Fizika-matematika fanlari doktori, professor **Salim Otakulov** 1975-yildan buyon Samarqand davlat universiteti, Toshkent axborot texnologiyalari universiteti Samarqand filiali va Jizzax politexnika institutida malakali mutaxassis va ilmiy-pedagogik kadrlar tayyorlashda samarali faoliyat olib bormoqda.

S.Otakulovning optimal boshqaruv matematik nazariyasi, iqtisodiy-matematik modellar, qaror qabul qilishning matematik usullari va oliy matematikani o‘qitish bo‘yiha ko‘p yillik tadqiqotlari natijasida 280 dan ziyod ilmiy va o‘quv-uslubiy ishlar, jumladan, 2 ta monografiya, 20 dan ortiq o‘quv-uslubiy qo‘llanmalar va uslubiy ko‘rsatmalar tayyorlandi. Jumladan, O‘zbekiston Respublikasi oliy va o‘rta maxsus ta’lim vazirligi tavsiyasi bilan 2007-yilda “Исследование операций”, 2009-yilda “Kombinatorika va graflar nazariyasi”, 2020-yilda “Iqtisodiyotdagi matematik usullar”, 2020-yilda “Chiziqli algebra asoslari va uning tatbiqlari” nomli o‘quv qo‘llanmalari nashr etildi, 2019-yilda Germaniyanng “LAP – Lambert academic publishing” nashriyotida “Задачи управления ансамблями траекторий дифференциальных включений” nomli monografiyasi chop etildi va bir qator chet tillarga tarjima gilindi.

Fizika-matematika fanlari nomzodi, dotsent **Abdumannon Ochilovich Musayev** 1983-yildan buyon Jizzax politexnika institutida olib borayotgan ilmiy-pedagogik faoliyati davrida «Oliy matematika» kafedrasi o‘qituvchisi, kafedra mudiri, dotsent lavozimlarida oliy ma’lumotli kadrlar tayyorlashga o‘z hissasini qo‘shib kelmoqda.

Ilmiy-tadqiqot ishlari matematikaning fundamental yo‘nalishlaridan biri bo‘lgan funksiyalar nazariyasi va uning tatbiqlariga bagishlangan. Matematik usullarning iqtisodiyotdagi tatbiqlari, oily matematikaning algebra, matematik tahlil va analitik geometriya kabi bo‘limlari bo'yicha ham ilmiy-uslubiy izlanishlar olib bormoqda. Ilmiy - pedagogik faoliyati natijalari bo‘yicha 50 dan ortiq ilmiy-uslubiy ishlari, shu jumladan, 10 dan ortiq o‘quv-uslubiy ishlari chop etilgan.

ISBN 978-9943-8123-6-9

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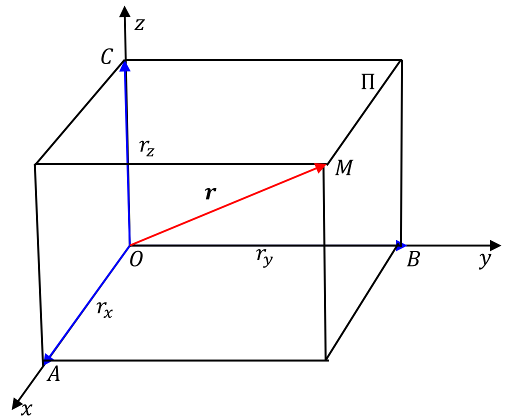
$$(\boldsymbol{a},\boldsymbol{b}) = |\boldsymbol{a}||\boldsymbol{b}|\cos\varphi$$

$$\boldsymbol{a}\times\boldsymbol{b} = \boldsymbol{i}\begin{vmatrix} a_y & a_z \\ b_y & b_z\end{vmatrix} - \boldsymbol{j}\begin{vmatrix} a_x & a_z \\ b_x & b_z\end{vmatrix} + \boldsymbol{k}\begin{vmatrix} a_x & a_y \\ b_x & b_y\end{vmatrix}$$



