosilalarni hisoblash qoidalari

Hosilalarni topishning bir necha foydali qoidalari mavjud, bu qoidalarni biz teoremalar shaklida ifodalaymiz.

1-teorema. **Agar u va v funksiyalar x_0 nuqtadan differensiallanuvchi bo‘lsa, u holda bularning yig‘indisi ham shu nuqtada differensiallanuvchi, yani $(u+v)'=u'+v'$.**

Yoki qisqacha: **yig‘indining hosilasi hosilalar yig‘indisiga teng.**

Isbot. Funksiyalar yig‘indisining orttirmasini topamiz:

$$\Delta(u+v)=(u(x_0+\Delta x)+v(x_0+\Delta x))-(u(x_0)+v(x_0))=(u(x_0+\Delta x)-u(x_0))+$$
$$+(v(x_0+\Delta x)-v(x_0))=\Delta u+\Delta v.$$

Yig‘indining limiti haqidagi teoremadan, shuningdek u va v funksiyalarning x_0 nuqtada differensiallanuvchiligidan foydalanib, yig‘indining hosilasini topamiz:

$$(u+v)' = \lim_{\Delta x\to 0}\frac{\Delta(u+v)}{\Delta x} = \lim_{\Delta x\to 0}\frac{\Delta u+\Delta v}{\Delta x} = \lim_{\Delta x\to 0}\left(\frac{\Delta u}{\Delta x}+\frac{\Delta v}{\Delta x}\right) =$$

$$= \lim_{\Delta x\to 0}\frac{\Delta u}{\Delta x} + \lim_{\Delta x\to 0}\frac{\Delta v}{\Delta x} = u' + v'.$$

Demak, $$(u+v)' = u' + v'. \qquad (1)$$

Funksiyalar ayirmasining hosilasi hosilalar ayirmasiga teng ekanligi ham yuqoridagi kabi isbotlanadi, ya'ni

$$(u-v)' = u' - v' \qquad (2)$$

Eslatma $(u_1+u_2+...+u_n)' = u_1'+u_2'+...+u_n'$ ekanini ham isbotlash mumkin.

1-misol. $\left(x^2+\frac{1}{x}\right)'$ ni topamiz.

Buning uchun 23-§ dagi hosilasi topilgan. Misollar va 1-teoremadan foydalanamiz:

$$\left(x^2+\frac{1}{x}\right)' = (x_2)' + \left(\frac{1}{x}\right)' = 2x + \left(-\frac{1}{x^2}\right) = 2x - \frac{1}{x^2}.$$

2-misol. $\left(\sqrt{x}-7x\right)'$ ni hisoblaymiz.

Yechish. $\left(\sqrt{x}-7x\right)' = \left(\sqrt{x}\right)' - (7x)' = \frac{1}{2\sqrt{x}} - 7.$

2-teorema. **Agar *u* va *v* funksiyalar x_0 nuqtada differensiallanuvchi bo'lsa, u holda bu funksiyalarning ko'paytmasi ham shu nuqtada differensiallanuvchi yani** $(uv)' = u'\cdot v + u\cdot v'$ formula bo'yicha topiladi.

Isbot. *u* va *v* funksiyalar differensiallanuvchi bo'lgani uchun ularning ko'paytmasi ham differensiallanishini isbotlaymiz.

Funksiyalar ko'paytmasining orttirmasini topamiz:

$\Delta(uv) = u(x_0+\Delta x)\, v(x_0+\Delta x) - u(x_0)\, v(x_0)$

Funksiyalar orttirmasi

$\Delta u = u(x_0+\Delta x) - u(x_0)$ lardan $u(x_0+\Delta x) = u(x_0) + \Delta u.$

$\Delta v = v(x_0+\Delta x) - v(x_0)$ $\qquad v(x_0+\Delta x) = v(x_0) + \Delta v.$

bularni $\Delta(uv)$ orttirmaga qo'yamiz:

$\Delta(uv)=(u(x_0)+\Delta u)(v(x_0)+\Delta v)-u(x_0)v(x_0)$ bundagi $u(x_0)=u$, $v(x_0)=v$ deb belgilaymiz.

$\Delta(uv)=(u+\Delta u)(v+\Delta v)-u\cdot v=uv+u\cdot\Delta v+v\Delta u+\Delta u\cdot\Delta v-uv=u\cdot\Delta v$ $+ +v\Delta u+\Delta u\cdot\Delta v.$

$\Delta(uv)=v\Delta u+u\cdot\Delta v+\Delta u\cdot\Delta v$ buni Δx ga bo'lib va $\Delta x\to 0$ qilib limit olamiz:

$$\lim_{\Delta x\to 0}\frac{\Delta(u\cdot v)}{\Delta x}=\lim_{\Delta x\to 0}\frac{\Delta u\cdot v}{\Delta x}+\lim_{\Delta x\to 0}\frac{u\cdot\Delta v}{\Delta x}+\lim_{\Delta x\to 0}\frac{\Delta u\cdot\Delta v}{\Delta x}.$$

$$(u+v)'=u'v+u\cdot v'+\lim_{\Delta x\to 0}\frac{\Delta u\cdot\Delta v\cdot\Delta x}{\Delta x\cdot\Delta x}$$

Bundagi $\lim\limits_{\Delta x\to 0}\frac{\Delta u\cdot\Delta v\cdot\Delta x}{\Delta x\cdot\Delta x}=\lim\limits_{\Delta x\to 0}\frac{\Delta u}{\Delta x}\cdot\lim\limits_{\Delta x\to 0}\frac{\Delta v}{\Delta x}\cdot\lim\limits_{\Delta x\to 0}\Delta x=u'\cdot v'\cdot 0=0.$

bo'lgani uchun $\quad\quad$ $\mathbf{(u'\cdot v')'=u'v+u\cdot v'.}$ $\quad\quad$ (3)

Natija. **O'zgarmas ko'paytuvchi hosila belgisidan tashqariga chiqariladi, ya'ni** $(cu)'=c\cdot u'$.

Isbot. $(cu)'$ ni ko'paytmaning hosilasiga asosan (2-teorema) topamiz: $(cu)'=c'\cdot u+c\cdot u'=o\cdot u+c\cdot u'=c\cdot u'$.

Demak, $\quad\quad$ $\mathbf{(cu)'=c\cdot u'}$ $\quad\quad$ (4)

3-misol. $\left(\sqrt{x}\cdot(3-5x^2)\right)$ ko'paytmaning hosilasini topamiz.

Yechish. $\sqrt{x}=u$, $3-5x^2=v$ deb olib, 2-teoremaga asosan hosilasini topamiz:

$$\left(\sqrt{x}\cdot(3-5x^2)\right)'=\left(\sqrt{x}\right)'\cdot(3-5x^2)+\sqrt{x}(3-5x^2)'=\frac{1}{2\sqrt{x}}(3-5x^2)+$$

$$+\sqrt{x}\left(3'-(5x^2)'\right)=\frac{1}{2\sqrt{x}}(3-5x^2)+\sqrt{x}(0-10x)=\frac{1}{2\sqrt{x}}(3-5x^2)+$$

$$+\sqrt{x}(-10x)=\frac{\sqrt{x}}{2x}(3-5x^2-20x^2)=\frac{\sqrt{x}}{2x}(3-25x^2).$$

3-teorema. **Agar u va υ funksiyalar x_0 nuqtada differensiallanuvchi va funksiya shu nuqtada nolga teng bo'lmasa, u holda $\frac{u}{\upsilon}$ bo'linma ham shu nuqtada differensiallanuvchi, yani** $\left(\frac{u}{\upsilon}\right)' = \frac{u'\upsilon - u\upsilon'}{\upsilon^2}$.

Isbot. Avval $\left(\frac{u}{\upsilon}\right)' = -\frac{u\upsilon' - u\upsilon'}{\upsilon^2}$ formulani chiqaramiz.

Buning uchun $\frac{1}{\upsilon}$ funksiyaning orttirmasini topamiz:

$$\Delta\left(\frac{1}{\upsilon}\right) = \frac{1}{\upsilon(x_0+\Delta x)} - \frac{1}{\upsilon(x_0)} = \frac{\upsilon(x_0)-\upsilon(x_0+\Delta x)}{\upsilon(x_0)\cdot\upsilon(x_0+\Delta x)} = \frac{-\Delta\upsilon}{\upsilon(\upsilon+\Delta\upsilon)}$$ (bunda $\upsilon=\upsilon(x_0)$ – son). Bundagi $\lim\limits_{\Delta x\to 0}\Delta\upsilon = \lim\limits_{\Delta x\to 0}\left(\frac{\Delta\upsilon}{\Delta x}\cdot\Delta x\right) = \upsilon'\cdot 0 = 0.$ bo'lganidan

$$\left(\frac{1}{\upsilon}\right)' = \lim_{\Delta x\to 0}\frac{\left(\Delta\frac{1}{\upsilon}\right)}{\Delta x} = \lim_{\Delta x\to 0}\frac{-\Delta\upsilon}{\upsilon\cdot(\upsilon+\Delta\upsilon)\Delta x} = -\frac{\lim\limits_{\Delta x\to 0}\frac{\Delta\upsilon}{\Delta x}}{\lim\limits_{\Delta x\to 0}\upsilon\cdot(\upsilon+\Delta\upsilon)} = \frac{\upsilon'}{\lim\limits_{\Delta x\to 0}\upsilon\cdot\lim\limits_{\Delta x\to 0}(\upsilon+\Delta\upsilon)} = -\frac{\upsilon'}{\upsilon^2}$$

2-teoremaga ko'ra: $\left(\frac{u}{\upsilon}\right)' = \left(u\cdot\frac{1}{\upsilon}\right)' = u'\cdot\frac{1}{\upsilon} + u\cdot\left(\frac{1}{\upsilon}\right)' = \frac{u'}{\upsilon} + u\cdot\frac{(-\upsilon')}{\upsilon^2} =$
$= \frac{u'\cdot\upsilon - u\cdot\upsilon'}{\upsilon^2}$.

$$\textbf{Demak, } \left(\frac{u}{\upsilon}\right)' = \frac{u'\cdot\upsilon - u\cdot\upsilon'}{\upsilon^2} \qquad (5)$$

4-misol. $\left(\frac{3x-4}{\sqrt{x}}\right)'$ funksiyaning hosilasini topamiz.

Yechish. 3-teoremaga asosan bo'linmaning hosilasini topamiz:

$$\left(\frac{3x-4}{\sqrt{x}}\right)' = \frac{(3x-4)'\cdot\sqrt{x} - (3x-4)\cdot(\sqrt{x})'}{(\sqrt{x})^2} = \frac{3\cdot\sqrt{x} - (3x-4)\cdot\frac{1}{2\sqrt{x}}}{x} = \frac{6x-3x+4}{2x\sqrt{x}} = \frac{3x-4}{2x\sqrt{x}}.$$

Endi x^n darajali funksiyaning hosilasi uchun umumiy formula chiqaramiz. x^2 funksiyaning hosilasi $(x^2)'=2x$ edi.

Ko'paytmaning hosilasi formulasidan foydalanib, $(x^3)'$ va $(x^4)'$ larni topamiz:

$$(x^3)^1 = (x^2\cdot x)^1 = (x^2)^1\cdot x + x^2\cdot x^1 = 2x\cdot x + x^2 = 3x^2.$$

$$(x^4)^1=(x^3\cdot x)^1=(x^3)^1\cdot x+x^3\cdot x^1=3x^2\cdot x+x^3=4x^3.$$

Bu $2x$, $3x^2$ va $4x^3$ larni boshqa ko‘rinishda yozamiz: $(x^2)^1=2x=2\cdot x^{2-1}$; $(x^3)^1=3x^2=3\cdot x^{3-1}$; $(x^4)^1=4x^3=4\cdot x^{4-1}$.

Endi boshqa natural n larda ham $\mathbf{(x^n)^1=n\cdot x^{n-1}}$ ekanligini ko‘rsatamiz.

Isbot. Bu formulaning isbotini matematik induksiya metodi bilan isbotlaymiz.

Bu formula $n=k$ da to‘g‘ri bo‘lsin, ya’ni $(x^k)'=kx^{k-1}$ deymiz.

Bu formulaning to‘g‘riligini $n=k+1$ da ham to‘g‘ri bo‘lishini ko‘rsatamiz. Haqiqatan, $(x^{k+1})'=(x^k\cdot x)'=(x^k)'\cdot x+x^k\cdot x'=k\cdot x^{k-1}\cdot x+x^k\cdot 1=$ $=k\cdot x^k+x^k=x^k(k+1)=(k+1)\cdot x^k$.

Demak, formula $n=k+1$ da $(x^{k-1})'=(k+1)x^k$ to‘g‘ri.

Demak, $\mathbf{(x^k)'=k\cdot x^{k-1}}$.

Bu formulaga ko‘ra $(x)'=1\cdot x^{1-1}=1\cdot x^0=1\cdot 1=1$.

$(x^0)'=0\cdot x^{0-1}=0\cdot x^{-1}=0$. Bu natijalar avvalgi topilgan natijalar bilan bir xil.

$(x^{31})'=31\cdot x^{30-1}=31\cdot x^{30}$ va hokazo.

x^n funksiyadagi n – butun manfiy son bo‘lgan holini ko‘rib chiqamiz.

n – manfiy butun sonni $n=-m$ ko‘rinishda yozib, m – natural son qilib olamiz.

$$(x^n)'=(x^{-m})'=\left(\frac{1}{x^m}\right)'=\frac{-(x^m)'}{(x^m)^2}=\frac{-mx^{m-1}}{x^{2m}}=-m\cdot\frac{1}{x^{m+1}}=-m\cdot x^{-m-1}=n\cdot x^{n-1}$$

Demak, $$\mathbf{(x^n)'=nx^{n-1}} \qquad (6)$$

Shunday qilib, biz quyidagi 4-teoremani isbotladik.

4-teorema. **Har qanday butun n va har qanday x uchun (n – butun, $x\neq 0$) $(x^n)'=nx^{n-1}$.**

5-misol. a) $3x^{-5}$; b) $8x^{11}-\frac{7}{x^5}$ funksiyalarning hosilalarini topamiz.

Yechish: a) $(3x^{-5})'=3\cdot(x^{-5})'=3\cdot(-5)x^{-5-1}=-15x^{-6}$.

b) $\left(8x^{11}-\frac{7}{x^5}\right)'=(8x^{11})'-(7x^{-5})'=8\cdot 11\cdot x^{10}-7\cdot(-5)x^{-6}=88x^{10}+35x^{-6}=$ $=88x^{10}+\frac{35}{x^6}$.

Oldingi yechilgan misollarni ko‘zdan kechirib $(x^n)'=nx^{n-1}$ formula ratsional ko‘rsatkichlar uchun ham to‘g‘ri ekanini ko‘rish mumkin.

Haqiqatan, $\left(x^{\frac{1}{2}}\right)'=\left(\sqrt{x}\right)'=\frac{1}{2\sqrt{x}}$ yoki $\left(x^{\frac{1}{2}}\right)'=\frac{1}{2}x^{\frac{1}{2}-1}=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\cdot x^{\frac{1}{2}}}=$

$=\frac{1}{2\sqrt{x}}$. $\left(x^{\frac{2}{3}}\right)'=\left(\sqrt[3]{x^2}\right)'=\frac{2}{3\sqrt[3]{x}}$ yoki $\left(x^{\frac{2}{3}}\right)'=\frac{2}{3}\cdot x^{\frac{2}{3}-1}=\frac{2}{3}x^{-\frac{1}{3}}=\frac{2}{3\sqrt[3]{x}}$.

$(x^n)'=nx^{n-1}$ formulani har qanday n uchun $x>0$ da to‘g‘riligini «Darajali funksiyaning hosilasi» mavzusida isbotlanadi.

6-misol. $\left(\sqrt[7]{x3}\right)'$ hosilani topamiz.

Yechish. $\left(\sqrt[7]{x^3}\right)'=\left(x^{\frac{3}{7}}\right)'=\frac{3}{7}x^{\frac{3}{7}-1}=\frac{3}{7}x^{-\frac{4}{7}}=\frac{3}{7\sqrt[7]{x^4}}$.

TAKRORLASH UCHUN SAVOLLAR

1. Funksiyalar yig‘indisining hosilasi haqidagi 1-teoremani ayting va formulasini yozing.
2. Funksiyalar ayirmasining hosilasi haqidagi xossani ayting va formulasini yozing.
3. Funksiyalar ko‘paytmasining hosilasi haqidagi 2-teoremani ayting va formulasini yozing.
4. Funksiyalar bo‘linmasining (kasrning) hosilasi haqidagi 3-teoremani ayting va formulasini yozing.
5. Darajali funksiyaning hosilasi haqidagi 4-teoremani ayting va formulasini yozing.
6. a) $(x^7)'$; b) $(x^{-5})'$ larni toping.

FUNKSIYANING HOSILALARINI TOPING

142. a) x^2-3x; b) $3x+\sqrt{x}$; d) $7x^4-\frac{1}{x}$;

e) $2\sqrt{x}-7x^{-3}$; f) $\sqrt[3]{x^3}-\frac{4}{x^2}$.

143. a) $4x^2\sqrt{x}-\frac{1}{\sqrt[3]{x^5}}$; b) $x^5+4x^3-7x^2+6$.

144. a) $\frac{x}{3}-\frac{7}{2x^2}+10$; b) $\frac{\sqrt[4]{x^3}}{5}-\frac{2}{\sqrt[3]{x}}+x\sqrt[3]{x}+9$.

145. a) $\frac{1+2x}{3-5x}$; b) $\frac{x}{1+x^2}$; d) $\frac{\sqrt{x}}{4+x}$.

146. a) $(3+x)(2-\sqrt{x}+x^2)$; b) $\left(2-\frac{x}{3}+\sqrt[4]{x^3}\right)(7-x^2)$.

147. $f(x)=x^2-3x$ bo'lsa, $f'(0), f'(-1), f'(2), f'(t+1)$ larni toping.

148. $f(x)=\frac{3-x}{x+2}$ bo'lsa, $f'(4), f'(9), f'(0{,}01), f'(2-t)$ larni toping.

3-misol. $f(x)=x^2-3x$ va $g(x)=\sqrt{x-3}$ funksiyalar berilgan:

a) $f(g(x))$; b) $g(f(x))$ ko'rinishda ifodalang.

Yechish. a) $f(g(x))$ murakkab funksiyada $g(x)=\sqrt{x-3}$ funksiya $f(x)$ ning argumenti bo'lgani uchun $f(g(x))=(\sqrt{x-3})^2-3\sqrt{x-3}=x-3-$ $-3\sqrt{x-3}=x-3\sqrt{x-3}-3$, ya'ni $f(g(x))=x-3\sqrt{x-3}-3$.

b) $g(f(x))$ murakkab funksiyada $f(x)=x^2-3x$ funksiya $g(x)$ ning argumenti bo'lgani uchun $g(f(x))=\sqrt{x^2-3x-3}$.

MASALALARNI YECHING

149. Har bir funksiyani ikkita sodda funksiya ko'rinishida tasvirlang:

a) $y=\sqrt{x^2+2x}$; e) $f(x)=\cos^3 t$;

b) $y=2\sqrt[4]{x^3-2x^2+5}$; f) $g(x)=\lg(7-x^3)$;

d) $y=(x^3-2x+5)^3$; g) $h(x)=2\sqrt{\lg(x-3)}$.

27-§. Murakkab funksiyaning hosilasi

Berilgan $h(x)=g(f(x))$ murakkab funksiyani soddaroq va tanish funksiyalardan tuzilgan funksiya ko'rinishida tasvirlash funksiyalarning hosilasini topishni qulaylashtiradi.

$h(x)=g(f(x))$ murakkab funksiya hosilasini topishda $f(x)=y$ bilan belgilab, $h(x)=g(y)$ funksiyani hosil qilamiz.

Funksiya orttirmasi Δh ni Δx ga nisbati $\frac{\Delta h}{\Delta x}$ ni olamiz. Bu nisbatning surat va maxrajini Δy ga ko'paytirib, $\frac{\Delta h}{\Delta x}=\frac{\Delta h\cdot\Delta y}{\Delta x\cdot\Delta y}=\frac{\Delta h}{\Delta x}\cdot\frac{\Delta y}{\Delta x}$ ko'paytmani hosil qilamiz. Bundan ko'paytmani limitini topish qoidasiga asosan limit olamiz:

$$\lim_{\Delta x\to 0}\frac{\Delta h}{\Delta x}=\lim_{\Delta y\to 0}\frac{\Delta h}{\Delta y}\cdot\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=h'(y)\cdot y'(x).$$

$$h'(x)=h'(y)\cdot y'(x) \text{ yoki } h'(x)=g'(f(x))\cdot f'(x) \quad (7)$$

Demak, $h(x)=g(f(x))$ murakkab funksiyadan hosila olish uchun dastlab $g'(f(x))$ hosila va so'ngra $f'(x)$ hosilalar olinib, natijalar o'zaro ko'paytiriladi.

1-misol. $y=(3x^2+2x+5)^4$ funksiyaning hosilasini topamiz.

Yechish. Bu funksiyani ikkita sodda funksiya shaklida tasvirlaymiz:

$$u=3x^2+2x+5 \text{ va } y=u^4.$$

Dastlab, $y'(u)=(u^4)=4u^3$ ni undan keyin $u'(x)=(3x^2+2x+5)'=6x+2.$

Izlangan hosila: $y'=y'(u)\cdot u'(x)=4\cdot u^3\cdot(6x+2)=4\cdot(3x^2+2x+5)^3\times$ $\times(6x+2)=4(3x^2+2x+5)^3\cdot(3x+1).$

Demak, $y'=4(3x^2+2x+5)^3(3x+1)$

Agar $y(x)=u^m(x)$ darajali murakkab funksiya bo'lsa, u holda bu qoidaga asosan uning hosilasi quyidagicha yoziladi.

$$(u^m(x))'=m\cdot u^{m-1}(x)\cdot u'(x) \quad (8)$$

2-misol. $y=(x^3-4x+2)^4$ funksiyaning hosilasini topamiz.

Yechish. Bu misolda $u(x)=x^3-4x+2$ deb olamiz. (8) formulaga asosan hosilasini topamiz: $y=u^4$ dan

$$y'=4u^3u'=4(x^3-4x+2)^3\cdot(x^3-4x+2)'=4(x^3-4x+2)^3\cdot(3x^2-4).$$

Bu hosilani qisqacha bundan keyin

$y'=((x^3-4x+2)^4)'=4\cdot(x^3-4x+2)^3\cdot(x^3-4x+2)'=4(x^3-4x+2)^3\cdot(3x^2-4)$ ko'rinishda yoziladi.

3-misol. $y=\sqrt{x^4-5x^3+8}$ funksiyaning hosilasini topamiz.

Yechish. $y'=\left(\left(x^4-5x^3+8\right)^{\frac{1}{2}}\right)'=\frac{1}{2}\cdot\left(x^4-5x^3+8\right)^{\frac{1}{2}-1}\cdot\left(x^4-5x^3+8\right)'=$

$=\frac{1}{2}\left(x^4-5x^3+8\right)^{-\frac{1}{2}}(4x^3-15x^2)=\frac{x^2(4x-15)}{2\sqrt{x^4-5x^3+8}}$; $\quad y'=\frac{x^2(4x-15x)}{2\sqrt{x^4-5x^3+8}}$.

QUYIDAGI FUNKSIYALARNING HOSILALARINI TOPING

150. a) $y=(2x-7)^4$; b) $y=(3+5x)^{11}$.

151. a) $y=\sqrt{5x-8}$; b) $y=\sqrt[3]{(2x+3)^2}$.

152. a) $y=(3x-1)^{15}+(2x+2)^4$; b) $y=(5x-2)^{13}-(3x+7)^{20}$.

153. a) $y=\sqrt{4x^2-1}$; b) $y=\sqrt[3]{9x^3-15}$.

154. a) $y=(x-1)\sqrt{x^2+1}$; b) $y=(x+1)^2\sqrt{x-1}$.

155. a) $y=\frac{(2x^2-1)^2}{x^2}$; b) $y=\frac{2x-1}{\sqrt{x^2+1}}$.

28-§. Trigonometrik funksiyalarning hosilalari. Sinusning hosilasi

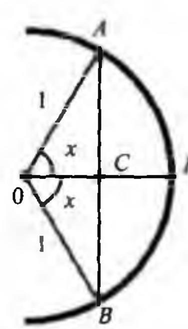
Trigonometrik funksiyalardan sin$x$ ning hosilasini topamiz, ya'ni $(\sin x)'=\cos x$ ekanligini ko'rsatamiz.

1) Bizga ma'lumki, cos$x$ funksiya argumentning barcha qiymatlarida uzluksiz, ya'ni $\lim\limits_{\Delta x\to 0}\cos(x+\Delta x)=\cos x$.

2) $\lim\limits_{x\to 0}\frac{\sin x}{x}=1$ ekanligini ko'rsatamiz.

Bu tenglikning to'g'riligini geometrik mulohazalar yordamida ishonarli qilib ko'rsatamiz.

Haqiqatan ham x ni musbat deb olib, birlik aylanada (26-chizma) P_0 nuqtadan uzunligi x ga teng bo'lgan P_0A va P_0B yoylar ($\angle x=\cup P_0A$)ni ikki tomonga qo'yamiz. Butun $\cup AB=2x$ bo'ladi. $AC=BC=\sin x$. $|AB|=|AC|+|CB|=2\sin x$ bo'ladi.



26-чизма.

Shunday qilib, AB vatar uzunligining $\overset{\frown}{AB}$ yoy uzunligiga nisbati $\frac{2\sin x}{2x}=\frac{\sin x}{x}$ teng. 28-chizmadan x kichik bo'lganda vatar uzunligi, bilan yoy uzunligi deyarli teng bo'lib, ya'ni $\frac{\sin x}{x}$ nisbat birga teng bo'ladi.

Demak, $\lim\limits_{x\to 0}\frac{\sin x}{x}=1$ **(9)** bo'ladi.

Hosilaning ta'rifiga asosan $\lim\limits_{\Delta x\to 0}\frac{\sin x}{x}=1$ limitni topamiz. Bunda $\Delta\sin x=\sin(x+\Delta x)-\sin x$.

Ushbu $\sin\alpha-\sin\beta=2\cos\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$ formula bo'yicha $\Delta\sin x=$

$=\sin(x+\Delta x)-\sin x=2\cos\frac{x+\Delta x+x}{2}\cdot\sin\frac{x+\Delta x-x}{2}=2\cos\left(x+\frac{\Delta x}{2}\right)\sin\frac{\Delta x}{2}.$

Yuqoridagi ma'lumotlar va limitlar haqidagi teoremalardan foydalanib, $\sin' x$ ni topamiz:

$$\sin' x=\lim_{\Delta x\to 0}\frac{\Delta\sin x}{\Delta x}=\lim_{\Delta x\to 0}\frac{2\cos\left(x+\frac{\Delta x}{2}\right)\cdot\sin\frac{\Delta x}{2}}{\Delta x}=$$

$$=\lim_{\Delta x\to 0}\cos\left(x+\frac{\Delta x}{2}\right)\cdot\lim_{\Delta x\to 0}\frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}=\cos x\cdot 1=\cos x$$

Demak, $$(\sin x)'=\sin' x=\cos x. \qquad (10a)$$

1-misol. Murakkab funksiyani differensiyallash formulasiga asosan $(\sin(ax+b))'$ ni topamiz, ya'ni

$$(\sin(ax+b)'=\cos(ax+b)\cdot(ax+b)'=\cos(ax+b)\cdot a=a\cdot\cos(ax+b).$$

$$(\sin(ax+b)'=a\cdot\cos(ax+b) \qquad (10b)$$

2-misol. $5\sin^2(3x-1)$ ning hosilasini topamiz.

Yechish. $(5\sin^2(3x-1))'=5\cdot 2\cdot\sin(3x-1)\cdot(\sin(3x-1))'=10\sin(3x-1)\times$ $\times\cos(3x-1)\cdot(3x-1)'=5\cdot 2\sin(3x-1)\cdot\cos(3x-1)\cdot 3=15\sin(6x-2).$

3-misol. $\cos 2u\cdot\cos\frac{\pi}{2}-\sin 2u\cdot\sin\frac{\pi}{2}$ ning hosilasini topamiz.

Yechish. $\left(\cos 2u \cdot \cos\frac{\pi}{2} - \sin 2u \cdot \sin\frac{\pi}{2}\right)' = \left(\cos\left(2u + \frac{\pi}{2}\right)\right)' = (-\sin 2u)' =$

$= -\cos 2u \cdot (2u)' = -2\cos 2u.$

TAKRORLASH UCHUN SAVOLLAR

1. $\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$ ning to'g'riligini geometrik mulohazalar yordamida tushuntiring.
2. $\sin\alpha - \sin\beta$ ayirmani ko'paytma shakliga keltiradigan formulani yozing.
3. $(\sin x)'$ nimaga teng?
4. $(\sin(ax+b))'$ nimaga teng?
5. $(\sin 5x)'$ ni toping.

FUNKSIYALARNING HOSILASINI TOPING

156. a) $y=\sin 3x$; b) $y=\sin(5x+2)$.

157. a) $y=\frac{1}{2}\sin 3x$; b) $y=0{,}3\sin^3 x$.

158. a) $y=\sqrt{\sin x}$; b) $y=\sqrt[3]{\sin x}$.

159. a) $y=\sin 2x\cdot\cos x-\sin x\cdot\cos 2x+5$;

b) $y=2x+3{,}6\sin 5(\pi-x)$.

160. a) $y=\sqrt{x}\sin x$; b) $y=\frac{\sqrt{x}}{\sin x}$.

29-§. Kosinus, tangens va kotangensning hosilalari

Kosinus, tangens va kotangenslarni differensiallash formulasini chiqarish uchun sinusning hosilasidan foydalanamiz.

1) $\cos x = \sin\left(\frac{\pi}{2} - x\right)$ bo'lgani uchun murakkab funksiyaning formulasidan foydalanamiz:

$$\cos' x = \left(\sin\left(\frac{\pi}{2} - x\right)\right)' = \cos\left(\frac{\pi}{2} - x\right)\cdot\left(\frac{\pi}{2} - x\right)' = \sin x \cdot (-1) = -\sin x.$$

Demak, $\mathbf{(\cos x)' = \cos' x = -\sin x}$ (11)

2) $\operatorname{tg}x=\frac{\sin x}{\cos x}$ dan $(\operatorname{tg}x)'=\left(\frac{\sin x}{\cos x}\right)'=\frac{\sin' x\cdot\cos x-\sin x\cdot\cos' x}{\cos^2 x}=$

$$=\frac{\cos x\cdot\cos x-\sin x\cdot(-\sin x)}{\cos^2 x}=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}.$$

Demak, $$(\mathbf{tg}x)'=\mathbf{tg}'(x)=\frac{1}{\cos^2 x} \quad (12)$$

3) $\operatorname{ctg}x=\frac{\cos x}{\sin x}$ dan $(\operatorname{ctg}x)'=\left(\frac{\cos x}{\sin x}\right)'=\frac{\cos' x\cdot\sin x-\cos x\cdot\sin' x}{\sin^2 x}=$

$$=\frac{-\sin x\cdot\sin x-\cos x\cdot\cos x}{\sin^2 x}=\frac{-(\sin^2 x+\cos^2 x)}{\sin^2 x}=-\frac{1}{\sin^2 x}.$$

Demak, $$(\mathbf{ctg}x)'=\mathbf{ctg}'x=-\frac{1}{\sin^2 x} \quad (13)$$

Masalan, $(7\operatorname{ctg}(5x-3\pi))'=(7\operatorname{ctg}5x)'=-\frac{7}{\sin^2 5x}\cdot(5x)'=-\frac{35}{\sin^2 5x}$.

FUNKSIYALARNING HOSILASINI TOPING

161. a) $y=2{,}5\cos x$; b) $y=\operatorname{tg}(3x-2)$; d) $y=2\cdot\operatorname{ctg}3x$.

162. a) $y=3\cos(2{,}3x-6\pi)$; b) $y=5\operatorname{tg}(2x+3\pi)$; d) $y=7\operatorname{ctg}(-2x-\pi)$.

163. a) $y=\frac{\cos x}{x}$; b) $y=\frac{2\sin(6x-2)}{\cos(6x-2)}+\operatorname{tg}(6x-2)$.

164. a) $y=\sin^3 x\cdot\cos x$; b) $y=2\sin^2 x\cdot\cos^2 x$.

165. a) $y=\sqrt[3]{\sin 2x}$; b) $y=\frac{1}{3}\operatorname{tg}^3\frac{1}{x}$.

30-§. Ko‘rsatkichli funksiyaning hosilasi

$y=a^x$ ko‘rsatkichli funksiya grafigini o‘rganganimizda grafik hech bir siniqsiz silliq shaklida tasvirlanganligini bilamiz. Shu sababli ko‘rsatkichli funksiya barcha nuqtalarda differensiallanuvchi deb hisoblaymiz.

Matematikada shunday 2 dan katta, ammo 3 dan kichik bo‘lgan irratsional e soni mavjud.

y son $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \approx 2{,}718...$ bo‘lib, bu tenglikning isboti matematik analizning to‘la kursida o‘rganiladi.

e asosli $y=e^x$ ko‘rsatkichli funksiyaning 0 nuqtadagi hosilasi 1 ga teng, ya’ni $y' = \lim\limits_{\Delta x\to 0}\frac{\Delta e^x}{\Delta x} = \lim\limits_{\Delta x\to 0}\frac{e^{0+\Delta x}-e^0}{\Delta x} = \lim\limits_{\Delta x\to 0}\frac{e^{\Delta x}-1}{\Delta x} = 1.$

Demak, $\lim\limits_{\Delta x\to 0}\left(\frac{e^{\Delta x}-1}{\Delta x}\right)=1\,(A)$ (Kichik miqdorlar nisbating limitini 1 ga teng deb qabul qilingan).

1-teorema. **e^x ko‘rsatkichli funksiya har bir nuqtada differensiallanuvchi va $(e^x)'=e^x$.**

Isbot. $y=e^x$ funksiyaning x_0 nuqtadagi orttirmasini topamiz:

$\Delta y=e^{x_0+\Delta x}-e^{x_0}=e^{x_0}\cdot e^{\Delta x}-e^{x_0}=e^{x_0}(e^{\Delta x}-1).$

(A) tenglikdan foydalanib, quyidagi hosilani topamiz:

$$y' = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to \infty}\frac{e^{x_0}(e^{\Delta x}-1)}{\Delta x} = e^{x_0}\cdot\lim_{\Delta x\to 0}\frac{e^{\Delta x}-1}{\Delta x} = e^{x_0}\cdot 1 = e^{x_0}.$$

Demak, $y'=(e^x)'=e^x$(14).

1-misol. e^{3x} funksiyaning hosilasini topamiz.

$(e^{3x})'=e^{3x}\cdot(3x)'=3e^{3x}.$

Ta’rif. ***e* asosga ko‘ra logarifm natural logarifm deyiladi.** Uni $\log_e x=\ln x$ ko‘rinishda yoziladi.

Natural logarifmda asos e bo‘lgani uchun logarifmning asosiy ayniyatini $e^{\ln a}=a$ ko‘rinishda yozamiz.

Bu ayniyatdan foydalanib har qanday a^x ko‘rsatkichli funksiyani $a^x=(e^{\ln a})^x$ ko‘rinishda yozamiz.

2-teorema. **Har qanday musbat *a* da a^x funksiya har bir *x* nuqtada differensiallanuvchi va $(a^x)'=a^x\cdot\ln a$.**

Isbot. Ko‘rsatkichli funksiyasining hosilasini topishda $a^x=(e^{\ln a})^x$ ayniyatdan va murakkab funksiya hosilasidan foydalanamiz: $(a^x)'=((e^{\ln a})^x)'= =(e^{x\ln a})'=e^{x\ln a}\cdot\ln a=a^x\cdot\ln a$. Demak, $y'=(a^x)'=a^x\cdot\ln a$ (15).

2-misol. $y=5^{-3x}$ funksiyaning hosilasini topamiz.

Yechish. (15) formulaga asoslanib, funksiya hosilasini topamiz:

$$y'=(5^{-3x})'=5^{-3x}\cdot\ln 5\cdot(-3x)'=-3\cdot 5^{-3x}\ln 5.$$

3-misol. $y=3^{\frac{x}{2}}\cos 8x$ funksiyaning hosilasini topamiz:

Yechish. $y'=\left(3^{\frac{x}{2}}\cos 8x\right)'=\left(3^{\frac{x}{2}}\right)'\cdot\cos 8x+3^{\frac{x}{2}}\cdot(\cos 8x)'=3^{\frac{x}{2}}\cdot\ln 3\cdot\left(\frac{x}{2}\right)'\times$

$\times\cos 8x+3^{\frac{x}{2}}\cdot(-\sin 8x)\cdot(8x)'=3^{\frac{x}{2}}\left(\ln 3\cdot\frac{1}{2}\cos 8x-\sin 8x\cdot 8\right)=$

$=3^{\frac{x}{2}}\left(\frac{1}{2}\ln 3\cos 8x-8\sin 8x\right)$

TAKRORLASH UCHUN SAVOLLAR

1. Irratsional e soni qanday hosil qilingan va y nimaga teng?
2. 1-teoremani ayting ($y=e^x$ ning hosilasi nimaga teng?).
3. Qanday logarifmni natural logarifm deyiladi?
4. 2-teoremani ayting ($y=a^x$ ning hosilasi nimaga teng?).
5. $y=e^{2x}$; $y=e^{-\frac{1}{3}x}$; $y=2^{3x}$; $y=0{,}3^{-0{,}5x}$ larning hosilalarini toping.

FUNKSIYALARNING HOSILASINI TOPING

166. a) $y=e^{5x}$; b) $y=e^{-\frac{1}{3}x}$; d) $y=e^{\frac{4}{x}}$; e) $y=e^{\sqrt{x}}$.

167. a) $y=e^{x^2}$; b) $y=e^{3-5x}$; d) $y=x^2e^{3x}$; e) $y=\frac{e^{8x}}{x^3}$.

168. a) $y=e^{\frac{x}{2}}-3e^{91x}$; b) $y=e^{5x}+4e^{\frac{x}{3}}$.

169. a) $y=3^{5x}$; b) $y=2^{5-3x}$; d) $y=5\cdot 3^{7-3x}$; e) $1{,}7^{\frac{x}{7}}+15$.

170. a) $y=2^x\cos x$; b) $y=7^{\frac{x}{2}}\cdot\mathrm{tg}3x$.

171. a) $y=\frac{0{,}3^{-x}}{\sqrt{x}+0{,}5}$; b) $y=\frac{3^x}{2^x+5^x}$.

31-§. Logarifmik funksiyaning hosilasi

Logarifmik funksiya hosilasining formulasini keltirib chiqarish uchun teskari funksiyaning hosilasi formulasidan foydalanamiz.

Teorema. **f va g funksiyalar o'zaro teskari bo'lib, f funksiyaning x_0 nuqtadagi hosilasi mavjud va nolga teng bo'lmasin. U holda g funksiyaning $y_0=f(x_0)$ nuqtadagi hosilasi mavjud va $\frac{1}{f'(x_0)}$ ga teng bo'ladi, ya'ni $g'(x)=\frac{1}{f'(x_0)}$.**

Bu teoremaning isbotiga to'xtalmay logarifmik funksiyaning hosilasini topishni o'rganamiz.

$y=\log_a x$ bilan $x=a^y$ funksiyalar o'zaro teskari bo'lgani uchun yuqoridagi teoremaga asosan quyidagilarni topamiz:

$$(\log_a x)' = \frac{1}{(a^y)'} = \frac{1}{(a^{\log_a x})'} = \frac{1}{a^{\log_a x}\cdot\ln a} = \frac{1}{x\ln a}.$$

Demak, $y'=(\log_a x)'=\frac{1}{x\ln a}$ (16).

Agar asos $a=e$ bo'lsa, u holda $(\ln x)'=\frac{1}{x\ln e}=\frac{1}{x}$.

Demak, $y'=(\ln x)'=\frac{1}{x}$ (17).

1-misol. a) $y=\log_3 x$; b) $y=\log_7(2x-5)$; d) $y=\ln(x^2-3)$ larni hosilasini topamiz.

Yechish. a) $y'=(\log_3 x)'=\frac{1}{x\ln 3}$; b) $y'=((\log_7(2x-5)'=\frac{1}{(2x-5)\ln 7}\cdot(2x-5)'=$

$=\frac{2}{(2x-5)\ln 7}$. d) $y'=\left(\ln(x^2-3)\right)'=\frac{1}{x^2-3}\cdot(x^2-3)'=\frac{2x}{x^2-3}$.

2-misol. $y=\lg(\text{tg}3x+3^{5x})$ funksiyaning hosilasini topamiz.

Yechish. $y'=\left(\lg(\text{tg}3x+3^{5x})\right)'=\frac{(\text{tg}3x+3^{5x})'}{(\text{tg}3x+3^{5x})\ln 10}=\frac{\frac{1}{\cos^2 3x}\cdot(3x)'+3^{5x}\cdot\ln 3\cdot(5x)'}{(\text{tg}3x+3^{5x})\cdot\frac{\lg 10}{\lg e}}=$

$$=\frac{\lg e\left(\frac{3}{\cos^2 3x}+5\ln 3\cdot 3^{5x}\right)}{\text{tg}3x+3^{5x}}=\frac{\lg e\left(3+5\ln 3\cdot 3^{5x}\cdot\cos^2 3x\right)}{\cos^2 3x(\text{tg}3x+3^{5x})}.$$

TAKRORLASH UCHUN SAVOLLAR

1. O'zaro teskari $f(x)$ va $g(x)$ larning hosilalari o'zaro qanday tenglikka ega?
2. $y=\log_a x$ funksiyaning hosilasi nimaga teng?
3. $y=\ln x$ funksiyaning hosilasi nimaga teng?
4. a) $y=\log_5 x$; b) $y=\log_3(4x)$; d) $y=\ln 3x$; e) $y=\ln(3x-1)$ funksiyalarning hosilasini toping.

FUNKSIYALARNING HOSILASINI TOPING

172. a) $y=\log_3 x$; b) $y=\log_{0,3} x$; d) $y=\ln 2x$; e) $y=\ln\sqrt{x}$.

173. a) $y=\log_3 7x$; b) $y=\log_5(2x+3)$; d) $y=\ln(3x-4)$; e) $y=\ln\frac{1}{x}$.

174. a) $y=x^3\ln x$; b) $y=\frac{\ln x}{x}$;

175. a) $y=\sqrt{x}\cdot\ln x$; b) $y=\frac{\ln(5+3x)}{x^2+1}$;

176. a) $y=3^{2x}\cdot\ln 5x$; b) $y=\sin^3 2x\cdot\ln\sin 2x$.

IV bob. HOSILANI TAQRIBIY HISOBLASHLARGA, GEOMETRIYAGA VA FIZIKAGA TATBIQI

32-§. Funksiya orttirmasining bosh qismi

$f(x)$ funksiyaning x_0 nuqtadagi hosilasi $f'(x_0)=\lim\limits_{\Delta x\to 0}\frac{\Delta f(x_0)}{\Delta x}$ bo'lib, bundagi x ning yetarlicha kichik qiymatlarida $\frac{\Delta f(x_0)}{\Delta x}\approx f'(x_0)$ tenglik o'rinli. Bundan $f(x_0+\Delta x)-f(x_0)=f'(x_0)\cdot\Delta x$ yoki $\boldsymbol{f(x_0+\Delta x)=f(x_0)+f'(x_0)\cdot\Delta x}$. (1) Bu formula taqribiy hisoblashlar uchun asosiy formuladir.