TOSHKENT

O‘ZBEKISTON RESPUBLIKASI
OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

S. OTAKULOV, A. O. MUSAYEV

ANALITIK GEOMETRIYA VA VEKTORLAR ALGEBRASI

O‘zbekiston Respublikasi Oliy va o‘rta maxsus ta’lim vazirligi tomonidan talabalar uchun o‘quv qo‘llanma sifatida tavsiya etilgan

TOSHKENT–2022

UO‘K 516.0+517.2
KBK 22.151.5ya73
O-86

S. Otakulov, A. O. Musayev. Analitik geometriya va vektorlar algebrasi. O‘quv qo‘llanma.-T.: «Fan va texnologiyalar nashriyot –matbaa uyi», 2022. -168 b.

ISBN 978-9943-8123–6–9

Ushbu o‘quv qo‘llanmada quyidagi mavzular yoritilgan: vektorlar va ular ustida chiziqli algebraik amallar; vektorlar uzunligi va yo‘naltiruvchi kosinuslari haqida ma’lumotlar; vektorlarning skalyar, vektorli va aralash ko‘paytmalari; to‘g‘ri burchakli Dekart koordinatalar sistemasi tushunchasi va uning sodda masalalarga tatbiqlari; tekislikdagi chiziqlar haqida zarur tushunchalar; to‘g‘ri chiziqlar va ularning tekislikda joylashuviga oid masalalar; ikkinchi tartibli sirtlar va ularning xossalari; fazodagi sirt va chiziqlar tenglamalariga doir umumiy tushunchalar; fazodagi tekislik va chiziq tenglamalari; tekislik va to‘g‘ri chiziqlarning o‘zaro joylashuvi masalalari; to‘g‘ri chiziq va tekislik tenglamalarini tuzish masalalari; ikkinchi tartibli sirtlar haqidagi asosiy tushuncha va ma’lumotlar.

Bayon etilgan mavzularga mos amaliy misol va masalalar keltirilgan. Mavzularni chuqurroq o‘rganishga yordamlashuvchi nazorat savollari, amaliy masalalar va mashqlar keltirilgan. Talabalar bilmni baholash uchun mo‘ljallangan test topshiriqlari ham berilgan.

Oliy matematika fanining analitik geometriya va vektorlar algebrasi bo‘limlariga bag‘ishlangan o‘quv qo‘llanma texnika va iqtisodiyot yo‘nalishida tahsil olayotgan talabalar, jumladan, 5230200 – “Menejment”, 5230900 – “Buxgalteriya hisobi va audit”, 5340400 –“Muhandislik kommunikatsiyalari qurilishi va montaji, 5310700 – Elektr mexanikasi va elektr texnologiyalari, 5620100 – “Tashishlarni tashkil etish va transport logistikasi, 5230100 – “Iqtisodiyot”, 5330200 – “Informatika va axborot texnologiyalari (iqtisodiyot)” kabi bakalavriyat mutaxassisliklari uchun mo‘ljallangan. O‘quv qo‘llan-madan texnika va iqtisodiyot yo‘nalishidagi barcha mutaxassislar hamda oily matematikani mustaqil o‘rganuvchilar ham foydalanishlari mumkin.

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В учебном пособии изложены: понятие вектора и линейные алгебраические операции над ними; длина и направляющие косинусы векторов; сведения о скалярном, векторном и смешанном произведении векторов; прямоугольная декартовая система координат на плоскости и ее применение к простейшим задачам; необходимые сведения о линиях на плоскости, уравнения прямой и задачи, связанные с расположением прямых; линии второго порядка и их свойства; общие понятия об уравнениях поверхностей и линий в пространстве; уравнения плоскости и прямой в пространстве; задачи взаимного расположения плоскостей и прямых; задачи на составление уравнений прямой и плоскости; необходимые понятия и сведения о поверхностях второго порядка.

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The paper presents: basic concepts about vectors and linear algebraic operations on them; length and direction cosines of vectors; information about scalar, vector and mixed product of vectors; the concept of a rectangular Cartesian coordinate system on a plane and the application of this system to the simplest problems; necessary information about lines on a plane, about the equations of a straight line and problems related to the location of lines; about second-order lines and their properties; general concepts about the equations of surfaces and lines in space; the equations of a plane and a straight line in space; problems about the mutual arrangement of planes and straight lines; problems of composing equations of a straight line and a plane; necessary information about second-order surfaces.

UO‘K 516.0+517.2
KBK 22.11

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ISBN 978-9943-8123–6–9

KIRISH

Barcha matematik tadqiqotlar obyektlari real dunyoning *miqdoriy munosabatlari va fazoviy formalarining* inson abstraksiyasi natijasi sifatida ifodalanishidir. Qandaydir predmet, hodisa yoki jarayonni matematik usullar bilan tadqiq etish uchun uning barcha sifatiy xususiyatlaridan faqatgina miqdor va formani tavsiflovchi xossalariga asosiy e'tiborni qaratish zarur bo'ladi.

Matematika faninning tarixi ko'rsatadiki, moddiy voqe'lik talablari uning rivojlanishini belgilovchi bosh omil bo'lib keldi. Matematikaning paydo bo'lish davrlaridanoq yangi tushuncha, usul va g'oyalarning shakllanishida matematik usullar tatbiqi imkoniyatiga ega bo'lgan barcha tabiatshunoslik fanlari kompleksi katta ahamiyatga ega bo'ldi. Matematikaning taraqqiyotiga boshqa fanlardan ko'ra astronomiya, mexanika va fizika muhim ta'sirini ko'rsatdi.

XVI asr oxiriga kelib matematikaning *elementar matematika* (arifmetika va boshlang'ich algebra, elementar geometriya) deb ataluvchi uzoq davom etgan rivojlanish tarixiga yakun yasaldi. Bu davr uchun asosiy belgilovchi xususiyatlar sifatida qaralayotgan obyektlarning qo'zgalmasligini, cheksiz katta yoki cheksiz kichik miqdor tushunchalarining yo'qligini, umumiy yechish usullarining rivojlanmaganligini ko'rsatish mumkin. XVII va XVIII asrlarda ishlab chiqarish, texnika va tabiatshunoslik fanlaridagi jadal rivojlanish natijasida funksional bog'lanishda bo'lgan miqdorlarni o'rganishga imkon beruvchi matematik apparatni yaratish talabi qo'yildi. O'zgaruvchi miqdorlar tushunchasining kiritilishi va ular orasidagi bog'liqlikni cheksiz kichik miqdor go'yasi orqali o'rganishga asoslangan matematik apparat yaratilishi matematika fani taraqqiyotida yangi davr – *oliy matematikaning* rivojlanish davrini boshlab berdi. Analitik geometriya, oily algebra, differensial va integral hisob, differensial tenglamalar nazariyasi va boshqalar paydo bo'ldi va rivojlandi.

Matematika na faqat boshqa fanlar ta'sirida rivojlanadi. U o'z navbatida boshqa fanlarga matematik tadqiqot usullarini joriy etadi. Zamonaviy fanlar taraqqiyotidagi tavsiflovchi belgi – mumkin bo'lgan barcha murakkab fizik, kimyoviy, biologik, iqtisodiy, ijtimoiy va boshqa jarayon va hodisalarni tizimli o‘rganishdek muhim talabning mavjudligidir. Matematik tadqiqotlarning texnika, iqtisodiyot va boshqaruv muammolariga tatbiq doirasi yanada kengaymoqda. Shu sababli matematika fanining turli yo‘nalish va sohalari rivojlanmoqda: *kombinatorik tahlil, differensial va integral tenglamalarni taqribiy yechish usullari, axborot uzatish nazariyasi, qaroq qabul qilish nazariyasi, optimal jarayonlar nazariyasi, differensial o‘yinlar, “fuzzy” to‘plamlar nazariyasi, “fazzy” mantiq asosida hisoblashlar* va boshqalar.