1-misol. $f(x)=\sqrt[n]{x}$ ni hisoblash uchun formula chiqaramiz ($x\neq 0$).

$$f'(x)=\left(\sqrt[n]{x}\right)'=\left(x^{\frac{1}{n}}\right)'=\frac{1}{n}\cdot x^{\frac{1}{n}-1}=\frac{1}{n}x^{\frac{1}{n}}\cdot x^{-1}=\frac{\sqrt[n]{x}}{nx}$$

Bularni (1) tenglikka qo'yib,

Masalan: $\sqrt[n]{x_0+\Delta x}=\sqrt[n]{x_0}+\frac{\sqrt[n]{x_0}}{n\cdot x_0}\cdot\Delta x.$ (2) topildi.

a) $\sqrt[3]{27,03}$ ning qiymatini (2) formula bo'yicha hisoblaymiz: Bunda $x_0+\Delta x=27+0,03$; $n=3$, $\Delta x=0,03$ deb olamiz:

$$\sqrt[3]{27,03}=\sqrt[3]{27+0,03}=\sqrt[3]{27}+\frac{\sqrt[3]{27}}{3\cdot 27}\cdot 0,03=3+\frac{0,03}{27}=3+\frac{0,01}{9}\approx 3,0011.$$

b) $\sqrt[10]{1000}=\sqrt[10]{1024-24}=\sqrt[10]{1024+(-24)}=\sqrt[10]{1024}+\frac{\sqrt[10]{1024}}{10\cdot 1024}\cdot(-24)=$

$$=2+\frac{2}{10\cdot 1024}\cdot(-24)=2-\frac{\cancel{48}^{\,3}}{\cancel{10240}_{\,640}}=2-\frac{3}{640}\approx 2-0,005=1,995.$$

Demak, $\sqrt[10]{1000}\approx 1,995$.

TAKRORLASH UCHUN SAVOLLAR

1. $y=f(x)$ funksiya orttirmasining bosh qismi formulasini yozing.
2. $y=\sqrt[k]{x}$ funksiya orttirmasining bosh qismi formulasini yozing.
3. $f(x)$ funksiyaning x_0 nuqtadagi hosilasini topish formulasini yozing.

HISOBLASHLARNI BAJARING

177. a) $\sqrt{2}$, $\sqrt[3]{3}$ va $\sqrt[4]{3}$ ning qiymatlarini jadval bo‘yicha toping.

Ko‘rsatma. $\sqrt[3]{3}$ va $\sqrt[4]{3}$ larni logarifm jadvalidan foydalanib hisoblang.

b) $\sqrt{2}$, $\sqrt[3]{3}$ va $\sqrt[4]{3}$ ning qiymatlarini (2) formula yordamida hisoblang.

Ko‘rsatma. $2=1,4^2+0,04$; $3=(1,4)^3+0,256$ va $3=(1,3)^4+0,1439$ ekanidan foydalaning.

178. (2) formuladan foydalanib, $\sqrt[3]{8\cdot3}$, $\sqrt[3]{81}$, $\sqrt[4]{625\cdot3}$, $\sqrt[4]{48}$ larni hisoblang (avvalgi misoldagi $\sqrt[3]{3}\approx1,443$; $\sqrt[4]{3}=1,316$ lardan foydalaning).

179. Quyidagilarni hisoblang:

a) $10^{\lg\sqrt[3]{24}}$; b) $10^{\lg\sqrt[3]{24}}$; d) $10^{2\lg\sqrt[4]{3}}$.

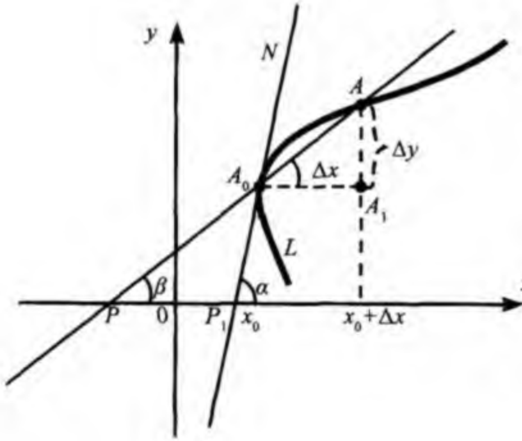
180. Taqribiy qiymatlarini toping:

a) $\sqrt[3]{30}$; b) $\sqrt[4]{90}$; d) $\sqrt[5]{33}$.

33-§. Funksiyaning grafigiga urinma

Hosila tushunchasi ayoniy geometrik ma’noga ega. $f(x)$ funksiyaning biror oraliqda uzluksiz bo‘lgan grafigini qaraymiz (27-chizma). Hosil bo‘lgan L egri chiziqda $A_0(x_0;\ y_0)$ va $A=M(x_0+\Delta x;\ f(x_0+\Delta x))$ nuqtalarni olamiz va kesuvchi A_0A to‘g‘ri chiziqni o‘tkazamiz.

Δx ga nolga yaqinlashuvchi qiymatlar berib, A nuqta A_0 nuqtaga yaqinlashishini, AA_0 to‘g‘ri chiziq esa A_0 nuqta atrofida burilishini ko‘ramiz (27-chizmaga qarang).



27-chizma.

Bunda A_0A kesuvchi biror A_0N limit holatga intilishini ko'ramiz. A_0N to'g'ri chiziq L egri chiziqqa A_0 nuqtada urinma deyiladi.

Bunda: α – A_0N urinmaning OX o'qi bilan hosil qilgan burchagi, β – AA_0 kesuvchini OX o'qi bilan hosil qilgan burchagi, ya'ni $\angle A_0P_1X=\alpha$, $\angle A_0PP_1=\beta$.

ΔAA_0A_1 da: $A_0A_1=\Delta x$; $AA_1=\Delta y$, $\angle AA_0A_1=\beta$ bo'lib, $\Delta y=\Delta x\cdot \mathrm{tg}\beta$ yoki $\frac{\Delta y}{\Delta x}=tg\beta$ bo'ladi.

Δx qanchalik kichik bo'lsa, β burchakning kattaligi α ga juda yaqin bo'ladi, ya'ni $\lim\limits_{\Delta x\to 0} tg\beta = tg\alpha$ yoki $\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=tg\alpha$ bunda $\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=f'(x_0)$ bo'lgani uchun $\boldsymbol{f'(x_0)=\mathrm{tg}\alpha}$ kelib chiqadi.

Demak, $f(x)$ funksiyaning $A_0(x_0;\ y_0)$ nuqtasida o'tkazilgan urinmaning burchak koeffitsiyenti $f(x)$ funksiyaning x_0 nuqtadagi hosilasiga teng.

Bu xulosa hosilaning geometrik ma'nosini anglatadi.

Endi $f(x)$ funksiyaning grafigiga $A_0=M(x_0;\ f(x_0))$ nuqtadagi urinma tenglamasini chiqaramiz. To'g'ri chiziqning $y=kx+b$ tenglamasidagi burchak koeffitsiyent $\boldsymbol{k=\mathrm{tg}=f'(x_0)}$ bo'lganidan $y=f'(x_0)x+b$ tenglik to'g'ri bo'ladi.

Bu to‘g‘ri chiziqdagi b ni hisoblash uchun urinma $A_0(x_0;\ f(x_0)$ nuqtadan o‘tishidan foydalanamiz:

$y=f'(x_0)x+b$ ga $(x_0;\ f(x_0))$ koordinatni qo‘yamiz. $f(x_0)=f'(x_0)x_0+b$, bundan $b=f(x_0)-f'(x_0)\cdot x_0$ topilib, buni to‘g‘ri chiziq tenglamasi $y=kx+b$ ga qo‘yib, urinma tenglamasini topamiz:

$$y=f'(x_0)\cdot x+f(x_0)-f'(x_0)\cdot x_0) \quad \text{yoki}$$

$$\boldsymbol{y=f(x)+f'(x_0)(x-x_0)} \qquad (3)$$

1-misol. $y=x^2$ parabolaga absissasi x_0 ga teng nuqtada urinma tenglamasini topamiz.

Yechish. $y=x^2$ ning x_0 nuqtadagi hosilasini topamiz $y'=2x$; $y'(x_0)=2x_0$.

$y(x_0)=x_0^2$; bularni (3) tenglamaga qo‘yib, urinma tenglamasini topamiz:

$y=x_0^2+2x_0(x-x_0)=x_0^2+2x_0x-2x_0^2=2x_0x-x_0^2$

Demak, $y=2x_0x-x_0^2$

Agar $x_0=1$ deb olsak, $y=2x-1$ bo‘ladi.

2-misol. $y=\sqrt{x}$ funksiya grafigining qaysi nuqtasida urinma absissalar o‘qiga, nisbatan 45° burchak ostida qiyalangan bo‘ladi?

Yechish. Urinmani absissalar o‘qiga og‘ish burchagining tangensi $\operatorname{tg}\alpha=f'(x_0)$ ga tengligidan foydalanib x_0 ni topamiz:

$\operatorname{tg}45°=f'(x_0)$, bundagi $f'(x_0)=\left(\sqrt{x_0}\right)'=\frac{1}{2\sqrt{x_0}}$ ekanligidan $1=\frac{1}{2\sqrt{x_0}}$; $2\sqrt{x_0}=1$; $\sqrt{x_0}=\frac{1}{2}$; $x_0=\frac{1}{4}$.

Berilgan $y=\sqrt{x}$ funksiyadan $y_0=\sqrt{x_0}=\sqrt{\frac{1}{4}}=\pm\frac{1}{2}$ $y=\sqrt{x}$ ning qiymati $y>0$ bo‘lgani uchun $y_0=\frac{1}{2}$. *Javob:* Urinish nuqtasi $\left(\frac{1}{4};\frac{1}{2}\right)$.

3-misol. $y=x^2-4x+1$ parabolaga M(–1; –3) nuqtadan o‘tuvchi urinmalar tenglamasini yozing.

Yechish. $y'=2x-4$; $M(-1;\ -3)$ nuqtadan o‘tuvchi urinma $y=x^2-4x+1$ parabolaning biror x_0 nuqtasida urinsin deb faraz qilamiz. Shu x_0 nuqtada urinuvchi urinmaning tenglamasini tuzamiz:

$y(x_0)=x_0^2-4x_0+1$; $y'(x_0)=2x_0-4$ bularni $y=f(x_0)+f'(x_0)(x-x_0)$ tenglamaga qo‘yamiz: $y=x_0^2-4x_0+1+(2x_0-4)(x-x_0)$ni soddalashtirib, $y=-x_0^2+2x_0x-4x+1$. Bu urinma $M(-1;\ 3)$ dan o‘tgani uchun $x=-1$ va $y=-3$ larni urinma tenglamasiga qo‘yib, x_0 ni topamiz:

$-3=-x_0^2+2x_0\cdot(-1)-4\cdot(-1)+1;$

$x_0^2+2x_0-8=0$, bundan $x_{01}=-4;\ x_{02}=2.$

Bularni $y=-x_0^2+2x_0x-4x+1$ ga qo‘yib:

1) $x_{01}=-4$ da $y_1=-(-4)^2+2\cdot(-4)x-4x+1=-12x-15.$

2) $x_{02}=2$ da $y_2=-2^2+2\cdot2\cdot x-4x+1=-3.$

Javob: $y_1=-12x-15$ va $y_2=-3$.

TAKRORLASH UCHUN SAVOLLAR

1. $f(x)$ funksiyaning x_0 nuqtadagi hosilasining qiymati nimaga teng?
2. Hosilaning geometrik ma’nosi nimadan iborat?
3. Egri chiziqqa o‘tkazilgan urinmaning burchak koeffitsiyenti nimaga teng?
4. $f(x)$ funksiyaning grafigiga $M(x_0;\ y_0)$ nuqtada o‘tkazilgan urinma tenglamasini yozing.

MASALALARNI YECHING

181. a) $f(x)=x^2$ funksiyaning grafigiga $M(-3;\ 9)$ nuqtada; b) $f(x)=x^3$ funksiya grafigiga $M(-2;\ -8)$ nuqtada o‘tkazilgan urinmaning burchak koeffitsiyentini toping.

182. $f(x)=\frac{4x-x^2}{4}$ funksiyaning grafigiga:

a) $M(2;\ 1)$; b) $M(4;\ 0)$ nuqtada urinma qiyalik burchagining tangensini toping.

183. $f(x)=x^3$ funksiyaning grafigiga:

a) $x=2$; b) $x=0,2$ nuqtalardagi urinma tenglamasini yozing.

184. $f(x)=\frac{3}{x}$ funksiyaning grafigiga:

a) $x=1$; b) $x=-1$ nuqtalardagi urinma tenglamasini yozing.

Quyidagi egri chiziqlar OX to‘g‘ri chiziq bilan kesishish nuqtalarining har birida qanday burchak hosil qiladi (burchak tangensini ko‘rsating).

185. $f(x)=x^3-3x$; **186.** $f(x)=x^3-3x+2$;

187. $y=x^2-4x+1$ parabola $M(0;\ 0)$ nuqtadan o‘tuvchi urinmalarning tenglamalarini yozing.

34-§. Tezlik va tezlanish

Jism to‘g‘ri chiziqli tekis harakat qilayotgan bo‘lsin. Tekis harakat qonuni $S=vt+S_0$ (1) formula bilan ifoda etiladi, bu formula birinchi darajali chiziqli funksiyani, geometrik jihatdan to‘g‘ri chiziqni tasvir etadi.

Bu formulada v hamda S_0 – o‘zgarmas sonlar. Bunga ishonch hosil qilish uchun, jismning t_1, va t_2 vaqtlar orasida o‘tgan yo‘llari:

$S_1=vt_1+S_0$

$S_2=vt_2+S_0$ topilib, $\Delta S=S_2-S_1=(vt_2+S_0)-(vt_1+S_0)=v(t_2-t_1)$ ni hosil qilamiz. $t_2-t_1=\Delta t$ vaqt orttirma topilib, $\frac{S_2-S_1}{t_2-t_1}=\frac{\Delta S}{\Delta t}=v$ nisbatni hosil qilamiz.

Demak, $\frac{\Delta S}{\Delta t}=v$ (2) tenglik tekis harakatning tezligini tasvirlaydi.

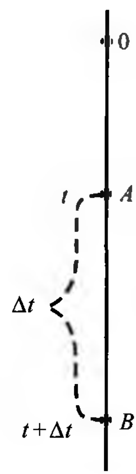
1-misol. $S=2t^2$ qonun bo‘yicha harakat qiluvchi jismning $t_1=2$ dan $t_2=4$ gacha vaqt oralig‘idagi o‘rtacha tezligini toping.

Yechish. $S_1=2t_1^2=2\cdot 2^2=8$; $S_2=2t_2^2=2\cdot 4^2=32$.

$\Delta S=S_2-S_1=32-8=24$.

$\Delta t=t_2=t_1=4-2=2$.

$v=\frac{\Delta S}{\Delta t}=\frac{24}{2}=12$ *Javob:* $v_{\text{o‘rta}}=12$.



28-чизма.

Tabiatda tekis harakatdan tashqari notekis (tekismas) harakat ham uchrab turadi. Masalan, jismning erkin tushish qonunini ifodalovchi $\boldsymbol{S=4{,}9t^2}$**(3)** formula berilgan bo‘lsin.

Jismning erkin tushishi notekis harakat bo‘lganligidan vaqtning biror paytida uning tezligini qanday aniqlash kerak degan savol tug‘iladi.

Tushish boshlanish paytida jism O nuqtada turgan bo‘lsin (28-chizma). t vaqt o‘tgandan keyin jism $S_1=4{,}9t^2$

masofani bosib, A nuqtaga kelib qoladi. Harakat boshlanishidan $t+\Delta t$ vaqt o'tgandan keyin u $S_2=4{,}9(t+\Delta t)^2$ yo'lni bosib o'tadi va B nuqtaga yetib keladi.

Jismning Δt vaqt ichida o'tgan yo'li $\Delta S=S_2-S_1=4{,}9(t+\Delta t)^2-4{,}9t^2=$ $=4{,}9t^2+9{,}8t(\Delta t)+4{,}9(\Delta t)^2-4{,}9t^2=9{,}8t(\Delta t)+4{,}9(\Delta t)^2$ tenglikni hosil qilamiz.

$\frac{\Delta S}{\Delta t}$ nisbatni ΔS yo'l oralig'ida jism erkin tushishining **o'rta tezligi** deyiladi.

Lekin jism harakatining o'rtacha tezligi ixtiyoriy paytdagi haqiqiy tezlikni ifoda etmaydi.

O'rtacha tezlik aniqlanadigan yo'l oralig'i qancha kichik bo'lsa, u harakatni shuncha aniqroq xarakterlab beradi.

Agar $\Delta t\to 0$ deb faraz etsak, u holda $\Delta t+t\to t$; $\frac{\Delta S}{\Delta t}$ nisbat esa A nuqtadagi tezlikka mos keluvchi t paytdagi tezlik deb ataluvchi miqdorga intiladi.

Bu tezlikni υ bilan belgilasak, u holda $\boldsymbol{\upsilon=\lim\limits_{\Delta t\to 0}\frac{\Delta S}{\Delta t}=S'(t)}$ (3) formula topiladi.

$S=4{,}9t^2$ bo'lganda $\upsilon=S'=(4{,}9t^2)'=9{,}8t$ hosil bo'ladi.

Harakatning tezlanishi haqida ham shunga o'xshash fikrni aytish mumkin.

Agar tezlikni $\upsilon(t)$, tezlanishni a desak, u holda $\upsilon'(t)=a$ kelib chiqadi. $a=\upsilon'(t)=(9{,}8t)'\approx 9{,}8$ yoki $a=g\approx 9{,}8$.

Tezlanishni qisqacha: **tezlikdan vaqt bo'yicha olingan hosila tezlanishdir**.

Agar moddiy nuqtaning erkin tushishini umumiy holda $s(t)=\frac{gt^2}{2}$ deb olsak, u holda nuqtaning t vaqtdagi tushish tezligi $\vartheta(t)=\left(\frac{gt^2}{2}\right)'=gt$ ga, tezlanishi esa, $a=\upsilon'(t)=(gt)'=g$ – o'zgarmas miqdordir.

2-misol. To'g'ri chiziq bo'ylab harakat qilayotgan nuqtaning o'tgan masofasi $s(t)=\frac{a}{2}t^2+\upsilon_0 t+x_0$ formula bilan berilgan bo'lsin, bunda $a\neq 0$, υ_0 va x_0 – o'zgarmaslar. Harakatning tezligini va tezlanishini topamiz.

Yechish. Bu harakatning tezligi: $\upsilon(t)=s'(t)=\left(\frac{a}{2}t^2+\upsilon_0 t+x_0\right)'=2\cdot\frac{a}{2}t+$ $+\upsilon_0=at+\upsilon_0$. Harakatning tezligi vaqtning funksiyasi bo'lganligidan bu harakatning tezlanishini topamiz:

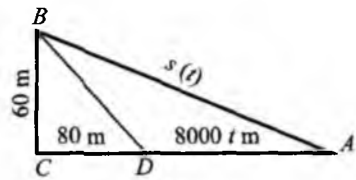
$$\upsilon'(t)=(at+\upsilon_0)'=a.$$

Agar $a>0$ bo'lsa, u holda biz tekis tezlanuvchan harakatni kuzatamiz.

Agar $a<0$ bo'lsa, u holda tekis sekinlanuvchan harakatni kuzatamiz.

3-masala. Bir kishi balandligi 60 m bo'lgan minora tagidan 8 km/soat tezlik bilan uzoqlashmoqda. Shu kishi asosidan 80 m narida bo'lganda minora uchidan uzoqlashish tezligi qanday bo'ladi?

Yechish. Shu kishi AD masofani t soatda bosib o'tsin deylik, u holda AD masofa $8\text{ km}\cdot t=8000\cdot t$ m bo'ladi (29-chizma).



29-chizma.

$AC=80+8000\,t$(m), $AB=s(t)$ desak.

ΔABC dan:

$$s(t)=\sqrt{60^2+(80+8000t)^2}=\sqrt{10000+1280000t+64000000t^2}.$$

$$\upsilon(t)=S'(t)=\frac{1280000+128000000t}{2\sqrt{10000+1280000t+64000000t^2}}$$

AD ni t soatda o'tgani uchun D nuqta harakat boshlanishi, ya'ni $t=0$ bo'ladi.

$$\upsilon_D=S'(0)=\frac{1280000}{2\sqrt{10000}}=\frac{1280000}{2\cdot100}=6400\text{ (m/soat)}=6{,}4\text{ km/soat}.$$

Javob: 6,4 km/soat.

? TAKRORLASH UCHUN SAVOLLAR

1. Tekis harakat qanday formula bilan beriladi?
2. Tekis harakatning tezligi qanday topiladi?
3. Notekis harakatga misollar keltiring.
4. Notekis harakatning tezligi nimaga teng?
5. Notekis harakatning tezlanishi qanday topiladi?

MASALALARNI YECHING

188. $S=0{,}5t^2+3t$ qonun bo‘yicha harakat qiluvchi jismning $t_1=1$ dan $t_2=3$ gacha vaqt oralig‘idagi o‘rtacha tezligini toping.

189. Agar jismning harakat tezligi $S=4t^2-3$ formula bilan berilgan bo‘lsa, uning $t=2$ vaqtdagi tezligini toping.

190. Jism o‘q atrofida $\varphi(t)=3t^2-4t+2$ (rad) qonun bo‘yicha aylanadi. Vaqtning t paytidagi va $t=4$s bo‘lgandagi $\omega(t)$ burchak tezligini toping.

191. Tormozlanayotgan moxovik t sekundda $\varphi(t)=4t-0{,}3t^2$ rad burchakka buriladi.

1) Moxovik aylanishining $t=2$c paytdagi tezligi $\omega(t)$ ni toping; 2) Moxovik qaysi paytda to‘xtaydi?

192. Nuqta $s(t)=2t^3+t-1$ (sm) qonun bo‘yicha to‘g‘ri chiziqli harakat qilsin. t sekund paytdagi tezlanishni toping. Qaysi paytda tezlanish: a) 1 sm/s^2; b) 2 sm/s^2 bo‘ladi.

193. Massasi 2 kg bo‘lgan jism $s(t)=t^2+t+1$ qonun bo‘yicha to‘g‘ri chiziqli harakat qiladi.

Masofa sm bilan, t vaqt sekund bilan o‘lchanadi.

1) Ta’sir qiluvchi kuchni toping; 2) jism harakat boshlagandan 2 sekund keyingi kinetik energiyasi E ni toping.

Ko‘rsatma. Kuch $F=ma\left(1\text{ H}=\frac{\text{kg}\cdot\text{m}}{\text{sek}^2}\right)$ kinetik energiya $E=\frac{m\upsilon^2}{2}\left(1\text{ erg}=\frac{\text{g}\cdot\text{sm}}{\text{sek}^2}\right)$ lardan foydalaniladi.

194. Nuqta $s(t)=\sqrt{t}$ qonun bo‘yicha to‘g‘ri chiziqli harakat qiladi. Uning tezlanishi tezligining kubiga proporsional ekanini ko‘rsating.

35-§. Funksiya o'sishining (kamayishining) yetarli sharti

Funksiyalarni tekshirish paytida paydo bo'ladigan asosiy masalalardan biri, bu funksiyani o'sadigan yoki kamayadigan oraliqlarini topish masalasidir. Bu punktda funksiya o'sishining (kamayishining) yetarli shartini ifodalab, uni isbotlaymiz. Isbotlashda matematik analiz kurslarida muhim ahamiyatga ega bo'lgan quyidagi teoremadan foydalanamiz.

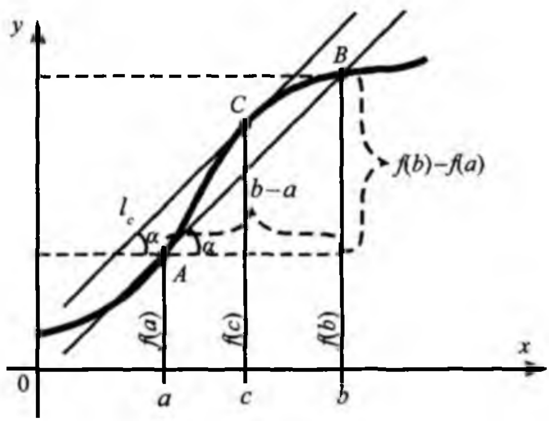
1-teorema **(Lagranj teoremasi).** $f(x)$ **funksiya biror oraliqning har bir nuqtasida differensiallanuvchi bo'lsin. U holda bu oraliqning istalgan ikkita** a **va** b **nuqtalari orasida shunday c nuqta topiladiki, buning uchun:**

$$f(b)-f(a)=f'(c)(b-a) \tag{1}$$

Bu formula ***Lagranj formulasi*** deb ataladi.

Bu teoremaning isboti maktab kursi doirasidan tashqarida bo'lgani uchun ayoniy tushunchalar bilan cheklanamiz.

$f(x)$ funksiyaning grafigida $A=M(a;f(a))$ va $B=N(b;f(b))$ nuqtalarni olamiz AB to'g'ri chiziqning burchak koeffitsiyenti $tg\alpha=\frac{f(b)-f(a)}{b-a}$ ga teng (30-chizma).



30-chizma.

Ikkinchi tomondan $f(x)$ funksiya grafigiga $C(c; f(c))$ nuqtada urinma l_c ning burchak koeffitsiyenti $f'(c)=\text{tg}\alpha$ va $\frac{f(b)-f(a)}{b-a}=\text{tg}\alpha$ tengliklardan $\frac{f(b)-f(a)}{b-a}=f'(c)$ yoki $\boldsymbol{f(b)-f(a)=f'(c)(b-a)}$ kelib chiqadi.

Bu tenglik l_c va AB to'g'ri chiziqlarning burchak koeffitsiyentlari teng, ya'ni $l_c \| AB$ ekanini bildiradi.

2-teorema. **1) Agar $f(x)$ funksiya I oraliqning har bir nuqtasida musbat hosilaga ega bo'lsa, u holda $f(x)$ funksiya shu oraliqda o'sadi.**

2) **Agar $f(x)$ funksiya I oraliqning har bir nuqtasida manfiy hosilaga ega bo'lsa, u holda bu funksiya shu oraliqda kamayadi.**

Isbot. $f(x)$ funksiyaning I oralig'iga tegishli ixtiyoriy ikkita x_1 va x_2 nuqta olamiz. Aniqlik uchun $x_2>x_1$ bo'lsin.

Lagranj teoremasiga ko'ra bu nuqtalar orasida shunday c nuqta topiladiki, $\boldsymbol{f(x_2)-f(x_1)=f'(c)(x_2-x_1)}$, $\boldsymbol{x_1<c<x_2}$ (2) tenglik o'rinli bo'ladi.

$c\in]x_1; x_2[$ bo'lgani uchun teorema shartidan

1) holda $f'(c)>0$; 2) holda $f'(c)<0$ bo'ladi.

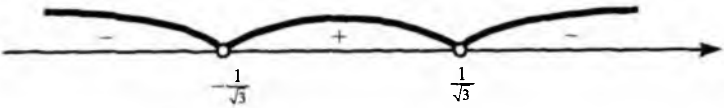
1) $f'(c)>0$ va $x_2-x_1>0$ bo'lgani uchun (2) tenglikdan $f(x_2)>f(x_1)$ ekanini topamiz. Bu esa $f(x)$ funksiya I oraliqda o'sishini bildiradi.

2) $f'(c)<0$ va $x_2-x_1>0$ bo'lgani uchun, (2) tenglikdan $f(x_2)-f(x_1)<0$ bo'ladi, ya'ni $x_2>x_1$ bo'lsa, $f(x_2)<f(x_1)$ ekanini topamiz. Bu esa $f(x)$ funksiya I oraliqda kamayishini bildiradi.

1-misol. $f(x)=x-x^3$ funksiyaning o'sish va kamayish oraliqlarini topamiz va uning grafigini yasaymiz.

Yechish. Bu funksiya barcha haqiqiy sonlarda aniqlangan. $f'(x)=(x-x^3)'=1-3x^2$.

Bu funksiya $f'(x)>0$ da o'suvchi bo'lgani uchun $1-3x^2>0$ ning yechimida o'sadi. $1-3x^2>0$ tengsizlikni intervallar metodi bilan yechamiz: $1-3x^2=0$; $x^2=\frac{1}{3}$; $x_1=-\frac{1}{\sqrt{3}}$; $x_2=\frac{1}{\sqrt{3}}$



31-chizma.

Haqiqiy sonlarni $\left(-\infty; -\frac{1}{3}\right]$; $\left]-\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}\right[$ va $\left]-\frac{1}{\sqrt{3}}; +\infty\right)$ oraliqlarga ajratdik (31-chizma).

$\left(-\infty; -\frac{1}{\sqrt{3}}\right[$ dan $x=-1$ da $f'(-1)=1-3\cdot(-1)^2=-2<0.$

$\left]-\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}\right[$ dan $x=0$ da $f'(0)=1-3\cdot 0^2=1>0.$

$\left]-\frac{1}{\sqrt{3}}; +\infty\right)$ dan $x=1$ da $f'(1)=1-3\cdot 1^2=-2<0.$

Demak, $f(x)=x-x^3$ funksiya $\left(-\infty; -\frac{1}{\sqrt{3}}\right[$ va $\left]\frac{1}{\sqrt{3}}; +\infty\right)$ oraliqlarda kamayadi, $\left]-\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}\right[$ oraliqda o‘sadi.

Bu funksiyaning grafigini aniq tasvirlash uchun topilgan oraliqlarning oxirlarida uning qiymatlarini hisoblaymiz:

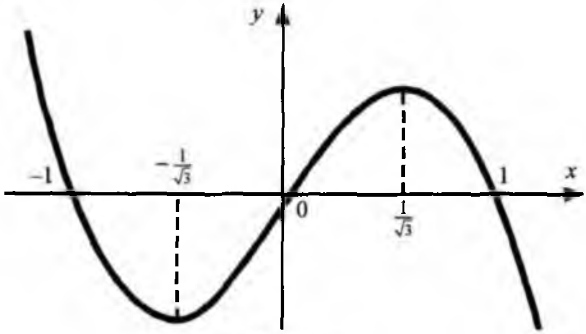
$$f\left(-\frac{1}{\sqrt{3}}\right)=-\frac{1}{\sqrt{3}}-\left(-\frac{1}{\sqrt{3}}\right)^3=-\frac{2}{3\sqrt{3}};$$

$$f\left(\frac{1}{\sqrt{3}}\right)=\frac{1}{\sqrt{3}}-\left(\frac{1}{\sqrt{3}}\right)^3=\frac{2}{3\sqrt{3}}.$$

Koordinata tekisligida $M_1\left(-\frac{1}{\sqrt{3}}; -\frac{2}{3\sqrt{3}}\right)$ va $M_2\left(-\frac{1}{\sqrt{3}}; \frac{2}{3\sqrt{3}}\right)$ nuqtalarni belgilab va $\left]-\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}\right[$ oraliqda o‘suvchi hamda $\left(-\infty; -\frac{1}{\sqrt{3}}\right[$ va $\left]\frac{1}{\sqrt{3}}; +\infty\right)$ oraliqlarda kamayuvchi funksiyaning grafigini yasaymiz (32-chizma).

Eslatma. 1-misoldagi $f(x)$ funksiya $-\frac{1}{\sqrt{3}}$ va $\frac{1}{\sqrt{3}}$ nuqtalarda uzluksiz bo‘lsa, uni shu oraliqqa birlashtirish mumkin, ya’ni $\left[-\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}\right]$

oraliqda o'sadi, $\left(-\infty; -\frac{1}{\sqrt{3}}\right]$ va $\left[\frac{1}{\sqrt{3}}; +\infty\right)$ oraliqlarda kamayuvchi bo'ladi.



32-chizma.

2-misol. $f(x)=2x+\frac{1}{x^2}$ funksiyaning o'sish (kamayish) oraliqlarini topamiz va uning grafigini chizamiz.

Yechish. Bu funksiyaning aniqlanish sohasi $x\neq0$ barcha sonlar yoki $(-\infty; 0)$ va $(0; +\infty)$ lar. $f'(x)=\left(2x+\frac{1}{x^2}\right)'=2-\frac{2}{x^3}$ bo'lgani uchun $f'(x)=0$ deb olib, $2-\frac{2}{x^3}=0$ tenglamani yechamiz: $2x^3=2$; $x=1$. Demak, 0 va 1 nuqtalar $f(x)$ funksiyaning aniqlanish sohasini uchta $(-\infty; 0[$, $]0; 1]$ va $[1; +\infty)$ oraliqqa bo'ladi.

33-chizma.

$(-\infty; 0[$ dagi $x=-1$ da $f'(1)=2-\frac{2}{(-1)^3}=4>0.$

$]0; 1[$ dagi $x=0{,}5$ da $f'(0{,}5)=2-\frac{2}{0{,}5^3}=2-16=-14<0.$

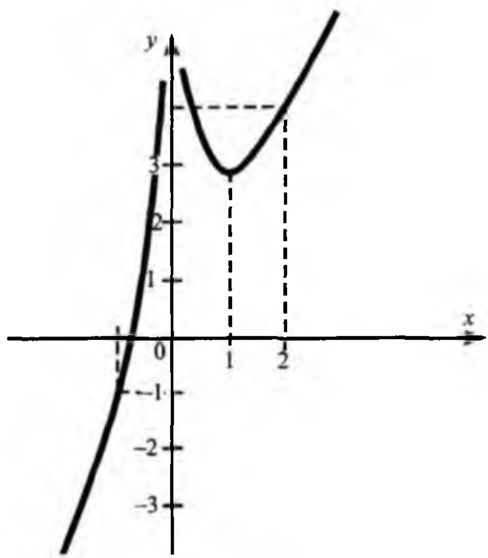
$[1; +\infty)$ dagi $x=2$ da $f'(2)=2-\frac{2}{2^3}=1{,}75>0.$

Demak, berilgan funksiya $(-\infty; 0[$ va $[1; +\infty)$ oraliqlarda o'sadi, $]0; 1]$ oraliqda kamayadi.

$x=0$ nuqtada $f(0)$ – mavjud emas,

$x=1$ nuqtada $f(1)=3$ ga teng. Funksiyaning grafigini chizamiz (34-chizma):

$(-\infty; 0[$ oraliqda o'suvchi bo'lib, grafigi OY o'qiga intilib boradi; $x=-1$ da $f(-1)=-1$; $]0; 1]$ oraliqda $f(1)=3$ gacha kamayadi, $[1; +\infty)$ da o'sadi $\left(f(2)=2\cdot 2+\frac{1}{2^2}=4,25\right)$.



34-chizma.

TAKRORLASH UCHUN SAVOLLAR

1. Lagranj teoremasi (1-teorema)ning formulasini yozing.
2. $f(x)$ funksiya I oraliqda qachon o'suvchi bo'ladi?
3. $f(x)$ funksiya I oraliqda qachon kamayuvchi bo'ladi?
4. Agar $f(x)$ funksiya o'sish (kamayish) oralig'i oxirlaridan birortasida uzluksiz bo'lsa, u shu oraliqqa tegishlimi (eslatma)?
5. a) $y=5x-7$; b) $y=-2x+8$ funksiyalar o'suvchimi yoki kamayuvchimi?

MASALALARNI YECHING

Quyidagi funksiyalarning o'sish va kamayish oraliqlarini aniqlang:

195. a) $f(x)=3x+1$; b) $g(x)=-4x+2$.

196. a) $f(x)=\frac{2}{x}$; b) $g(x)=\frac{2}{3-x}$.

197. a) $f(x)=x^2$; b) $u(x)=(x-1)^2$.

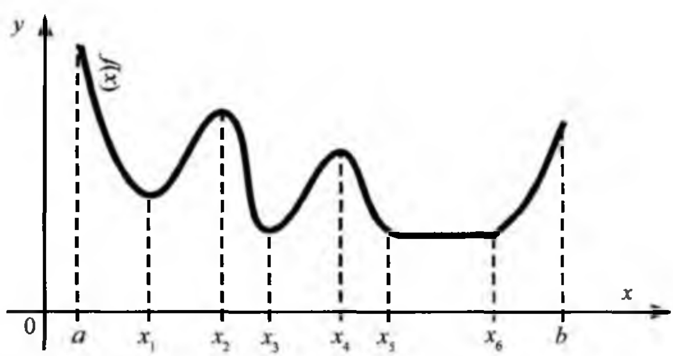
198. a) $y(x)=5x^2-3x+1$; b) $f(x)=x^2-2x+5$.

199. a) $h(x)=x^3-27x$; b) $g(x)=x^2(x-3)$.

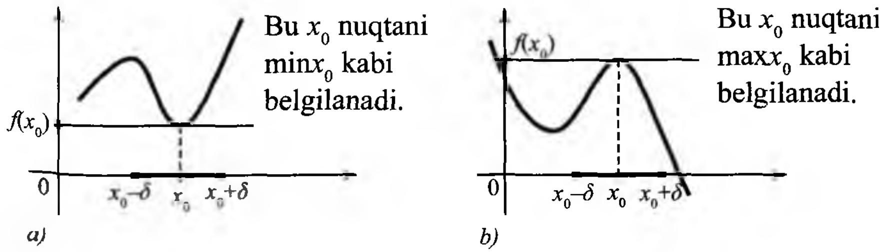
36-§. Funksiyaning kritik nuqtalari, uning maksimum va minimumlari

Bundan oldingi punktda funksiyalarni tekshirganda aniqlanish sohalarining hosila, mavjud bo'lmaydigan yoki nolga teng bo'ladigan ikki nuqtalari muhim ekanligini ko'rdik. Bunday nuqtalar funksiyaning **kritik nuqtalari** deyiladi.

$f(x)$ funksiyaning 35-chizmada tasvirlangan grafigini qarab chiqamiz. Bu funksiyaning x_1, x_2, x_3 va x_4 kritik nuqtalarining quyidagi xossalarini aytib o'tish mumkin. $f(x)$ funksiyaning x_1 va x_3 nuqtalarga yetarlicha yaqin barcha nuqtalardagi qiymatlari mos ravishda $f(x_1)$ va $f(x_3)$ qiymatlardan kichik emas, shu $f(x)$ funksiyaning x_2 va x_4 nuqtalarga yetarlicha yaqin nuqtalardagi qiymatlari mos ravishda $f(x_2)$ va $f(x_4)$ qiymatlaridan katta emas, ya'ni:



35-chizma.



36-chizma.

1-ta'rif. **Agar x_0 nuqtaning shunday atrofi topilsa, shu atrofdan bo'lgan barcha x lar uchun (36-*a*, chizma).**

$f(x_0) \leq f(x)$ bo'lsa, x_0 nuqta $f(x)$ funksiyaning minimum nuqtasi deyiladi.

2-ta'rif. **Agar x_0 nuqtaning shunday atrofi topilsa, bu atrofdan bo'lgan barcha x lar uchun (36-*b*, chizma) $f(x_0) \geq f(x)$ bo'lsa, x_0 nuqta $f(x)$ funksiyaning maksimum nuqtasi deyiladi.**

Funksiyaning minimum va maksimum nuqtalari shu funksiyaning ***ekstremum*** nuqtalari, funksiyaning shu nuqtalardagi qiymatlari esa funksiyaning ***ekstremumlari*** deyiladi.

Shunday qilib, x_1 va x_3 nuqtalar $f(x)$ funksiyaning minimum nuqtalari, x_2 va x_4 nuqtalar esa shu funksiyaning maksimum nuqtalaridir (35-chizma).

Bu ekstremum nuqtalarni funksiyaning kritik nuqtalari ekanini ko'rsatamiz.

1-teorema (Ferma teoremasi). **Agar x_0 nuqta $f(x)$ funksiyaning ekstremum nuqtasi bo'lib, bu nuqtada hosila mavjud bo'lsa, bu hosila nolga teng bo'ladi, ya'ni $f'(x_0)=0$.**

Bu teoremaning isboti maktab programmasiga kirmaydi, shu sababli biz geometrik tasvirlash bilan cheklanamiz. 36-chizmada ekstremal nuqtalar (x_0) ga o'tkazilgan urinmalar absissa o'qi bilan $0°$ yoki $180°$ li burchak hosil qiladi. Bunda $k=\text{tg}\alpha=f'(x_0)$ ga asosan $k=\text{tg}0°=\text{tg}180°=f'(x_0)=0$.

Demak, ekstremal nuqtaning hosilasi nol bo'ladi, ya'ni $f'(x_0)=0$.

2-teorema. **Agar $f(x)$ funksiya x_0 nuqtada uzluksiz bo'lib, $]a; x_0[$ intervalda $f'(x)>0$ va $]x_0; b[$ intervalda $f'(x)<0$ bo'lsa, u holda x_0 nuqta $f(x)$ funksiyaning maksimum nuqtasidir.**

Bu teoremaning qulay ifodasi: **x_0 nuqtada hosila ishorasini plyusdan minusga o'zgartirsa, u holda x_0 maksimum nuqtasidir.**

Isbot. $]a; x_0]$ oraliqda $f'(x)>0$, $f(x)$ funksiya esa x_0 nuqtada uzluksiz bo'lgani uchun, funksiya o'sishining yetarli alomatidan $f(x)$ funksiya $]a; x_0]$ oraliqda o'suvchi ekanligi, demak, $]a; x_0]$ oraliqda o'suvchi ekanligi, demak, $]a; x_0]$ oraliqdagi barcha x larda $f(x_0)\geq f(x)$ tengsizlik o'rinli bo'ladi.

$[x_0; b[$ oraliqda $f(x)$ funksiya kamayadi (isboti yuqoridagidek bajarildi). Demak, $[x_0; b[$ oraliqdagi barcha x lar uchun $f(x_0)\geq f(x)$ o'rinli.

Shunday qilib, $]a; b[$ intervalda bo'lgan barcha $x\neq x_0$ lar uchun $f(x_0)\geq f(x)$ bajariladi, ya'ni x_0 nuqta $f(x)$ funksiyaning maksimum nuqtasi.

3-teorema. **Agar $f(x)$ funksiya x_0 nuqtada uzluksiz bo'lib, $]a; x_0]$ intervalda $f'(x)<0$, $[x_0; b]$ intervalda $f'(x)>0$ bo'lsa, u holda x_0 nuqta $f(x)$ funksiyaning minimum nuqtasidir.**

Bu teoremaning qulay ifodasi: x_0 nuqtada hosila o'z ishorasini minusda plyusga o'zgartirsa, u holda x_0 minimum nuqtasi bo'ladi.

1-misol. $f(x)=3x-x^3$ funksiyaning ekstremum nuqtalarini topamiz.

Yechish. Bu funksiyaning hosilasi $(3x-x^3)'=3-3x^2$ barcha nuqtalarda aniqlangan. Bu funksiyaning ekstremumlarini topamiz (Ferma teoremasiga ko'ra): $3-3x^2=0$; $3(1-x^2)=0$; $3(1-x)(1+x)=0$.

Bu hosila $x_1=-1$ va $x_2=1$ nuqtalarda nolga aylanadi.

Hosilaning aniqlanish sohasini oraliqlarga ajratib, oraliqlar ishorasini aniqlaymiz (37-chizma).

$x=-2$ da $f'(-2)=3-3\cdot(-2)^2=3-12=-9<0$.

$x=0$ da $f'(0)=3-3\cdot 0^2=3>0$.

$x=2$ da $f'(2)=3-3\cdot 2^2=3-12=-9<0$.

37-chizma.

2 va 3-teoremalarga asosan $x=-1$ nuqta $f(x)$ funksiyaning minimum nuqtasi (minusdan plyusga o'tgan), $x=1$ nuqta $f(x)$ funksiyaning maksimum nuqtasi (plyusdan minusga o'tgan).