Hozirgi kunda amaliy matematika va axborot texnologiyalari ilmiy-texnik taraqqiyotning muhim belgilovchi omillaridan hisoblanadi. Ular murakkab tizimlarni modellashtirishda va texnologik garayonlar optimal parametrlarini tanlagan holda loyihalashda yangi imkoniyatlarni ochib bermoqda.

Matematika bo‘yicha mutaxassislargina emas, balki barcha texnik mutaxassislar, iqtisodchilarning ham matematik bilimi oily matematikaning uch asosiy bo‘limi: *matematik tahlil, analitik geometriya* va *oliy algebra* asosida shakllanadi.

Geometrik mazmunli masalalar dastlab qadimgi dunyoning buyuk matematik olimlari Yevklid, Arximed va Appoloniy (eramizdan oldingi IV–II asrlar) asarlarida paydo boldi, o‘rganildi va rivojlantirildi. Yevklidning “Boshlang‘ichlar” asarida *yevklid geometriyasi* asoslari berildi. Taniqli rus matematigi N.I.Lobachevskiy (1792-1856) tomonidan yangi geometriya – *noyevklid “Lobachevskiy geometriyasi”*ning yaratilishi matematika fani o‘rganadigan miqdoriy munosabatlar va fazoviy formalar doirasinig yanada kengayishida muhim qadam bo‘ldi. Ammo noyevklid geometriyasi olimlar orasida uzoq davom etgan kurash, bahs-munozaralardan keyin, faqat XIX asr oxirida tan olindi va rivojlandi. Noyevkld geometriyasi faqatgina *nisbiylik nazariyasi* paydo bo‘lgandan so‘ng, fazoviy-vaqt munosabatlarining real tabiati haqidagi fizik tadqiqotlar matematik asosinig muhim qismiga aylangandan keyingina o‘z o‘rnini topdi va rivojlandi.

Analitik geometriya matematikaning geometrik figuralar xossalarini algebraik taxlil yordamida o‘rganish bilan shug‘ullanuvchi bo‘limidan iborat. Analitik geometriya asoslari fransuz matematigi va faylasufi R. Dekart (1596-1650) tomonidan berildi. Analitik geometriyaning rivojlanishida I. Nyuton (1642-1727), L. Eyler (1707-1783), G. Monj (1746-1816), J. Lagrang (1736–1813) va boshqa olimlar katta hissa qo‘shdilar. XVIII asrda analitik geometriyaning fan sifatida shakllanishi yakunlandi, oliy matematik va texnik ma’lumot olishning klassik asosining tarkibiy qismi sifatida muhim o‘quv predmetlaridan biriga aylandi.

Analitik geometriya uchun tadqiqot predmeti emas, balki usul – koordinatalar metodi hal qiluvchi ahamiyatga ega. Uchbu metodning mohiyati shundaki, tekislik va fazodagi nuqta koordinatalari tushunchasi yordamida geometrik obyektlarga qandaydir standart usulda tenglamalar (yoki tengalamalar sistemalari) mos qo‘yiladiki, obyektlardagi geometrik munosabatlar ularning tenglamalari xossalrida ifodalanadi.

Aytish mumkinki, analitik geometriya fani geometriyani algebra va matematik tahlil bilan shunday birlashtirdiki, bu esa matematikaning ushbu uchta bo‘limining rivojlanishiga samarali ta’sir ko‘rsatdi. Turli masalalarni yechishga yondashuvda universallik xususiyatiga egaligi sababli analitik geometriya metodi geometrik tadqiqotlarning asosiy usuliga aylandi va tabiatshunoslikning boshqa sohalarida, ayniqsa, mexanika va fizikada keng qo‘llaniladi. Shu o‘rinda analitik geometriya usulining bayoni va uning tatbiqlari vektorlar algebrasi bilan uzviy bog‘liqligini ta’kidlash joyizdir.