Endi $f(x)=3x-x^3$ funksiyaning grafigini chizamiz:

1) $f(-1)=3\cdot(-1)-(-1)^3=-3+1=-2; f(1)=3\cdot 1-1^3=2.$

2) Grafikni OX o'qi bilan kesishish nuqtalarini topamiz:

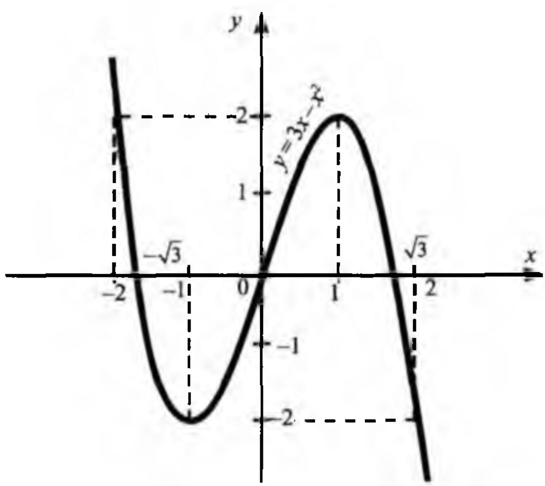
$3x-x^3=0;\ x(3-x^2)=0;\ x_1=0;\ x_{2/3}=\pm\sqrt{3}.$

Funksiya OX o'qini $(-\sqrt{3};\ 0)$, $(0;\ 0)$ va $(\sqrt{3};\ 0)$ nuqtalarda kesib o'tadi.

3) Funksiya $(-\infty;\ -1)$ va $(1;\ +\infty)$ oraliqlarda kamayishini, $(-1;\ 1)$ oraliqda o'sishini e'tiborga olib, grafigini chizamiz (38-chizma). Grafik aniq chiqishi uchun uning ixtiyoriy ikki chetki nuqtasini topamiz. $f(-2)=3\cdot(-2)-(-2)^3=2;\ f(2)=3\cdot 2-2^3=-2.$

2-misol. $f(x)=\frac{1}{3}x^3-\frac{3}{2}x^2-4x+6$ funksiyaning ekstremum nuqtalarini toping va grafigini chizing.

Yechish. Bu funksiya barcha haqiqiy sonlarda aniqlangan. Funksiyaning ekstremal nuqtalarini topamiz:



38-chizma.

39-chizma.

$$\left(\frac{1}{3}x^3-\frac{3}{2}x^2-4x+6\right)'=x^2-3x-4.$$

$x^2-3x-4=0$ bundan $x_{1/2}=1,5\pm\sqrt{2,25+4}=1,5\pm2,5.$

$x_1=-1$; $x_2=4$. Funksiyaning o'sish va kamayish oraliqlarini tekshiramiz:

$x=-1$ kritik nuqtani tekshiramiz:

$x=-2$ da $f'(-2)=(-2)^2-3\cdot(-2)-4=6>0$, $x=0$ da $f'(0)=-4<0$.

$x=5$ da $f'(5)=5^2-3\cdot5-4=6>0$.

Demak, $x=-1$ atrofida hosila ishorasi plyusdan minusga o'zgardi. Bu nuqtada funksiya maksimumga erishadi, ya'ni:

$$f(-1)=\frac{1}{3}(-1)^3-\frac{3}{2}(-1)^2-4\cdot(-1)+6=8\frac{1}{6}.$$

$x=4$ bo'lganda funksiya minimumga ega bo'ladi, ya'ni

$$f(4)=\frac{1}{3}\cdot4^3-\frac{3}{2}\cdot4^2-4\cdot4+6=-12\frac{2}{3}.$$

Koordinata tekisligida maksimum va minimumga mos kelgan $A\left(-1;8\frac{1}{6}\right)$ va $B\left(4;-12\frac{2}{3}\right)$ nuqtalarni belgilaymiz.

Grafikni aniqroq chizish uchun –1 dan kichik $x=-2$ da

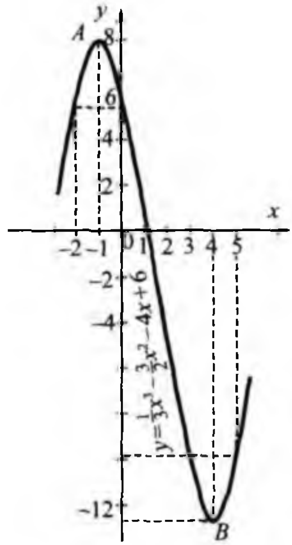
$f(-2)=\frac{1}{3}(-2)^3-\frac{3}{2}\cdot(-2)^2-4\cdot(-2)+6=-2\frac{2}{3}-6+8+6=5\frac{1}{3}$ qiymatni va

4 dan katta $x=5$ da $f(5)=\frac{1}{3}\cdot5^3-\frac{3}{2}\cdot5^2-4\cdot5+6=41\frac{2}{3}-37\frac{1}{2}-20+6=-9\frac{5}{6}$

qiymatni topamiz. Topilganlar bo'yicha grafik chizamiz.

TAKRORLASH UCHUN SAVOLLAR

1. Qanday nuqtalarni funksiyaning kritik nuqtalari deyiladi?
2. $f(x)$ funksiyaning minimum nuqtasi deb nimaga aytiladi?
3. $f(x)$ funksiyaning maksimum nuqtasi deb nimaga aytiladi?
4. Funksiyaning ekstremum nuqtalari deb nimaga aytiladi?
5. Ferma teoremasini ayting *(1-teorema)*.
6. x_0 nuqta $f(x)$ funksiyaning maksimum nuqtasi haqidagi teoremani ayting *(2-teorema)*.
7. x_0 nuqta $f(x)$ funksiyaning minimum nuqtasi haqidagi teoremani ayting *(3-teorema)*.



40-чизма.

MASALALARNI YECHING

Quyidagi funksiyalarning kritik nuqtalarini toping, ularning qaysilari maksimum va minimum ekanini aniqlang:

200. a) $f(x)=\frac{1}{2}x^2-3x$; b) $g(x)=x^2-\frac{1}{2}x$.

201. a) $h(x)=\frac{x}{3}+\frac{3}{x}$; b) $y(x)=2x^3+6x^2-18x+120$.

202. a) $u(x)=3x^4-4x^3$; b) $v(x)=\sqrt{x}$.

Quyidagi funksiyalarning ekstremumlarini topib, grafiklarini chizing:

203. a) $f(x)=4x^2-6x$; b) $g(x)=\frac{1}{2}x^2-3x$.

204. a) $s(x)=6x^5+15x^4+10x^3$; b) $h(x)=x^2(x-12)^2$;

205. a) $u(x)=\frac{x^2}{x^2+3}$; b) $y(x)=\frac{x^2-2x+2}{x-1}$.

37-§. Funksiyalarni tekshirish sxemasi

Biz quyi sinflarda funksiyalarning grafiklarini nuqtalar bo‘yicha yasab keldik. Ammo bu usulda ekstremum nuqtalarini o‘tkazib yuborish yoki noto‘g‘ri tasvirlash mumkin. Shuning uchun grafik yasashni funksiyani tekshirishdan boshlash ma’qul. Bu tekshirish quyidagi tartibda bajariladi:

1. Funksiyaning aniqlanish sohasi topiladi;
2. Funksiyaning hosilasi olinadi;
3. Kritik nuqtalari topiladi;
4. Funksiyaning kritik nuqtadagi qiymatlari topiladi;
5. O‘sish va kamayish oraliqlari topilib, jadvalga kiritiladi;
6. Ekstremumlari topiladi;
7. Topilganlar bo‘yicha jadval to‘ldirilib, grafik chiziladi.
8. Ba’zan $f(x)$ funksiya grafigining koordinata o‘qlari bilan kesishi nuqtalarini topish (bunda $f(0)$ ni va $f(x)=0$ hisoblab topiladi).

1-misol. $f(x)=3x^5-5x^3+2$ funksiyani tekshirib grafigini chizamiz:

1) Funksiyani aniqlanish sohasi barcha haqiqiy sonlar, ya’ni $(D(f)=R$. R – barcha haqiqiy sonlar.

2) $f'(x)=(3x^5-5x^3+2)'=15x^4-15x^2$.

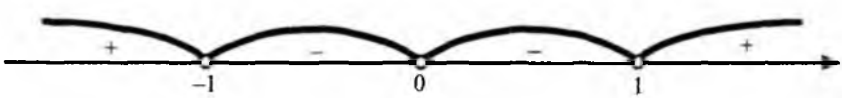
3) $f'(x)=0$ yoki $15x^4-15x^2=0$; $15x^2(x^2-1)=0$.

$x_1=0$; $x^2-1=0$; $x_{3/4}=\pm 1$. Kritik nuqtalar: –1; 0; 1.

4) Funksiyaning kritik qiymatlari:

$f(-1)=3\cdot(-1)^5-5\cdot(-1)^3+2=4$; $f(0)=2; f(1)=0$.

5) –1; 0; 1 kritik nuqtalar bo‘yicha o‘sish va kamayishi oraliqlarni topamiz (41-chizma).



41-chizma.

Jadvalning 1-qatoriga o‘sish va kamayish oraliqlari va kritik nuqtalar yoziladi.

x	$(-\infty; -1[$	-1	$]-1; 0[$	0	$]0; 1[$	1	$]1; +\infty)$
$f'(x)$	+	0	–	0	–	0	+
$f(x)$	↗	4	↘	2	↘	0	↗
ekstremum		max		–		min	

6) Hosilaning shu oraliqlardagi ishoralari aniqlaniladi:

$$f'(-2)=15\cdot(-2)^2((-2)^2-1)=60(4-1)=180>0.$$

$$f'\left(-\frac{1}{2}\right)=15\cdot\left(-\frac{1}{2}\right)^2\left(\left(-\frac{1}{2}\right)^2-1\right)=\frac{15}{4}\cdot\left(-\frac{3}{4}\right)<0.$$

$$f'\left(\frac{1}{2}\right)=15\cdot\left(\frac{1}{2}\right)^2\left(\left(\frac{1}{2}\right)^2-1\right)=\frac{15}{4}\cdot\left(-\frac{3}{4}\right)<0.$$

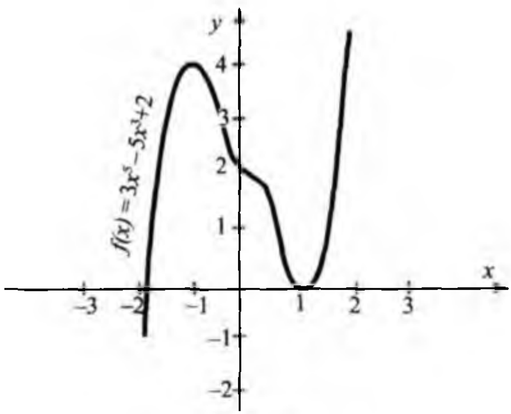
$$f'(2)=15\cdot 2^2(2^2-1)=60\cdot 3=180>0.$$

Oraliqlarning ishoralari ikkinchi qatorga yoziladi.

Jadvalning uchinchi qatoriga funksiyaning qanday o‘zgarishi (o‘sishi yoki kamayishi) strelka bilan ko‘rsatiladi. Shu qatorga funksiyaning kritik qiymatlari ham yoziladi.

7) To‘rtinchi qatorda funksiyaning ekstremumlari yoziladi.

8) Grafik jadvaldan foydalanib chiziladi (42-chizma). Grafik aniqroq bo‘lishi uchun chetki oraliqlardan funksiyaning bittadan qiymatlari topiladi:



42-chizma.

$f(-2)=3\cdot(-2)^5-5\cdot(-2)^3+2=-96+42=-54.$

$f(2)=3\cdot 2^5-5\cdot 2^3+2=96-38=58.$

2-misol. $y(x)=\frac{6(x-1)}{x^2+3}$ funksiyani tekshirib, grafigini chizamiz.

Yechish. 1) Aniqlanish sohasi $D(y(x))=R$ – haqiqiy son.

2) $y'(x)=\left(\frac{6(x-1)}{x^2+3}\right)'=\frac{6\cdot(x^2+3)-(6x-6)\cdot 2x}{(x^2+3)^2}=\frac{-6x^2+12x+18}{(x^2+3)^2};$

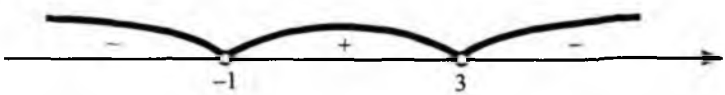
3) Kritik nuqtalarni topamiz:

$\frac{-6x^2+12x+18}{(x^2+3)^2}=0;\ -6x^2+12x+18=0;\ x^2-2x-3=0.$

$x_1=-1$ va $x_2=3.$

4) Kritik qiymatlari: $y(-1)=\frac{6(-1-1)}{(-1)^2+3}=\frac{-12}{4}=-3.\quad y(3)=\frac{6(3-1)}{3^2+3}=1.$

5) O‘sish va kamayish oraliqlari (43-chizma).



43-chizma.

$y'(-2)=\frac{-6\cdot(-2)^2+12\cdot(-2)+18}{((-2)^2+3)^2}=\frac{-24-24+18}{49}<0;$

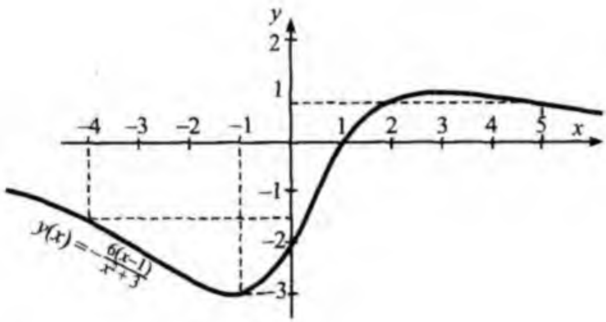
$y'(0)=\frac{-6\cdot 0^2+12\cdot 0+18}{(0^2+3)^2}=2>0;\qquad y'(4)=\frac{-6\cdot 4^2+12\cdot 4+18}{(4^2+3)^2}<0.$

6) Jadvalni tuzamiz:

x	$(-\infty;-1[$	-1	$]-1;3[$	3	$]3;+\infty)$
$y'(x)$	–	0	+	0	–
$y(x)$	↘	–3	↗	1	↘
ekstremum		min		max	

7) Funksiyaning chetki oraliqlaridan funksiyaning bittadan qiymatlarini topamiz (44-chizma):

$y(-4)=\frac{6(-4-1)}{(-4)^2+3}=\frac{-30}{19}\approx-1,6$



44-chizma.

$$y(5)=\frac{6(5-1)}{5^2+3}=\frac{24}{28}\approx 0{,}86.$$

Grafik *OX* o'qini (1; 0) va *OY* o'qini (0; –2) nuqtalarda keladi.

TAKRORLASH UCHUN SAVOLLAR

1. Funksiyalar nima maqsadda tekshiriladi?
2. Funksiya qanday bosqichda tekshiriladi?
3. Jadval qanday qilib tuziladi?
4. Funksiyaning kritik nuqtalari qanday topiladi?
5. Kritik nuqtalar qanday shartlarda maksimum yoki minimum nuqtalar bo'ladi?
6. Funksiya grafigining *OX* va *OY* o'qlar bilan kesishgan nuqtalari qanday topiladi?
7. Grafikni yasashda nima uchun chetki oraliqlardan funksiyaning bittadan qiymati topiladi?

MASALALARNI YECHING

Funksiyalarni tekshiring va grafiklarini yasang:

206. a) $f(x)=x^2-2x-8$; b) $y(x)=-x^2-5x+4$.

207. $\varphi(x)=-x^3+3x-2$.

208. $y(x)=3x^2-x^3$.

209. $u(x)=x^4-2x^3+3$.

210. $h(x)=x\sqrt{2-x}$.

211. $\varphi(x)=-x^2(3-x)$.

212. $\upsilon(x)=\frac{2x}{1+x^2}$

Quyidagi funksiyalarni o'sish va kamayish oraliqlarini toping:

213. $f(x)=-2{,}5x^3-3x^2-1{,}5x+1.$

214. $g(x)=x^3-6x^2+15x-2.$

215. $h(x)=x^5-\frac{4}{3}x^3+x-2.$

Ko'rsatma. Hosila musbat bo'lganda o'suvchi yoki hosila manfiy bo'lganda kamayuvchiligidan foydalaning.

38-§. Funksiyaning eng katta va eng kichik qiymatlari

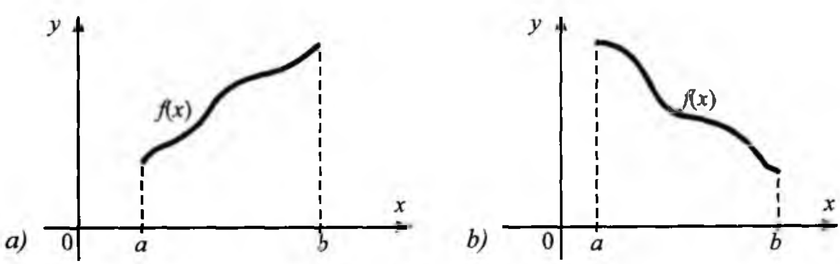
Ko'pgina masalalarni yechish ko'pincha kesmada uzluksiz bo'lgan funksiyaning eng katta va eng kichik qiymatlarini topishga keltiriladi.

$f(x)$ funksiya $[a;\ b]$ kesmada uzluksiz bo'libgina qolmay, balki shu kesmada chekli sonda kritik nuqtalarga ega bo'lgan hol uchun f(x) funksiyaning eng katta va eng kichik qiymatlarini topish qoidasini o'rganamiz.

Dastlab $f(x)$ funksiya $[a;\ b]$ kesmada kritik nuqtalarga ega emas deb faraz qilamiz. U holda funksiya shu oraliqda o'sadi (43-*a*, chizma) yoki kamayadi (43-*b*, chizma).

Demak $f(x)$ funksiya $[a;\ b]$ kesmadagi eng katta va eng kichik qiymatlari shu $f(x)$ funksiyaning kesmani a va b oxirlaridagi qiymatlariga teng, ya'ni $\min\limits_{[a;b]} f(x)=f(a);\ \max\limits_{[a;b]} f(x)=f(b)$ ko'rinishda yoziladi.

Endi $f(x)$ funksiya $[a;\ b]$ kesmada chekli sonda kritik nuqtalarga ega bo'lsin. Shu nuqtalar $[a;\ b]$ kesmani ichlarida kritik nuqtalari bo'lmagan chekli sondagi kesmalarga bo'ladi. Shuning uchun $f(x)$ funksiyaning



45-chizma.

eng katta va eng kichik qiymatlari yuqorida aytilgandek ularning har ikki uchida, ya'ni $f(x)$ funksiyaning kritik nuqtalarida yoki a va b nuqtalarda olinadi.

Shunday qilib, **kesmada chekli sonda kritik nuqtalarga ega bo'lgan funksiyaning eng katta va eng kichik qiymatlarini topish uchun, funksiyaning barcha kritik nuqtalaridagi va kesmaning oxirlaridagi qiymatlarini hisoblash hamda, hosil bo'lgan sonlardan eng kattasini va eng kichigini olish kerak.**

1-masala. $y(x)=x^3-1{,}5x^2-6x+1$ funksiyaning $[-2;\ 0]$ kesmadagi eng katta va eng kichik qiymatlarini topamiz.

Yechish. 1) Kritik nuqtalarni topamiz:

$y'(x)=3x^2-3x-6$; $y'(x)=0$ tenglama, ya'ni $3x^2-3x-6=0$ ni yechib, $x_1=-1$ va $x_2=2$ kritik nuqtalarni topamiz.

2) Kritik nuqtalar va $[-2;\ 0]$ kesmaning chetki nuqtalaridagi funksiyaning qiymatlarini topamiz:

Bundagi $x_2=2$ kritik nuqta $[-2;\ 0]$ kesmaga tegishli emas. Kritik nuqta $x_0=-1$.

$$y(-2)=(-2)^3-1{,}5\cdot(-2)^2-6\cdot(-2)+1=-1;\quad y(0)=1;$$

$$y(-1)=(-1)^3-1{,}5\cdot(-1)^2-6\cdot(-1)+1=4{,}5.$$

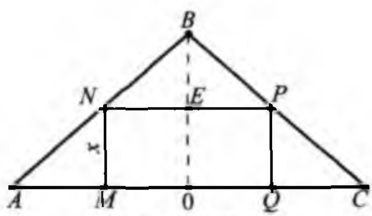
Bulardan eng katta qiymati 4,5, eng kichigi – 1 ga teng.

Bularni qisqacha quyidagidek yozamiz:

$$\max_{[-2;0]} y(x)=y(-1)=4{,}5;\qquad \min_{[-2;0]} y(x)=y(-2)=-1.$$

2-masala. Asosi 20 sm va balandligi 8 sm bo'lgan teng yonli uchburchakka to'g'ri to'rtburchak ichki chizilgan bo'lib, uning bir tomoni uchburchak asosida yotadi. To'g'ri to'rtburchakning yuzi eng katta bo'lishi uchun uning balandligi qanday bo'lishi kerak? (46-chizma)

Berilgan: ΔABC – teng yonli.
$AC=20$ sm; $BO=h=8$ sm.
$MNPQ$ to'g'ri to'rtburchak
ΔABC ga ichki chizilgan.
S_{MNPQ} eng katta bo'lsin.
Topish kerak: MN kesmani.



46-chizma.

Yechish. $MN=x$ bo'lsin. MQ masofani topish kerak bo'ladi. Bunda $AO=\frac{AC}{2}=10$ sm, $BE=8-x$. $\Delta ABO \backsim \Delta BNE$ dan $\frac{AO}{NE}=\frac{BO}{BE}$; $\frac{10}{NE}=\frac{8}{8-x}$;

$NE=\frac{10(8-x)}{8}=\frac{5}{4}(8-x)$; $NP=2\cdot NE=2\cdot\frac{5}{4}(8-x)=2{,}5(8-x)=20-2{,}5x$.

$S(x)=NP\cdot MN=(20-2{,}5x)\cdot x=20x-2{,}5x^2$.

$S'(x)=20-5x$; Kritik nuqta $20-5x=0$ dan

$x_0=4$. x ning aniqlanish sohasi $[0; 8]$.

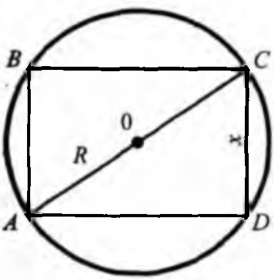
$S(0)=20\cdot 0-2{,}5\cdot 0^2=0$; $S(8)=20\cdot 8-2{,}5\cdot 8^2=160-160=0$.

$S(4)=20\cdot 4-2{,}5\cdot 4^2=80-40=40$; $x<x_0=4$ da $S'(x)>0$ va $x>x_0=4$ da $S'(x)<0$. Demak, $\max x_0=4$. *Javob:* 4 sm.

$\max\limits_{[0;8]} S(4)=40$. $\max\limits_{[0;8]} x_0=4$ (sm). *Javob:* 4 sm.

3-masala. Aylanaga ichki chizilgan barcha to'g'ri to'rtburchaklardan yuzi eng katta to'g'ri to'rtburchakni toping.

Berilgan R radiusli aylana, to'g'ri to'rtburchak aylanaga ichki chizilgan (47-chizma).



47-chizma.

Topish kerak: max S_{ABCD} bo'lgan tomonlarni.

Yechish. $CD=x$ bo'lsin.

To'g'ri to'rtburchakning AD tomonini

ΔACD dan, $AD=\sqrt{(2R)^2-x^2}$ topamiz.

$S(x)=AD\cdot CD=\sqrt{4R^2-x^2}\cdot x$ topiladi.

$$S'(x)=1\cdot\sqrt{4R^2-x^2}+\frac{-2x\cdot x}{2\sqrt{4R^2-x^2}}=\frac{4R^2-x^2-x^2}{\sqrt{4R^2-x^2}}=\frac{4R^2-2x^2}{\sqrt{4R^2-x^2}};$$

Kritik nuqtasi: $4R^2-2x^2=0$, $2x^2=4R^2$; $x_0=\sqrt{2}R$. x ning o'zgarish sohasi $0\le x\le 2R$ dan iborat.

$S(0)=\sqrt{4R^2-0^2}\cdot 0=0$; $S(2R)=\sqrt{4R^2-(2R)^2}\cdot 2R=0$.

Demak, $x=x_0=\sqrt{2}R$ da $S'(x)>0$ va $x>x_0=\sqrt{2}R$ da $S'(x)<0$.

$$\max_{[0;\,2R]} S(x_0) = S(\sqrt{2}R) = 2R^2; \quad AD = \sqrt{4R^2 - (\sqrt{2}R)^2} = \sqrt{2}R.$$

Demak, $CD = AD = \sqrt{2}R$.

Javob: $ABCD$ – kvadrat ($AD = CD = \sqrt{2}R$).

TAKRORLASH UCHUN SAVOLLAR

1. Funksiyaning o‘suvchi yoki kamayuvchi ekanligini hosila yordamida qanday aniqlaymiz?
2. Funksiyaning kritik nuqtalari hosila yordamida qanday topiladi?
3. Kritik nuqtalarni maksimum yoki minimum ekanligini qanday aniqlaymiz?
4. Biror $[a;\ b]$ oraliqdagi $f(x)$ funksiyaning eng katta yoki eng kichik qiymatlari qanday topiladi?

MASALALARNI YECHING

216. $f(x)=x^4-8x^2-9$ funksiyaning: a) $[-1;\ 1]$; b) $[0;\ 3]$ oraliqlardagi eng kichik va eng katta qiymatlarini toping.

217. Moddiy nuqta $S(t)=5t+2t^2-\frac{2}{3}t^3$ qonun bo‘yicha to‘g‘ri chiziqli harakat qiladi, bunda $S(t)$–yo‘l (metr bilan), t – vaqt (sekund bilan). Qaysi paytda nuqta harakatining tezligi eng katta bo‘ladi va bu eng katta tezlikning miqdori qanday bo‘ladi?

218. Berilgan doiraga ichki chizilgan teng yonli uchburchaklar ichida teng tomonli uchburchak eng katta yuzaga ega ekanini ko‘rsating.

Ko‘rsatma. *Doira radiusini* R, *diametrini* $2R$ *deb olib*, uchburchak balandligini x bilan belgilang. Balandlikni diametrga to‘ldiruvchi qismini $2R-x$ deb oling. Uchburchak asosining yarmini o‘rta proporsional kesma yordamida topib, $S_\Delta(x)$ funksiyaning kritik *nuqtasi topiladi.* $x_0=1{,}5R$ balandlik teng tomonli uchburchakning balandligi (uchburchak R radiusli aylanaga ichki chizilgan).

219. Gipotenuzasi berilgan barcha to‘g‘ri burchakli uchburchaklar ichida teng yonli uchburchak eng katta yuzaga ega ekanini ko‘rsating.

220. Berilgan musbat sonni ikki qo'shiluvchiga shunday ajratingki, bu qo'shiluvchilarning ko'paytmasi eng katta bo'lsin.

221*. Asosi kvadrat shaklida bo'lgan ochiq bakka V litr suyuqlik ketadi. Shu bak qanday o'lchovlarda yasalganda eng kam material ketadi (o'lchovlar 1:2 nisbatda olinganda)?

Ko'rsatma. $h:x=1:2$ kabi, bunda x – bak asosining tomoni, h – bak balandligi, $S(x)$ – to'la sirti. $S(x)=x^2+\frac{4V}{x}$ funksiyaning minimumi topiladi. x_0 orqali h topilib, $h:x$ nisbat topiladi.

222. Berilgan $2P$ perimetrli teng yonli uchburchaklardan qandayining yuzi eng katta bo'ladi?

223. Berilgan hajmdagi barcha silindrlar ichidan to'la sirti eng kichik bo'lganini toping.

Ko'rsatma. V – hajm, $r=x$ – radius deb belgilab, $V_s=\pi x^2\cdot h$ dan $h=\frac{V}{\pi x^2}$ topiladi, $S_y(x)=2\pi x^2+2\pi x\cdot h$ ga h ni qo'yib, $S_y(x)$ funksiyaning minimumi topiladi. Nihoyat, $h=2x=2r$ ekanligi ko'rsatiladi.

39-§. Boshlang'ich funksiya va integral

Avval ko'rganimizdek vaqtning $t=0$ momentida jismning tezligi 0, ya'ni $v(0)=0$ bo'lsa, u holda jism erkin tushishida vaqtning t momentida $S(t)=\frac{g}{2}t^2$ (1) yo'lni o'tadi. Jismning tezligi esa $S'(t)=v(t)=gt$(2) bo'ladi.

Tezlanish esa, $v'(t)=a(t)=g$ (3) o'zgarmas tezlikka ega bo'ladi.

Biz yo'l formulasi $(S(t))$dan hosila olib, tezlik $(v(t))$ni topdik, tezlik formulasi $(v(t))$dan hosila olib, tezlanish $(a(t))$ni topdik.

Endi ishni aksincha qilib, tezlanish $a(t)$dan tezlik $v(t)$ni va tezlik $v(t)$dan o'tilgan yo'l $S(t)$ni topishni qarab chiqamiz, ya'ni $a(t)$dan $v(t)$ni topishni, $v(t)$dan $s(t)$ni topishni ko'rib chiqamiz.

Bunday masalalarni yechish uchun differensiallash amaliga teskari bo'lgan **integrallash** amali xizmat qiladi.

Ta'rif. **Agar berilgan oraliqdan olingan barcha x lar uchun $F'(x)=f(x)$ tenglik bajarilsa, u holda $F(x)$ funksiya shu oraliqda $f(x)$ funksiyaning boshlang'ich funksiyasi deyiladi.**

1-misol. $F(x)=\frac{x^3}{3}$ funksiya $(-\infty;+\infty)$ oraliqda $f(x)=x^2$ funksiyaning boshlang'ich funksiyasidir, chunki $(-\infty;+\infty)$ oraliqdagi barcha x lar uchun: $F'(x)=\left(\frac{x^3}{3}\right)'=\frac{1}{3}(x^3)'=\frac{1}{3}\cdot 3x^2=x^2=f(x)$. $\frac{x^3}{3}+7$ ham xuddi shu x^2 hosilaga ega.

Shuning uchun $\frac{x^3}{3}+7$ funksiya ham x^2 ning $(-\infty;+\infty)$ oraliqda boshlang'ich funksiyasidir. Bundagi 7ning o'rniga istagan o'zgarmas sonni qo'yish mumkin. Demak, boshlang'ich funksiyani topish masalasi cheksiz ko'p yechimga ega.

2-misol. $F(x)=2\sqrt{x}$ funksiya $f(x)=\frac{1}{\sqrt{x}}$ funksiyaning $]0;\ +\infty)$ oraliqda boshlang‘ich funksiyasi bo‘ladi, chunki shu oraliqdan olingan barcha x lar uchun:

$$F'(x)=(2\sqrt{x})'=2\cdot\frac{1}{2\sqrt{x}}=\frac{1}{\sqrt{x}}=f(x).$$

$2\sqrt{x}+c$ (c – istagan o‘zgarmas son) funksiya shu $]0;\ +\infty)$ oraliqda $\frac{1}{\sqrt{x}}$ funksiyaning boshlang‘ich funksiyasidir.

3-misol. $F(x)=\frac{3}{x}$ funksiya $(-\infty;\ +\infty)$ oraliqda $f(x)=-\frac{1}{x^2}$ funksiyaning boshlang‘ich funksiyasi bo‘lmaydi, chunki $F'(x)=f(x)$ tenglik 0 nuqtada bajarilmaydi.

Ammo $(-\infty;\ 0[$ va $]0;\ +\infty)$ oraliqlarning har birida $F(x)$ funksiya $f(x)$ funksiyaning boshlang‘ich funksiyasidir.

4-misol. $F(x)=\frac{2}{3}x^{\frac{3}{2}}$ funksiya $[0;\ +\infty)$ oraliqda $f(x)=\sqrt{x}$ funksiyaning boshlang‘ich funksiyasi ekanligini ko‘rsatamiz.

Haqiqatan, $F'(x)=\left(\frac{2}{3}x^{\frac{3}{2}}\right)'=\frac{2}{3}\cdot\frac{3}{2}x^{\frac{3}{2}-1}=x^{\frac{1}{2}}=\sqrt{x}=f(x).$

5-misol. $f(x)=x^4$ funksiyaning $(-\infty;\ +\infty)$ oraliqda boshlang‘ich funksiyasini topamiz.

Yechish. $F(x)=\frac{1}{5}x^5+c$ bo‘lsin deylik, chunki $F'(x)=\left(\frac{1}{5}x^5+c\right)'=\frac{1}{5}\cdot 5x^4+$ $+0=x^4=f(x)$. *Javob:* $F(x)=\frac{1}{5}x^5+c$.

TAKRORLASH UCHUN SAVOLLAR

1. Jismning erkin tushishda o‘tgan yo‘li formulasidan:
   a) Jismning tezligini; b) tezlanishini toping.
2. Integrallash amali qanday amal?
3. Qanday funksiyani boshlang‘ich funksiya deyiladi?
4. $F(x)=2x^3$ funksiya $f(x)=6x^2$ funksiyaga $(-\infty;\ +\infty)$ oraliqda boshlang‘ich funksiya bo‘ladimi?

MASALALARNI YECHING

$F(x)$ funksiya ko‘rsatilgan oraliqda $f(x)$ funksiyaning boshlang‘ich funksiyasi ekanini isbotlang:

224. a) $F(x)=x^5$; $f(x)=5x^4$; $x\in(-\infty;+\infty)$.

b) $F(x)=\frac{1}{3}x^{-3}$; $f(x)=-x^4$; $x\in]0;+\infty)$.

225. a) $F(x)=\sin x+3$; $f(x)=\cos x$; $x\in(-\infty;+\infty)$.

b) $F(x)=-\cos x+4$; $f(x)=\sin x$; $x\in(-\infty;+\infty)$.

226. a) $F(x)=x\in]0;+\infty)=4x^{\frac{1}{4}}$; $f(x)=x^{-\frac{3}{4}}$.

b) $F(x)=\frac{3}{2}\sqrt[3]{x^2}$; $f(x)=\frac{1}{\sqrt[3]{x}}$; $x\in]0;+\infty)$.

227. a) $F(x)=\mathrm{tg}x-\sqrt{2}$; $f(x)=\frac{1}{\cos^2 x}$; $x\in\left]-\frac{\pi}{2};\frac{\pi}{2}\right[$.

b) $F(x)=3-\mathrm{ctg}x$; $f(x)=\frac{1}{\sin^2 x}$; $x\in(-\infty;+\infty)$.

228. a) $F(x)=\sin^2 x$; $f(x)=\sin 2x$; $x\in(-\infty;+\infty)$.

b) $F(x)=\sin 3x$; $f(x)=3\cos 3x$; $x\in(-\infty;+\infty)$.

229. $F(x)=9-\frac{1}{x}$; $f(x)=\frac{1}{x^2}$; $x\in]0;+\infty)$.

230. $f(x)$ ning $(-\infty;+\infty)$ oraliqdagi boshlang‘ich funksiyasini toping:

a) $f(x)=2{,}3$; b) $f(x)=x$; d) $f(x)=x^4$; e) $f(x)=3x^3$.

40-§. Boshlang‘ich funksiyaning asosiy xossasi

Integrallash masalasi berilgan funksiyaning barcha boshlang‘ichlarini topishdan iborat. Bu masalani yechishda quyidagi lemma asosiy vazifani bajaradi.

Lemma (funksiyaning o‘zgarmaslik alomati). Agar biror J oraliqda $F'(x)=0$ bo‘lsa, u holda $F(x)$ funksiya shu oraliqda o‘zgarmasdir.

Isbot. J oraliqdan biror x_0 ni olamiz. U vaqtda shu oraliqqa tegishli har qanday x son uchun, Lagranj teoremasiga ko‘ra x va x_0 lar orasida shunday c sonni ko'rsatish mumkinki, bunda $F(x)-F(x_0)=F'(c)(x-x_0)$ tenglik o'rinli bo'ladi: $c\in J$ bo'lgani uchun lemma shartiga ko'ra, $F'(c)=0$ ekanligidan yuqoridagi tenglik $F(x)-F(x_0)=0$ bo'ladi. Shunday qilib, J oraliqqa tegishli barcha x lar uchun $F(x)-F(x_0)$ bajariladi, ya'ni $F(x)$ funksiya o'z qiymatini o'zgartirmay saqlaydi.

Endi boshlang'ich funksiyalarning **asosiy xossasini** isbotlaymiz:

Teorema. **$F(x)$ funksiya J oraliqda $f(x)$ funksiyaning boshlang'ich funksiyasi bo'lsin. U holda:**

1) Har qanday o'zgarmas C da $F(x)+C$ funksiya ham J oraliqda $f(x)$ funksiyaning boshlang'ich funksiyasi bo'ladi;

2) $f(x)$ funksiyaning J oraliqdagi istalgan boshlang'ich funksiyasi $F(x)+C$ ko'rinishda yozilishi mumkin.

Isbot. Shartga ko'ra, $F(x)$ funksiya $f(x)$ funksiyaning J oraliqda boshlang'ich funksiyasidir. Demak, istalgan $x\in J$ uchun $F'(x)=f(x)$. Shu sababli $(F(x)+C)'=F'(x)+C'=f(x)+0=f(x)$ bajariladi.

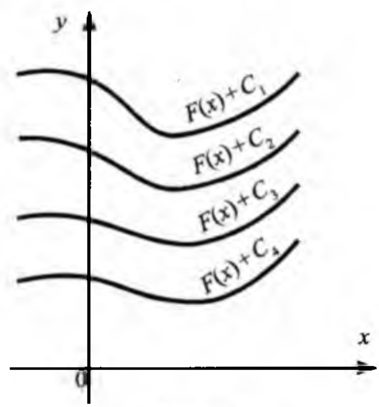
Demak, $F(x)+C$ funksiya $f(x)$ning boshlang'ich funksiyasi.

2) $g(x)$ funksiya $f(x)$ funksiyaning o'sha J oraliqdan olingan yana bitta boshlang'ich funksiyasi bo'lsin. Ya'ni, barcha $x\in J$ lar uchun $g'(x)=f(x)$ bo'lsin. U holda $(g(x)-F(x))'=g'(x)-F'(x)=f(x)-f(x)=0$.

Funksiyaning o'zgarmaslik alomatiga ko'ra, $g(x)-F(x)$ ayirma J oraliqda o'zgarmas funksiya ekani kelib chiqadi.

Demak, $g(x)=F(x)=C$ bo'lib, bundan $g(x)-F(x)=C$ kelib chiqadi.

Bulardan boshlang'ich funksiyalarning asosiy xossasiga geometrik ma'no beramiz: $f(x)$ funksiyaning istagan ikkita boshlang'ich funksiyasining grafiklari OY o'q bo'ylab ($y=C_n$ bo'lib), parallel ko'chirish natijasida hosil bo'ladi (48-chizma).



48-chizma.

1-misol. $\frac{1}{2\sqrt{x}}$ funksiya uchun grafigi (4; 3) nuqtadan o'tadigan boshlang'ich funksiyasini topamiz.

Yechish. $\frac{1}{2\sqrt{x}}$ ning istagan boshlang'ich funksiyasi $F(x)=\sqrt{x}+C$ ko'rinishda yoziladi. Bunda $x=4$; $y=F(x)=3$ bo'lganidan $3=\sqrt{4}+C$ $C=3-2=1$. Boshlang'ich funksiya $F(x)=\sqrt{x}+1$ topiladi.

$F(x)$ funksiya $f(x)$ funksiyaning qandaydir boshlang'ich funksiyasi bo'lsin, u holda $F(x)+C$ funksiya $f(x)$ funksiyaning umumiy ko'rinishdagi boshlang'ich funksiyasi bo'ladi.

Masalan, $\sin x+C$ ifoda $\cos x$ funksiya uchun umumiy ko'rinishdagi boshlang'ich funksiya bo'ladi.

Quyida darajali va ba'zi trigonometrik funksiyalarning boshlang'ich funksiyalari jadvali keltirilgan.

Funksiya $f(x)$	k (o'zgarmas)	x^n $(n\neq-1)$	$\sin x$	$\cos x$	$\frac{1}{\cos^2 x}$	$\frac{1}{\sin^2 x}$
Boshlang'ich funksiyaning umumiy ko'rinishi $F(x)+C$	$kx+C$	$\frac{x^{n+1}}{n+1}+C$	$-\cos x+C$	$\sin x+C$	$\mathrm{tg}x+C$	$-\mathrm{ctg}x+C$

2-misol. $\frac{1}{\sqrt[3]{x^2}}$ funksiyaning boshlang'ich funksiyalaridan birining grafigi (1; 2) nuqtadan, boshqasining grafigi esa (8; 4) nuqtadan o'tadi. Bulardan qaysi birining grafigi yuqoriroq joylashgan? Bu boshlang'ich funksiyalarning ayirmasi qanday?

Yechish. Bunda $f(x)=\frac{1}{\sqrt[3]{x^2}}=x^{-\frac{2}{3}}$ bo'lganidan unga boshlang'ich

funksiya: $F(x)=\frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}+C=\frac{x^{\frac{1}{3}}}{\frac{1}{3}}+C=3\sqrt[3]{x}+C.$ $F(x)=3\sqrt[3]{x}+C.$

1) (1; 2) nuqtadan o'tadigan boshlang'ich funksiyani topamiz: Bunda $x=1$; $y=F(x)=2$ bo'lganidan $2=3\cdot\sqrt[3]{1}+C_1$; $C_1=-1$. Boshlang'ich funksiya $F_1(x)=3\sqrt[3]{x}-1$ bo'ladi.

2) (8; 4) nuqtadan o'tadigan boshlang'ich funksiyani topamiz:

Bunda $x=8$, $y=F(x)=4$ bo‘lganidan $4=3\sqrt[3]{8}+C_2$; $C_2=4-6=-2$; Boshlang‘ich funksiya $F_2(x)=3\sqrt[3]{x}-2$ bo‘ladi.

OY o‘qida –1 soni –2 dan yuqorida turgani uchun $F_1(x)=3\sqrt[3]{x}-1$ funksiyaning grafigi $F_2(x)=3\sqrt[3]{x}-2$ funksiyaning grafigidan yuqorida joylashadi.

Bu funksiyalar ayirmasi $F_1(x)-F_2(x)=3\sqrt[3]{x}-1-(3\sqrt[3]{x}-2)=3\sqrt[3]{x}-$ $-1-3\sqrt[3]{x}+2=1$ bo‘ladi, ya’ni $F_1(x)-F_2(x)=1$.

TAKRORLASH UCHUN SAVOLLAR

1. Funksiyaning o‘zgarmasligi haqidagi lemmani ayting.
2. $F(x)$ boshlang‘ich funksiya haqidagi teoremani ayting.
3. $f(x)$ funksiyaning istalgan birinchi boshlang‘ich funksiyalarining grafiklari o‘zaro qanday joylashgan bo‘ladi?
4. Berilgan funksiyalarga boshlang‘ich funksiyani topish jadvalni yozib bering.

MASALALARNI YECHING

$f(x)$ funksiya uchun grafigi berilgan A nuqtadan o‘tuvchi boshlang‘ich funksiyani toping.

231. $f(x)=x^3$; $A(2;\ 1)$.

232. $f(x)=\dfrac{1}{\cos^2 x}$; $A\left(\dfrac{\pi}{4};\ 0\right)$.

233. $f(x)=\sin x$; $A(0;\ 3)$.

234. $f(x)=-2$; $A(3;\ 5)$.

235. $f(x)=\dfrac{1}{x^3}$; $A\left(-\dfrac{1}{2};\ 3\right)$.

236. $f(x)=\cos x$; $A\left(\dfrac{\pi}{2};\ 0\right)$.

237. $f(x)=-\dfrac{1}{x^2}$ funksiyaning boshlang‘ich funksiyalaridan birining grafigi (2; –4) nuqtadan o‘tadi, boshqasining grafigi (–3; 5) nuqtadan o‘tadi. Grafiklarning qaysi biri yuqoriroq joylashgan. Bu boshlang‘ich funksiyalarning yig‘indisi qanday?

$f(x)$ funksiyaning shunday $F(x)$ boshlang‘ich funksiyasini topingki, u berilgan qiymatni ko‘rsatilgan nuqtada qabul qilsin.

238 $f(x)=x^2$; $F(3)=0$.

239. $f(x)=-\frac{1}{x^2}$; $F(1)=-1$.

240. $f(x)=\sin x$; $F(\pi)=7$.

241. $f(x)=\frac{1}{\cos^2 x}$; $F\left(\frac{\pi}{4}\right)=-1$.

41-§. Boshlang‘ich funksiyalarni topishning uch qoidasi

Boshlang‘ich funksiyalarni izlash qoidalari differensiallashning tegishli qoidalariga o‘xshaydi.

1-teorema. **Agar $F(x)$ funksiya $f(x)$ning boshlang‘ich funksiyasi, $H(x)$ funksiya $h(x)$ning boshlang‘ich funksiyasi bo‘lsa, u holda $F(x)+H(x)$ yig‘indi $f(x)+h(x)$ning boshlang‘ich funksiyasi bo‘ladi.**

Haqiqatan, $F'(x)=f(x)$ va $H'(x)=h(x)$ bo‘lgani uchun yig‘indining hosilasini hisoblash qoidasiga binoan ushbuga ega bo‘lamiz: $(F(x)+H(x))'= =F'(x)+H'(x)=f(x)+h(x)$.

2-teorema. **Agar $F(x)$ funksiya $f(x)$ning boshlang‘ich funksiyasi, k esa o‘zgarmas bo‘lsa, $kF(x)$ funksiya $kf(x)$ning boshlang‘ich funksiyasi bo‘ladi.**

Haqiqatan, o‘zgarmas ko‘paytuvchini hosila belgisidan tashqariga chiqarish mumkin, shu sababli $(kF(x))'=kF'(x)=k\cdot f(x)$.

3-teorema. **Agar $F(x)$ funksiya $f(x)$ning boshlang‘ich funksiyasi, k va b o‘zgarmaslar (bunda $k\neq0$) bo‘lsa, $\frac{1}{k}F(kx+b)$ funksiya $f(kx+b)$ ning boshlang‘ich funksiyasi bo‘ladi.**

Haqiqatan, murakkab funksiyaning hosilasini hisoblash qoidasiga ko‘ra $\left(\frac{1}{k}F(kx+b)\right)'=\frac{1}{k}F'(kx+b)\cdot(kx+b)'=\frac{1}{k}f(kx+b)\cdot k=f(kx+b)$.

Teoremalarning qo‘llanishiga doir misollar keltiramiz.

1-misol. $f(x)=x^3+\frac{1}{x^2}$ funksiyaning boshlang‘ich funksiyalarining umumiy ko‘rinishini topamiz.

x^3 funksiyaning boshlang'ich funksiyalaridan biri $\frac{x^4}{4}$ ga teng, $\frac{1}{x^2}$ funksiyaning boshlang'ich funksiyalaridan biri esa $-\frac{1}{x}$ ga teng. 1 – teoremaga asosan $x^3+\frac{1}{x^2}$ funksiyaning boshlang'ich funksiyalaridan biri $\frac{x^4}{4}-\frac{1}{x}$ ga teng.

Javob: $F(x)=\frac{x^4}{4}-\frac{1}{x}+C.$

2-misol. $f(x)=7\cos x$ funksiyaning boshlang'ichlaridan birini topamiz.

$\cos x$ funksiyaning boshlang'ich funksiyalaridan biri $\sin x$ bo'lgani uchun 2-teoremaga asosan $f(x)=7\cos x$ ga boshlang'ich funksiyaning umumiy ko'rinishi $F(x)=7\sin x+C$ bo'ladi.

3-misol. $f(x)=\sin(5x+3)$ funksiyaning boshlang'ichlaridan birini topamiz.

$\sin x$ funksiyaning boshlang'ich funksiyalaridan biri $-\cos x$ bo'l-ganidan 3-teoremaga asosan izlanayotgan boshlang'ich funksiyalar-dan biri $-\frac{1}{5}\cos(5x+3)$ ga teng.

Javob: $F(x)=-\frac{1}{5}\cos(5x+3)+C.$

4-misol. $f(x)=\frac{1}{(7-3x)^5}$ funksiyaning boshlang'ich funksiyalaridan birini topamiz.

$\frac{1}{x^5}$ funksiyaning boshlang'ich funksiyasi

$-\frac{1}{4x^4}$ bo'lgani uchun 3-teoremaga asosan $\frac{1}{-3}\cdot\frac{-1}{4(7-3x)}=\frac{1}{12(7-3x)}.$

Javob: $F(x)=\frac{1}{12(7-3x)}+C.$

5-misol. $f(x)=x^3-5\sqrt{x}+\frac{7}{\cos^2 3x}$ funksiyaning boshlang'ich funksiyasini topamiz. x^3 ga boshlang'ich funksiya $\frac{x^4}{4}$ ga teng, $5\sqrt{x}$ ga boshlang‘ich funksiya $5\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}=\frac{10}{3}x\sqrt{x}$ ga teng va $\frac{7}{\cos^2 3x}$ ga boshlang‘ich funksiya $\frac{7}{3}\,\text{tg}3x$.

Javob: $F(x)=\frac{x^4}{4}-\frac{10}{3}x\sqrt{x}+\frac{7}{3}\,\text{tg}3x+C.$

TAKRORLASH UCHUN SAVOLLAR

1. Boshlang‘ich funksiyalarni topishga doir 1-teoremani ayting.
2. Boshlang‘ich funksiyalarni topishga doir 2-teoremani ayting.
3. Boshlang‘ich funksiyalarni topishga doir 3-teoremani ayting.
4. a) $f(x)=x$; b) $f(x)=\frac{1}{x}$; d) $f(x)=\sqrt{x}$ funksiyalarning boshlang‘ich funksiyalarini ayting.

MASALALARNI YECHING

Funksiyaning boshlang‘ich funksiyalarining umumiy ko‘rinishini toping:

242. $f(x)=5x^2-1.$

243. $f(x)=\frac{1}{x^2}-4\sin x.$

244. $f(x)=kx+b.$

245. $f(x)=ax^2+bx+c.$

246. $f(x)=1-\cos 3x.$

247. $f(x)= f(x)=\frac{2}{\sin^2 3x}.$

248. $f(x)=7\sin\frac{x}{3}+\frac{2}{\cos^2 4x}.$

249. $f(x)=\sqrt[3]{5x-2}.$

250. $f(x)=\frac{5}{\sqrt{2x+7}}.$

251. $f(x)=-\frac{3}{\cos^2 5x}+\frac{2}{\sqrt{2x}}.$

252. Yer sirtidan yuqoriga qaratib tosh otilgan. Havoning qarshiligini hisobga olmay va og‘irlik kuchining tezlanishi $g\approx 9{,}8$ ga teng deb hisoblab:

1) toshning boshlang‘ich tezligi v_0 ga bog‘liq holda eng katta ko‘tarilish balandligini toping;

2) toshning eng yuqori holatdagi tezligini toping;

3) qancha vaqtdan keyin tosh yerga tushadi?

Yechish. Yuqoriga otilgan toshning tezlanishi $a=-9{,}8$ bo‘ladi. $v'(t)=a=-9{,}8$ ekanidan $v(\mathrm{t})=-9{,}8t+C_1$. Bunda boshlang‘ich tezlik $v(0)=v_0$ bo‘lganidan $v_0=-9{,}8\cdot 0+C_1$; $C_1=v_0$.

$v(t)=-9{,}8t+v_0$ topiladi.

$h'(t)=v(t)$ ekanligidan, $h(t)=-\frac{9{,}8}{2}t^2+v_0t+C_2$

yoki $h(t)=-4{,}9t^2+v_0t+C_2$. $h(0)=0$ (boshlang‘ich balandlik) $0=-4{,}9\cdot 0^2+v_0\cdot 0+C_2$ dan $C_2=0$ bo‘ladi.

Bundan $h(t)=-4{,}9t^2+v_0t$ balandlik topiladi.

Eng katta balandlikni hosila yordamida topamiz: $h'(t)=-9{,}8t+v_0$ bo‘lib, kritik nuqta $-9{,}8t+v_0=0$; $t_0=\frac{v_0}{9.8}$.

Bu $t_0=\frac{v_0}{9.8}$ da balandlik eng katta qiymatga erishadi, ya’ni

$$\max h(t_0)=-4{,}9\cdot\left(\frac{v_0}{9{,}8}\right)^2+v_0\cdot\frac{v_0}{9{,}8}=-\frac{v_0^2}{19{,}6}+\frac{v^2}{9{,}8}=\frac{v^2}{19{,}6}.$$

2) Topishning eng yuqori holatdagi tezligi O bo‘ladi, ya’ni

$$v(t_0)=v\left(\frac{v_0}{9{,}8}\right)=-9{,}8\cdot\frac{v_0}{9{,}8}+v_0=-v_0+v_0=0.$$

3) Toshning yerga tushish vaqti $t=2t_0=2\cdot\frac{v_0}{9.8}=\frac{v_0}{4{,}9}$.

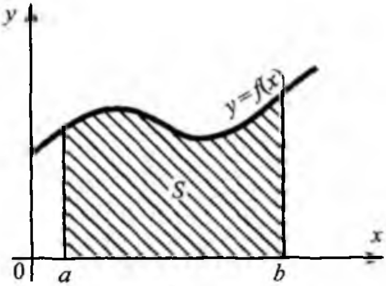
Javob: 1) $\frac{v_0^2}{19{,}6}$; 2) 0; 3) $\frac{v_0}{4{,}9}$.

253*. Agar nuqtaning tezligi $v=9{,}8t-0{,}003t^2$ qonun bo‘yicha o‘zgaradigan bo‘lsa, $t=0$ dan $t=5$ gacha bo‘lgan vaqt oralig‘ida nuqta o‘tgan yo‘lni toping. Bu nuqta yo‘lining oxiridagi (ya’ni $t=5$ dagi) tezlanishini toping.

254*. Harakatlanayotgan nuqtaning tezligi $v=Rt+a\sqrt{t}$ qonun bo‘yicha o‘zgaradi. Shu nuqtaning $t=0$ dan $t=4$ gacha bo‘lgan vaqt oralig‘ida o‘tgan yo‘lni, yo‘lning oxiridagi tezlanishini toping.

42-§. Egri chiziqli trapetsiyaning yuzi

$[a;\ b]$ kesmada ishorasini o‘zgartirmaydigan uzluksiz $f(x)$ funksiya berilgan bo‘lsin.



49-чизма.

Ta'rif. **$f(x)$ funksiyaning grafigi, OX o'qining $[a;\ b]$ kesmasi va $x=a$ hamda $x=b$ to'g'ri chiziqlar bilan chegaralangan figura egri chiziqli trapetsiya deyiladi** (49-chizma).

Egri chiziqli trapetsiyalarning yuzlarini hisoblash uchun ko'pincha quyidagi teorema qo'llaniladi.

Teorema. **$f(x)$ funksiya $[a;\ b]$ kesmada uzluksiz va musbat bo'lsin, S – tegishli egri chiziqli trapetsiyaning yuzi bo'lsin. Agar $F(x)$ funksiya $[a;\ b]$ kesmada $f(x)$ning boshlang'ich funksiyasi bo'lsa, u holda $S=F(b)-F(a)$.**

Isbot. $[a;\ b]$ kesmada aniqlangan $S(x)$ funksiyani qarab chiqamiz. Agar $x=a$ bo'lsa, u holda $S(a)=0$ bo'ladi. Agar $a<x\leq b$ bo'lsa OX o'qidagi vertikal x to'g'ri chiziqdan chapda joylashgan egri chiziqli trapetsiya qismining yuzi $S(x)$ bo'ladi (50-chizma).

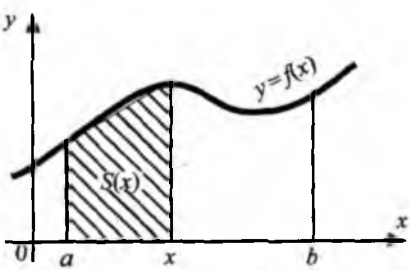
$S(a)=0$ bo'lganidan $S(b)=S$ bo'ladi, (S – egri chiziqli trapetsiyaning yuzi).

Avval biz $S'(x)=f(x)$ ekanligini isbotlaymiz.

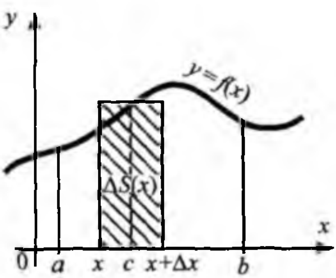
Hosila ta'rifiga ko'ra, $S'(x)=\lim\limits_{\Delta x\to 0}\dfrac{\Delta S(x)}{\Delta x}$ bo'ladi.

Bundagi $\Delta S(x)=S(x+\Delta x)-S(x)$.

Bu $\Delta S(x)$ning geometrik ma'nosi 51-chizmadagi shtrixlangan yuzadan iborat. $[x;\ x+\Delta x]$ da ixtiyoriy C nuqta olamiz va $f(c)$ topamiz.



50-chizma.



51-chizma.

Bu shtrixlangan $\Delta S(x)$ yuza $f(c)\cdot\Delta x$ ga teng, ya'ni $\Delta S(x)=f(c)\cdot\Delta x$.

Bu $\Delta S(x)$ ni hosiladagi limitga qo'yib, $S'(x)=\lim\limits_{\Delta x\to 0}\frac{\Delta S(x)}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{f(c)\cdot\Delta x}{\Delta x}=$

$=\lim\limits_{\Delta x\to 0} f(c)$.

C nuqta x va $x+\Delta x$ orasida yotgani uchun u $\Delta x\to 0$ da x ga intiladi, natijada $\lim\limits_{\Delta x\to 0} f(c)=f(x)$ tenglik bajariladi.

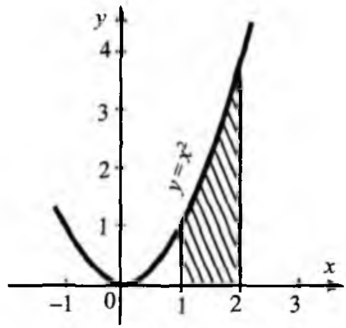
Demak, $S'(x)=f(x)$ kelib chiqadi.

Bundan $S(x)$ funksiya $f(x)$ funksiyaning boshlang'ich funksiyasi ekani topildi. Boshlang'ich funksiyani $[a;\ b]$ oraliqda topish qoidasiga ko'ra $S(x)=F(x)+C$ ni yozamiz (C – o'zgarmas son). Bunda $x=a$ deb olib, Cni topamiz: $S(a)=F(a)+C$ ($S(a)=0$ edi); $F(a)+C=0$; $C=-F(a)$.

Topilgan C ni topilgan boshlang'ich funksiyaga qo'yib, $S(x)=F(x)-F(a)$ ni hosil qilamiz.

Egri chiziqli trapetsiyaning yuzi $S(b)$ ga teng bo'lganidan x ning o'rniga b ni qo'yib, $S(b)=F(b)-F(a)$ formulani hosil qilamiz va $S(b)=S$ ekanligidan $\boldsymbol{S=F(b)-F(a)}$ hosil bo'ladi.

1-misol. $y=x^2$ funksiya grafigi bilan chegaralangan va $[1;\ 2]$ kesmaga tayangan egri chiziqli trapetsiyaning yuzini topamiz (52-chizma).



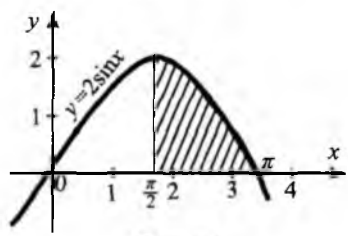
52-chizma.

Yechish. $y=x^2$ funksiyaning boshlang'ich funksiyasi $F(x)=\frac{x^3}{3}$ bo'ladi. $S=F(b)-F(a)$ formulaga ko'ra

$S=\frac{2^3}{3}-\frac{1^3}{3}=\frac{8}{3}-\frac{1}{3}=\frac{7}{3}=2\frac{1}{3}$ (kv.bir).

Javob: $S=2\frac{1}{3}$ kv. birlik.

2-misol. $y=2\sin x$, $y=0$ va $\frac{\pi}{2}\le x\le\pi$ chiziqlar bilan chegaralangan figuraning yuzini toping (53-chizma).



53-чизма.

Yechish. $y=2\sin x$ funksiyaning boshlang'ich funksiyasi $F(x)=-2\cos x$;

$$S=-2\left(\cos\pi-\cos\frac{\pi}{2}\right)=-2\cdot(-1-0)=2.$$

Javob: $S=2$ kv. birlik.

TAKRORLASH UCHUN SAVOLLAR

1. Qanday figurani egri chiziqli trapetsiya deyiladi?
2. Egri chiziqli trapetsiyaning yuzini topadigan teoremani ayting.
3. $[a; b]$ oraliqda uzluksiz bo'lgan $f(x)$ funksiyaning eng katta qiymati qanday topiladi?
4. *Eng kichik qiymati qanday topiladi?*

MASALALARNI YECHING

Quyidagi chiziqlar bilan chegaralangan figuraning yuzini toping:

255. $y=x^3$; $y=0$; $0\le x\le 2$.

256. $y=\cos x$; $y=0$; $x=0$; $x=\frac{\pi}{2}$.

257. $y=3\sin x$; $y=0$; $0\le x\le \pi$.

258. $y=\frac{1}{x^2}$; $y=0$; $x=1$; $x=2$.

259. $y=2x-x^2$; $y=0$.

260. $y=(x+2)^2$; $y=0$; $x=0$.

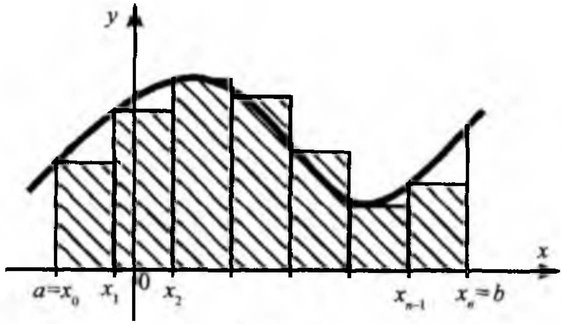
43-§. Integral. Nyuton–Leybnits formulasi

Egri chiziqli trapetsiya yuzini hisoblashning boshqacha usuli ham bor, bu paragrafda biz shu usulni ko'rib chiqamiz.

Soddalik uchun $f(x)$ funksiyani $[a; b]$ kesmada musbat va uzluksiz deb hisoblaymiz, u holda tegishli egri chiziqli trapetsiyaning yuzini taqriban quyidagicha hisoblash mumkin.

$[a; b]$ kesmani bir xil uzunlikdagi n ta kesmaga $x_0=a<x_1<x_2<...<$ $<x_{n-1}<x_n=b$ nuqtalar yordamida bo'lamiz va $\Delta x=\frac{b-a}{n}=x_k-x_{k-1}$ bo'lsin, bunda $k=1, 2, ..., n-1, n$.

$[x_{k-1}; x_k]$ kesmalarning har birida, shu kesmani asos qilib, balandligi $f(x_{k-1})$ga teng bo'lgan to'g'ri to'rtburchak yasaymiz. Bu to'g'ri to'rtburchakning yuzi $S_k=f(x_{k-1})\cdot\Delta x=\frac{b-a}{n}f(x_{k-1})$ ga teng, bunday to'g'ri to'rtburchaklar yuzlarining yig'indisi esa ushbuga teng (54-chizma).



54-chizma.

$$\frac{b-a}{n}(f(x_0)+f(x_1)+\ldots+f(x_{n-1})).$$

Bu yig'indini $\Sigma_n(a;\, b)$ kabi belgilaymiz, ya'ni

$$\Sigma_n(a;\, b)=\frac{b-a}{n}(f(x_0)+f(x_1)+\ldots+f(x_{n-1})).$$

Σ – grek harfi «sigma» deb o'qiladi. $f(x)$ funksiyaning uzluksizligi sababli yasalgan to'g'ri to'rtburchaklar yig'indisi n kattalashgan sari yoki Δx kichiklashganda egri chiziqli trapetsiya bilan deyarli ustma-ust tushadi. Bunda:

$\lim\limits_{n\to\infty}\Sigma_n(a;\, b)=S$ hosil bo'ladi.

Bu limit $f(x)$ funksiyaning grafigi bilan chegaralangan va $[a;\, b]$ kesmaga tayanuvchi egri chiziqli trapetsiyaning yuzi bo'ladi. Bu limitni boshqacha nom bilan **«$f(x)$ funksiyaning a dan b gacha olingan integrali»** deyiladi va u $\int\limits_a^b f(x)dx$ bilan belgilanadi.

Bu integral quyidagicha «a dan b gacha integral ef iks de iks» kabi o'qiladi. $\int$ – integral belgisi, $f(x)$ – integral ostidagi funksiya, x – integrallash o'zgaruvchisi. Shunday qilib, $[a;\, b]$ kesmada $f(x)\geq 0$ bo'lsa, $S=\lim\limits_{n\to\infty}\Sigma_n(a;\, b)=\int\limits_a^b f(x)dx$ formulaga ega bo'ldik. Bu formula bilan avvalgi 42-§ dagi egri chiziqli trapetsiyaning yuzi $S=F(b)-F(a)$ solishtirib, $\boldsymbol{S=\int\limits_a^b f(x)dx=F(b)-F(a)}$ formulaga ega bo'lamiz.

Bu formulani **Nyuton–Leybnits formulasi** deyiladi.

Eslatma. Agar $a \geq b$ bo‘lsa, integralni $\int_a^b f(x)dx = -\int_b^a f(x)dx$ yozish mumkin.

Ya’ni $\int_a^b f(x)dx = F(b) - F(a) = -(F(a) - F(b)) = -\int_a^b f(x)dx.$

1-misol. $\int_{-1}^{2} x^2 dx$ ni hisoblaymiz.

Yechish. x^2 ga boshlang‘ich funksiyaning topamiz. $F(x) = \frac{x^3}{3}$ bo‘ladi. $\int_{-1}^{2} x^2 dx = \frac{x^3}{3}\Big/_{-1}^{2} = \frac{2^3}{3} - \frac{(-1)^3}{3} = \frac{8}{3} + \frac{1}{3} = 3.$ Misollarni hisoblashda yozuvni qulaylashtirish uchun $\boldsymbol{F(b) - F(a) = F(x)\Big/_{a}^{b}}$ yozuv bilan almashtiriladi.

Bu belgilashdan foydalanib, Nyuton–Leybnits formulasini $\boldsymbol{\int_a^b f(x)dx = F(x)\Big/_{a}^{b}}$ ko‘rinishda yoziladi.

2-misol. Kiritilgan belgilashlardan foydalanib, integralni hisoblaymiz.

$$\int_0^{\pi} \sin x dx = -\cos x\Big/_0^{\pi} = -(\cos\pi - \cos 0) = -(-1-1) = 2.$$

3-misol. $\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$ ekanligini isbotlang.

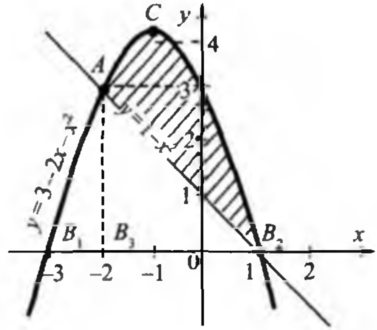
Isbot. $f(x)$ ning boshlang‘ich funksiyasi $F(x)$, $g(x)$ ning boshlang‘ich funksiyasi $G(x)$ bo‘lsin. U holda $F(x)+G(x)$ funksiya $f(x)+g(x)$ ning boshlang‘ich funksiyasi bo‘ladi. Shuning uchun

$$\int_a^b (f(x) + g(x))dx = \left(F(x) + G(x)\right)\Big/_a^b = F(b) + G(b) - F(a) - G(a) =$$

$$) = F(b) - F(a) + G(b) - G(a) = \int_a^b f(x)dx + \int_a^b g(x)d.$$

4-misol. $y=1-x$ va $y=3-2x-x^2$ chiziqlar bilan chegaralangan figuraning yuzini hisoblaymiz.

Yechish. Bu funksiyalarning grafiklarini yasaymiz: 1) $y=1-x$ to'g'ri chiziq bo'lib, u grafiklarning kesishgan nuqtasidan o'tadi. $1-x=3-2x-x^2$; $x^2+x-2=0$. Tenglamani yechib $x_1=-2$ va $x_2=1$ absissalar topiladi. $y_1=1-(-2)=3$; $y_2=1-1=0$. Kesishish nuqtalari (–2; 3) va (1; 0) (55-chizma).



55-chizma.

2) $y=3-2x-x^2$ parabolaning uchi $x_0=-\frac{b}{2a}=-\frac{-2}{-1\cdot 2}=-1$.

$y_0=3-2\cdot(-1)-(-1)^2=4$

Parabola uchi $C(-1;\ 4)$ OX o'qini kesadigan nuqtalari $-x^2-2x+3=0$. bundan $x_1=-3$; $x_2=1$. Kesish nuqtalari (–3; 0) va (1; 0).

Izlanayotgan yuza B_3ACB_2 egri chiziqli trapetsiya yuzidan ΔB_3AB_2 yuzasini ayirish orqali topiladi.

$$S_{B_3ACB_2}=\int_{-2}^{1}(3-2x-x^2)dx=\left(3x-x^2-\frac{x^3}{3}\right)\Big/_{-2}^{1}=$$

$$=3-1-\frac{1}{3}-\left(3\cdot(-2)-(-2)^2-\frac{(-2)^3}{3}\right)=1\frac{2}{3}-\left(-7\frac{1}{3}\right)=9.$$

$$S_{\Delta B_3AB_2}=\frac{1}{2}\cdot 3\cdot 3=4{,}5.$$

Demak, $S=S_{B_3ACB_2}-S_{\Delta B_3AB_2}=9-4{,}5=4{,}5$.

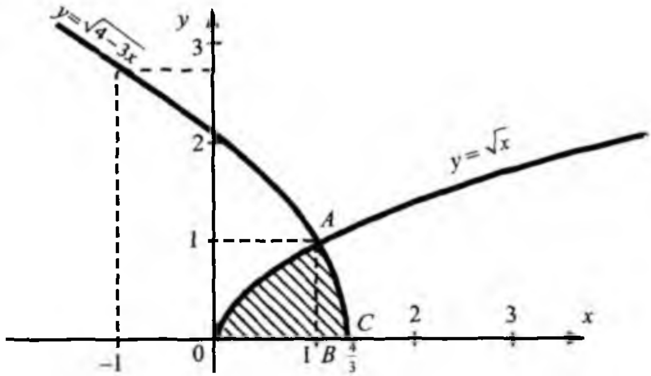
Javob: $S=4{,}5$ kv. birlik.

5-misol. $y=\sqrt{x}$ va $y=\sqrt{4-3x}$ chiziqlar bilan chegaralangan figuraning yuzini toping.

Yechish. Chiziqlarning umumiy nuqtasini topamiz: $\sqrt{x}=\sqrt{4-3x}$ (bunda $x\geq 0$); $x=4-3x$; $x=1$ va $4-3x\geq 0$; $x<\frac{4}{3}=1\frac{1}{3}$ $y=\sqrt{4-3x}$ grafigini chizamiz.

x	$\frac{4}{3}$	1	0,5	0	−1	−2
$\sqrt{4-3x}$	0	1	1,6	2	2,6	3,2

Bu jadval bo'yicha 56-chizmali grafikni chizamiz.



56-chizma.

Izlanayotgan yuza shtrixlab ko'rsatilgan. Bu yuzani egri chiziqli *OAB* va *ABC* uchburchaklar yuzalarining yig'indisiga teng.

OAB ning yuzi $S_1=\int_0^1\sqrt{x}dx=\frac{2}{3}x\sqrt{x}\Big/_0^1=\frac{2}{3}-0=\frac{2}{3}$;

ABC ning yuzi

$$S_2=\int_1^{\frac{4}{3}}\sqrt{4-3x'}dx=\frac{\frac{2}{3}(4-3x)\sqrt{4-3x}}{-3}\Big/_1^{\frac{4}{3}}=-\frac{2}{9}\left((4-4)\sqrt{4-4}-(1\cdot1)\right)=-\frac{2}{9}\cdot(-1)=\frac{2}{9};$$

$S=S_1+S_2=\frac{2}{3}+\frac{2}{9}=\frac{8}{9}$. *Javob:* $\frac{8}{9}$ kv. birlik.

6-misol. Integralni hisoblang: $\int_0^{28}\frac{7+x}{\sqrt[3]{1+\frac{x}{4}}}dx$

$\int_0^{28}\frac{7+x}{\sqrt[3]{1+\frac{x}{4}}}dx$ ni hisoblash uchun $7+x$ ni $1+\frac{x}{4}$ ifoda qatnashgan ifodaga keltiramiz $7+x=3+4\left(1+\frac{x}{4}\right)$.

$$\int_0^{28}\frac{7+x}{\sqrt[3]{1+\frac{x}{4}}}dx=\int_0^{28}\frac{3+4\left(1+\frac{x}{4}\right)}{\sqrt[3]{1+\frac{x}{4}}}dx=\int_0^{28}\frac{3}{\sqrt[3]{1+\frac{x}{4}}}dx+\int_0^{28}\frac{4\left(1+\frac{x}{4}\right)}{\sqrt[3]{1+\frac{x}{4}}}dx=$$

$$=\frac{3\left(1+\frac{x}{4}\right)^{\frac{2}{3}}}{\frac{2}{3}\cdot\frac{1}{4}}\Bigg/_0^{28}+4\int_0^{28}\left(1+\frac{x}{4}\right)^{\frac{2}{3}}dx=18\cdot\left(8^{\frac{2}{3}}-1\right)+\frac{4\left(1+\frac{x}{4}\right)^{\frac{5}{3}}}{\frac{5}{3}\cdot\frac{1}{4}}\Bigg/_0^{28}=$$

$$=54+\frac{48}{5}\cdot\left(8^{\frac{5}{3}}-1\right)=54+\frac{48}{5}\cdot 31=54+297,6=351,6.$$

7-misol. $\int_0^{\pi}16\sin 3x\cdot\cos 5x dx$ integralni hisoblang.

Yechish. $\sin 3x\cdot\cos 5x$ ifodani $\sin\alpha\cdot\cos\beta=\frac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta))$ formula bo‘yicha yig‘indi ko‘rinishga keltirib, so‘ngra integralni hisoblaymiz.

$$\int_0^{\pi}16\sin 3x\cdot\cos 5x dx=16\int_0^{\pi}\frac{1}{2}(\sin 8x-\sin 2x)dx=8\left(\int_0^{\pi}\sin 8x dx-\int_0^{\pi}\sin 2x dx\right)=$$

$$=8\cdot\left(\frac{-\cos 8x}{8}-\frac{-\cos 2x}{2}\right)\Bigg/_0^{\pi}=-8\left(\frac{\cos 8\pi}{8}-\frac{\cos 2\pi}{2}-\left(\frac{\cos 0}{8}-\frac{\cos 0}{2}\right)\right)=$$

$$=-8\left(\frac{1}{8}-\frac{1}{2}-\left(\frac{1}{8}-\frac{1}{2}\right)\right)=-8\cdot 0=0.$$

TAKRORLASH UCHUN SAVOLLAR

1. Egri chiziqli trapetsiyaning yuza qanday limit orqali topiladi?
2. $f(x)$ funksiyani a dan b gacha olingan integrali deb nimaga aytiladi va u qanday yoziladi?
3. Nyuton–Leybnits formulasini yozing.
4. a) $\int_2^5 c dx$ (c – o‘zgarmas son); $\int_{-1}^{3} x dx$ integrallarni hisoblang.

261. $\int_{-1}^{1} x^4 dx.$ **262.** $\int_{0}^{\frac{\pi}{2}} \cos x dx.$ **263.** $\int_{0}^{\frac{\pi}{4}} \frac{dx}{\cos^2 x}.$

264. $\int_{2}^{\frac{1}{3}} \frac{dx}{x^2}.$ **265.** $\int_{\frac{1}{2}}^{1} \frac{dx}{x^3}.$ **266.** $\int_{0,008}^{1} \sqrt[3]{x} dx.$

267. $\int_{1}^{4} \frac{dx}{\sqrt{x}}.$ **268.** $\int_{1}^{8} \frac{dx}{\sqrt[3]{x^2}}.$ **269.** $\int_{0}^{\frac{\pi}{12}} \sin x \cos x dx.$

Quyidagi chiziqlar bilan tasvirlangan figuraning yuzini hisoblang (avval chizmasini chizib oling).

270. $y=x^3;\ x=1;\ x=3;\ y=0.$

271. $y=2+x-x^2;\ y=0.$

272. $y=\sqrt{x};\ y=0;\ x=4.$

273. $y=x^2;\ y=2x.$

274. $y=x^2;\ y=\sqrt[3]{x}.$

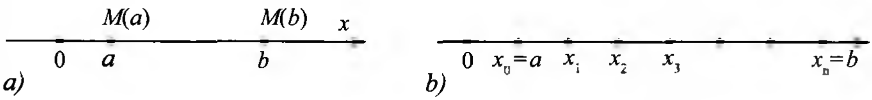
275. Agar $f(x)$ funksiya $[a;\ b]$ kesmada uzluksiz bo‘lsa va $f(x)\leq 0$ ekanligidan $\int_{a}^{b} f(x)dx=-S$ bo‘lishini isbotlang.

276 $\int_{a}^{b} f(x)dx=\int_{a}^{c} f(x)dx+\int_{c}^{b} f(x)dx$ ekanligini isbotlang.

44-§. O‘zgaruvchi kuchning ishi

Moddiy nuqta P kuch ta’sirida to‘g‘ri chiziq bo‘ylab harakat qilsin. Agar moddiy nuqtaga ta’sir qiluvchi kuch o‘zgarmas, o‘tilgan yo‘l esa S ga teng bo‘lsa, u holda, fizikadan ma’lumki, bu kuch bajargan A ish, P kuch bilan o‘tilgan S yo‘lning ko‘paytmasiga teng, ya’ni $A=P\cdot S$. Endi o‘zgaruvchi kuch bajaradigan ishni hisoblash formulasini chiqaramiz.

Nuqta OX o'q bo'ylab OX o'qdagi proyeksiyasi x ning funksiyasi (bu funksiyani $f(x)$ bilan belgilaymiz) bo'lgan kuch ta'sirida harakat qilayotgan bo'lsin. Bunda biz $f(x)$ ni uzluksiz deb hisoblaymiz. Shu kuch ta'sirida moddiy nuqta $M(a)$ nuqtadan $M(b)$ nuqtaga o'tadi (57-*a*, chizma). Bunday holda A ish $\int_a^b f(x)dx$ formula bilan hisoblanishini ko'rsatamiz. $[a;\ b]$ kesmani uzunliklari bir xil $\Delta x = \frac{b-a}{n}$ ga teng n ta kesmaga bo'lamiz (57-*b*, chizma):



57-chizma.

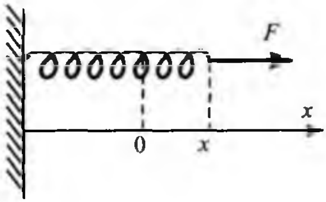
Kuchning butun $[a;\ b]$ kesmada bajargan ishi hosil bo'lgan kesmalarda shu kuch bajargan ishlar yig'indisiga teng. $f(x)$ funksiya x ning uzluksiz funksiyasi bo'lganidan yetarlicha kichik $[a;\ x_1]$ kesmada kuch bajargan ish taqriban $f(a)(x_1-a)$ga teng bo'ladi, ikkinchi $[x_1;\ x_2]$ kesmada bajargan ishi taqriban $f(x_1)(x_2-x_1)$ga teng va hokazo, kuchning n – kesmada bajargan ishi taqriban $f(x_{n-1})(b-x_{n-1})$ga teng. Kuchning butun $[a;\ b]$ kesmada bajargan ishi taqriban $A \approx A_n = f(a)\Delta x + f(x_1)\Delta x + \ldots +$ $+ f(x_{n-1}) \cdot \Delta x$. $[a;\ b]$ kesma bo'lingan kesmalar uzunligi qancha qisqa bo'lsa, taqribiy tenglikning aniqligi shuncha to'g'ri bo'ladi.

Tabiiyki, n cheksizlikka intilganda limitga o'tilsa, bu taqribiy tenglik aniq tenglikka o'tadi, ya'ni $A = \lim\limits_{n\to\infty} An = \lim\limits_{x\to\infty} \sum\limits_{n} (a;\ b) = \int_a^b f(x)dx$ (integral ta'rifiga asosan).

Demak, $A = \int_a^b f(x)dx.$

1-misol. 5 sm cho'zilgan prujinaning elastiklik kuchi 3H ga teng. Prujinani shu 5 sm ga cho'zish uchun qanday ish bajarish kerak?

Yechish. Guk qonuniga ko'ra prujinani x kattalikka cho'zuvchi F kuch $\boldsymbol{F=kx}$ formula bilan hisoblanadi, bunda k – o'zgarmas proporsionallik koeffitsiyenti (58-chizma).



58-чизма.

O nuqta prujinaning erkin holatiga to‘g‘ri keladi. Masalaning shartidan $3=k\cdot 0{,}05$, bundan $k=60$. Prujinaning cho‘zilishida $F=60x$ kuch sarflanadi. Bajarilgan ish $A=\int_0^{0,05} 60x dx = 30x^2 \Big/_0^{0,05} =$

$= 30\cdot 0{,}05^2 = 0{,}075$ j (1 j = 1 H · m).

Javob: 0,075 j.

TAKRORLASH UCHUN SAVOLLAR

1. Moddiy nuqtaga ta’sir qiluvchi kuch o‘zgarmas bo‘lganda bajarilgan ish qanday formula bilan topiladi?
2. Moddiy nuqtaga ta’sir qiluvchi kuch o‘zgaruvchan bo‘lganda bajarilgan ish qanday formula bilan topiladi?
3. Prujinani cho‘zilishi yoki siqilishidagi Guk qonuni nimadan iborat?

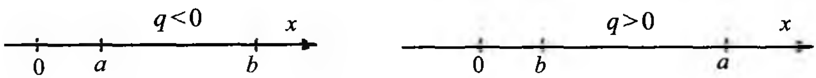
MASALALARNI YECHING

277. Agar 2 H kuch prujinani 1 sm qisishi ma’lum bo‘lsa, shu prujinani 4 sm qisish uchun qancha ish bajarish kerak?

278. 4 H kuch prujinani 8 sm cho‘zadi. Prujinani 6 sm cho‘zish uchun qanday ish bajarish kerak?

279. 6 H kuch prujinani 2 sm cho‘zadi. Prujinani 6 sm cho‘zish uchun qanday ish bajarish kerak?

280. O nuqtaga joylashgan q zaryad ta’sirida elektron a masofadan b masofaga to‘g‘ri chiziq bo‘ylab ko‘chadi. Zaryadlarning o‘zaro ta’siridagi bajarilgan ishni toping (59-chizma).



59-chizma.

(Kulon qonunini ifodalovchi formula – $\left(F=-\gamma\frac{q}{x^2}\right)$dagi proporsionallik koeffitsiyentini γ ga teng deb hisoblang).

45-§. Egri chiziqli trapetsiyani aylantirish natijasida hosil bo'lgan figuraning hajmi

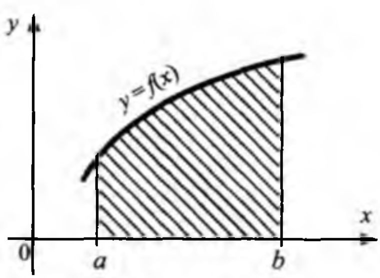
Tekislikdagi to'g'ri burchakli koordinatalar sistemasida uzluksiz musbat $f(x)$ funksiya, absissalar o'qi va $x=a$, $x=b$ $(a<b)$ to'g'ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyani ko'rib chiqamiz (60-chizma). Egri chiziqli trapetsiyani absissalar o'qi atrofida aylanishi natijasida Φ aylanish figurasi hosil bo'ladi (61-chizma). Φ figuraning absissalar o'qiga perpendikulyar tekislik bilan ixtiyoriy kesimi doira bo'ladi. Absissasi $x\in[a; b]$ bo'lgan nuqta orqali o'tkazilgan kesimini ko'rib chiqamiz. U ning $S(x)$ yuzi $\pi f^2(x)$ ga teng. Φ figuraning a va x nuqtalar orqali o'tkazilgan kesimlar orasidagi qismini $V(x)$ bilan belgilaymiz. $S(x)$ va $V(x)$ funksiyalar $[a; b]$ kesmada qaraladi.

$f(x)$ funksiya $[a; b]$ kesmada o'suvchi (yoki kamayuvchi) bo'ladi deb faraz qilib, Φ figuraning hajmi $V=\int_a^b S(x)dx$ formula bilan topilishini isbotlaymiz.

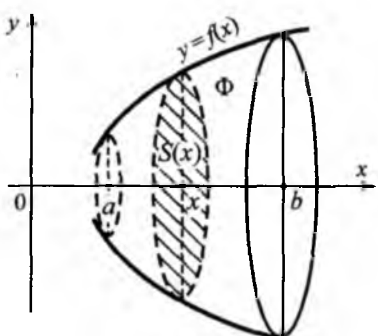
x ga $\Delta x>0(x+\Delta x\leq b)$ orttirma beramiz. Bu holda $V(x)$ hajm ΔV orttirmani oladi (62-chizma).

Umumiy balandligi Δx bo'lgan ikkita silindrni qaraymiz: ulardan birinchisi asosining yuzi $S(x)$ bo'lgan doira, ikkinchisining asosi esa yuzi $S(x+\Delta x)$ bo'lgan doiradan iborat. ΔV orttirmali hajm quyidagi tengsizlik bilan yoziladi, ya'ni $S(x)\Delta x\leq\Delta V\leq S(x+\Delta x)\cdot\Delta x$, bundan

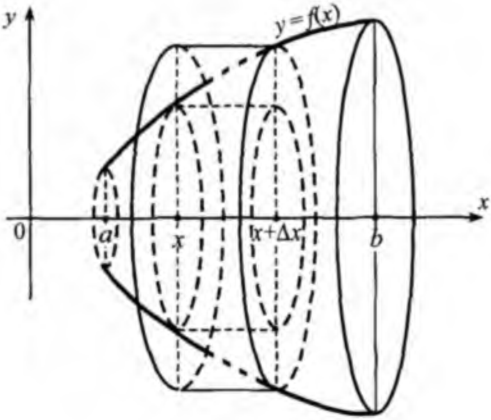
$$S(x)\leq\frac{\Delta V}{\Delta x}\leq S(x+\Delta x) \qquad (1).$$



60-chizma.



61-chizma.



62-chizma.

Biz $f(x)$ funksiya $[a;\ b]$ kesmada uzluksiz, deb shartlashdik, shuning uchun $S(x)=\pi f^2(x)$ funksiya ham uzluksiz bo‘ladi. Demak, $\lim\limits_{\Delta x\to 0} S(x+\Delta x)=S(x)$. U holda (1) tengsizlik limitidan $\lim\limits_{\Delta x\to 0}\frac{\Delta V}{\Delta x}=S(x)$ kelib chiqadi. Bu limitdan $V'(x)=S(x)$ni yozamiz. Shundan qilib, $V(x)$ hajm $S(x)$ kesimning boshlang‘ich funksiyasi bo‘ladi. Kesim $S(x)=\pi f^2(x)$ ni $V'(x)=S(x)$ga quyib, $V'(x)=\pi f^2(x)$ni hosil qilamiz.

Integral ta’rifiga asosan $V(x)=\int\limits_a^b \pi f^2(x)dx$ ni yozamiz. Bunda $V(a)=0$; $V(b)=V$ bo‘lgani uchun

$$V=\pi\int\limits_a^b f^2(x)dx \qquad (2).$$

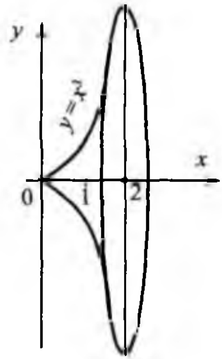
Shunday qilib, **egri chiziqli trapetsiyani absissa o‘qi atrofida aylantirish natijasida hosil bo‘lgan figuraning hajmi (2) formula bo‘yicha topiladi.**

Masala. Chegaralari $y=x^3$, $x=2$, $y=0$ tenglamalar bilan berilgan egri chiziqli trapetsiyani OX o‘qi atrofida aylantirishdan hosil bo‘lgan figuraning hajmini toping.

Yechish. Hosil bo‘lgan figuraning hajmi $V=\pi\int\limits_0^2 y^2dx=\pi\int\limits_0^2 (x^3)^2dx=$ $=\pi\int\limits_0^2 x^6dx=\frac{\pi x^7}{7}\Big/_0^2=\frac{\pi\cdot 2^7}{7}-0=\frac{128\pi}{7}$. *Javob:* $V=\frac{128\pi}{7}$.

TAKRORLASH UCHUN SAVOLLAR

1. Egri chiziqli trapetsiyani OX o'qi atrofida aylantirilganda qanday figura hosil bo'ladi?
2. Absissasi x nuqtadan o'tuvchi perpendikulyar kesim nimaga teng ($S(x)$ nimaga teng)?
3. ΔV (hajm orttirmasi) nimalar oralig'ida?
4. $\lim\limits_{\Delta x \to 0} \frac{\Delta V}{\Delta x}$ nimaga teng?
5. Egri chiziqli trapetsiyani OX o'q atrofida aylantirishdan hosil bo'lgan figuraning hajmi nimaga teng?



63-чизма.

MASALALARNI YECHING

281. $y=2x+1$, $x=1$, $x=4$, $y=0$ chiziqlar bilan chegaralangan figurani OX o'q atrofida aylanishidan hosil bo'lgan hajmni toping.

282. $y=x^2$, $x=1$, $y=0$ bilan chegaralangan figurani OX o'q atrofida aylanishidan hosil bo'lgan hajmni toping.

283. $y=2x-x^2$, $y=0$ bilan chegaralangan figurani OX o'q atrofida aylanishidan hosil bo'lgan hajmni toping.

284. $y=1+x^3$, $x=2$, $y=0$ bilan chegaralangan figurani OX o'q atrofida aylanishidan hosil bo'lgan hajmni toping.

285. $y=x^2+1$ va $y=2$ chiziqlar bilan chegaralangan figuraning OX o'q atrofida aylanishidan hosil bo'lgan hajmni toping.

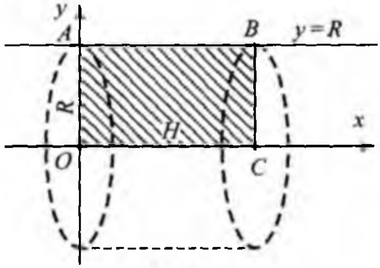
46-§. Egri chiziqli trapetsiyani aylantirish natijasida hosil bo'lgan silindr, konus va kesik konuslarning hajmi

1. Silindrning hajmi

1-teorema. **Silindrning hajmi asosining yuzi bilan balandligi ko'paytmasiga teng.**

Isbot qilish kerak: $V_s=\pi R^2 H$.

Isbot. Koordinatalar sistemasida bir uchi koordinatalar boshida, asosi OX o'qda bo'lgan $OABC$ to'g'ri to'rtburchakni olamiz. Bu to'g'ri to'rtburchak OX o'q atrofida aylanib, asos radiusi R va balandligi H bo'lgan silindrni hosil qiladi. Bunda $f(x)=y=R$ dan iborat (64-chizma).



64-chizma.

$V=\pi\int_a^b f^2(x)dx$ formulaga asosan

$V_s=\pi\int_a^H y^2dx=\pi R^2 x\Big|_0^H=\pi R^2H.$

Demak, $\boldsymbol{V_s=\pi R^2H}$

(bunda πR^2 – silindr asosining yuzi).

1-masala. Po'lat valning uzunligi 97 sm va diametri 8,4 sm bo'lib, uni yo'nish natijasida diametri 0,20 sm kamaygan. Yo'nish natijasida valning massasi qanchaga kamaygan? (Po'latning zichligi $\rho=7{,}4$ g/sm³).

Berilgan: Valning uzunligi $H=97$ sm, $D_1=8{,}4$ sm ($R_1=4{,}2$ sm), $D_2=8{,}4-0{,}2=8{,}2$($R_2=4{,}1$ sm), $\rho_{\text{po'lat}}=7{,}4$ g/sm³.

Yechish. Po'lat valni yo'nilmasdan oldingi hajmi $V_{s1}=\pi R_1^2H=\pi\cdot$ $\cdot 4{,}2^2\cdot 97=\pi\cdot 17{,}64\cdot 97=1711{,}08$ π. Po'lat valning yo'nilgandan keyingi hajmi $V_{s2}=\pi R_2H=\pi\cdot 4{,}1^2\cdot 97=\pi\cdot 16{,}81\cdot 97=1630{,}57$ π. Yo'nish natijasida valning kamaygan hajmi: $V=V_{s1}-V_{s2}=1711{,}08\pi-1630{,}57\pi=$ $=80{,}51\pi=252{,}8$ (sm³). Og'irlik $P=V\cdot\rho$ formula bo'yicha topiladi, ya'ni $P=252{,}8\cdot 7{,}4=1870$ g $\approx 1{,}9$ kg. *Javob:* 1,9 kg kamaygan.

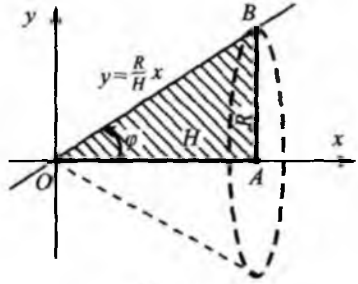
2. Konusning hajmi

2-teorema. **Konusning hajmi asosining yuzi bilan balandligi ko'paytmasining uchdan biriga teng.**

Isbot qilish kerak: $V_k=\frac{1}{3}\pi R^2H.$

Isbot. To'g'ri burchakli koordinatalar sistemasida to'g'ri burchakli $OAB(\angle A=90°)$ uchburchakli OA kateti atrofida aylantirish natijasida konus hosil qilingan bo'lsin (65-chizma). Bunda $AB=R$, $OA=H$ va OB to'g'ri chiziqning tenglamasi $y=kx$ ko'rinishda bo'ladi. Bu to'g'ri chiziq OX o'qi bilan φ burchak hosil qilsin. $y=kx$ dagi

$k = \text{tg}\,\varphi = \frac{R}{H}$ bo‘ladi. U holda OB to‘g‘ri chiziq $y = \frac{R}{H}x$ ko‘rinishda bo‘ladi.



65-чизма.

ΔOBA egri chiziqli trapetsiyaning xususiy holi bo‘ladi (bu OX o‘qi, $y = \frac{R}{H}x$ funksiya grafigi va $x = H$ to‘g‘ri chiziq bilan chegaralangan. Konus hajmini $V = \pi\int_a^b f^2(x)dx$ formula bo‘yicha topamiz.

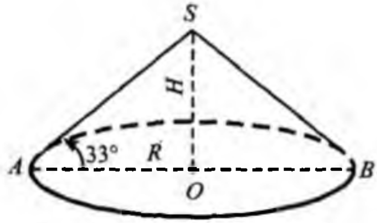
$$V_k = \pi\int_0^H \left(\frac{R}{H}x\right)^2 dx = \pi\frac{R^2}{H^2}\int_0^H x^2 dx = \pi\frac{R^2}{H^2}\cdot\frac{x^3}{3}\Big/_0^H = \frac{\pi R^2 H}{3}$$

Demak, $\boldsymbol{V_k = \frac{1}{3}\pi R^2 H}$.

2-masala. Shag‘al qiyalik burchagi 33° bo‘lgan konus shaklida to‘kilgan. Uyumning hajmi 10 m^3 bo‘lishi uchun uning balandligi qancha bo‘lishi kerak?

Berilgan. SAB – konus,
$\angle OAS = 33°$.
$V_k = 10$ m^3.

Topish kerak: $SO = H$ (balandlik) (66-chizma).



66-chizma.

Yechish. $V_k = \frac{1}{3}\pi R^2 H$ formuladan H ni topishimiz kerak.

ΔAOS dan $AO = SO\cdot\text{ctg}33°$; $AO = R = H\cdot 1{,}54 = 1{,}54\,H$.

Bu topilganlarni $V_k = \frac{1}{3}\pi R^2 H$ ga qo‘yamiz:

$10 = \frac{1}{3}\pi\cdot(1{,}54H)^2\cdot H$; $10 = \frac{1}{3}\cdot 3{,}14\cdot 2{,}37H^3$

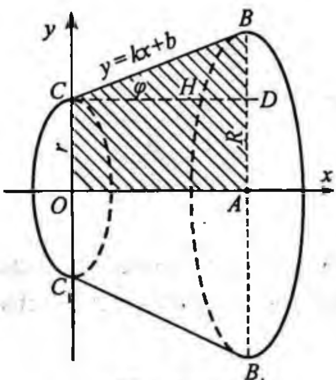
$10=2{,}48H^3$; $H^3=\frac{10}{2{,}48}=4{,}03$; $H^3=4{,}03$ buni lg · lab H ni topamiz.

$\lg H^3=4{,}03$; $3\lg H=0{,}6053$; $\lg H=0{,}2017$; $H\approx1{,}592$ (m).

Javob: $H\approx1{,}6$ m.

3. Kesik konusning hajmi

3-teorema. **Kesik konusning hajmi $V_{k.k}=\frac{1}{3}\pi H(R^2+Rr+r^2)$ formula bo'yicha topiladi, bunda: R – ostki, r – ustki asoslarining radiuslari, H – konus balandligi, $\pi\approx3{,}14$ – doimiy son.**



67-a, чизма.

Isbot. $OABC$ ($\angle O=\angle A=90°$) to'g'ri burchakli trapetsiyani OA tomoni atrofida aylantirishda kesik konus hosil qilingan bo'lsin (67-*a*, chizma). Ushbu $OA=H$, $AB=R$, $OC=r$ belgilashni qabul qilamiz. Trapetsiya tekisligida boshi O va absissalar o'qi OA bo'lgan to'g'ri burchakli koordinatalar sistemasini kiritamiz.

CB to'g'ri chiziqning tenglamasi $y=kx+b$ ko'rinishda bo'ladi, bunda $b=r$, $BD=R-r$, $k=\operatorname{tg}\varphi(\angle DCB=\varphi)=$, ya'ni hosil bo'ladi. Aylanish figuralar hajmini topish formulasiga ko'ra

$$V=\pi\int_0^H\left(\frac{R-r}{H}x+r\right)^2dx=\pi\int_0^H\frac{(R-r)^2}{H^2}x^2+2\cdot\frac{R-r}{H}rx+r^2)dx=$$

$$=\pi\left(\frac{(R-r)^2}{H^2}\cdot\frac{x^3}{3}+2r\cdot\frac{R-r}{H}\cdot\frac{x^2}{2}+r^2x\right)\Big/_0^H=$$

$$=\pi\left(\frac{(R-r)^2}{H^2}\cdot\frac{H^3}{3}+2r\cdot\frac{R-r}{H}\cdot\frac{H^2}{2}+r^2H\right)=$$

$$=\frac{\pi H}{3}((R-r)^2+3r(R-r)+3r^2)=\frac{1}{3}\pi H(R^2+Rr+r^2).$$

Demak, $V_{k.k}=\frac{1}{3}\pi H(R^2+Rr+r^2)$

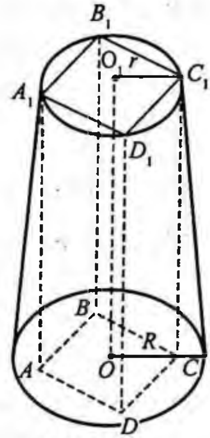
3-masala. Xoda uchlarining diametrlari 32 sm va 26 sm, uning uzunligi 5,3 m. Shu g'o'ladan eng katta kvadrat kesimli to'sin tayyorlangan. To'sinning hajmi xoda hajmining necha protsentini tashkil qiladi?

Berilgan: Uzunligi 5,3 m xoda ($OO_1 = H$),

Katta asos diametri 32 sm$=2R$.

Ustki asos diametri 26 sm$=2r$.

Xodadan kesimi kvadrat bo'lgan to'sin olingan (67-*b*, chizma).



67-b, chizma.

Topish kerak: $\frac{V_{\text{to'sin}}}{V_{\text{xoda}}} \cdot 100\%$ ekanligini.

Yechish. $V_{\text{to'sin}} = S_{ABCD} \cdot H$ dagi S_{ABCD} ni topamiz. $A_1B_1C_1D_1$ ichki chizilgan muntazam to'rtburchakning

$a_4 = \sqrt{2}r = \sqrt{2} \cdot 13 \approx 1{,}41 \cdot 13 \approx 18{,}3$ (sm).

$S_{ABCD} = a^4_2 = 18{,}3^2 \approx 334{,}9$ (sm^2).

$V_{\text{to'sin}} = S_{ABCD} \cdot H = 334{,}9 \cdot 530 = 177497$ (sm^3).

$$V_{\text{xoda}} = V_{\text{k.k.}} = \frac{1}{3}\pi H(R^2 + Rr + r^2) = \frac{1}{3}\pi \cdot 530(16^2 + 16 \cdot 13 + 13^2) =$$

$$= \frac{1}{3} \cdot 3{,}14 \cdot 530\,(256 + 208 + 169) \approx 351146 \text{ (sm}^3\text{)}.$$

$\frac{V_{\text{to'sin}}}{V_{\text{xoda}}} = \frac{17749{,}7}{351146} \cdot 100\% \approx 50{,}55\%.$ *Javob:* 50,55%.

TAKRORLASH UCHUN SAVOLLAR

1. Silindrning hajmi nimaga teng?
2. Konusning hajmi nimaga teng?
3. Kesik konusning hajmi nimaga teng (formulasini yozing)?
4. Egri chiziqli trapetsiyani *OX* o'qi atrofida aylantirish natijasida hosil bo'lgan figuraning hajmini topish formulasini yozing.

MASALALARNI YECHING

286. Yog'ochdan yasalgan silindr asosining diametri 1,6 m, uzunligi 6 m bo'lsa, uning og'irligini toping (yog'ochning solishtirma og'irligi 0,8 $\frac{\text{g}}{\text{sm}^3}$).

287. 25 m mis simning og'irligi 100,7 g. Simning diametrini toping (misning solishtirma og'irligi 8,9 $\frac{\text{g}}{\text{sm}^3}$).

288. Termometrdagi simob ustunining uzunligi 15,6 sm bo‘lib, og‘irligi 5,2 g keladi (simobning solishtirma og‘irligi 13,6 $\frac{g}{sm^3}$). Simob ustuni ko‘ndalang kesimining yuzini toping.

289. Silindrning o‘q kesimi – kvadrat bo‘lib, uning diagonali 4 dm ga teng. Silindrning hajmini toping.

290. Silindrning yon sirti S, asos aylanasining uzunligi C. Hajmini toping.

291. Konusning balandligi 3 sm, yasovchisi 5 sm. Hajmini toping.

292. 122 millimetrli bomba yorilganda diametri 4 m va chuqurligi 1,5 m li konus shaklidagi chuqur yasaydi. Shu bomba qancha miqdordagi tuproqni qo‘porib tashlaydi? (1 m^3 tuproq – tuproqning og‘irligi 1650 kg).

293. Konusning o‘q kesimi teng yonli to‘g‘ri burchakli uchburchakdan iborat, uning yuzi 9 m^2. Konusning hajmini toping.

294. Uchburchakning asosi b, balandligi h. Uchburchakning o‘z asosi atrofida aylanishidan hosil bo‘lgan jismning hajmini toping.

295. Tomonlari 10 sm, 17 sm va 21 sm li uchburchak katta tomoni tevaragida aylanadi. Hosil bo‘lgan jismning hajmini va sirtini aniqlang.

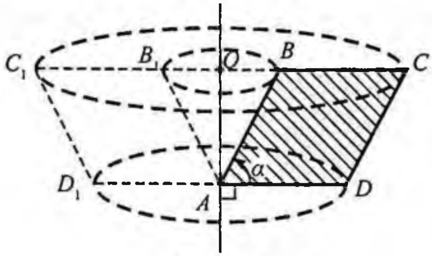
296. Kesik konus shaklidagi jism asoslarining diametrlari 6 sm va 10 sm, balandligi 12 sm bo‘lsa, uning hajmini toping.

297. Qarag‘aydan kesilgan 15,5 m uzunlikdagi to‘sin uchlarining diametrlari: $d_1=42$ sm, $d_2=25$ sm. To‘sinning hajmini, o‘rta ko‘ndalang kesimining yuzini uning uzunligiga ko‘paytirish bilan hisoblaganimizdagi qilinadigan xatoning protsentini aniqlang.

298. Kesik konus asoslarining radiuslari 1 dm va 9 dm, yasovchisi 1 m. Hajmini toping.

299*. Kesik konus asoslarining radiuslari R va r ($R>r$). Kesik konus hajmining shu kesik konus bir qismi bo‘lgan butun konus hajmiga nisbatini toping.

300*. Tomoni a va o‘tkir burchagi α bo‘lgan romb o‘tkir burchagi uchidan o‘tib, rombning tomoniga perpendikulyar bo‘lgan o‘q atrofida aylanadi. Aylanish figurasining hajmini toping.



68-chizma.

Ko‘rsatma. Aylanma jismning hajmini DCC_1D_1 o‘q kesimli kesik konus hajmidan ΔABB_1 o‘q kesimli konus hajmini ayirish bilan topiladi (68-chizma).

1) ΔABB_1 o‘q kesimli konus hajmi topiladi.

$OB = a \cdot \sin(90° - \alpha) = a\cos\alpha.$

$AO = a\cos(90° - \alpha) = a\sin\alpha.$

$$V_k = \frac{1}{3}\pi \cdot (OB)^2 \cdot AO = \frac{1}{3}\pi \cdot (a\cos\alpha)^2 \times a\sin\alpha = \frac{1}{3}\pi a^3 \sin\alpha \cdot \cos^2\alpha.$$

2) Kesik piramidada: $R = OB + BC = a + a\cos\alpha.$

$r = a;\ H = AO = a\sin\alpha.$

$V_{k.k} = \frac{1}{3}\pi \cdot H(R^2 + r^2 + Rr)$ topiladi.

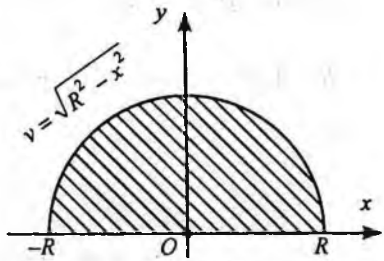
3) $V = V_{k.k} - V_k$ topiladi.

47-§. Shar va shar bo‘laklarining hajmi

1. Sharning hajmi

1-teorema. **R radiusli sharning hajmi $V = \frac{4}{3}\pi R^3$ formula bo‘yicha hisoblanadi.**

Isbot. To‘g‘ri burchakli koordinatalar sistemasida markazi O nuqtada radiusi R bo‘lgan yarim doirani olamiz. Markazi O nuqtada radiusi R bo‘lgan aylana tenglamasi $x^2 + y^2 = R^2$ dan $y = \sqrt{R^2 - x^2}$ ni olamiz. Bu yarim doira egri chiziqli trapetsiyaning xususiy holi



69-чизма.

bo'ladi. U absissalar o'qi va $y=\sqrt{R^2-x^2}$ funksiyaning grafigi bilan chegaralangan (69-chizma).

Berilgan yarim doiraning absissalar o'qi atrofida aylanishi natijasida shar hosil bo'ladi. Aylanish figuraning formulasidan foydalanib uning hajmini topamiz:

$$V=\pi\int_a^b y^2dx=\pi\int_{-R}^{R}(R^2-x^2)dx=\pi\left(R^2x-\frac{x^3}{3}\right)\Big/_{-R}^{R}=$$

$$=\pi\left(R^3-\frac{R^3}{3}-\left(-R^3+\frac{R^3}{3}\right)\right)=\pi\left(2R^3-\frac{2}{3}R^3\right)=\frac{4}{3}\pi R^3.$$

Demak, $V=\frac{4}{3}\pi R^3$.

1-masala. Temir sharning massasi 4 kg. Uning diametrini toping (temirning zichligi 7,9 $\frac{\text{g}}{\text{sm}^3}$).

Berilgan: Temir sharning massasi 4 kg,

Zichligi $\rho=7,9\ \frac{\text{g}}{\text{sm}^3}$.

Topish kerak: $d=2R$.

Yechish. $P=V\cdot\rho$ formula bilan massa topiladi. Bunda $V_{sh.}=\frac{4}{3}\pi R^3$.

$4000=\frac{4}{3}\pi R^3\cdot 7,9$ dan R ni topamiz.

$3000=\pi R^3\cdot 7,9;\ R^3=\frac{3000}{3,14\cdot 7,9}=\frac{3000}{24,81}=120,5.$

$R^3=120,9$ dan R ni lg. lash orqali topamiz:

$\lg R^3=\lg 120,5.$

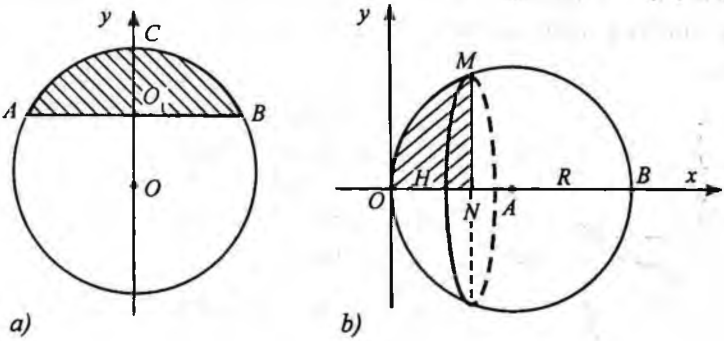
$3\lg R=2,0810;\ \lg R=0,6937.$

Antilogarifm jadvalidan $R\approx 4,940$.

$d=2R\approx 2\cdot 4,940=9,880\approx 9,9$ (sm). *Javob:* $\approx$ 9,9 sm.

2. Shar segmentining hajmi

Doiraviy segmentni uning vatariga perpendikulyar bo'lgan o'q atrofida aylantirish natijasida hosil bo'lgan figura shar segmenti deyiladi (70-*a*, *b* chizma).



70-chizma.

A markazli va $OB=2R$ diametrli yarim doirani ko'rib chiqamiz. Yarim doira tekisligida boshi *O* nuqtada va absissalar o'qi *OA* bo'lgan to'g'ri burchakli koordinatalar sistemasini kiritamiz (70-*b*, chizma). *N* – yarim aylana *M* nuqtasining *OA* to'g'ri chiziqdagi proyeksiyasi bo'lsin. Berilgan yarim doirani absissalar o'qi atrofida aylantirish natijasida shar hosil qilamiz, egri chiziqli *OMN* trapetsiyani aylantirish natijasida esa, *H* balandligi *ON* ga teng bo'lgan shar segmentini hosil qilamiz. *A* markazli, *R* radiusli aylananing tenglamasi $(x-R)^2+y^2=R^2$ yoki $y^2=2Rx-x^2$ ko'rinishda bo'ladi.

Bu tenglama va aylanish figuralarining hajmini topish formulasidan foydalanib, shar segmentining hajmini topamiz:

$$V=\pi\int_0^H y^2dx=\pi\int_0^H\left(2Rx-x^2\right)dx=\pi\left(2R\frac{x^2}{2}-\frac{x^3}{3}\right)\Big/_0^H=$$

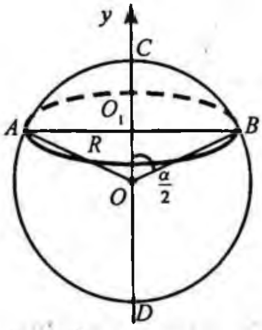
$$=\pi\left(RH^2-\frac{H^3}{3}\right)=\frac{1}{3}\pi H^2(3R-H).$$

Demak, $V_{seg}=\frac{1}{3}\pi H^2(3R-H)$.

Bunda: R – shar radiusi,

H – segment balandligi.

2-masala. O'q kesimida a uzunlikdagi vatari α yoyni tortib turadi. Shar segmentining hajmini toping. $a=6$ sm, $\alpha=120°$ bo'lgan hol uchun hisoblang.



71-чизма.

Berilgan: O markazli $R=OA=OB$ radiusli shar, $AB=a$ – o'qkesim vatari.

$\angle AOB=\alpha$, $a=6$ sm, $\alpha=120°$ da hisoblang.

Topish kerak ABC shar segmentining hajmini (71-chizma).

Yechish. $\angle BOC=\frac{\alpha}{2}$ ga teng, $BO_1=\frac{a}{2}$,

ΔBOO_1 dan $BO_1=OB\cdot\sin\frac{\alpha}{2}$;

$$R=OB=\frac{BO_1}{\sin\frac{\alpha}{2}}=\frac{a}{2\sin\frac{\alpha}{2}},$$

$$OO_1=OB\cdot\cos\frac{\alpha}{2}=R\cos\frac{\alpha}{2}=\frac{a}{2\sin\frac{\alpha}{2}}\cdot\cos\frac{\alpha}{2}=\frac{a}{2}\operatorname{ctg}\frac{\alpha}{2}.$$

$$H=CO_1=OC-OO_1=R-\frac{a}{2}\operatorname{ctg}\frac{\alpha}{2}=\frac{a}{2\sin\frac{\alpha}{2}}-\frac{a\cdot\cos\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}}=\frac{a}{2}\cdot\frac{1-\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}=\frac{a}{2}\times$$

$$\times\frac{2\sin^2\frac{\alpha}{4}}{2\cdot\sin\frac{\alpha}{4}\cdot\cos\frac{\alpha}{4}}=\frac{a\cdot\sin\frac{\alpha}{4}}{2\cos\frac{\alpha}{4}}=\frac{a}{2}\operatorname{tg}\frac{\alpha}{4}.$$

Demak, $R=\frac{a}{2\sin\frac{\alpha}{2}}$; $H=\frac{a}{2}\operatorname{tg}\frac{\alpha}{4}$.

$$V_{seg}=\frac{1}{3}\pi H^2(3R-H)=\frac{1}{3}\pi\cdot\frac{a^2}{4}\cdot\operatorname{tg}\frac{\alpha}{4}\cdot\operatorname{tg}\frac{\alpha}{4}\cdot\left(3\cdot\frac{a}{2\sin\frac{\alpha}{2}}-\frac{a}{2}\operatorname{tg}\frac{\alpha}{4}\right)=$$

$$=\frac{1}{3}\pi\cdot\frac{a^2}{4}\cdot tg\frac{\alpha}{4}\cdot\frac{\sin\frac{\alpha}{4}}{\cos\frac{\alpha}{4}}\left(\frac{3a}{2\sin\frac{\alpha}{2}}-\frac{a\sin\frac{\alpha}{4}}{2\cos\frac{\alpha}{4}}\right)=\frac{\pi a^2\operatorname{tg}\frac{\alpha}{4}}{12}\cdot\frac{\sin\frac{\alpha}{4}}{\cos\frac{\alpha}{4}}\times$$

$$\times\left(\frac{3a}{2\cdot 2\sin\frac{\alpha}{4}\cdot\cos\frac{\alpha}{4}}-\frac{a\sin\frac{\alpha}{4}}{2\cos\frac{\alpha}{4}}\right)=\frac{\pi a^3\operatorname{tg}\frac{\alpha}{4}\cdot\sin\frac{\alpha}{4}}{12\cos\frac{\alpha}{4}}\cdot\frac{3-2\sin^2\frac{\alpha}{4}}{4\sin\frac{\alpha}{4}\cdot\cos\frac{\alpha}{4}}=\frac{\pi a^3\operatorname{tg}\frac{\alpha}{4}}{12\cos\frac{\alpha}{4}}\times$$

$$\times\frac{3-\left(1-\cos\frac{\alpha}{2}\right)}{4\cos\frac{\alpha}{4}}=\frac{\pi a^3\operatorname{tg}\frac{\alpha}{4}}{12\cdot\cos\frac{\alpha}{4}}\cdot\frac{2+\cos\frac{\alpha}{2}}{4\cos\frac{\alpha}{4}}=\frac{\pi a^3\operatorname{tg}\frac{\alpha}{4}\cdot\left(2+\cos\frac{\alpha}{2}\right)}{24\cdot 2\cos^2\frac{\alpha}{4}}=\frac{\pi a^3\operatorname{tg}\frac{\alpha}{4}\left(2+\cos\frac{\alpha}{2}\right)}{24\left(1+\cos\frac{\alpha}{2}\right)}.$$

Javob: $V_{seg}=\dfrac{\pi a^3\operatorname{tg}\frac{\alpha}{4}\left(2+\cos\frac{\alpha}{2}\right)}{24\left(1+\cos\frac{\alpha}{2}\right)}.$

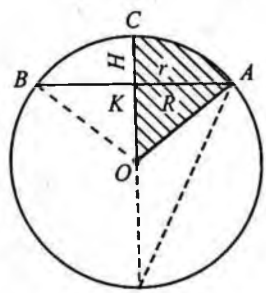
$a=6$ sm; $\alpha=120°$ da $V_{seg}=\dfrac{3{,}14\cdot 6^3\cdot\operatorname{tg}30°(2+\cos 60°)}{24(1+\cos 60°)}=$

$$=\frac{3{,}14\cdot 216\cdot\frac{\sqrt{3}}{3}\cdot(2+0{,}5)}{24\left(1+\frac{1}{2}\right)}=\frac{678{,}24\cdot 0{,}577\cdot 2{,}5}{24\cdot 1{,}5}=\frac{978{,}36}{36}=27{,}176\approx 27{,}2\ (\text{sm}^3).$$

Javob: $\approx 27{,}2$ sm^3.

3. Shar sektorining hajmi

Doiraviy sektorni uning chegaraviy radiuslaridan biri yotgan o'q atrofida aylantirish natijasida hosil bo'lgan figura **shar sektori** deyiladi (72-chizma).



72-chizma.

Bu aylantirishda doiraviy sektorning yoyi ($\cup AC$) segment sirtini hosil qiladi.

Shar radiusi R va segmentning balandligi H ni bilgan holda shar sektorining hajmini topamiz.

AOC doiraviy sektorni CD diametri atrofida aylantirish natijasida shar sektori hosil qilingan bo'lsin. $OA=R$, $CK=H$, $AK=r$ kabi belgilashni kiritamiz. Yarim sharda joylashgan shar sektorining hajmi konus bilan shar segmenti hajmlarining yig'indisiga teng, ya'ni

$$V_{sek}=V_{kon}+V_{seg}=\frac{1}{3}\pi r^2(R-H)+\frac{1}{3}\pi H^2(3R-H).$$

To'g'ri burchakli ACD uchburchakda r o'rta proporsional kesma bo'lgani uchun $r^2=H\cdot(2R-H)$ ga teng. Buni yuqoridagi sektor hajmiga qo'yib:

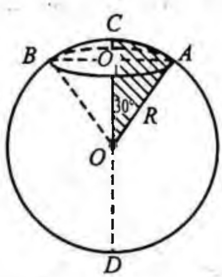
$$V_{sek}=\frac{1}{3}\pi H(2R-H)\cdot(R-H)+\frac{1}{3}\pi H^2(3R-H)=$$

$$=\frac{1}{3}\pi H(2R^2-2RH-RH+H^2+3RH-H^2)=\frac{1}{3}\pi H\cdot 2R^2=\frac{2}{3}\pi R^2 H.$$

Demak, $\boldsymbol{V_{seg}=\frac{2}{3}\pi R^2 H}$.

Bunda: R – shar radiusi, H – segment balandligi.

3-masala. Burchagi 30° va radiusi R ga teng doiraviy sektor yon radiuslarining biri atrofida aylanadi. Hosil qilingan jismning hajmini toping.



73-чизма.

Berilgan: AOC – doiraviy sektor,

$OA=R$, $\angle AOC=30°$,

AOC sektor CD atrofida aylanadi (73-chizma).

Topish kerak: $AOBC$ sektor hajmini yoki V_{sek} ni.

Yechish. $V_{sek}=\frac{2}{3}\pi R^2 H$ formula bo'yicha sektor hajmini topamiz:

$$\Delta OO_1A \text{ dan } OO_1=R\cos30°=R\cdot\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{2}R.$$

$$H=R-OO_1=R-\frac{\sqrt{3}}{2}R=\frac{1}{2}R(2-\sqrt{3}).$$

$$V_{sek}=\frac{2}{3}\pi R^2 H=\frac{2}{3}\pi R^2\cdot\frac{1}{2}R(2-\sqrt{3})=\frac{1}{3}\pi R^3(2-\sqrt{3}).$$

Javob: $\frac{1}{3}\pi R^3(2-\sqrt{3})$.

TAKRORLASH UCHUN SAVOLLAR

1. Shar qanday hosil qilinadi?
2. Sharning hajmi nimaga teng?
3. Shar segmenti qanday hosil qilinadi?
4. Shar segmentining hajmi nimaga teng?
5. Shar sektori qanday hosil qilinadi?
6. Shar sektorining hajmi nimaga teng?

MASALALARNI YECHING

301. Sharning radiusi 3 marta orttirilsa, uning hajmi necha marta ortadi? 2 marta shar radiusi kamaytirilsa, shar hajmi necha marta kamayadi?

302. a) Diametrlari 25 sm va 35 sm boʻlgan ikki choʻyan sharni qaytadan bir shar qilib quyish kerak. Yangi sharning diametrini toping.

b) Uchta sharning radiuslari 3 sm, 4 sm va 5 sm. Hajmi uchala shar hajmlarining yigʻindisiga teng boʻlgan sharning radiusini toping.

303. Ogʻirligi 1 kg boʻlgan qoʻrgʻoshin berilgan. Shundan diametri 1 sm li qancha shar quyish mumkin? Qoʻrgʻoshinning solishtirma ogʻirligi 11,4 g/sm^3.

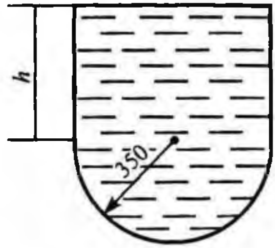
304. Diametri 3 sm li qoʻrgʻoshin shar quyish kerak. Buning uchun diametri 5 mm li qoʻrgʻoshin sharchalardan nechtasini olish kerak?

305. a) Balandligi asosining diametriga teng boʻlgan yogʻoch silindr (teng tomonli silindr)dan eng katta shar yasalgan. Materialning necha protsenti yoʻnilganligini aniqlang.

b) Kub yoʻnilib, eng katta shar yasalgan. Materialning necha protsenti yoʻnilgan?

306. Kovak sharning tashqi diametri 18 sm, devorning qalinligi 3 sm. Devorning hajmini toping.

307. Suv idishi radiusi 350 sm bo‘lgan yarim shar va asosining radiusi o‘shanday bo‘lgan silindrdan iborat (74-chizma). Idishning butun hajmi 200 m^3 ga teng bo‘lishi uchun silindr bo‘lagining balandligi h qancha bo‘lishi kerak? (O‘lchovlar santimetr bilan berilgan).



74-chizma.

308. Shar berilgan. Diametrga perpendikulyar bo‘lgan tekislik diametrni 3 sm va 9 sm li bo‘laklarga bo‘ladi. Sharning hajmi qanday bo‘laklarga bo‘linadi?

309. Balandligi shar diametri d ning 0,3 qismiga teng bo‘lgan shar segmentining hajmini toping.

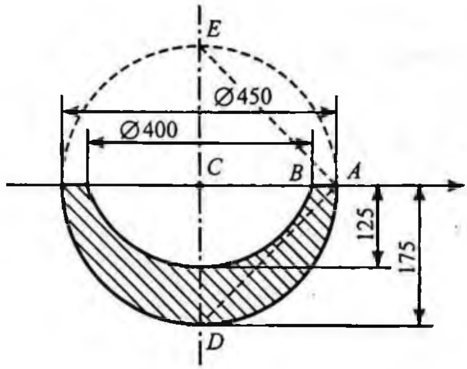
310. Bir-biriga teng ikki shar shunday qo‘yilganki, birining markazi ikkinchisining sirtida. Sharlarning umumiy bo‘lagi hajmining butun shar hajmiga nisbati qanday?

311. Metallquyar cho‘michning uzunasiga olingan qismi (75-chizma) chizmada ko‘rsatilgan. Ichki va tashqi sirtlari – sferik (o‘lchamlar millimetr hisobida berilgan).

Solishtirma og‘irligi 7,9 $\frac{g}{sm^3}$.

Cho‘michning og‘irligini toping.

Ko‘rsatma. Masalani yechishda cho‘michning tashqi radiusi R ni, ichki radiusi r lar topiladi. Buning uchun yoylarni aylanaga to‘ldirib, o‘rta proporsional kesmadan foydalanamiz (75-chizma). Tashqi yoyning A nuqtasidan o‘tkazilgan o‘rta perpendikulyar $AC=22{,}5$ sm, diametrning kesmalari $CD=17{,}5$ sm, $EC=2R-17{,}5$ bo‘lganidan $(AC)^2=CD\cdot(2R-CD)$ yoki $22{,}5^2=17{,}5(2R-17{,}5)$ dan R topiladi.



75-chizma.

Xuddi shu kabi r ham topiladi. $V = V_{\text{kat.seg}} - V_{\text{kich.seg}}$ topiladi. $P = V \cdot \rho$ dan og'irlik topiladi.

312. Shar sektorining radiusi R, o'q kesimining burchagi 120° bo'lsa, hajmini toping.

313. Shar sektori asosi aylanasining radiusi 60 sm, sharning radiusi 75 sm. Shar sektorining hajmini toping.

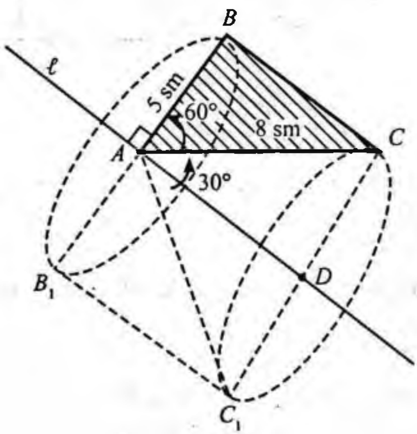
314. Radiusi R bo'lgan yarim doira ikki radius bilan teng uch bo'lakka bo'lingan va diametri atrofida aylanadi. Har bir bo'lagining aylanishidan hosil bo'lgan jismlarning hajmlarini toping.

315. Agar sferik sektorda o'q kesimining yuzi katta doira yuzining $\frac{1}{3}$ qismiga teng bo'lsa, uning hajmi shar hajmining $\frac{1}{4}$ qismiga teng bo'lishini isbotlang.

316*. Yuzi Q va tomoni a ga teng bo'lgan romb o'z tomoni atrofida aylanadi. Hosil bo'lgan jismning hajmini toping.

317*. Tomonlari 8 sm va 5 sm hamda ular orasidagi burchagi 60° bo'lgan uchburchak, shu burchak uchidan uning kichik tomoniga perpendikulyar holda o'tgan o'q atrofida aylanadi. Hosil bo'lgan jismning hajmini toping.

Ko'rsatma. ΔABC ni ℓ o'q ($\ell \perp AB$) atrofida aylantirishdan hosil bo'lgan BCC_1B_1 o'q kesimli kesik konus hajmidan ACC_1 o'q kesimli konus hajmini ayirib, izlangan jismning hajmi topiladi (76-chizma).



76-chizma.

VI bob. BUTUN KURSNI TAKRORLASHGA DOIR QO'SHIMCHA MASALALAR

Hisoblang:

318. $2,25:1,5+3:1\frac{2}{3}+\frac{25}{128}\cdot 615-\frac{3}{32}:4;$

319. $\dfrac{25,839:4,05}{8\frac{1}{35}-5\frac{4}{7}:104}+\left(78,54:25,5-0,1:\frac{3}{38}\right)\cdot\frac{25}{68};$

320. $\dfrac{((7-6,35):6,5+9,8999...)\cdot\frac{1}{12,8}}{\left((1,2:36)+\left(1\frac{1}{5}:0,25\right)-1,8(3)\right)\cdot 1\frac{1}{4}}:0,125;$

321. Ombordan birinchi kuni kartoshkaning $\frac{5}{12}$ qismni, ikkinchi kuni birinchi kunda jo'natilganning $\frac{6}{7}$ qismini jo'natgandan keyin omborda 38 t kartoshka qoldi. Dastlab omborda qancha kartoshka bo'lgan?

322. Ikki bo'lak jezning massasi 30 kg. Birinchi bo'lakda 5 kg, ikkinchi bo'lakda 4 kg sof mis bor. Agar ikkinchi bo'lak jezda misning miqdori birinchi bo'lakdagi mis miqdoridan 15% ortiq bo'lsa, birinchi bo'lakda necha foiz mis bor?

323. Ichida 40 g tuz bo'lgan eritmaga 200 g suv qo'shildi, shundan keyin uning konsentratsiyasi 10% kamaydi. Eritmada qancha suv bo'lgan va uning konsentratsiyasi qanday bo'lgan?

Algebraik ifodalarni aynan almashtirish:

324. $\left(\frac{3}{2x-y}-\frac{2}{2x+y}-\frac{1}{2x-5y}\right):\frac{y^2}{4x^2-y^2};$

325. $\dfrac{\sqrt[4]{x^5}+\sqrt[4]{xy^4}-\sqrt[4]{x^4y}-\sqrt[4]{y^5}}{\sqrt{x}-\sqrt{y}}\cdot\left(\sqrt[4]{x}+\sqrt[4]{y}\right);$

326. $\dfrac{a^{-1}-b^{-1}}{a^{-3}+b^{-3}}:\dfrac{a^2b^2}{(a+b)^2-3ab}\cdot\left(\dfrac{a^2-b^2}{ab}\right)^{-1}$; $a=1-\sqrt{2}$, $b=1+\sqrt{2}$ bo'lganda hisoblang.

Kasrning maxrajidagi irratsionallikni yo'qoting:

327. $\dfrac{14}{\sqrt[4]{3}+\sqrt[8]{2}}.$ **328.** $\dfrac{3+\sqrt{2}+\sqrt{3}}{3-\sqrt{2}-\sqrt{3}}.$

329. Soddalashtiring:

a) $\sqrt{180+26\sqrt{11}}$; b) $\sqrt{161{,}25-25\sqrt{5}}$; d) $\sqrt{5\frac{9}{16}+\frac{3}{2}\sqrt{5}}$.

330. a) $\sqrt{A+\sqrt{B}}=\sqrt{\dfrac{A+\sqrt{A^2-B}}{2}}+\sqrt{\dfrac{A-\sqrt{A^2-B}}{2}}$;

b) $\sqrt{A-\sqrt{B}}=\sqrt{\dfrac{A+\sqrt{A^2-B}}{2}}-\sqrt{\dfrac{A-\sqrt{A^2-B}}{2}}$.

331. Ifodani soddalashtiring:

a) $\sqrt{67-42\sqrt{2}}+\sqrt{19-6\sqrt{2}}$;

b) $\sqrt{51-4\sqrt{77}}-\sqrt{47-4\sqrt{33}}$.

Tenglamalarni yeching:

332. $\dfrac{x-3}{x-1}+\dfrac{x+3}{x+1}=\dfrac{x+6}{x+2}+\dfrac{x-6}{x-2}.$

333. $\dfrac{6}{(x+1)(x+2)}+\dfrac{8}{(x-1)(x+4)}=1.$

334. $\sqrt{x}+\dfrac{2x+1}{x+2}=2.$

335. $\frac{y}{y+1}-2\sqrt{\frac{y+1}{y}}=3.$

336. Agar $ax^2+bx+c=0$ tenglamaning a, b va c koeffitsiyentlari $2b^2-9ac=0$ shart bilan bog'langan bo'lsa, u holda tenglama ildizlarining nisbati 2 ga teng bo'lishini ko'rsating.

337. Agar $x^2+px+q=0$ tenglama ildizlarining ayirmasi 5 ga, ular kublarining ayirmasi esa 35 ga teng bo'lsa, tenglamaning ildizlarini topmasdan, p va q larni aniqlang.

338. Arifmetik progressiyada:
$a_5+a_{10}=-9$ va $a_4+a_6=-4$ bo'lsa, uning dastlabki o'nta hadining yig'indisini toping.

339 Qanday natural son o'zidan oldingi barcha natural sonlarning yig'indisiga teng bo'ladi?

340. Parashyutchi samolyotdan sakragandan 7 sek o'tgach, yerdan 400 m yuqorida parashyut ochiladi. Parashyutchi necha metr balandlikdan sakragan?
(*Izoh.* Parashyutchi 1-sekundda 4,9 m, keyingi har bir sekundda esa 9,8 m tushgan).

341. Oralaridagi masofa 445 m bo'lgan *A* va *B* joylardan ikki kishi bir-biriga qarab harakat qildi. *A* dan chiqqan kishi 1 minutda 5 m yurdi, har bir keyingi minutda oldingi minutdagidan 3 m ortiq yurdi. *B* dan chiqqan kishi 1 minutda 15 m yuradi, har bir keyingi minutda oldingi minutdagidek 2 m ortiq yurdi. Necha minutdan keyin ular orasidagi masofa 200 m bo'ladi.

342. Geometrik progressiyada:
$b_5=1\frac{1}{2}$ ga, $q=-\frac{1}{2}$ bo'lsa, uning dastlabki beshta hadining yig'indisini toping.

343. Geometrik progressiyada $a_1+a_2=9$ va $a_1-q_1=2{,}75$ bo‘lsa, a_3 ni toping.

344. Geometrik progressiya tashkil etgan uchta natural sonlar yig‘indisi 26 ga teng. U sonlarga teskari sonlar yig‘indisi esa $\frac{13}{18}$ ga teng. Shu sonlarni toping.

345. Arifmetik va geometrik progressiyaning 7 tadan hadlari bo‘lib, 1 – va 7 – hadlari o‘zaro teng. Arifmetik progressiyaning 2 – hadi 34,5 ga, 7 – hadi esa 192 ga teng bo‘lsa, geometrik progressiyani toping.

Tenglamalarni yeching:

346. $81\sqrt[3]{3^{x^2-8x}}=1.$ **347.** $4^{\sqrt{3x^2-2x}+1}+2=9\cdot 2^{\sqrt{3x^2-2x}}.$

348. $x^2\cdot 3^{x-2}+3^{\sqrt{x}+2}=3^x+x^2\cdot 3^{\sqrt{x}}.$ **349.** $3\sqrt{\log_3 x}-2\log_3\sqrt{3x}=1.$

350. $\log_3\sqrt{130-7^{\log_x(6-x)}}=2.$ **351.** $\log_3\frac{1}{\sqrt{\log_3 x}}=\log_9\log_9\frac{x}{3}.$

Ko‘rsatma. $\log_3 x=y$ olib, tenglama yechiladi.

Tengsizliklarni yeching:

352. $\left(\frac{1}{3}\right)^{x^2-x}<\frac{1}{9}.$ **353.** $\left(\sqrt{5}\right)^x-4\left(\sqrt{5}\right)^{-x}<3.$

354. $(1{,}25)^{1-(\log_2 x)^2}<0{,}64^{2+\log_{\sqrt{2}} x}.$ **355.** $4^x<2^{x+1}+3.$

356. $\log_2\sqrt{x}-2\log^2_{\frac{1}{4}}x+1>0.$ **357.** $\log_{(x-2)}(9x+20)>\log_{(x-2)}(3x+2).$

Trigonometrik ifodalarni soddalashtiring va ayniyatni isbotlang:

358. $\cos 70°+\cos 50°$ ifodani soddalashtiring.

359. $\sin\alpha+\operatorname{tg}\alpha$ ni ko‘paytma shakliga keltiring.

360*. $\cos 47°-\cos 61°-\cos 11°+\cos 25°$ ifodani soddalashtiring.

361. $\mathrm{tg}10^\circ \cdot \mathrm{tg}20^\circ + \mathrm{tg}20^\circ \cdot \mathrm{tg}60^\circ + \mathrm{tg}60^\circ \cdot \mathrm{tg}10^\circ = 1$ ekanligini isbotlang.

362. $\frac{1-\sin^6\alpha-\cos^6\alpha}{1-\sin^4\alpha-\cos^4\alpha}=\frac{3}{2}$ tenglikni isbotlang.

Trigonometrik tenglamalarni yeching:

363. $6\cos^2 x+5\sin x-7=0.$

364. $\sin x+\mathrm{ctg}\frac{x}{2}=2\cos\frac{x}{2}.$

365. $\cos 2x-3\sin x\cdot\cos x+1=0.$

366. $4\sin x\cdot\cos x\cdot\cos 2x=1.$

367. $\cos^2 x-\cos^2 2x=\cos^2 3x-\cos^2 4x.$

Trigonometrik tengsizliklarni yeching:

368. $\sin 7x\cdot\cos 3x>\sin 3x\cdot\cos 7x.$

369. $2\sin^2\left(x-\frac{\pi}{4}\right)+2<5\sin\left(x-\frac{\pi}{4}\right).$

370. $\mathrm{tg}^2 x+0{,}75\mathrm{tg}x>\frac{1}{4}.$

371. $\left|\sin\frac{x}{2}\cdot\cos\frac{x}{2}\right|<\frac{1}{2\sqrt{2}}.$

372. Funksiyalarning eng kichik musbat davrini toping:

a) $y=4\sin\left(3x-\frac{2\pi}{5}\right);$ b) $f(x)=\sin 1{,}5x+5\cos 0{,}75x.$

$y=\sin kx$, $y=\cos kx$ larning davri $T=\frac{2\pi}{k}$ kabi topiladi.

Masalan $y=\sin\left(3x-\frac{\pi}{5}\right)$ ning davri $T=\frac{2\pi}{3}.$

373. Funksiyaning juft (toq)ligini aniqlang:

a) $y=5x^3\sin x;$ b) $f(x)=\sin\frac{x^3-x^2}{x-1}.$

Funksiyalarning hosilasini toping:

374. a) $y=2x^6-3{,}8x^5+x-\sqrt{2};$ b) $y=\frac{3-2x}{x+1}.$

375. a) $f(x)=\frac{8x\sqrt{x}+2}{x}$ bo'lsa, $f'(1)$ ni toping;

b) $f(x)=(3x^2+x)\cos 2x$ bo'lsa, $f'(0)+f'\left(-\frac{\pi}{2}\right)$ ni toping.

376. a) $y=\cos(x^2+5)$ bo‘lsa, $f'(x)$ ni toping;

b) $y=\sin^2 x\cdot\cos x$ bo‘lsa, $f'\left(-\frac{\pi}{3}\right)$ ni toping.

377. $y=e^{\text{ctg}3x}$ bo‘lsa, y' ni toping.

378. $y=5^{\sin(2x-1)}$ bo‘lsa, y' ni toping.

379. $y=\ln(1-\cos x)$ bo‘lsa, y' ni toping.

380. $y=\log_5 \sin^3 2x$ bo‘lsa, y' ni toping.

Berilgan funksiyaga teskari funksiyani topib, uning o‘sish va kamayish oraliqlarini aniqlang:

381. a) $y=2^x+1$; b) $y=\lg\frac{1+x}{1-x}$.

382. O‘sish (kamayish) oraliqlarini hamda maksimum va minimum nuqtalarini toping:

a) $y=x-\ln x$; b) $y=\frac{6\ln x}{x}$.

383. $y=-2x^4+3x^2-6$.

384. Qanday musbat son o‘ziga teskari songa qo‘shilganda eng kichik yig‘indini beradi?

385. $y=x^2+2x$ funksiyaning grafigiga shu grafikning absissalar o‘qi bilan kesishish nuqtalarida va $x=1,5$ absissali nuqtada urinma tenglamasini yozing.

386. a ning qanday qiymatida $y=a\ln(3x-1)$ funksiya grafigiga $x_0=2$ nuqtada o‘tkazilgan urinma absissa o‘qi bilan 45° li burchak hosil qiladi.

387. $A(1;\ 4)$ nuqtadan $y=-2-\frac{2}{x}$ funksiyaning grafigiga o‘tkazilgan urinmalar tenglamasini yozing.

388. Hajmi 72 sm^3, asosining tomonlari 1:2 nisbatda bo‘ladigan qopqoqli yashik tayyorlash kerak. Yashikning to‘la sirti eng kichik bo‘lishi uchun uning barcha tomonlarining o‘lchami qanday bo‘lishi kerak?

389. Deraza yarim doira bilan tugallangan to‘g‘ri to‘rtburchak shaklida. Berilgan perimetrda derazaning eng katta yuzaga ega bo‘ladigan o‘lchovlarini toping.

Funksiyaning boshlang‘ich funksiyalarining umumiy ko‘rinishini toping:

390. a) $f(x)=8x^3-\cos 3x$; b) $f(x)=x^{\sqrt{3}}-\sin(2x+1)-\frac{1}{(2x+1)^2}$.

391*. $f(x)=(x+3)\sqrt{5x-2}$.

Integralni hisoblang:

392. a) $\int\limits_{-1}^{2}\frac{5\sqrt[3]{9}}{\sqrt[3]{(5x+2)^2}}dx$; b) $\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{8}}\left(\frac{\cos 4x}{2}+\frac{5}{\sin^2 2x}\right)dx$.

393*. $\int\limits_{0}^{2}\frac{3x-4}{\sqrt{5x+2}}dx$.

394. $y=\frac{5}{x}$ va $y=6-x$ chiziqlar bilan chegaralangan figuraning yuzini toping.

395. $y=x^2$, $y=4x-4$, $x=0$ chiziqlar bilan chegaralangan figuraning yuzini toping.

396*. R radiusli yarim sharga tashqi chizilgan eng kichik hajmli konusning H balandligini toping, bunda konus asosining markazi shar markazida yotadi.

397*. Sahifada matn 384 sm^2 yuzni egallashi kerak. Ustki va pastki hoshiyalar 3 sm dan, chap va o‘ng hoshiyalar esa 2 sm dan bo‘lishi kerak. Agar e’tiborga faqat qog‘ozni tejash olinsa, u holda sahifaning eng qulay o‘lchamlari qanday bo‘lishi kerak (qog‘ozning o‘lchamlari)?

398. $y=0, y=\frac{1}{x}, x=1$ va $x=3$ chiziqlar bilan chegaralangan figurani OX o‘qi atrofida aylanishidan hosil bo‘lgan figuraning hajmini toping.

399. $y=(x-1)^2$ va $y=x-1$ chiziqlar bilan chegaralangan figurani OX o‘qi atrofida aylanishidan hosil bo‘lgan figuraning hajmini toping.

400. $y=x^2+1$ va $y=2$ chiziqlar bilan chegaralangan figurani OX o‘qi atrofida aylanishidan hosil bo‘lgan figuraning hajmini hisoblang.

401*. $\left(\sqrt{7+\sqrt{48}}\right)^x+\left(\sqrt{7-\sqrt{48}}\right)^x=14$ tenglamani yeching.

402. $\underbrace{\sqrt[3]{x^3\sqrt{x\sqrt[3]{x\ldots}}}}_{n \text{ ta ildiz}}=3$ tenglamani yeching.

ALGEBRANING ASOSIY MA'LUMOTI VA FORMULALARI

Natural ko'rsatkichli daraja.

$\underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n} = a^n$ (n – natural son)

1. $a^m \cdot a^n = a^{m+n}$; 2. $a^m : a^n = a^{m-n}$(m va n – natural sonlar). $a^0 = 1$.

3. $(a \cdot b)^n = a^n \cdot b^n$; 4. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$; 5. $(a^m)^n = a^{m \cdot n}$.

(bunda m va n – natural sonlar).

Qisqa ko'paytirish formulalari:

1. $(a+b)^2 = a^2 + 2ab + b^2$.
2. $(a-b)^2 = a^2 - 2ab + b^2$.
3. $(a+b) \cdot (a-b) = a^2 - b^2$.
4. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.
5. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.
6. $(a+b)(a^2 - ab + b^2) = a^3 + b^3$.
7. $(a-b)(a^2 + ab + b^2) = a^3 - b^3$.

Birinchi darajali bir noma'lumli tenglama deb, $\boldsymbol{ax+b=0}$ ko'rinishdagi tenglamaga aytiladi. Bunda: x – noma'lum son, a (koeffitsiyent) – $a \neq 0$ har qanday son, b (ozod had) – har qanday son.

$ax+b=0$ tenglama quyidagicha yechiladi:

$ax+b=0$;

$ax=-b$;

$x=-\frac{b}{a}$. $-\frac{b}{a}$ – tenglamaning ildizi.

$\boldsymbol{ax+by=c}$ – ikki o'zgaruvchili chiziqli tenglama. Bunda: $a \neq 0$, $b \neq 0$ va c – ixtiyoriy sonlar. $\boldsymbol{y=kx+b}$ ni chiziqli funksiya deyiladi.

Tenglamani to'g'ri tenglikka aylantiradigan har qanday qo'sh son (x_0; y_0) tenglamaning yechimi deyiladi. Bu chiziqli tenglama cheksiz ko'p yechimga ega. Bu tenglamaning yechimi $y=\frac{c-ax}{b}$ dagi x ga ixtiyoriy qiymatlar berish bilan topiladi.

$ax+by=c$ chiziqli tenglamaning grafigi to‘g‘ri chiziqdan iborat bo‘ladi.

Ikki noma’lumli birinchi darajali ikki tenglama sistemasi.

Bunday sistema umumiy holda $\begin{cases} a_1x_1+b_1y_1=c_1 \\ a_2x+b_2y=c_2 \end{cases}$ ko‘rinishda bo‘ladi.

Bunda: x va y – noma’lumlar, a_1, a_2, b_1, b_2, c_1 va c_2 – berilgan sonlar. Sistemadagi ikkala tenglamani qanoatlantiradigan $(x_0;\ y_0)$ qo‘sh son sistemaning yechimi deyiladi.

Bunday sistema uch xil usulda yechiladi:

1) O‘rniga qo‘yish usuli;

2) Qo‘shish usuli;

3) Grafik usuli.

Masalan: $\begin{cases} 8x-3y=46 \\ 5x+6y=13 \end{cases}$

1-usul. $\begin{cases} 3y=8x-46 \\ 5x+6y=13 \end{cases}$; $\begin{cases} y=\frac{8x-46}{3} \\ 5x+6\cdot\frac{8x-46}{3}=13 \end{cases}$; $5x+16x-92=13$;

$21x=105$; $x=\frac{105}{21}=5$; $y=\frac{8\cdot5-46}{3}=-2$. *Yechim:* (5; –2).

2-usul. $\begin{cases} 8x-3y=46 \\ 5x+6y=13 \end{cases}\Bigg|\,2 \quad + \begin{cases} 16x-6y=92 \\ 5x+6y=13 \end{cases}$

$$21x=105.$$

$$x=\frac{105}{21}=5;$$

$x=5$ da, $5\cdot5+6y=13$;

$6y=-12$; $y=-2$. *Yechim:* (5; –2).

3-usul. Berilgan tenglamalar grafiklarining (to‘g‘ri chiziqlarning) kesishgan nuqtasi koordinatalari sistemaning yechimi bo‘ladi, ya’ni (5; –2).

Tenglikka kiruvchi o‘zgaruvchilarning barcha qabul qiladigan qiymatlarida to‘g‘ri bo‘ladigan tenglik **ayniyat** deyiladi.

Kasr $\frac{ac}{bc}=\frac{a}{b}$ kabi qisqartiriladi.

Kasr $\frac{-a}{b}=\frac{a}{-b}=-\frac{a}{b}$; $\frac{a}{b}=-\frac{-a}{b}=-\frac{a}{-b}$ yoziladi.

Kasrlar bir xil maxrajli bo'lganda:

1) $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$; 2) $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$.

Kasrlar har xil maxrajli bo'lganda:

1) $\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$; 2) $\frac{a}{b}-\frac{c}{d}=\frac{ad}{bd}-\frac{bc}{bd}=\frac{ad-bc}{bd}$;

3) $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$; 4) $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$; 5) $\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}(bcd\neq 0)$.

Kvadrat ildizlar:

$\sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b}(a\geq 0;\ b\geq 0)$; $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}(a\geq 0;\ b>0)$.

$\sqrt{x^2}=|x|$; x – barcha sonlar; $\sqrt{(-6)^2}=|-6|=6$.

yoki $\sqrt{x^2}=|x|=\begin{cases} x, & \text{agar } x\geq 0; \\ -x, & \text{agar } x<0. \end{cases}$

Umumiy shakldagi kvadrat tenglama $\boldsymbol{ax^2+bx+c=0}$ ko'rinishda bo'ladi ($a\neq 0$). Xususiy holda b va c yoki ikkalasi ham nolga teng bo'lishi mumkin:

$$\left.\begin{array}{lll} 1)\ b=0 & \text{bo'lsa,} & ax^2+c=0 \\ 2)\ c=0 & \text{bo'lsa,} & ax^2+bx=0 \\ 3)\ b=c=0 & \text{bo'lsa,} & ax^2=0 \end{array}\right\} \text{chala kvadrat tenglamalar.}$$

Agar normal shakldagi tenglamada $a=1$ bo'lsa, tenglama $\boldsymbol{x^2+px+q=0}$ ko'rinishga keltirilib, uni **keltirilgan kvadrat tenglama** deyiladi.

1) $ax^2+c=0$ ning ildizi: $x_{1/2}=\pm\sqrt{-\frac{c}{a}}$;

2) $ax^2+bx=0$ ning ildizi: $x_1=0$; $x_2=-\frac{b}{a}$.

3) $ax^2=0$ ning ildizi: $x=0$.

$x^2+px+q=0$ keltirilgan kvadrat tenglamaning ildizlari: $x_{1/2}=-\frac{p}{2}\pm\sqrt{\frac{p^2}{4}-q}$.

Umumiy shakldagi $ax^2+bx+c=0$ tenglamaning ildizlari: $x_{1/2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Bunda $b^2-4ac=D$ diskriminant deyiladi.

Bunda:

1) $b^2-4ac>0$ bo‘lsa, $x_1=\frac{-b-\sqrt{b^2-4ac}}{2a}$ va $x_2=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ ikkita ildizga ega.

2) $b^2-4ac=0$ bo‘lsa, $x=-\frac{b}{2a}$ bitta ildizga ega.

3) $b^2-4ac<0$ bo‘lsa, tenglama ildizga ega emas.

Agar $ax^2+bx+c=0$ tenglamada $b=2k$ (juft) bo‘lsa, $x_{1/2}=\frac{-k\pm\sqrt{k^2-ac}}{a}$ kabi oson topiladi.

$ax^2+bx+c=a(x-x_1)(x-x_2)$ ko‘paytuvchilarga ajraladi. Bunda x_1 va x_2 kvadrat uchhadning ildizlari.

Viet teoremasi: 1) $x^2+px+q=0$ keltirilgan kvadrat tenglamada $x_1+x_2=-p$ va $x_1\cdot x_2=q$ bo‘ladi.

2) $ax^2+bx+c=0$ da $x_1+x_2=-\frac{b}{a}$ va $x_1\cdot x_2=\frac{c}{a}$

Tengsizlikning xossalari:

1) Agar $a<b$ va c istalgan son bo‘lsa, u holda $a\pm c<b\pm c$ bo‘ladi.

2) Agar $a<b$ va c musbat son bo‘lsa, u holda $ac<bc$ bo‘ladi. Agar $a<b$ va c manfiy son bo‘lsa, u holda $ac>bc$ bo‘ladi.

3) Agar $a<b$ ni biror c songa bo‘lish uchun tengsizlikni bo‘luvchining teskarisiga ko‘paytirib topiladi:

agar $c>0$ bo‘lsa, $a<b$ ni c ga bo‘lish uchun $a\cdot\frac{1}{c}<b\cdot\frac{1}{c}$ kabi bajariladi.

agar $c<0$ bo‘lsa, $a<b$ ni c ga bo‘lish uchun $a\cdot\frac{1}{c}>b\cdot\frac{1}{c}$ kabi bajariladi (tengsizlik ishorasi almashtiriladi).

4) Agar $a<b$ va $c<d$ bo'lsa, u holda $a+c<b+d$ bo'ladi.

5) $a<b$ dan $c<d$ tengsizlikni ayirish uchun $c<d$ tengsizlikni $-d<-c$ kabi yozib, ularni qo'shamiz, ya'ni $\begin{array}{r} + \begin{array}{c} a<b \\ -d<-c \end{array} \\ \hline a-d<b-c \end{array}$ kabi bajariladi.

6) Agar $a<b$ va $c<d$ (bunda a, b, c, d – musbat sonlar) bo'lsa, u holda $ac<bd$ bo'ladi.

7) $a<b$ tengsizlikni $c<d$ ga bo'lish uchun $a<b$ ni $c<d$ ning teskarisi $\frac{1}{d}<\frac{1}{c}$ ga ko'paytirib topiladi, ya'ni

$$(a<b):(c<d)=(a<b)\cdot\left(\frac{1}{d}<\frac{1}{c}\right)=\begin{cases} a<b \\ \frac{1}{d}<\frac{1}{c}=\frac{a}{d}<\frac{b}{c}. \end{cases}$$

Birinchi darajali tengsizliklar: $\boldsymbol{ax+b>0}$ yoki $\boldsymbol{ax+b<0}$ (bunda: $a\neq 0$, b – ixtiyoriy sonlar; x – noma'lum o'zgaruvchi) birinchi darajali tengsizliklar.

$x>-\frac{b}{a}$ yoki $x<-\frac{b}{a}$ tengsizlikning yechimi deyiladi.

Arifmetik progressiya (a_n) da:

$a_{n+1}=a_n+d$, bunda a_n n – hadi soni; d – ayirmasi;

$d=a_{n+1}-a_n$; $\boldsymbol{a_n=a_1+(n-1)d}$. Bu n – hadi formulasi. Yoki $a_n=a_1+(n-1)d=a_1+nd-d=nd+\underbrace{(a_1-d)}_{k \text{ bo'lsa}}=nd+k$, ya'ni $\boldsymbol{a_n=nd+k}$ – arifmetik progressiyaning boshqa ko'rinishdagi n – hadi formulasi.

$S_n=\frac{a_1+a_n}{2}\cdot n$ yoki $S_n=\frac{2a_1+d(n-1)}{2}\cdot n$ arifmetik progressiyaning hadlari yig'indisining formulasi.

Geometrik progressiya (b_n) da:

$\boldsymbol{b_n=b_1q^n}$. n – hadining formulasi.

$\boldsymbol{S_n=\frac{b_nq^n-b_1}{q-1}}$ yoki $\boldsymbol{S_n=\frac{b_1(q^n-1)}{q-1}\cdot(q\neq 1)}$.

$\boldsymbol{S=\frac{b_1}{1-q}}$ – cheksiz kamayuvchi geometrik progressiya hadlari yig'indisining formulasi. n – darajali ildizning xossalari:

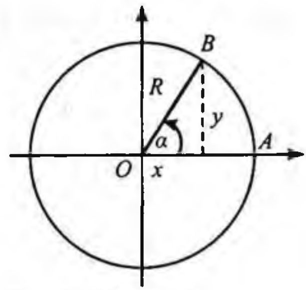
1) $\sqrt[n]{a\cdot b}=\sqrt[n]{a}\cdot\sqrt[n]{b}$; 2) $\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$; 3) $\sqrt[m]{\sqrt[k]{a}}=\sqrt[mk]{a}$ yoki $\sqrt[nk]{a^{mk}}=\sqrt[n]{a^m}$.

Ta'rif. **Istalgan $a>0$ son uchun, $\frac{m}{n}$ – kasr son (m – butun, n – natural bo'lsa, u holda $a^{\frac{m}{n}}=\sqrt[n]{a^m}$ bo'ladi.**

$a>0$ va $b>0$; p va q ratsional sonlar uchun:

1. $a^p\cdot a^q=a^{p+q}$;
2. $a^p:a^q=a^{p-q}$;
3. $(a^p)^q=a^{pq}$;
4. $(a\cdot b)^p=a^p\cdot b^p$;
5. $\left(\frac{a}{b}\right)^p=\frac{a^p}{b^p}$.

Trigonometrik funksiyalarning ta'rifi



77-чизма.

1. B nuqta ordinatasining radiusga nisbati α burchakning sinusi deyiladi ya'ni $\frac{y}{R}=\sin\alpha$ (77-chizma).

2. B nuqta absissasining radiusga nisbati α burchakning kosinusi deyiladi, ya'ni $\frac{x}{R}=\cos\alpha$.

3. B nuqta ordinatasining shu nuqta absissasiga nisbati α burchakning tangensi deyiladi, ya'ni $\frac{y}{x}=\mathrm{tg}\alpha$.

4. B nuqta absissasining shu nuqta ordinatasiga nisbati α burchakning kotangensi deyiladi, ya'ni $\frac{x}{y}=\mathrm{ctg}\alpha$.

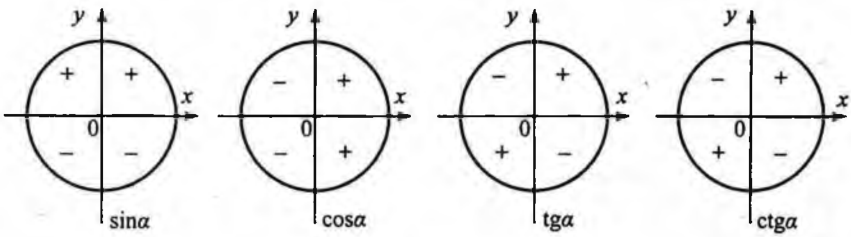
5. Radiusning B nuqta absissasiga nisbati α burchakning sekansi deyiladi, ya'ni $\frac{R}{x}=\sec\alpha$.

6. Radiusning B nuqta ordinatasiga nisbati α burchakning kosekansi deyiladi, ya'ni $\frac{R}{y}=\mathrm{cosec}\alpha$.

Trigonometrik funksiyalar ba'zi burchaklari qiymatlarining jadvali

α	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	–1
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	–1	0
$\text{tg}\,\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	mavjud emas	0	mavjud emas
$\text{ctg}\,\alpha$	–	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	mavjud emas	0

Trigonometrik funksiyalarning ishoralari



78-chizma.

Trigonometrik funksiyalarning juft va toqligi.

$\cos(-\alpha)=\cos\alpha$ – juft funksiya.

$\sin(-\alpha)=-\sin\alpha$; $\text{tg}(-\alpha)=-\text{tg}\,\alpha$; $\text{ctg}(-\alpha)=-\text{ctg}\,\alpha$ – toq funksiyalar.

Trigonometrik funksiyalarning davriyligi.

$$\left.\begin{aligned}\sin(\alpha\pm2\pi)=\sin\alpha\\ \cos(\alpha\pm2\pi)=\cos\alpha\end{aligned}\right\}$$ larning eng kichik musbat davri 2π dan iborat.

Sinus va kosinuslarning umumiy davri $2n\pi$ dan iborat, ya'ni

$\sin(\alpha+2n\pi)=\sin\alpha$;
$\cos(\alpha+2n\pi)=\cos\alpha$ bunda n – butun son.

$$\left.\begin{aligned}\text{tg}(\alpha\pm\pi)=\text{tg}\,\alpha\\ \text{ctg}(\alpha\pm\pi)=\text{ctg}\,\alpha\end{aligned}\right\}$$ larning eng kichik musbat davri π dan iborat.

Tangens va kotangenslarning umumiy davri $n\pi$ dan iborat, ya'ni

$\text{tg}(\alpha+n\pi)=\text{tg}\,\alpha$;
$\text{ctg}(\alpha+n\pi)=\text{ctg}\,\alpha$ bunda n – butun son.

Asosiy trigonometrik ayniyatlar:

$$\mathrm{tg}\alpha = \frac{\sin\alpha}{\cos\alpha};\ \mathrm{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha};\ \ \mathrm{tg}\alpha \cdot \mathrm{ctg}\alpha = 1;\ \sin^2\alpha + \cos^2\alpha = 1;\ 1 + \mathrm{tg}^2\alpha =$$

$$= \frac{1}{\cos\alpha}.\ 1 + \mathrm{ctg}^2\alpha = \frac{1}{\sin^2\alpha}.$$

Burchakning radian o‘lchovi:

$$1\ \text{radian} = \frac{180^\circ}{\pi} \approx 57^\circ 17' 45''\ n_{\text{radian}} = \frac{180^\circ}{\pi} \cdot n;\ \pi = 180^\circ.$$

$$1^\circ = \frac{\pi}{180^\circ} \approx 0{,}017453;\ n^\circ = \frac{\pi}{180^\circ} \cdot n;\ \ 36^\circ = \frac{\pi}{180^\circ} \cdot 36^\circ = \frac{\pi}{5} \approx 0{,}628.$$

Keltirish formulalari jadvali

α	$-\alpha$	$\frac{\pi}{2}+\alpha$ $(90^\circ+\alpha)$	$\frac{\pi}{2}-\alpha$ $(90^\circ+\alpha)$	$\pi+\alpha$ $(180^\circ+\alpha)$	$\pi-\alpha$ $(180^\circ+\alpha)$	$\frac{3\pi}{2}+\alpha$ $(270^\circ+\alpha)$	$\frac{3\pi}{2}-\alpha$ $(270^\circ-\alpha)$	$2\pi+\alpha$ $(360^\circ+\alpha)$	$2\pi-\alpha$ $(180^\circ+\alpha)$
$\sin\alpha$	$-\sin\alpha$	$\cos\alpha$	$\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$\sin\alpha$	$-\sin\alpha$
$\cos\alpha$	$\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$\cos\alpha$	$\cos\alpha$
$\mathrm{tg}\alpha$	$-\mathrm{tg}\alpha$	$-\mathrm{ctg}\alpha$	$\mathrm{ctg}\alpha$	$\mathrm{tg}\alpha$	$-\mathrm{tg}\alpha$	$-\mathrm{ctg}\alpha$	$\mathrm{ctg}\alpha$	$\mathrm{tg}\alpha$	$-\mathrm{tg}\alpha$
$\mathrm{ctg}\alpha$	$-\mathrm{ctg}\alpha$	$-\mathrm{tg}\alpha$	$\mathrm{tg}\alpha$	$\mathrm{ctg}\alpha$	$-\mathrm{ctg}\alpha$	$-\mathrm{tg}\alpha$	$\mathrm{tg}\alpha$	$\mathrm{ctg}\alpha$	$-\mathrm{ctg}\alpha$

Qo‘shish formulalari:

$\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta.$

$\sin(\alpha-\beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta.$

$\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta.$

$\cos(\alpha-\beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta.$

$$\mathrm{tg}(\alpha+\beta) = \frac{\mathrm{tg}\alpha + \mathrm{tg}\beta}{1 - \mathrm{tg}\alpha \cdot \mathrm{tg}\beta};\quad \mathrm{tg}(\alpha-\beta) = \frac{\mathrm{tg}\alpha - \mathrm{tg}\beta}{1 + \mathrm{tg}\alpha \cdot \mathrm{tg}\beta}.$$

$$\mathrm{ctg}(\alpha+\beta) = \frac{\mathrm{ctg}\alpha \cdot \mathrm{ctg}\beta - 1}{\mathrm{ctg}\alpha + \mathrm{ctg}\beta};\quad \mathrm{ctg}(\alpha-\beta) = \frac{1 + \mathrm{ctg}\alpha \cdot \mathrm{ctg}\beta}{\mathrm{ctg}\beta - \mathrm{ctg}\alpha}.$$

Ikkilangan burchakning formulalari:

$\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha;\ \cos 2\alpha = \cos^2\alpha - \sin^2\alpha.$

$$\mathrm{tg}2\alpha = \frac{2\mathrm{tg}\alpha}{1 - \mathrm{tg}^2\alpha};\ \mathrm{ctg}2\alpha = \frac{\mathrm{ctg}^2\alpha - 1}{2\mathrm{ctg}\alpha} = \frac{1 - \mathrm{tg}^2\alpha}{2\mathrm{tg}\alpha}.$$

$1 - \cos 2\alpha = 2\sin^2\alpha;\quad 1 + \cos 2\alpha = 2\cos^2\alpha.$

$$\sin\alpha = \pm\sqrt{\frac{1-\cos 2\alpha}{2}};\ \cos\alpha = \pm\sqrt{\frac{1+\cos 2\alpha}{2}};\ \operatorname{tg}\alpha = \pm\sqrt{\frac{1-\cos 2\alpha}{1+\cos 2\alpha}}.$$

Trigonometrik funksiyalar yig'indisi va ayirmasining formulalari:

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2};\quad \sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}.$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2};\quad \cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}.$$

$$\operatorname{tg}\alpha + \operatorname{tg}\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta};\quad \operatorname{tg}\alpha - \operatorname{tg}\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}.$$

$$\operatorname{ctg}\alpha + \operatorname{ctg}\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta};\quad \operatorname{ctg}\alpha - \operatorname{ctg}\beta = -\frac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}.$$

Argumentni ikkiga bo'lish formulalari:

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}};\quad \sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}.$$

$$\operatorname{tg}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}};\quad \operatorname{ctg}\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}.$$

$\operatorname{tg}\frac{\alpha}{2}$ ni kvadrat ildiz qatnashmagan formulalari: $\operatorname{tg}\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}$, yoki $\operatorname{tg}\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$.

$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$; $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$; $\operatorname{tg} 3\alpha = \frac{3\operatorname{tg}\alpha - \operatorname{tg}^3\alpha}{1-3\operatorname{tg}^2\alpha}$; $\sin\alpha$, $\cos\alpha$ va $\operatorname{tg}\alpha$ larni $\operatorname{tg}\frac{\alpha}{2}$ orqali ifodalash formulasi:

$$\sin\alpha = \frac{2\operatorname{tg}\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}};\quad \cos\alpha = \frac{1-\operatorname{tg}^2\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}};\quad \operatorname{tg}\alpha = \frac{2\operatorname{tg}\frac{\alpha}{2}}{1-\operatorname{tg}^2\frac{\alpha}{2}}.$$

Trigonometrik funksiyalar ko'paytmasini yig'indi shakliga keltirish formulalari:

$$\cos\alpha\cdot\cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta));$$

$$\sin\alpha\cdot\sin\beta = \frac{1}{2}(\cos(\alpha-\beta) - \cos(\alpha+\beta));$$

$\sin\alpha\cdot\cos\beta=\frac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta))$.

Trigonometrik tenglamalarning ildizlari:

1. $\sin x=a$ da: a) $|a|>1$ da tenglama ildizga ega emas.

b) $a=1$ da $\sin x=1$ bo'lib, $x=\frac{\pi}{2}+2k\pi$, $k\in z$.

d) $a=-1$ da $\sin x=-1$ bo'lib, $x=-\frac{\pi}{2}+2k\pi$, $k\in z$.

e) $a=0$ da $\sin x=0$ bo'lib, $x=n\pi$.

f) $-1<a<1$ da $\sin x=a$ bo'lib, $x=(-1)^k\arcsin a+k\pi$, $k\in z$.

2. $\cos x=a$ da: a) $|a|>1$ da tenglama ildizga ega emas.

b) $a=1$ da $\cos x=1$ bo'lib, $x=2n\pi$, $k\in z$.

d) $a=-1$ da $\cos x=-1$ bo'lib, $x=\pi+2k\pi=\pi(2k+1)$, $k\in z$.

e) $a=0$ da $\cos x=0$ bo'lib, $x=\frac{\pi}{2}+k\pi$, $k\in z$.

f) $-1<a<1$ da $\cos x=a$ bo'lib, $x=\pm\arccos a+2k\pi$, $k\in z$.

3. $\mathrm{tg}x=a$ da: a – ixtiyoriy sonda ildiz mavjud.

a) $a=0$ da, $\mathrm{tg}x=0$ bo'lib, $x=k\pi$, $k\in z$.

b) a – ixtiyoriy son, $\mathrm{tg}x=a$ bo'lib, $x=\mathrm{arctg}a+k\pi$, $k\in z$.

4. $\mathrm{ctg}x=a$ da: a – ixtiyoriy sonda ildiz mavjud.

a) $a=0$ da, $\mathrm{ctg}x=0$ bo'lib, $x=\frac{\pi}{2}+k\pi$, $k\in z$. Eng sodda trigonometrik tengsizliklarning yechimi:

1) $\sin x>0$ yechimi $2k\pi<x<(2k+1)\pi$, $k\in z$,
   $\sin x<0$ yechimi $(2k+1)\pi<x<2(k+1)\pi$, $k\in z$.

2) $\cos x>0$ yechimi $-\frac{\pi}{2}+2k\pi<x<\frac{\pi}{2}+2k\pi$, $k\in z$,
   $\cos x<0$ yechimi $\frac{\pi}{2}+2k\pi<x<\frac{3\pi}{2}+2k\pi$, $k\in z$.

3) $\mathrm{tg}x>0$ yechimi $k\pi<x<\frac{\pi}{2}+k\pi$, $k\in z$,
   $\mathrm{tg}x<0$ yechimi $\frac{\pi}{2}+k\pi<x<\pi+k\pi$, $k\in z$.

4) ctg$x>0$ yechimi $k\pi<x<\frac{\pi}{2}+k\pi,\ k\in z$.

ctg$x<0$ yechimi $\frac{\pi}{2}+k\pi<x<\pi+k\pi,\ k\in z$.

Ko'rsatkichli va logarifmik funksiyalar

$y=a^x$ – ko'rsatkichli funksiya. Bunda: $a\neq 1$ – musbat son, x – o'zgaruvchi (ixtiyoriy son), y – funksiya. $a>1$ da o'suvchi funksiya, $0<a<1$ da kamayuvchi funksiya bo'ladi.

Eng sodda ko'rsatkichli tenglama $a^x=b$ ko'rinishda bo'ladi. Bunda $a\neq 1$ – musbat son, b – musbat son, x – noma'lum son. Masalan, $5^{3x-4}=125$, $9^{\frac{1}{5}x+3}=\frac{1}{27}$ va hokazo.

Logarifmik funksiya $y=\log_a x$ ko'rinishda bo'ladi. Bunda: $a\neq 1$ – musbat son, x – o'zgaruvchi, y – funksiya.

$a>1$ da o'suvchi bo'ladi, $0<a<1$ da kamayuvchi bo'ladi. Masalan: $\log_3(2x-1)>\log_3 7$ tengsizlikni $2x-1>7$ kabi (asos $3>1$ bo'lgani uchun), $\log_{0,2}(2x-11)>\log_{0,2}0,25$ tengsizlik $2x-11<0,25$ kabi yoziladi (asos $0<0,2<1$ bo'lgani uchun).

Logarifm formulalari:

1. $a^{\log_a M}=M$ **(1)**. Bunda $a\neq 1$ – musbat son, $M>0$ son. (1) formula logarifmning asosiy ayniyati.

2. $a\neq 1$ – musbat son, $x>0$ va $y>0$ da $\mathbf{\log_a(x\cdot y)=\log_a x+\log_a y}$ (2) (ko'paytmaning logarifmi).

3. $a\neq 1$ – musbat son, $x>0$ va $y>0$ da $\mathbf{\log_a\left(\frac{x}{y}\right)=\log_a x-\log_a y}$ (3) (bo'linmaning logarifmi).

4. $a\neq 1$ – musbat son, $x>0$ va r – ixtiyoriy sonda $\mathbf{\log_a x^p=p\cdot\log_a x}$ (darajaning logarifmi).

5. $a\neq 1$ – musbat son, $x>0$ va n – ixtiyoriy sonda $\mathbf{\log_a\sqrt[n]{x^k}=\frac{k}{n}\log_a x}$ (ildizning logarifmi).

6. $a\neq 1$ – musbat son, $x>0$; k, n – ixtiyoriy sonlar. $\mathbf{\log_{a^n}x=\frac{1}{n}\log_a x}$ (asosi a^n bo'lgan logarifm).

7. $a\neq 1$, – musbat son, $b>0$ va $c>0$ da $\mathbf{\log_a b=\frac{\log_c b}{\log_c a}}$ (boshqa c asosga o'tish formulasi).

8. $a \neq 1$ – musbat son, $b > 0$; da $\log_a b = \frac{1}{\log_b a}$ (asosni almashtirish formulasi).

Funksiyalarning limitlari haqidagi asosiy teoremalar

1) $\lim\limits_{x \to a} c = c$ (o‘zgarmas sonning limiti).

2) $\lim\limits_{x \to a}(k \cdot f(x)) = k \cdot \lim\limits_{x \to a} f(x)$ (o‘zgarmas ko‘paytuvchini limit ishorasidan tashqariga chiqarish).

3) $\lim\limits_{x \to a}(f(x) \pm g(x)) = \lim\limits_{x \to a} f(x) \pm \lim\limits_{x \to a} g(x)$ (funksiyalar yig‘indisining (ayirmasining) limiti).

4) $\lim\limits_{x \to a}(f(x) \cdot g(x)) = \lim\limits_{x \to a} f(x) \cdot \lim\limits_{x \to a} g(x)$ (funksiyalar ko‘paytmasining limiti).

5) $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)}$; ($\lim\limits_{x \to a} g(x) \neq 0$, (funksiyalar nisbatining limiti).

Funksiyaning orttirmasi:

x va x_0 erkli o‘zgaruvchilar bo‘lsa, $x - x_0 = \Delta x$ – erkli o‘zgaruvchining (argumentning) orttirmasi deyiladi.

$f(x + \Delta x) - f(x_0) = \Delta f(x_0)$ funksiyaning x_0 nuqtadagi orttirmasi deyiladi.

$\lim\limits_{\Delta x \to a} \frac{\Delta f(x)}{\Delta(x)} = f'(x) = y'(x)$ $f(x)$ funksiyaning hosilasi.

Hosilani topish qoidalari

1. c – o‘zgarmas son. $c' = 0$;
2. x – o‘zgaruvchi. $x' = 1$;
3. $(u + v)' = u' + v'$; (yig‘indining hosilasi hosilalar yig‘indisiga teng).
4. $(u - v)' = u' - v'$ (ayirmaning hosilasi hosilalar ayirmasiga teng).
5. $(u \cdot v)' = u' \cdot v + u \cdot v'$ (funksiyalar ko‘paytmasining hosilasi).
6. $(c \cdot u)' = c \cdot v'$ (o‘zgarmas ko‘paytuvchi hosila belgisidan tashqariga chiqariladi.

7. $\left(\frac{u}{\upsilon}\right)' = \frac{u'\cdot\upsilon - u\cdot\upsilon'}{\upsilon^2}$ (funksiyalar bo‘linmasining hosilasi).

8. $(x^n)' = n\cdot x^{n-1}$ (darajaning hosilasi).

9. $\left(\sqrt[n]{x^k}\right)' = \left(x^{\frac{k}{n}}\right)' = \frac{k}{n}x^{\frac{k}{n}-1} = \frac{k}{n}x^{\frac{k-n}{n}} = \frac{k}{n}x^{-\frac{n-k}{n}} = \frac{k}{n}\cdot\frac{1}{\sqrt[n]{x^{n-k}}}$.

10. Murakkab funksiyaning hosilasi

$(h(f(x)))' = h'(f(x))\cdot f'(x)$ yoki $(u^m(x))' = m\cdot u^{m-1}(x)\cdot u'(x)$.

Masalan, $y=(3x-2x^3+5)^3$, $y'=((3x^4-2x^3+5)^3)'=3(3x^4-2x^3+5)^2\times$ $\times(3x^4-2x^3+5)'=3(3x^4-2x^3+5)^2(12x^3-6x^2)$.

Trigonometrik funksiyalarning hosilalari

$\lim\limits_{x\to 0}\frac{\sin x}{x}=1$. $(\sin x)'=\cos x$; $(\sin(ax+b))'=a\cdot\cos(ax+b)$.

$(\cos x)'=-\sin x$; $(\cos(ax+b))'=-a\sin(ax+b)$.

$(\mathrm{tg}x)'=\frac{1}{\cos^2x}$; $(\mathrm{ctg}x)'=-\frac{1}{\sin^2x}$.

$\lim\left(1+\frac{1}{n}\right)^n = l \approx 2{,}718...$ ayniyat o‘rinli.

$\log_{10}a=\ln a$ ko‘rinishda yozilib, uni natural logarifm deyiladi.

Ko‘rsatkichli funksiyaning hosilasi:

$(e^x)'=e^x$. Masalan, $(e^{5x+3})'=e^{5x+3}\cdot(5x+3)'=5e^{5x+3}$.

$(a^x)'=a^x\cdot\ln a$. Masalan, $(7^{\sin 2x})'=7^{\sin 2x}\cdot(\sin 2x)'\ln 7=7^{\sin 2x}\cos 2x\cdot(2x)'\cdot\ln 7=$ $=2\ln 7\cdot 7^{\sin 2x}\cdot\cos 2x$.

Logarifmik funksiyaning hosilasi

Agar $f(x)$ va $g(x)$ funksiyalar o‘zaro teskari funksiyalar bo‘lsa, u holda tenglik o‘rinli bo‘ladi.

$f'(x)=\frac{1}{g'(x)}$ tenglik o‘rinli bo‘ladi ($f(x)$ va $g'(x)$lar o‘zaro teskari).

$$(\log_a x)'=\frac{1}{(a^y)'}=\frac{1}{(a^{\log_a x})'}=\frac{1}{a^{\log_a x}\cdot\ln a}=\frac{1}{x\cdot\ln a}$$

$(\log_a x)'=\frac{1}{x\ln a}$. Agar $a=l$ bo‘lsa, $(\ln x)'=\frac{1}{x}$. Masalan, $(\log_7(2x-5))'=\frac{(2x-5)'}{(2x-5)\ln 7}=\frac{2}{(2x-5)\ln 7}$.

$f(x_0+x)=f(x_0)+f'(x_0)\cdot\Delta x$ – funksiya orttirmasining bosh qismi formulasi.

$\sqrt[n]{x_0+\Delta x}=\sqrt[n]{x_0}+\frac{\sqrt[n]{x_0}}{n\cdot x_0}\Delta x$ – n – darajali ildizni topish formulasi.

$\mathrm{tg}\alpha=k=f'(x_0)$ – urinmaning burchak koeffitsiyenti. $y=f(x)$ funksiyaga x_0 nuqtada o‘tkazilgan urinmaning tenglamasi $y=f(x_0)+f'(x_0)(x-x_0)$ ga teng.

Funksiyani tekshirib, uning grafigini chizish quyidagi tartibda bajariladi.

1. Funksiyaning aniqlanish sohasi topiladi.
2. Funksiyaning hosilasi olinadi.
3. Funksiyaning kritik nuqtalari topiladi.
4. Funksiyaning kritik nuqtalaridagi qiymatlari topiladi.
5. O‘sish va kamayish oraliqlari topilib, jadvalga kiritiladi.
6. Funksiyaning ekstremumlari topiladi.
7. Topilganlar bo‘yicha jadval to‘ldirilib, grafik chiziladi.
8. Funksiya grafigining koordinata o‘qlari bilan kesishish nuqtalarini topish (bunda $f(0)$ va $f(x)=0$ topiladi).

Masalan, $f(x)=3x^5-5x^3+2$ funksiyani tekshiramiz.

1) Funksiyani aniqlanish sohasi – $D(f(x)=R$.

2) $f'(x)=15x^4-15x^2$;

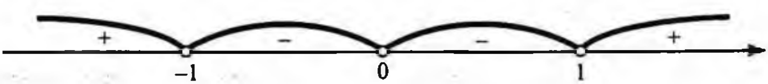
3) $f'(x)=0$ topiladi, $15x^4-15x^2=0$, bundan $x_1=0$; $x_{3/4}=\pm1$.

Kritik nuqtalar: –1; 0; 1.

4) Funksiyaning kritik qiymatlari:

$f(-1)=3\cdot(-1)^5-5\cdot(-1)^3+2=4$; $f(0)=2$; $f(1)=0$.

5) O‘sish va kamayish oraliqlari:



6) Jadval chiziladi va to‘ldiriladi.

x	(–∞; –1[	–1	]–1; 0[	0	]0; 1[	1	]1; +∞)
$f'(x)$	+	0	–	0	–	0	+
$f(x)$	↗	4	↘	2	↘	0	↗
Ekstremum		max		–		min	

7. Grafikni OY o'qi bilan kesishgan $f(0)=2$ nuqtasi topiladi.

8. Topilgan ma'lumotlar asosida grafik yasaladi (42-chizma).

Funksiyaning eng katta va eng kichik qiymatlari quyidagicha topiladi:

1) Berilgan kesmadagi kritik nuqtalar topiladi;
2) Funksiyaning kritik nuqtalardagi qiymatlari topiladi;
3) Berilgan kesmaning chetlaridagi qiymatlari topiladi;
4) Topilgan qiymatlarning eng kattasi va eng kichigi olinadi.

Ta'rif. **Agar berilgan oraliqda olingan barcha x lar uchun $F'(x)=f(x)$ tenglik bajarilsa, u holda $F(x)$ funksiya shu oraliqda $f(x)$ funksiyaning boshlang'ich funksiyasi bo'ladi.**

Ba'zi funksiyalarning boshlang'ich funksiyalari jadvali.

Funksiya $f(x)$	k	$x^n (n\neq-1)$	$\sin x$	$\cos x$	$\frac{1}{\cos^2 x}$	$\frac{1}{\sin^2 x}$
Boshlang'ich funksiya $F(x)$	$kx+c$	$\frac{x^{n+1}}{n+1}+c$	$-\cos x+c$	$\sin x+c$	$\operatorname{tg}x+c$	$-\operatorname{ctg}x+c$

Boshlang'ich funksiyalarni topishning uch qoidasi:

1. Agar $F(x)$ funksiya $f(x)$ ning, $H(x)$ funksiya $h(x)$ ning boshlang'ich funksiyasi bo'lsa, u holda $F(x)+H(x)$ yig'indi $f(x)+h(x)$ ning boshlang'ich funksiyasi bo'ladi.

2. Agar $F(x)$ funksiya $f(x)$ ning boshlang'ich funksiyasi, k o'zgarmas bo'lsa, $kF(x)$ funksiya $kf(x)$ ning boshlang'ich funksiyasi bo'ladi.

3. Agar $F(x)$ funksiya $f(x)$ ning boshlang'ich funksiyasi, k va b o'zgarmaslar (bunda $k\neq0$ bo'lsa, $\frac{1}{k}F(kx+b)$ funksiya $f(kx+b)$ning boshlang'ich funksiyasi bo'ladi.

Egri chiziqli trapetsiyaning yuzi:

$S=\int_a^b f(x)dx=F(b)-F(a)$. – **Nyuton–Leybnits formulasi.**

O'zgaruvchi kuchning ishi $A=\int_a^b f(x)dx$ formulada $f(x)=F$ kuchni ifodalovchi funksiya. ($A=F\cdot S$ formulaga asosan).

Egri chiziqli trapetsiyani absissalar o'qi atrofida aylantirish natijasida hosil bo'lgan jismning hajmi $V=\pi\int_a^b f^2(x)dx$ formula bilan topiladi.

1) Silindrning hajmi $V_s=\pi R^2H$ (bunda R – silindr asosining radiusi, H – balandligi);

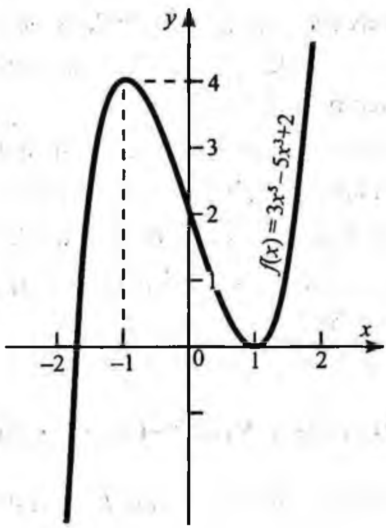
2) Konusning hajmi $V_k=\frac{1}{3}\pi R^2H$ (R – konus asosining radiusi, H – balandligi);

3) Kesik konusning hajmi $V_{k.k}=\frac{1}{3}\pi H(R^2+r^2+Rr)$. ($R$ – kesik konus ostki asosining radiusi, r – ustki asosining radiusi, H – balandligi);

4) Sharning hajmi $V_{sh.}=\frac{4}{3}\pi R^3$ (R – shar radiusi);

5) Shar segmentining hajmi $V_{seg}=\frac{1}{3}\pi H^2(3R-H)$ (bunda R – shar radiusi, H – segment balandligi);

6) Shar sektorining hajmi $V_{seg}=\frac{2}{3}\pi R^2H$ (bunda R – shar radiusi, H – segment balandligi).



79-chizma.

MATEMATIKA (arifmetika, algebra) TESTLARIDAN NAMUNALAR

1. Ushbu $y=\frac{-x^2+4x+3}{2}$ funksiyaning qiymatlar sohasini toping.

A) $(0;\ \infty)$; B) $(-\infty;\ 1{,}5]$; C) $(-\infty;\ 3{,}5]$ D) $[-2;\ \infty)$.

2. a ning qanday qiymatlarida $y=9x^2-12x-35a$ parabola absissalar o'qi bilan ikkita umumiy nuqtaga ega bo'ladi?

A) $a>\frac{18}{35}$; B) $a>-\frac{18}{35}$; C) $a<\frac{18}{35}$; D) $a>-\frac{4}{35}$.

3. a ning qanday qiymatida $y=ax^2-3x-5$ funksiya $x=-3{,}75$ nuqtada eng kichik qiymatga ega bo'ladi?

A) –0,4; B) 0,4; C) –0,5; D) 1,2.

4. Ushbu $y=\frac{3}{2-x}-1$ funksiyaga teskari funksiyani toping.

A) $y=\frac{x-2}{3}+1$; B) $y=\frac{3}{x+1}+2$; C) $y=2-\frac{3}{x+1}$; D) $y=\frac{1}{3}x-2$.

5. Quyidagilardan qaysi biri $y=\frac{x^2+3x-10}{x^2-x-2}$ funksiyaga teskari funksiya?

A) $y=\frac{5-x}{x-1}$; B) $y=\frac{x-5}{x+1}$; C) $y=1-\frac{4}{x-1}$; D) $y=\frac{x+5}{x-1}$.

6. Quyidagilardan qaysilari o'suvchi funksiyalar?

1) $y=3^{-x}$; 2) $y=0{,}2^{-x}$; 3) $y=\left(\sqrt[5]{10}\right)^x$; 4) $y=\left(\frac{5}{3}\right)^{2x}$; 5) $y=(2{,}3)^{-x}$.

A) 2; 3; 5; B) 2; 3; 4; C) 2; 5; D) 1; 2; 4.

7. Tenglamani yeching $(0{,}75)^{x-1}=\left(1\frac{1}{3}\right)^3$.

A) –1; B) 1; C) –2; D) 2.

8. Tenglamaning ildizlarining yig‘indisini toping.

$3\sqrt[x]{81}-10\sqrt[x]{9}+3=0$

A) –2; B) 0; C) 4; D) 5.

9. x son $4\sqrt[x]{81}-12\sqrt[x]{36}+9\sqrt[x]{16}=0$ tenglamaning ildizi bo‘lsa, $x-3$ ifoda nimaga teng.

A) 2; B) 1; C) 0; D) –1.

10. Tengsizlikning eng kichik butun yechimi 10 dan qancha kam?

$0{,}6^{x^2}\cdot 0{,}2^{x^2}>(0{,}12^{x})^4.$

A) 6 ta, B) 8 ta, C) 10 ta, D) 12 ta.

11. Tengsizlikni yeching. $0{,}2^{x^2+1}+0{,}2^{x^2-1}<1{,}04.$

A) $(-\infty;-1]\cup[1;\infty)$; B) $(-\infty;-1)\cup(1;\infty)$;

C) $(-\infty;-1]\cup(1;\infty)$; D) $[-\infty;1)$.

12. $\left(\cos\frac{\pi}{3}\right)^{x-0{,}5}>\sqrt{2}$ tengsizlikni yeching.

A) $(0;\infty)$; B) $(-\infty;0)$; C) $(0;0{,}5)$; D) $(0;1)$.

13. Argumentning nechta butun qiymati $f(x)=\frac{\sqrt{8-x}}{\lg(x-1)}$ funksiyaning aniqlanish sohasiga tegishli?

A) 4; B) 5; C) 6; D) 8.

14. $f(x)=\log_5\left(81^{-x}-3^{x^2+3}\right)$ funksiyaning aniqlanish sohasini toping.

A) $(0;\infty)$; B) $(-\infty;1)\cup(3;\infty)$; C) $(1;3)$; D) $(-3;-1)$.

15. Ushbu $y=-\log_5 x$ funksiyaning grafigi koordinatalar tekisligining qaysi choraklarida yotadi?

A) I; III; B) I; IV; C) I; II; D) II; III.

16. $\frac{\log_9 12}{\log_{36} 3}-\frac{\log_9 4}{\log_{108} 3}$ ni hisoblang.

A) 1; B) 2; C) 3; D) 4.

17. Ifodaning qiymatini toping. $49^{1-\log_7 2}+3^{-\log_3 4}$

A) 10; B) 12,5; C) 14; D) 25.

18. Tenglamani yeching. $(2x)^{\log_{2x}(x+4,5)^2}=25$.

A) –9,5; B) –0,5; –9,5; C) 0,5; D) yechimi yo‘q.

19. Tenglamani yeching. $\log_{\sqrt{2}} x+\frac{2}{\log_x 2}=4$

A) 2; B) –1; C) 3; D) 4.

20. Tenglamani yeching.

$\log_{\sqrt{5}} x+\log_{\sqrt[3]{5}} x+\log_{\sqrt[6]{5}} x+...+\log_{\sqrt[20]{5}} x=55$.

A) 1; B) 5; C) –5; D) $\sqrt{5}$.

21. Tenglamani yeching.

$\log_3 x+\log_3 x^2+\log_3 x^3+...+\log_3 x^8=54$.

A) –3; B) $\sqrt{2}$; C) $\sqrt{3}$; D) $3\sqrt{3}$.

22. Tenglamani yeching $\sqrt{1+\log_3\sqrt{x}}\cdot\log_x 9+\sqrt{2}=0$.

A) $\frac{1}{3}$; B) $\frac{1}{3}$; 9; C) 9; D) $\frac{1}{3}$; 3.

23. $\log_5(5-2x)\leq 1$ tengsizlikni yeching.

A) (–∞; –2,5); B) (0; 2,5); C) [0; 2,5) D) (–∞; 2,5].

24. Tengsizlikning yechimi bo‘lgan kesma o‘rtasining koordinatasini toping.

$\log_{0,3}(2x^2+4)\geq\log_{0,3}(x^2+20)$.

A) –2; B) –1; C) –0,3; D) 0.

25. Funksiyaning aniqlanish sohasini toping.

$\log_2\log_{0,5}\sqrt{4x-4x^2}$.

A) $\left(0;\frac{1}{2}\right)\cup\left(\frac{1}{2};1\right)$; B) (–1; 0); C) $(-\infty;0)\cup(1;\infty)$; D) $\left(-\frac{1}{2};\frac{1}{2}\right)$.

26. Nechta butun son $\begin{cases} \log_2 x^2 \geq 2 \\ \log_5 x^2 \leq 2 \end{cases}$ tengsizliklar sistemasini qanoatlantiradi?

A) 6; B) 7; C) 8; D) 10.

27. Qaysi javobda sin 910°, tg 220° va cos(–440°) larning ishoralari yozilish tartibida berilgan?

A) +; –; +; B) –; +; +; C) –; –; +; D) +; +; –.

28. $\sin 75° + \sqrt{3}\cos 75°$ ni hisoblang.

A) $\frac{\sqrt{3}}{2}$; B) $\frac{\sqrt{2}}{2}$; C) $\sqrt{3}$; D) $\sqrt{2}$.

29. Soddalashtiring.
$\sin^6\alpha + \cos^6\alpha + 3\sin^2\alpha \cdot \cos^2\alpha$.

A) –1; B) 0; C) 1; D) 2.

30. $\frac{\sin(\pi+\alpha)}{\sin\left(\frac{3\pi}{2}+\alpha\right)} + \frac{\cos(\pi-\alpha)}{\cos\left(\frac{\pi}{2}+\alpha\right)-1}$ ni soddalashtiring.

A) $\frac{1}{\sin\alpha}$; B) $\frac{1}{\cos\alpha}$; C) $-\cos\alpha$; D) –1.

31. Soddalashtiring. $\frac{\sin 56° \cdot \sin 124° - \sin 34° \cdot \cos 236°}{\cos 28° \cdot \sin 88° + \sin 178° \cdot \cos 242°}$

A) $\frac{1}{\sin 20°}$; B) tg28°; C) $-\frac{\sqrt{3}}{2}$; D) $\frac{2}{\sqrt{3}}$.

32. $\frac{\sin\alpha + 2\sin 2\alpha + \sin 3\alpha}{\cos\alpha + 2\cos 2\alpha + \cos 3\alpha}$ ni soddalashtiring.

A) $\mathrm{tg}2\alpha$; B) $\mathrm{tg}2\alpha + 1$; C) $\mathrm{tg}\alpha - 1$; D) $\mathrm{ctg}2\alpha$.

33. tg555° – ctg330° ni hisoblang.

A) $\sqrt{3}$; B) $\sqrt{3}-1$; C) 2; D) $2-\sqrt{3}$.

34. $2\cos^2 x-1=-\frac{1}{2}$ tenglamani yeching.

A) $\pm\frac{\pi}{6}+k\pi,\ k\in z;$ B) $\pm\frac{\pi}{3}+k\pi,\ k\in z;$

C) $\pm\frac{2\pi}{3}+k\pi,\ k\in z;$ D) $(-1)^k\frac{\pi}{6}+k\pi,\ k\in z.$

35. Tenglamani yeching. $\sin x\cdot\cos 3x+\cos x\cdot\sin 3x=1.$

A) $\frac{\pi}{4}n;\ n\in z;$ B) $\frac{\pi}{8}n;\ n\in z;$

C) $\frac{\pi}{2}+\frac{\pi}{8}n;\ n\in z;$ D) $\frac{\pi}{8}+\frac{1}{2}n\pi,\ n\in z.$

36. Tenglamani yeching. $\cos 2x-5\sin x-3=0.$

A) $(-1)^k\frac{\pi}{6}+2k\pi,\ k\in z;$ B) $(-1)^n\frac{\pi}{6}+k\pi,\ k\in z;$

C) $(-1)^{k+1}\frac{\pi}{6}+k\pi,\ k\in z;$ D) $(-1)^k\frac{\pi}{3}+k\pi,\ k\in z.$

37. $4^{\log_4(\sqrt{3}\cos x)}+5^{\log_5\sqrt{6}}=7^{\log_7(3\sin x)}$

A) $\frac{\pi}{4}+2n\pi,\ n\in z;$ B) $\pm\frac{3\pi}{4}-\frac{\pi}{3}+2k\pi,\ k\in z;$

C) $\frac{5\pi}{12}+2n\pi,\ n\in z;$ D) $\frac{7\pi}{12}+2n\pi,\ n\in z.$

38. $1-\sin 5x=\left(\cos\frac{3x}{2}-\sin\frac{3x}{2}\right)^2$ tenglamaning [360°; 450°] oraliqdagi ildizlari yig‘indisini toping.

A) 742°; B) 742°30′; C) 765°; D) 810°.

39. Ushbu $2\sin 2x\geq\operatorname{ctg}\frac{\pi}{4}$ tengsizlikni yeching.

A) $\left[\frac{\pi}{12}+n\pi;\ \frac{\pi}{12}+n\pi\right]n\in z;$ B) $\left[\frac{\pi}{6}+2n\pi;\ \frac{5\pi}{6}+2n\pi\right]n\in z;$

C) $\left[\frac{\pi}{12}+n\pi;\ \frac{5\pi}{12}+n\pi\right]n\in z;$ D) $\left[-\frac{\pi}{3}+2n\pi;\ \frac{\pi}{3}+2n\pi\right]n\in z.$

40. $1-2\cos 2x > \sin^2 2x$ tenglamani yeching.

A) $\left(\frac{\pi}{2}+n\pi;\ \pi+n\pi\right) n\in z$;   B) $\left(\frac{\pi}{3}+2n\pi;\ \frac{2\pi}{3}+2n\pi\right) n\in z$;

C) $\left(\frac{\pi}{4}+n\pi;\ \frac{3\pi}{4}+n\pi\right) n\in z$;   D) $\left(-\frac{\pi}{2}+n\pi;\ \frac{\pi}{2}+n\pi\right) n\in z$.

41. $[0;\ 2\pi]$ kesmaga tegishli nechta nuqta $y=\ln\left(2\sin 3x+3\cos 2x-\frac{17}{3}\right)$ funksiyaning aniqlanish sohasiga tegishli?

A) $\varnothing$,   B) 1;   C) 2;   D) 3.

42. $(\sin x-\cos x)^2<\sin 2x$ tengsizlikni yeching.

A) $\left(\frac{\pi}{6}+n\pi;\ \frac{5\pi}{6}+n\pi\right) n\in z$;   B) $\left(\frac{\pi}{3}+n\pi;\ \frac{2\pi}{3}+n\pi\right) n\in z$;

C) $\left(\frac{\pi}{12}+n\pi;\ \frac{5\pi}{12}+n\pi\right) n\in z$;   D) $\left(-\frac{7\pi}{6}+2n\pi;\ \frac{7\pi}{6}+2n\pi\right) n\in z$.

43. Agar $f(x)=x^3+5x^2+4x+2$ bo‘lsa, $f'(x)=f(1)$ tenglamaning eng kichik ildizini toping.

A) –6;   B) $-\frac{1}{3}$;   C) –4;   D) –6.

44. Agar $f(x)=l^{1-2x}\cdot\cos(2x-1)$ bo‘lsa, $f'\left(\frac{1}{2}\right)$ ning qiymatini toping.

A) $-2l$,   B) –2;   C) 0;   D) $2l$.

45. Agar $f(x)=\frac{8x\sqrt{x}+2}{x}$ bo‘lsa, $f'(1)$ni toping.

A) 2;   B) 4;   C) 0^x;   D) 10.

46. Ushbu $f(x)=\ln(x^2-3\sin x)$ funksiyaning hosilasini toping.

A) $\frac{2x}{x^2-3\sin x}$;   B) $\frac{3}{x^2-3\sin x}$;   C) $\frac{2x-3\cos x}{x^2-3\sin x}$;   D) $\frac{2x+3\cos x}{x^2-3\sin x}$.

47. $y=\log_2(4x)+\cos(x^2+3x)$ funksiyaning hosilasini toping.

A) $\frac{\ln 2}{x}-\sin(x^2+3x)(2x+3)$;   B) $\frac{1}{4x}-\sin(x^2+3x)(2x+3)$;

C) $\frac{1}{4x\ln 2}+(2x+3)\sin(x^2+3x)$; D) $\frac{1}{x\ln 2}-(2x+3)\sin(x^2+3x)$.

48. Ushbu $y=-\frac{1}{3}x^3-x^2+3x+5$ funksiyaning o'sish oraliqlarini toping.

A) $(-\infty; -1]$ va $[3; \infty)$; B) $[-1; 3]$;
C) $[-3; 1]$; D) $[1; 3]$.

49. $f(x)=\frac{x^2}{2}-12\ln(x-4)$ funksiyaning kamayish oraliqlarini toping.

A) $[6; \infty)$; B) $(4; 6]$; C) $(-2; 4)$; D) $(4; \infty)$.

50. Ushbu $f(x)=\frac{x^3}{3}-\frac{x^2}{2}-6x$ funksiyaning maksimum nuqtasidagi qiymatini toping.

A) 13,5; B) $12\frac{1}{3}$; C) $7\frac{1}{3}$; D) –3,5.

51. $y=\sqrt{x^2-6x+11}$ funksiyaning qiymatlari sohasini toping.

A) $(0; \infty)$; B) $[0; 11]$; C) $[\sqrt{2}; \infty)$; D) $(2; \infty)$.

52. $f(x)=-2x^3+18x^2+12$ funksiya o'sadigan kesmaning uzunligini aniqlang.

A) 4; B) 4,5; C) 6; D) 6,5.

53. Ushbu $g(x)=12x-x^3$ funksiyaning maksimumini toping.

A) –16; B) 16; C) 0; D) 24.

54. $f(x)=2^x+2^{2-x}$ funksiyaning $[0; 2]$ kesmadagi eng kichik qiymatini toping.

A) 2; B) 2,5; C) 3; D) 4.

55. Ushbu $f(x)=\frac{\sqrt{3}}{3}(x^3-1)$ funksiyaning grafigiga $x_0=1$ nuqtada o'tkazilgan urinmaning OX o'qi bilan hosil qilgan burchagini toping.

A) 30°; B) 45°; C) 60°; D) 120°.

56. $f(x)=\log_3(2x+1)$ funksiya grafigiga absissasi $x_0=1$ nuqtasida o‘tkazilgan urinma tenglamasini toping.

A) $y=\frac{2x}{3\ln 3}-\frac{2}{3\ln 3}+1$; B) $y=-x\ln 3$;

C) $y=-x\ln 3-1$; D) $y=\frac{x}{2\ln 3}+1$.

57. Ushbu $2\sin 3x$ funksiya uchun boshlang‘ich funksiyaning umumiy ko‘rinishini toping.

A) $-\frac{3}{2}\sin 3x+c$; B) $-\frac{2}{3}\cos 3x+c$; C) $\frac{3}{2}\cos 3x+c$; D) $\frac{2}{3}\sin 3x+c$.

58. Ushbu $F(x)=e^x-\frac{1}{3}\sin 3x+\mathrm{ctg}x+c$ funksiya quyidagi funksiyalardan qaysi birining boshlang‘ich funksiyasi?

A) $f(x)=e^x+\cos 3x-\frac{1}{\sin 2x}$; B) $f(x)=e^x+\cos 3x+\frac{1}{\sin^2 x}$;

C) $f(x)=e^x-\cos 3x-\frac{1}{\sin^2 x}$; D) $f(x)=e^x+\cos 3x+\frac{1}{\sin^2 x}$.

59. Ushbu $f(x)=\frac{1}{2\sqrt{(1-x)^3}}$ funksiyaning boshlang‘ich funksiyasini toping.

A) $F(x)=\frac{1}{\sqrt{1-x}}+C$; B) $F(x)=\frac{1}{4\sqrt{1-x}}+C$;

C) $F(x)=-\frac{2}{\sqrt{1-x}}+C$; D) $F(x)=\frac{3}{2\sqrt{1-x}}+C$.

60. Agar $F'(x)=e^x+\sin 2x$ va $F(0)=3{,}5$ bo‘lsa, $F(x)$ni toping.

A) $F(x)=e^x+\cos 2x+3$; B) $F(x)=e^x-\frac{1}{2}\cos 2x+3$;

C) $F(x)=e^x-\frac{1}{2}\cos x+4$; D) $F(x)=e^x-\frac{1}{2}\cos 2x-4$.

61. Hisoblang. $\int_{-1}^{0}(1+3x)^2\,dx$

A) -1; B) 0; C) 1; D) $-\frac{1}{3}$.

62. Integralni hisoblang.

$$\int_{0}^{\frac{\pi}{18}}(\cos x\cdot\cos 2x-\sin x\cdot\sin 2x)dx$$

A) $\frac{1}{2}$; B) $\frac{1}{6}$; C) $\frac{1}{3}$; D) $\frac{2}{3}$.

63. Hisoblang. $\int_{0}^{2\pi}\sin^4 x dx$

A) $\frac{3\pi}{4}$; B) $\frac{\pi}{4}$; C) $\frac{\pi}{8}$; D) $\frac{3\pi}{2}$.

64. $\int_{1}^{2}\frac{x}{x+1}dx$ ni hisoblang.

A) $1-\ln\frac{2}{3}$; B) $1+\ln\frac{2}{3}$; C) $3-\ln\frac{2}{3}$; D) $2-\ln\frac{2}{3}$.

65. $\int_{3}^{4}\frac{dx}{x^2-1}$ ni hisoblang.

A) $\ln\sqrt{\frac{2}{3}}$; B) $\ln\sqrt{\frac{3}{4}}$; C) $\ln\sqrt{\frac{15}{8}}$; D) $\ln\sqrt{\frac{6}{5}}$.

66. $\int_{0}^{1}\sqrt{x\sqrt[3]{x\sqrt[4]{x}}}\,dx$ ni hisoblang.

A) $\frac{17}{24}$; B) $\frac{8}{15}$; C) $\frac{24}{41}$; D) $\frac{12}{29}$.

67. $\int_1^2 \frac{x+1}{(2x-1)^3}dx$ ni hisoblang.

(Ko'rsatma. $\int_1^2 \frac{x+1}{(2x+1)^3}dx = \int_1^2 \frac{2x-1+3}{2(2x-1)^3}dx = \int_1^2 \frac{dx}{2\cdot(2x-1)^2} + \int_1^2 \frac{3dx}{2\cdot(2x-1)^3}$

integrallar hisoblanadi). $x+1 = \frac{1}{2}(2x-1+y)$ yozib, $\frac{1}{2}(-1+y)=1$ dan $y=3$ topiladi

A) $\frac{3}{8}$; B) $\frac{1}{2}$; C) $\frac{2}{3}$; D) $\frac{5}{2}$.

68. $y=x^2$ va $y=2x$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.

A) $1\frac{1}{3}$; B) 1; C) $1\frac{1}{4}$; D) $2\frac{1}{2}$.

69. Quyidagi chiziqlar blan chegaralangan figuraning yuzini toping. $y=\frac{1}{\sqrt{x}}, y=0, x=1, x=4.$

A) 5; B) 3; C) 2; D) 1,5.

70. $y=\sin 2x$, $y=0$, $x=0$ va $x=\frac{\pi}{2}$ chiziqlar bilan chegaralangan figuraning yuzini hisoblang.

A) $\frac{1}{2}$; B) 1; C) 2; D) $\sqrt{2}$.

71. $y=9-x^2$, $y=x^2+1$ va $x=0$ chiziqlar bilan chegaralangan sohaning yuzini toping.

A) $10\frac{1}{3}$; B) $10\frac{2}{3}$; C) 13; D) 24.

72. Agar $(x^3-x+1)^3+x$ ifoda standart shakldagi ko'phad ko'rinishida yozilsa, x ning toq darajalari oldidagi koeffitsiyentlarning yig'indisini toping.

A) 1; B) 3; C) 4; D) 7.

73. $\frac{x^2-x+1}{x^4+x^2+1}$ kasrni qisqartiring.

A) $\frac{1}{x^2-2x+1}$; B) $\frac{1}{x^2-x+1}$; C) $\frac{1}{x^2+x+1}$; D) $\frac{1}{x^2+2x-1}$.

74. $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+...+\frac{1}{\sqrt{1599}+\sqrt{1600}}$ ning qiymatini toping.

A) 28; B) 39; C) 41; D) 56.

75. $\sqrt{3\sqrt{3\sqrt{3\sqrt{3}}}}=9^{\frac{5}{4}x}$ tenglamani yeching.

A) $\frac{5}{8}$; B) $\frac{9}{20}$; C) $\frac{7}{20}$; D) $\frac{3}{8}$.

76. $x^3+2x^2-9x-18=0$ tenglamaning ildizlari yig'indisini toping.

A) 9; B) –2; C) 2; D) 6.

77. $5n^3-5n$ ifoda istalgan natural n da quyidagi sonlardan qaysi biriga qoldiqsiz bo'linadi?

A) 22; B) 25; C) 30; D) 60.

78. Tenglamaning natural sonlardagi yechimida y nimaga teng. $x+\frac{1}{y+\frac{1}{2}}=\frac{17}{15}$

A) 2; B) 3; C) 5; D) 7.

79. $y=x^2$ parabolaga $A(3;\ 5)$ nuqtadan o'tuvchi urinmalar tenglamasini toping.

A) $y=2x+1$ va $y=10x-25$; B) $y=2x-1$ va $y=10x-25$;

C) $y=2x-1$ va $y=10x+25$; D) $y=2x+1$ va $y=10x+25$.

80. Agar $\begin{cases} x^3-3x^2y=y^3+100 \\ 3xy^2=25 \end{cases}$ bo'lsa, $\frac{x-y}{10}$ ni hisoblang.

A) $-\frac{1}{3}$; B) $\frac{1}{2}$; C) 1; D) $\left(1\frac{1}{3}\right)$.

81. Agar $a^2+3ab+b^2=44$ va $a^2+ab+b^2=28$ bo'lsa, a^2-ab+b^2 ning qiymatini toping.

A) 12; B) 16; C) 18; D) 20.

82. $x^3-7x-6=0$ tenglamaning barcha haqiqiy ildizlari o'rta geometrigini toping.

A) $\sqrt{6}$; B) $2\sqrt{2}$; C) $\sqrt[3]{6}$; D) $-\sqrt[3]{6}$.

83. 72 va 96 sonlarining eng kichik umumiy karralisining eng katta umumiy bo'luvchisiga nisbatini toping.

A) 10; B) 0,1; C) 12; D) 9.

84. $(x^2+x-4)(x^2+x+4)=9$ tenglama ildizlarining ko'paytmasini toping.

A) 4; B) –5; C) 6; D) 5.

85. Agar a va b sonlari $x^2-8x+7=0$ kvadrat tenglamaning ildizlari bo'lsa, $\frac{1}{a^2}+\frac{1}{b^2}$ ni hisoblang.

A) $1\frac{1}{49}$; B) $1\frac{1}{50}$; C) $1\frac{1}{7}$; D) $2\frac{1}{49}$.

86. Ushbu $|3-|2+x||=1$ tenglamaning ildizlari ko'paytmasini toping.

A) 12; B) –12; C) –6; D) 0.

87. Hisoblang. $(\text{tg}60°\cos15°-\sin15°)\cdot7\sqrt{2}$.

A) $3\sqrt{2}$; B) 7; C) 14; D) $14\sqrt{2}$.

88. Funksiyaning boshlang'ich funksiyasining umumiy ko'rinishini yozing $y'=\frac{5}{3\sqrt{2x+7}}$.

A) $\frac{3}{5}\sqrt{2x+7}+C$; B) $5\sqrt{2x+7}+C$; C) $\frac{5}{3}\sqrt{2x+7}+C$; D) $\frac{15}{\sqrt{2x+7}}+C$.

89. $\int_{-\pi}^{\pi} \sin 5x \cdot \cos 3x dx$ integralni hisoblang.

A) 1; B) –2; C) 0; D) –1.

90. Hisoblang: $\dfrac{2\sin 54°+3\cos 36°-2\cos 144°}{\sin 70°\cdot\sin 74°-\sin 20°\cdot\sin 16°}$

A) 7; B) 3; C) 14; D) –7.

91. Hisoblang: $\sin 49\cdot\sin 11°+\cos^2 71°+1$.

A) –1; B) 0,5; C) 1,25; D) 2.

92. Agar $2\text{tg}\alpha-\sin\alpha+5\cos\alpha=10$ bo'lsa, $\text{tg}\alpha$ ni toping.

A) 2; B) 2,5; C) 10; D) 5.

93. Hisoblang: $2\arcsin\frac{1}{2}+\arccos\left(-\frac{1}{2}\right)+3\text{arctg}\frac{\sqrt{3}}{3}+\text{arcctg}(-1)$.

A) –1; B) 0,5; C) 2; D) .

94. $\text{tg}\left(\arcsin\frac{1}{4}\right)$ ni hisoblang.

A) –2; B) $\frac{2}{\sqrt{3}}$; C) $\sqrt{13}$; D) $\frac{1}{\sqrt{15}}$.

95. Hisoblang: $\cos\left(\arcsin\frac{12}{13}+\arcsin\frac{3}{5}\right)$.

A) $\frac{16}{65}$; B) $-\frac{16}{65}$; C) $\frac{56}{65}$; D) $\frac{51}{65}$.

96. Agar $x=0,6$ va $y=0,8$ bo'lsa, $\sin(\arcsin x-\arcsin y)$ ni hisoblang.

A) 0; B) 1; C) 0,5; D) 0,28.

Ko'rsatma. $\arcsin\left(-\frac{3}{5}\right)$ ni ark tangens orqali ifodalang (bunda $-\frac{\pi}{2}<\alpha<0$).

Ko'rsatma. $\arcsin\left(-\frac{3}{5}\right)=\alpha$ bo'lsin, bundan $\sin\alpha=-\frac{3}{5}$. $\cos\alpha=\pm\sqrt{1-\frac{9}{25}}=\frac{4}{5}$.

$$\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{4}; \quad \operatorname{tg}\alpha = -\frac{3}{4}$$ bundan $\alpha = \operatorname{arctg}\left(-\frac{3}{4}\right)$. Demak,

$\arcsin\left(-\frac{3}{5}\right) = \operatorname{arctg}\left(-\frac{3}{4}\right)$.

97. $\arccos\frac{\sqrt{5}}{3}$ ni ark kotangens orqali ifodalang.

A) $\arccos\frac{\sqrt{5}}{3} = \operatorname{arcctg}\left(-\frac{2}{3}\right)$; B) $\arccos\frac{\sqrt{5}}{3} = \operatorname{arcctg}\frac{2}{3}$;

C) $\arccos\frac{\sqrt{5}}{3} = \operatorname{arcctg}\frac{5}{\sqrt{3}}$; D) $\arccos\frac{\sqrt{5}}{3} = \operatorname{arcctg}\left(-\frac{5}{\sqrt{3}}\right)$.

98. $\arccos x - \arcsin x = \arccos\frac{\sqrt{2}}{2}$ tenglamani eching.

A) 1; B) $\frac{1}{2}$; C) $\frac{\sqrt{2}}{3}$; D) $\frac{\sqrt{3}}{2}$.

99. $3\arccos(x^2-7x+6{,}5)=$ tenglamaning ildizlari yig'indisini toping.

A) –7; B) –1; C) 3,5; D) 7.

4. d) $\frac{7\pi}{12}$; e) $\frac{3\pi}{4}$; f) $\frac{3\pi}{4}$; g) π. **5.** d) $\frac{\pi}{24}+(-1)^{k+1}\frac{\pi}{24}+\frac{k\pi}{4}, k\in z$; e) $\frac{\pi}{8}+(-1)^{k+1}\frac{\pi}{6}+\frac{k\pi}{2}$, $k\in z$; h) $\pm\frac{3\pi}{4}+6k\pi$, $k\in z$; i) $\frac{1}{2}\pm\frac{\pi}{3}+k\pi$, $k\in z$. **6.** b) $\frac{\pi}{6}+\frac{k\pi}{2}$, $k\in z$; d) $\frac{1}{3}x_0+\frac{k\pi}{3}$, $x_0=\text{arctg } 3{,}5$, $k\in z$; f) $\frac{\pi}{2}+2k\pi$, $k\in z$; g) $\frac{5\pi}{2}+5k\pi$, $k\in z$. **8.** d) $2k\pi$; $\pm\frac{\pi}{3}+2k\pi$; e) $\pm\frac{\pi}{2}+k\pi$; $(-1)^k\arcsin 0{,}75+k\pi$, $k\in z$. **9.** d) $-\frac{\pi}{4}+k\pi$; $59°2'+180°k$, $k\in z$; e) $\frac{\pi}{4}+k\pi$, $71°34'+180°k$, $k\in z$. **10.** d) $\frac{\pi}{2}+k\pi$, $-\frac{\pi}{4}+k\pi$, $k\in z$; e) $(-1)^k\pm\frac{\pi}{6}+k_1\pi$, $\frac{\pi}{2}+2k_2\pi$, $k_3\pi$, $k\in z$. **11.** d) $2k\pi$, $\pi+4k\pi$, $k\in z$; e) $(-1)^k\arcsin\frac{1}{3}+k\pi$, $k\in z$. **12.** a) $\frac{\pi}{2}+k\pi$, $k\in z$. b) $\pm\frac{\pi}{3}+2k\pi$, $k\in z$; d) $k\pi$, $k\in z$; e) $\frac{\pi}{2}+k\pi$, $k\in z$. **13.** b) $\frac{\pi}{8}(4k+1)$; d) $\frac{\pi}{2}+k\pi$; e) $\frac{\pi}{2}+k\pi$, $\pm\frac{\pi}{3}+k\pi$, $k\in z$. **14.** a) $\frac{\pi}{6}+k\pi$; b) $(-1)^k\frac{\pi}{4}-\frac{\pi}{6}+k\pi$; d) $\frac{\pi}{2}+\varphi+2k\pi$; $\varphi=\arccos\frac{12}{13}$, $k\in z$; e) $(-1)^{k+1}\frac{\pi}{6}-\varphi+k\pi$, $\varphi=\arccos 0{,}8$ $k\in z$. **15.** a) $\frac{\pi}{3}+k\pi$; b) $59°2'+180°k$ yoki $\text{arctg}\frac{5}{3}+k\pi$, $k\in z$; d) $\frac{\pi}{4}+k_1\pi$, $\text{arctg}7+k_2\pi$, $k\in z$; e) $k\pi$; $\text{arctg } 1{,}875+k\pi$, $k\in z$; f) $\frac{\pi}{4}+k_1\pi$, $\text{arctg}2+k_2\pi$, $k\in z$; g) $\frac{\pi}{4}+k\pi$, $\text{arctg}\frac{1}{2}+k\pi$, $k\in z$. **16.** a) $\frac{\pi}{4}+$ $+\frac{\pi}{2}k_1$, $\frac{\pi}{10}(1+4k_2)$, $\frac{\pi}{2}\cdot(1+4k_3)$, $k\in z$; b) $\frac{\pi}{2}k_1$ $\frac{\pi}{4}k_2$; d) $\pi(2k_1+1)$, $\frac{\pi}{8}(8k_2+1)$; $\frac{\pi}{12}\left((-1)^k+6k\right)$, $k\in z$. **Ko'rsatma 16.** e) $2\sin3x\cdot\cos x-2\sin x\cdot\cos x-$ $-\cos2x=0$. $2\cos x(\sin3x-\sin x)-\cos2x=0$; $2\cos x\cdot2\sin x\cdot\cos2x-\cos2x=0$;

$\cos 2x(2\sin 2x-1)=0$. Bundan: 1) $\cos 2x=0$; 2) $2\sin 2x-1=0$ tenglamalar yechiladi. **18.** d) $\frac{3\pi}{4}+k\pi<x<\frac{3\pi}{2}+k\pi,\ k\in z$; e) $\frac{5\pi}{6}+k\pi<x<\pi(k+1),\ k\in z$; **19.** b) $\frac{5\pi}{6}+2k\pi<x<\frac{7\pi}{6}+2k\pi$; d) $\frac{\pi}{6}+2k\pi\leq x\leq\frac{\pi}{3}+2k\pi$; e) $k\pi<x<\frac{\pi}{4}+k\pi$, $k\in z$. **20.** a) $\frac{\pi}{6}+k\pi<x<\frac{\pi}{2}+k\pi,\ k\in z$; b) $\frac{\pi}{2}+k\pi<x<\frac{3\pi}{4}+k\pi,\ k\in z$; d) 1) $\frac{\pi}{2}+k\pi<x<\frac{3\pi}{4}+k\pi$; 2) $\operatorname{arctg}\frac{1}{4}+k\pi<x<\frac{\pi}{2}+k\pi,\ k\in z$; e) $-\frac{\pi}{6}+k\pi<x<\frac{\pi}{6}+k\pi$, $k\in z$. **23.** a) $x\neq-1$; b) $x\geq0$; d) $x\leq0,5$; e) $x\leq0$, $x\leq1$. **24.** b) kamayuvchi, d) o'suvchi. **25.** e) $\frac{4}{3}$. **26** b) 4; d) 3. **27.** a) –1 va 2; b) $\frac{-1-\sqrt{13}}{2}$ va $\frac{-1\pm\sqrt{13}}{2}$; d) –2 va 3. **28.** a) 3; d) 1. **29.** a) 1 va 3; b) 3 va 11. **30** a) 2; b) –0,5; 1,5. **31.** a) 2; b) ±1. **32.** a) 7; b) 3. **34.** d) $x<2$; e) $x<\frac{4}{3}$. **35.** a) ixtiyoriy son; b) yechimi yo'q; d) $x\neq-1$; e) $x<2$ va $x>4$. **36.** a) $x<2$; b) $1\frac{1}{3}<x<3$. **37.** a) $-1,5<x<0$; b) $x<-\frac{1}{3}$ va $x>\frac{2}{3}$. **38.** a) $0<x<1$ va $x>2$; b) $0<x<1$ va $x>3$. **39.** a) $1,5<x<2$ va $x>4$; b) $-1,5<x<-1$. **40.** a) $-2<x<-1$; b) $1<x<2$. **49.** a) $m=0,01$; b) $m>10$; d) $m<10$; e) $m>10000$. **52.** e) $\frac{4}{3}$; f) –1,5; g) $-\frac{2}{3}$. **53.** a) 2,5; b) –2,5; d) –0,5; e) $1\frac{1}{3}$. **58.** d) $\lg a-2\lg b-\lg 2+1,5\lg\sin\alpha-$ $-0,5\lg\cos\alpha$; e) $-\frac{3}{7}\lg\cos\varphi$. **59.** d) $-\frac{4}{3}(2\lg a+\lg b+3\lg 3)$; e) $\lg 0,8+\frac{3}{4}\lg a$. **60.** a) $\frac{31}{72}\lg a$; b) $\frac{5}{8}(\lg a-\lg b)$. **62.** d) $\sqrt[10]{\frac{81}{7}}$; e) $\frac{b(a+b)^3}{(a-b)^2}$. **63.** a) $\frac{1}{\sqrt{a}}\times$ $\times\sqrt[24]{\frac{b^6(a-b)^4}{a^4(a+b)3}}$; b) $\sqrt[15]{\frac{a^4b^5}{a+b}}$; d) $\operatorname{tg}\varphi$. **67.** a) $\bar{3},1761$; b) 2,8751; d) 30,103. **69.** a) 1,4651; b) $\bar{5},9123$; d) $\bar{3},487$. **70.** a) 2,0554; b) $\bar{6},6784$; d) $\bar{1},7094$. **72.** a) 0,7112; b) 6,3055. **73.** d) 0,4048; e) 0,001061. **74.** e) ≈1,802. **75.**

e) $\approx 1374\cdot 10^6$. **76.** d) $\approx 0{,}04146$. **77.** a) $\approx 1{,}983$; b) $\approx 87{,}98$; d) $\approx 1{,}617$. **80.** d) 5; e) 5. **81.** a) 0,0001 va 10; b) 10 va 100; d) 0,1 va 0,001; e) $\sqrt[10]{10}$ va 100. **82.** a) $\frac{1}{3}$ va 3; b) 5; d) 3; e) 1. **83.** a) 0,001; 100; b) 0,01 va 0,1; d) $\frac{1}{9}$ va 3; e) $\frac{1}{16}$ va 2. **84.** a) 5; b) 1,55; d) 1 va 5; e) $\frac{1}{4}$ va $\frac{1}{2}$. **85.** a) 3; b) a^2. **86.** $\frac{1}{3}$ va 3. **87.** 2,5 va 4. **88.** 10000. **89.** 2. **90.** a) $x=1000$, $y=10$, b) $x=25$, $y=3$; d) $x_1=0{,}1$, $y_1=0{,}01$ va $x_2=100$, $y=10$. **91.** a) $x=4$, $y=2$; b) $x_1=1$, $y_1=7$, va $x_2=7$, $y_2=1$; d) $x_1=125$, $y_1=4$ va $x_2=625$, $y=3$. **92.** a) $x_1=\frac{1}{9}$, $y_1=81$ va $x_2=81$, $y_2=\frac{1}{9}$, b) $x_1=2$; $y_1=4$ va $x_2=4$, $y_2=2$; d) $x_1=16$, $y_1=2$ va $x_2=8\sqrt[3]{2}$, $y_2=2\sqrt[3]{4}$. **93.** a) $x_1=16$, $y_1=9$ va $x_2=9$, $y_2=16$; b) $x=3$, $y=4$. **95.** a) $2<x<5$; b) $7<x<7\frac{1}{16}$; d) $x>6$; e) $-3<x<-2$ va $x>3$. **96.** a) $-3<x<\frac{1-\sqrt{17}}{2}$ va $\frac{1+\sqrt{17}}{2}<x<4$; b) $-3<x<1$; d) $0<x<1$; e) $x<-1$ va $x>2$. **97.** a) $x<0{,}001$ va $x>10$; b) $0{,}01\le x\le 10000$; d) $x<\frac{\sqrt[4]{1000}}{10}$ va $x>\sqrt{10}$; e) $\frac{3+\sqrt{29}}{2}<x<5$. **98.** a) $0{,}5<x<1$; b) $-\frac{1}{3}<x<0$. **99.** $1\frac{1}{3}<x<2\frac{78}{83}$; **100.** $0<x<0{,}001$ va $1<x<10$. **101.** $x<\frac{1-\sqrt{37}}{6}$ va $x<\frac{1+\sqrt{37}}{6}$. **102.** $|x|\le 2$. **103.** a) 0,005; b) $\frac{1}{1500}=0{,}0007$. **104.** a) $\frac{1}{30}$; $\frac{1}{300}$; $\frac{1}{3000}$. **106.** 0. **107.** –10. **108.** 1. **109.** $\frac{1}{2}$. **110.** $\frac{7}{8}$. **111.** –2. **112.** $\frac{1}{4}$. **113.** $\frac{2}{3}$. **114.** 4. **115.** 3. **116.** 5. **117.** 2. **118.** –0,5. **119.** 0. **120.** a) $(-2; -0{,}5)$ va $(3; +\infty)$, b) $\left[-4; \frac{1}{3}\right]$. **121.** $(-\infty; -3]$, $[0; 3]$ va $(4; +\infty)$. **122.** $(-\infty; -3)$, $(1; 2)$ va $(4; +\infty)$. **123.** $(-\infty; -2)$. **124.** $(-\infty; -2]$, $[-1; 1]$ va $[2; +\infty)$. **125.** $[1; \infty)$. **126.** $(-\infty; -1)$,

$(1-\sqrt{2};\ 0)$ va $(1;\ 1+\sqrt{2}]$. **127.** $[3;\ 16]$. **129.** a) $(-\infty;\ 0]$ o‘sadi va $[0;\ +\infty)$ kamayadi; b) $(-\infty;\ 0]$ kamayadi va $[0;\ +\infty)$ o‘sadi. **130.** a) $(-\infty;\ 2]$ kamayadi; $[2;\ +\infty)$ o‘sadi, b) $(-\infty;\ 3]$ o‘sadi, $[3;\ +\infty)$ kamayadi. **131.** a) $(-\infty;\ -1{,}5]$ kamayadi, $[-1{,}5;\ \infty)$ o‘sadi; b) $(-\infty; -1{,}5]$ o‘sadi, $[1{,}5; \infty)$ kamayadi. **132.** a) $\left(-\infty;\ -1\frac{1}{3}\right]$ kamayadi, $\left[-1\frac{1}{3};+\infty\right)$ o‘sadi, b) $(-\infty; -0{,}9]$ o‘sadi; $[-0{,}9; +\infty)$ kamayadi. **133.** a) $(-\infty; 3)$ va $(3;\ +\infty)$ kamayadi; b) $(-\infty;\ -2)$ va $(-2;\ +\infty)$ o‘sadi. **134.** a) $(-2;\ +\infty)$ o‘sadi; b) $(1; +\infty)$ kamayadi. **135.** a) $(-\infty; +\infty)$ kamayadi; b) $(-\infty; +\infty)$ kamayadi. **136.** a) 3,2; 0,4, b) 4,06; 0,12, d) 0,2; e) 0,002. **137.** a) 0,5, 2,25, b) 0,15; 1,1475. **138.** a) $3x_0^2+6x_0\Delta x+3\Delta x^2$; $6x_0\cdot\Delta x+3\Delta x^2$; $6x_0+3\Delta x$; b) $\frac{2\Delta x}{\sqrt{x_0+\Delta x}+\sqrt{x_0}}$.

139. d) $6x$; e) $10x-1$. **141.** a) $\frac{2}{3}$; b) $\frac{1}{3}$; d) $\frac{4}{3}$. **142.** e) $\frac{2}{3\sqrt[3]{x^2}}+\frac{21}{x^4}=\frac{2x^3\sqrt[3]{x}+63}{3x^4}$,

f) $\frac{3}{5\sqrt[5]{x^2}}+\frac{8}{x^3}=\frac{3x^2\sqrt[5]{x^3}+40}{5x^3}$. **143.** a) $10x\sqrt{x}+\frac{5\sqrt[3]{x}}{3x^3}$; b) $5x^4+12x^2-14x$. **144.**

a) $\frac{1}{3}+\frac{7}{x^3}$; b) $\frac{3}{20\sqrt[4]{x}}+\frac{2}{3x\sqrt[3]{x}}+\frac{6}{5}\sqrt[5]{x}$. **145.** a) $\frac{11}{(3-5x)^2}$; b) $\frac{1-x^2}{(1+x^2)^2}$; d) $\frac{4-x}{2\sqrt{x}(4+x)^2}$.

146. a) $3x^2+6x-1{,}5\sqrt{x}+2-1{,}5\frac{1}{\sqrt{x}}$; b) $x^2-\frac{11}{4}x\sqrt[4]{x^3}-4x+\frac{21}{4\sqrt[4]{x}}-\frac{7}{3}$.

148. $-\frac{5}{36}$; $\frac{-5}{121}$; $-\frac{125}{101}$; $-\frac{5}{(4-t)^2}$. **150** a) $8(2x-7)^3$; b) $55(3+5x)^{10}$. **151.** a) $\frac{5}{2\sqrt{5x-8}}$;

b) $\frac{4}{3\sqrt[3]{2x+3}}$. **152.** a) $45(3x-1)^{14}+8(2x+2)^3$; b) $65(5x-2)^{12}-60(3x+7)^{19}$.

153. a) $\frac{4x}{\sqrt{4x^2-1}}$; b) $\frac{9x^2}{\sqrt[3]{(9x^3-15)^2}}$. **154.** a) $\frac{2x^2-x+1}{\sqrt{x^2+1}}$; b) $\frac{5x^2+2x-3}{2\sqrt{x-1}}$. **155.** a) $\frac{2(4x^4-1)}{x^3}$;

b) $\frac{x+2}{\sqrt{(x^2+1)^3}}$. **156.** a) $3\cos 3x$; b) $5\cos(5x+2)$. **157.** a) $1{,}5\cos 3x$; b) $0{,}9\sin 2x\cdot\cos x$.

158. a) $\frac{1}{2}\mathrm{ctg}x\sqrt{\sin x}$; b) $\frac{1}{3}\mathrm{ctg}x\sqrt[3]{\sin x}$. **159.** a) $\cos x$; b) $2+18\sin^4 x\cdot\cos x$.

160. a) $\frac{1}{2\sqrt{x}}(\sin x+2x\cos x)$; b) $\frac{\sin x-2x\cdot\cos x}{2\sqrt{x}\sin^2 x}$. **161** a) $-2{,}5\sin x$; b) $\frac{3}{\cos^2(3x-2)}$;

d) $-\frac{6}{\sin^2 3x}$. **162.** a) $-6{,}9\sin 2{,}3x$; b) $\frac{10}{\cos^2 2x}$; d) $\frac{14}{\sin^2 2x}$. **163.** a) $-\frac{x\sin x+\cos x}{x^2}$;

b) $\frac{18}{\cos^2(6x-2)}$. **164.** a) $\sin^2 x(3\cos^2 x-\sin^2 x)$; b) $\sin 4x$. **165.** a) $\frac{2}{3}\mathrm{ctg}2x\sqrt[3]{\sin 2x}$;

b) $-\frac{1}{x^2}\mathrm{tg}^2\frac{1}{x}\mathrm{ses}^2\frac{1}{x}$. **166.** d) $-\frac{4}{x^2}e^{\frac{4}{x}}$; e) $\frac{1}{2\sqrt{x}}e^{\sqrt{x}}$. **167.** d) $xe^{3x}(2+3x)$; e) $\frac{e^{8x}(8x-3)}{x^4}$.

168. a) $0{,}5e^{0{,}5x}-273e^{91x}$; b) $5e^{5x}+\frac{4}{3}e^{\frac{x}{3}}$. **169.** b) $-3\ln 2\cdot 2^{5-3x}$; d) $-15\ln 3\cdot 3^{7-3x}$;

e) $\frac{1}{7}\ln 1{,}7\cdot 1{,}7^{\frac{x}{7}}$. **170.** a) $2^x(\ln 2\cdot\cos x-\sin x)$; b) $7^{\frac{x}{2}}\left(0{,}5\ln 7\cdot tg3x+\frac{3}{\cos^2 3x}\right)$.

171. a) $\frac{-0{,}3^{-x}(2x\ln 0{,}3+\sqrt{x}\ln 0{,}3+1)}{2\sqrt{x}(\sqrt{x}+0{,}5)^2}$; b) $\frac{6^x\ln 1{,}5+15^x\ln 0{,}6}{(2^x+5^x)^2}$. **172.** d) $\frac{1}{x}$; e) $\frac{1}{2x}$.

173. d) $\frac{3}{3x-4}$; e) $-\frac{1}{x}$. **174.** a) $x^2(3\ln x+1)$; b) $\frac{1-\ln x}{x^2}$. **175.** a) $\frac{\ln x+2}{2\sqrt{x}}$;

b) $\frac{3(x^2+1)-2x(5+3x)\cdot\ln(5+3x)}{(5+3x)(x^2+1)^2}$. **176.** a) $3^{2x}\left(2\ln 3\cdot\ln 5x+\frac{1}{x}\right)$; b) $2\sin^2 2x\cdot\cos 2x\times$

$\times(3\lg\sin 2x+\ln 10)$. **179.** a) $\approx 14{,}443$; b) $\approx 2{,}886$; d) $\approx 1{,}732$. **180.** a) $\approx 3{,}11$;

b) $\approx 3{,}0803$; d) $\approx 2{,}013$. **181.** a) -6; b) 12. **182.** a) 0; b) -1. **183.** a) $y=12x-16$;

b) $y=0{,}12x-0{,}016$. **184.** a) $y=-3x+6$; b) $y=-3(x+2)$. **185.** $\mathrm{tg}\alpha_1=-3$;

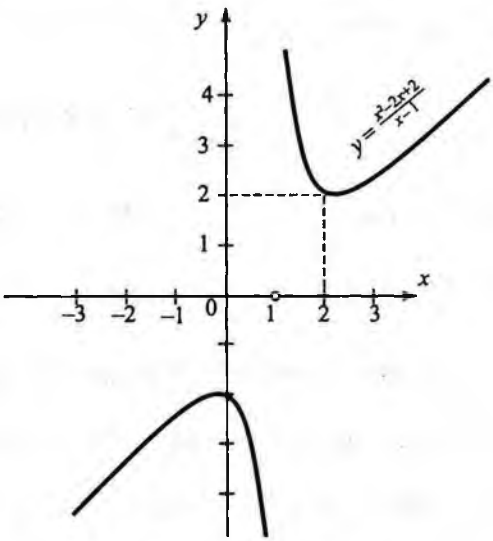
$\mathrm{tg}\alpha_{2/3}=6$. **186.** $\mathrm{tg}\alpha_1=0$; $\mathrm{tg}\alpha_2=9$. **187.** $y_1=-2x$ va $y_2=-6x$. **188.** 5. **189.** 16.

190. $6t-4$ rad/c; 20 rad/c. **191.** 1) 2,8 rad/c; 2) $6\frac{2}{3}$s. **192.** $12t$ sm/s; a) $\frac{1}{12}c$;

b) $\frac{1}{6}c$. **193.** 1) 0,04 n. 2) 25 000 erg. **194.** $\frac{a}{v^3}=-2$, yoki $a=-2v^3$. **195.**

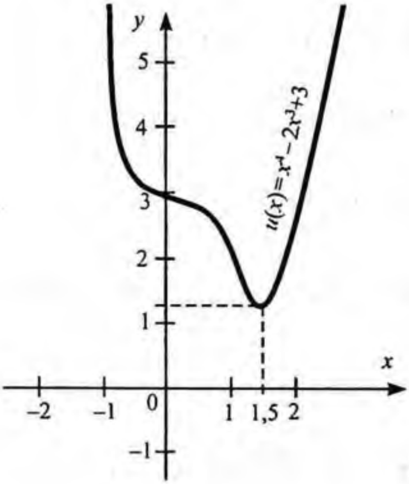
a) (–∞; +∞) o‘sadi, b) (–∞; +∞) kamaydi. **196.** a) (–∞; 0[va]0; +∞) kamayadi; b) (–∞; 3[va]3; +∞) o‘sadi. **197.** a) (–∞; 0] kamayadi; [0; +∞) o‘sadi; b) (–∞; 1] kamaydi [1; +∞) o‘sadi. **198.** a) (–∞; 0,3] kamaydi; [0,3; +∞) o‘sadi; b) (–∞; 1] kamayadi; [1; +∞) o‘sadi. **199.** a) (–∞; –3] va [3; +∞) o‘sadi; [–3; 3] kamayadi; b) (–∞; 0] va [2; +∞) o‘sadi; [0; 2] kamayadi. **200.** a) $\min x_0=3$; b) $\min x_0=\frac{1}{4}$. **201.** a) $\max x_1=-3$; $\min x_2=3$; b) $\max x_1=-3$; $\min x_2=1$. **202.** a) $\min x_1=1$; $x=0$ ekstrimum emas; b) kritik nuqtasi yo‘q. **203.** a) $\min x_0=\frac{3}{4}$; b) $\min x_0=3$; **204.** a) barcha haqiqiy sonlarda o‘sadi; b) $\min x_1=0$; $\min x_2=12$, $\max x_3=6$.

205. a) $\min x_0=0$; b) $\max x_1=0$; $\min x_2=2$. (230-betdagi chizma)

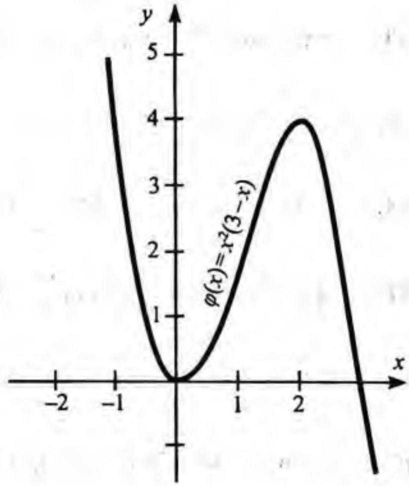


205-masala grafigi

206. a) $\min x_0=1$; (–∞; 1] kamayadi, [1; +∞) o‘sadi; b) $\max x_0=-2{,}5$; (–∞; –2,5] o‘sadi, [–2,5; +∞) kamayadi. **207.** $\min x_1=-1$; $\max x_2=1$; (–∞; 1]; [1; +∞;) kamayadi, [–1; 1] o‘sadi. **208.** $\min x_1=0$; $\max x_2=2$; (–∞; 0] va]2; +∞)



209-masala grafigi



211-masala grafigi

kamayadi; [0; 2] o'sadi. **210.** $\max x_0 = \frac{4}{3}$; $\left(-\infty; \frac{4}{3}\right]$ o'sadi va $\left[\frac{4}{3}; 2\right]$ kamayadi.

212. $\min x_1 = -1$; $\max x_2 = 1$; $(-\infty; -1]$ va $[1; +\infty)$ kamayadi; $[-1; 1]$ o'sadi.

213. $(-\infty; +\infty;)$ kamayadi. **214.** $(-\infty; +\infty)$ o'sadi. **215.** $(-\infty; +\infty)$ o'sadi.

216. a) $\min f(x) = f(\pm 1) = -16$; $\max f(x) = f(0) = -9$; b) $\min f$; $f(x) = f(2) = -25$; $\max f(x) = f(3) = 0$. **217.** 1 sek; 7 m/sek. **218.** $x_0 = h = 1{,}5R$ yoki $a_3 = \sqrt{3}R$. **219.** $x_0 = a = b = \frac{\sqrt{2}}{2}c$; $\max S\Delta = \frac{c^2}{4}$. **220.** $\max S(x) = \frac{a^2}{4}$($a>0$ son). **221.** $h:x = 1:2$.

222. teng tomonli uchburchak. **223.** $h = 2r$. $\min S_y = 6\pi\sqrt[3]{\frac{V^2}{4\pi^2}}$.

230. a) $2{,}3x+c$; b) $\frac{1}{2}x^2+c$; d) $\frac{1}{5}x^5+c$; e) $\frac{3}{4}x^4+c$. **231.** $\frac{x^4}{4}-3$. **232.** $\operatorname{tg}x-1$.

233. $-\cos x+4$. **234.** $-2x+11$. **235.** $-\frac{1}{2x^2}+5$. **236.** $\sin x-1$. **237.** $F_1(x) = \frac{1}{x}-4{,}5$; $F_2 = \frac{1}{x}+5\frac{1}{3}$; $F_2(x)$ grafigi yuqorida; $\frac{2}{x}+\frac{5}{6}$. **238.** $\frac{1}{3}x^3-9$. **239.** $\frac{1}{x}-2$;

240. $-\cos x+\cos 1+7$. **241.** $\mathrm{tg}x-2$. **242.** $\frac{5}{3}x^3-x+c$. **243.** $-\frac{1}{x}+4\cos x+c$.

244. $\frac{kx^2}{2}+bx+c$. **245.** $\frac{ax^3}{3}+\frac{bx^2}{2}+cx+c$. **246.** $x-\frac{1}{3}\sin 3x+c$. **247.** $-\frac{2}{3}\mathrm{ctg}3x+c$.

248. $-21\cos\frac{x}{3}+\frac{1}{2}\mathrm{tg}4x+c$. **249.** $\frac{3}{20}(5x-2)\sqrt[3]{5x-2}+c$. **250.** $5\sqrt{2x+5}+c$.

251. $-\frac{3}{5}\mathrm{tg}5x+2\sqrt{2x}+c$. **252.** 1) $\frac{\upsilon^2}{20}$; 2) 0; 3) $\frac{2\upsilon_0}{g}$. **254.** $8R+\frac{16}{3}a$; $R+\frac{a}{4}$. **255.** 4.

256. 1. **257.** 6. **258.** $\frac{1}{2}$. **259.** $1\frac{1}{3}$. **260.** $2\frac{2}{3}$. **261.** $\frac{2}{5}$. **262.** 1. **263.** 1. **264.** –2,5.

265. 1,5. **266.** 0,7488. **267.** 2. **268.** 3. **269.** $\frac{2-\sqrt{3}}{8}$. **270.** 20. **271.** 4,5. **272.** $5\frac{1}{3}$.

273. $1\frac{1}{3}$. **274.** $\frac{5}{12}$. **277.** 0,16 j. **278.** 0,16 j. **279.** 0,54 j. **280.** $\gamma q\left(\frac{1}{b}-\frac{1}{a}\right)$.

281. 117π. **282.** $\frac{\pi}{5}$. **283.** $\frac{16\pi}{15}$. **284.** $\frac{163\pi}{14}$. **285.** $\frac{64\pi}{15}$. **286.** ≈9,65 m. **287.** ≈0,74 mm.

288. ≈2,4 mm^2. **289.** $4\sqrt{2}\pi\approx 17,76$. **290.** $\frac{sc}{4\pi}$. **291.** 16π. **292.** ≈10 m. **293.** 9πm^3.

294. $\frac{1}{3}\pi bh^2$. **295.** 448 πsm^3; 216 πsm^2. **296.** 196 πsm^3. **297.** ≈2%.

298. 182πdm^3. **299.** $\frac{R^3-r^3}{R^3}$. **300.** $2\pi a^3\sin\alpha+\cos^2\frac{\alpha}{2}$. **301.** 27 marta; 8 marta.

302. a) ≈39 sm; b) 6 sm. **303.** ≈168 ta. **304.** 216 ta. **305.** a) $33\frac{1}{3}$%; b) ≈47,6%. **306.** ≈2148 sm^3. **307.** 290 sm. **308.** 45 πsm^3 va 243 πsm^3.

309. $\frac{9}{250}\pi d^3$. **310.** 5:16. **311.** ≈62 kg. **312.** $\frac{1}{3}\pi R^3$. **313.** 112,5 πdm^3.

314. $\frac{1}{3}\pi R^3$; $\frac{2}{3}\pi R^3$; $\frac{1}{3}\pi R^3$. **316.** $\frac{\pi Q^2}{a}$. **317.** $60\sqrt{3}\pi$ sm^3. **318.** $123\frac{63}{160}$; **319.** $1\frac{7}{15}$.

320. $1\frac{2}{3}$. **321.** 168 m. **322.** 25%. **323.** 200 g; 20%. **324.** $\frac{24}{5y-2x}$; **325.** $x+y$.

326. $\frac{1}{4}$. **327.** $2\left(\sqrt[4]{3}-\sqrt[8]{2}\right)\left(\sqrt{3}+\sqrt[4]{2}\right)\left(3+\sqrt{2}\right)$. **328.** $-\frac{1}{2}(4+3\sqrt{2})(5+3\sqrt{3})$.

329. a) $13+\sqrt{11}$; b) $12{,}5-\sqrt{5}$; d) $\frac{3}{4}+\sqrt{5}$. **331.** a) 6; b) $\sqrt{3}-\sqrt{7}$. **332.** 0. **333.** 0; –3; $\frac{-3\pm\sqrt{73}}{2}$. **334.** 1. **335.** $-\frac{4}{3}$. **337.** $p=\pm1$ va $q=-6$. **338.** –25. **339.** 3. **340.** 640,1 m. **341.** 7 minut. **342.** 16,5. **343.** –56,25 yoki 6,64. **344.** 2; 6; 18. **345.** 3; 6; 12; 24; 48; 96; 192 va 3; –6; 12; –24; 48; –96; 192. **346.** 2 va 6. **347.** 1 va $-\frac{1}{3}$; $\frac{16\pm\sqrt{304}}{48}$ **348.** 3 va 4. **Ko'rsatma.** Ko'paytuvchilarga ajratib yechiladi. **349.** 3 va 81. **350.** 2. **351.** 9. **Ko'rsatma.** Barcha logarifmlarni 3 asosga keltirib, $\log_3 x=y$ belgilab yechiladi. **352.** $x<-1$ va $x>2$. **353.** $-\infty<x<\frac{2\lg 4}{\lg\sqrt{5}}$. **354.** $0<x<\frac{1}{2}$ va $x>32$. **355.** $x<\log_2 3$. **356.** $\frac{1}{2}<x<4$. **357.** $x>3$. **358.** $\cos 10°$. **359.** $2\mathrm{tg}\alpha\cdot\cos^2\frac{\alpha}{2}$. **360*.** $\sin 7°$. **Ko'rsatma** $(\cos 47°+\cos 25°)-(\cos 61°+\cos 11°)$ ayirmani ko'paytmaga keltirib, ko'paytmani $\cos 18°$ ga ko'paytirib va bo'lib soddalashtiriladi. **361.** **Ko'rsatma.** $\frac{\sin 20°\cdot\sin 10°}{\cos 20°\cdot\cos 10°}+$

$$+\frac{\sqrt{3}\cdot\sin 20°}{\cos 20°}+\frac{\sqrt{3}\cdot\sin 10°}{\cos 10°}=\frac{\sin 20°\cdot\sin 10°+\sqrt{3}\sin 20°\cdot\cos 10°+\sqrt{3}\sin 10\cdot\cos 20°}{\cos 20°\cdot\cos 10°}=$$

$$=\frac{\frac{1}{2}(\cos 10°-\cos 30°)+\sqrt{3}(\sin 20°\cdot\cos 10°+\sin 10°\cdot\cos 20°)}{\cos 20°\cdot\cos 10°}=\frac{\frac{1}{2}(\cos 10°-\frac{\sqrt{3}}{2})+\sqrt{3}\cdot\sin 30°}{\cos 20°\cdot\cos 10°}=$$

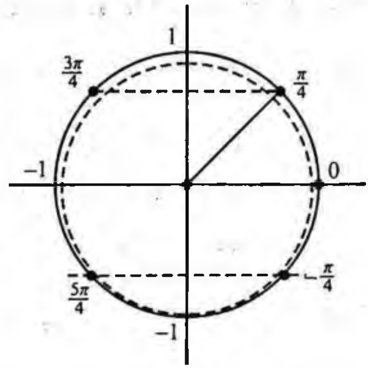
$$=\frac{\frac{1}{2}\cos 10°-\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{2}}{\cos 20°\cdot\cos 10°}=\frac{\frac{1}{2}\left(\cos 10°+\frac{\sqrt{3}}{2}\right)}{\cos 20°\cdot\cos 10°}=\frac{\cos 10°+\cos 30°}{2\cos 20°\cdot\cos 10°}=\frac{2\cos 20°\cdot\cos 10°}{2\cos 20°\cdot\cos 10°}=1.$$

363. $(-1)^k\frac{\pi}{6}+k_1\pi$; $(-1)^k\arcsin\frac{1}{3}+k_2\pi$, $k\in z$. **364.** $\pi(2k+1)$, $k\in z$. **365.** $\frac{\pi}{2}+k_1\pi$; $\mathrm{arctg}\frac{2}{3}+k_2\pi$, $k\in z$. **366.** $(1+4k)\frac{\pi}{8}$, $k\in z$. **367.** $\frac{k\pi}{2}$; $\frac{\pi}{10}(1+2k\pi)$, $k\in z$.

368. $\frac{n\pi}{2} < x < \frac{1}{4}(2n+1)\pi$, $k \in z$. **369.** $\frac{\pi}{12}(5+24k) < x < \frac{\pi}{12}(13+24k)$.

370. $\frac{\pi}{2}(2k-1) < x < \frac{\pi}{4}(4k-1)$ va $\operatorname{arctg}\frac{1}{4}+k\pi < x < \frac{\pi}{2}(2k+1)$, $k \in z$.

371*. $-\frac{\pi}{4}+k\pi < x < \frac{\pi}{4}+k\pi$, $k \in z$.



$\left|\sin\frac{x}{2}\cdot\cos\frac{x}{2}\right| < \frac{1}{2\sqrt{2}}$ da, $\left|\frac{1}{2}\sin x\right| < \frac{1}{2\sqrt{2}}$,

$|\sin x| < \frac{1}{\sqrt{2}}$ dan:

1) $\sin x < \frac{1}{\sqrt{2}}$; $\frac{3\pi}{4} < x < \frac{\pi}{4}+2\pi$;

$\frac{3\pi}{4}+2k\pi < x < \frac{9\pi}{4}+2k\pi$, yoki

$\frac{\pi}{4}(8k+3) < x < \frac{\pi}{4}(8k+9)$.

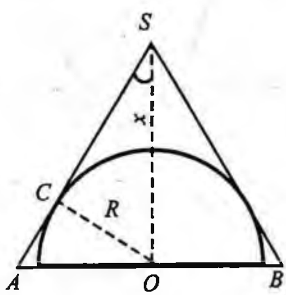
2) $-\sin x < \frac{1}{\sqrt{2}}$; $\sin x > -\frac{1}{\sqrt{2}}$; $-\frac{\pi}{4} < x < \frac{5\pi}{4}$; $-\frac{\pi}{4}+2k\pi < x < \frac{5\pi}{4}+2k\pi$ yoki $\frac{\pi}{4}(8k-1) < x < \frac{\pi}{4}(8k+5)$. Bu ikki yechimlarning umumiy yechimi: *Javob:* $\frac{\pi}{4}(8k-1) < x < \frac{\pi}{4}(8k+5)$, $k \in z$. **372.** a) $\frac{2\pi}{3}$; b) $\frac{8\pi}{3}$. **373.** a) juft; b) juft ham emas, toq ham emas. **374.** a) $12x^5-19x^4+1$; b) $-\frac{5}{(x+1)^2}$.

375. a) 2; b) 3π. **376.** $-2x\sin(x^2+5)$; b) $-\frac{5\sqrt{3}}{8}$. **377.** $-\frac{3}{\sin^2 3x}\cdot e^{\operatorname{ctg}3x}$. **378.** $2\ln 5\cdot 5^{\sin(2x-1)}\cos(2x-1)$. **379.** $\operatorname{ctg}\frac{x}{2}$. **380.** $\frac{6}{\ln 5}\operatorname{ctg}2x$. **381.** a) $y=\log_2(x-1)$; $(1; +\infty)$ da o'sadi. b) $y=\frac{10^x-1}{10^x+1}$; $(-\infty; +\infty)$ da o'sadi. **382.** a) $(0; 1)$ da kamayadi, $(1; +\infty)$ da o'sadi, $\min x_0=1$, b) $(0; \ell^6)$ da o'sadi, $(\ell^6; +\infty)$ da kamayadi, $\min x_0=e^6$. **383.** $\max x_0=\frac{\sqrt{3}}{2}$; $\max y_0=-4\frac{7}{8}$. **384.** 1. **385.** $y=2x$; $y=-2x-4$; $y=5x-2{,}25$. **386.** $a=1\frac{2}{3}$. **387.** $y=18x+10$; $y=18x-14$. **388.** 3 sm; 6 sm; 4 sm. **389.** $R=\frac{P}{\pi+4}$; $H=\frac{P}{\pi+4}$. **390.** a) $2x^4-\frac{1}{3}\sin 3x+c$, b) $\frac{1}{\sqrt{3}+1}x^{\sqrt{3}+1}+\frac{1}{2}\cos(2x+1)+\frac{1}{2(2x+1)}+c$. **391*.** $\frac{2}{125}(5x-2)^2\sqrt{5x-2}+\frac{34}{75}(5x-2)\sqrt{5x-2}+c$.

Ko'rsatma. $f(x)=(x+3)\sqrt{5x-2}=\left(\frac{1}{5}(5x-2)+3\frac{2}{5}\right)(5x-2)^{\frac{1}{2}}=\frac{1}{5}(5x-2)^{\frac{3}{2}}+$ $+\frac{17}{5}(5x-2)^{\frac{1}{2}}$; $\left(x+3=\frac{1}{5}(5x-2+y)=\frac{1}{5}\cdot 5x+\frac{1}{5}(-2+y)\right.$da $\frac{1}{5}(-2+y)=3$ dan $y=17$; $x+3=\frac{1}{5}(5x-2)+\frac{17}{5}$ga ajraldi$\left.\right)$ning boshlang'ich funksiyasi topiladi. **392.** a) $9\left(\sqrt[3]{4}+1\right)$; b) $-2\frac{3}{8}$. **393.** $\frac{8}{25}(6\sqrt{2}-7\sqrt{3})$.

Ko'rsatma. $3x-4=\frac{3}{5}(5x+2)-5\frac{1}{5}$ ko'rinishda yozilib, integralhisoblanadi. **394.** $12-5\ln 5$. **395.** $2\frac{2}{3}$. **396.** $H=\sqrt{3}R$.

Ko'rsatma. Shar radiusi R ga teng, Konus balandligi $H=x$ bo'lsin, ya'ni $SO=H=x$. $V_{\text{kon}}=\frac{1}{3}\pi R^2\cdot H$ formula bo'yicha topiladi, bunda $OA=R_1$, $SC=\sqrt{x^2-R^2}$ – chizmada $\Delta ASO\backsim\Delta OSC$ ($\angle ASO$ – umumiy burchak). $\frac{OA}{OC}=\frac{SO}{SC}$

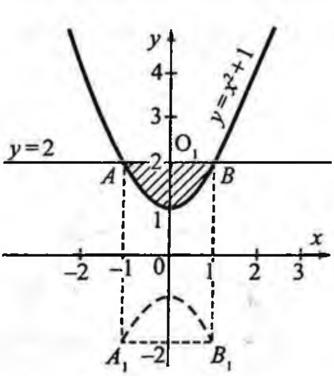


$\frac{OA}{R}=\frac{x}{\sqrt{x^2-R^2}}$; $OA=\frac{Rx}{\sqrt{x^2-R^2}}$; $(OA)^2=\frac{R^2x^2}{x^2-R^2}$. $V(x)=\frac{1}{3}\pi\cdot(OA)^2\cdot x=\frac{\pi}{3}\cdot\frac{R^2x^2}{x^2-R^2}\times$

$\times x=\frac{\pi R^2}{3}\cdot\frac{x^3}{x^2-R^2}$. $V(x)'=\frac{\pi R^2}{3}\cdot\frac{3x^2(x^2-R^2)-x^3\cdot 2x}{(x^2-R^2)^2}=\frac{\pi R^2(x^4-3R^2x^2)}{3(x^2-R^2)^2}$. $V'(x)=0$ da

$x^2-3R^2=0$; $x=\pm\sqrt{3}R$; $x_0=\sqrt{3}R$. $x<3\sqrt{R}$ da $V'(x)<0$, $x>\sqrt{3}R$ da $V'(x)>0$ bo'lganidan $\min x_0=\sqrt{3}R=H$. **397.** 30 sm va 20 sm. **Ko'rsatma.** Sahifaning bo'yi $AB=x$ bo'lsin. Matn bo'yi $x-6$, eni bo'ladi. Sahifaning eni $\frac{384}{x-6}$ bo'ladi. Sahifaning eni $\frac{384}{x-6}+4$ ga teng. Sahifaning yuzi $x\cdot\left(\frac{384}{x-6}+4\right)$, ya'ni $f(x)=\frac{x\cdot 384}{x-6}+4x$ funksiyaning eng kichik qiymatini $]6;+\infty)$ oraliqda topamiz. $f'(x)=\frac{384(x-6)-384x}{(x-6)^2}+4=\frac{-384\cdot 6}{(x-6)^2}+4=\frac{4x^2-48x-2160}{(x-6)^2}$; $f'(x)=0$ dan $4x^2-48x-2160=0$, $x_1=-18$; $x_2=30$. $x_0=30$ dan $x<30$ da $f'(x)<0$ va $x>30$ da $f'(x)>0$ ekanligidan $\min x_0=30$ bo'ladi. Sahifa bo'yi 30, eni $\frac{384}{x-6}+4=20$.

Javob: 30 va 20. **398.** $\frac{2}{3}\pi$. **399.** $\frac{2}{15}\pi$. **400.** $4\frac{4}{15}\pi$. **Ko'rsatma.** $y=x^2+1$ va $y=2$ grafiklarini chizamiz.



Shtrixlangan figurani OX o'qi atrofida aylanishidan hosil bo'lgan figurani $V_{sht.}$ bilan, ABB_1A_1 silindr hajmini $V_{sil.}$ bilan belgilaymiz $y=x^2+1$ ni aylanishidan hosil bo'lgan figuraning hajmini V_{egri} bilan belgilaymiz $[-1;\ 1]$ oraliqdagi).

1) $V_{\text{egri}}=\pi\int\limits_{-1}^{1}(x^2+1)^2dx=\pi\int\limits_{-1}^{1}(x^4+2x^2+1)dx=\pi\left(\frac{x^5}{5}+\frac{2}{3}x^3+x\right)\Big|_{-1}^{1}=$

$=\pi\left(\frac{1}{5}+\frac{2}{3}+1-\left(-\frac{1}{5}-\frac{2}{3}-1\right)\right)=\pi\cdot3\frac{11}{15}=3\frac{11}{15}\pi.$

2) $V_{\text{sil}}=\pi\cdot(OO_1)^2\cdot AB=\pi\cdot2^2\cdot2=8\pi.$

3) $V_{\text{sht.}}=V_{\text{sil}}-V_{\text{egri}}=8\pi-3\frac{11}{15}\pi=4\frac{4}{15}\pi.$ *Javob:* $4\frac{4}{15}\pi.$

401. ±2. **Ko'rsatma.** $\left(\sqrt{7-\sqrt{48}}\right)^x=\frac{\left(\left(\sqrt{7-\sqrt{48}}\right)^x\left(\sqrt{7+\sqrt{48}}\right)\right)^x}{\left(\sqrt{7+\sqrt{48}}\right)^x}=\frac{1}{\left(\sqrt{7+\sqrt{48}}\right)^x}$ ga

teng. $\left(\sqrt{7+\sqrt{48}}\right)^x=y$ bo'lsin. $y+\frac{1}{y}=14;$ $y^2-14y+1=0$ dan

$y_1=7-\sqrt{48};\ y_2=7+\sqrt{48}.$

1) $\left(\sqrt{7+\sqrt{48}}\right)^x=y_1;\ \left(7+\sqrt{48}\right)^{\frac{x}{2}}=7-\sqrt{48};\ \left(7+\sqrt{48}\right)^{\frac{x}{2}}=\frac{1}{7+\sqrt{48}};\ \left(7+\sqrt{48}\right)^{\frac{x}{2}}=$

$=\left(7+\sqrt{48}\right)^{-1};\ \frac{x}{2}=-1;\ x_1=-2.$

2) $\left(\sqrt{7+\sqrt{48}}\right)^x=y_2;\ \left(7+\sqrt{48}\right)^{\frac{x}{2}}=\left(7+\sqrt{48}\right)^1;\ \frac{x}{2}=1;\ x_2=2.$ *Javob:* ±2.

402. 9. **Ko'rsatma.** $\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x...}}}=3$ tenglamaning ikkala qismini kubga ko'taramiz. $x\underbrace{\sqrt[3]{x\sqrt[3]{x...}}}_{3}=27;\ x\cdot3=27;\ x=9.$ *Javob:* 9.

403. b) $\frac{2\pi}{3}$; f) $\frac{1}{2}$; g) $\frac{\pi}{2}$. **404.** a) 0; d) 0; e) $\frac{\sqrt{3}}{2}$. **405.** a) 0,8; b) $\frac{2\sqrt{2}}{3}$; d) $-\frac{2}{\sqrt{15}}$; e) $-\sqrt{15}$. **406.** b) $\frac{84}{85}$; d) $\frac{2}{9}$. **408.** a) $\frac{\sqrt{3}}{2}$; b) $\frac{\sqrt{2}}{2}$; d) 1; e) 2 va 3.

Matematika (arifmetika, algebra) testining javoblari

	0	1	2	3	4	5	6	7	8	9
0		C	B	D	C	C	C	B	C	C
1	B	C	A	C	D	B	D	B	A	C
2	D	B	C	D	C	D	C	C	A	D
3	B	C	A	D	B	C	B	A	D	C
4	D	B	D	C	A	B	C	B	B	A
5	C	B	A	D	B	C	A	B	C	D
6	B	A	C	C	A	D	B	A	C	C
7	B	C	A	C	B	C	B	D	A	C
8	B	C	D	A	C	A	B	C	B	D
9	A	B	D	C	D	B	D	A	B	D

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MUNDARIJA

III bob. **Funksiyaning limiti va hosilasi**

IV bob. **Hosilani taqribiy hisoblashlarga, geometriyaga va fizikaga tatbiqi**

V bob. **Boshlang'ich funksiya va integral**

M. ORTIQOV, Sh.M. YUSUPJONOVA

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(qo'llanma)

II qism

Muharrir *Sh. Ilhombekova*
Badiiy muharrir *J. Gurova*
Texnik muharrir *D. Salixova*
Kompyuterda sahifalovchi *B. Babaxodjayeva*

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