Ushbu o‘quv qo‘llanma oliy o‘quv yurtlarining texnika va iqtisodiyotga oid yo‘nalishlar bakalavriyat talabalari uchun oliy matematika fani dasturlari asosida yozilgan bo‘lib, u bir qator mutaxassisliklar, jumladan, 5230100 – “Iqtisodiyot”, 5230900– “Buxgalteriya hisobi va avdit”, 5230200 – “Menejment”, 5340400 –“Muhandislik kommunikatsiyalari qurilshi va montaji, 5310700 – Elektr mexanikasi va elektr texnologiyalari, 5620100 – “Tashishlarni tashkil etish va transport logistikasi, 5330200 – “Informatika va axborot texnologiyalari (iqtisodiyot)” uchun mo‘ljallangan.

Uchta bobdan iborat ushbu ishda analitik geometriya va vektorlar algebrasining asosiy tushuncha va ma’lumotlari bayon etilgan.

Birinchi bobda vektorlar algebrasiga oid zarur ma’lumotlar berilgan. Vektorlar va ular ustida chiziqli algebraik amallarga doir asosiy tushunchalar, vektorlarnng skalyar, vektorli va aralash ko‘paytmalari haqida ma’lumotlar keltirilgan.

Ikkinchi bob tekislikdagi analitik geometriya masalalariga bag‘ishlangan. Bu yerda to‘g‘ri burchakli Dekart koordinatalar sistemasi va uni almashtirish (parallel ko‘chirish, koordinata o‘qlarini burish) haqida ma’lumotlar bayon etilgan. To‘g‘ri burchakli koordinatalar sistemasining tekislikdagi sodda masalalari jumlasiga kiruvchi ikki nuqta orasidagi masofa, kesmani berilgan nisbatda bo‘laklarga ajratish, uchburchak yusini topish kabilarga qo‘llanishi berilgan. Tekislikdagi chiziqlar, to‘g‘ri chiziq tenglamalari va to‘g‘ri chiziqlarning o‘zaro joylashuvi bilan bog‘liq masalalar, ikkinchi tartibli chiziqlar va ularning xossalari haqida zarur ma’lumotlar keltirilgan.

Uchinchi bobda fazodagi analitik geometriyadan bir qator muhim ma’lumotlar keltirilgan. Fazodagi sirtlar va chiziqlar tenglamalari haqida umumiy tushunchalar berilgan. Tekislik va fazoda to‘g‘ri chiziq tenglamalari haqidagi masalalar bayon etilgan. To‘g‘ri chiziq va tekislikning o‘zaro joylashuvi, berilgan muayyan shartlarda to‘g‘ri chiziq va tekislik tenglamalarini tuzish kabi asosiy masalalar qaralgan. Ikkinchi tartibli sirtlar haqida ma’lumotlar berilgan. Ushbu bob materiallarini bayon etishda vektorlar algebrasi natijalaridan foydlanilgan.

Barcha boblarda har bir paragraph oxirida nazorat savollari, mustaqil ishlash uchun masala va topshiriqlar keltirilgan. Har bir bob yakunida talabalar bilimini sinashga mo‘ljallangan test savollari ham berilgan.

Mualliflar qo‘llanma bo‘yicha bildiriladigan barcha mulohaza va fikrlar uchun oldindan minnatdorlik bildiradilar.

I-BOB. VEKTORLAR ALGEBRASI ELEMENTLARI

§1. VEKTORLAR USTIDA CHIZIQLI AMALLAR

1.1. Skalyar va vektor miqdorlar

1-ta'rif. *Biror tanlab olingan birliklar sistemasida faqat o'zinig sonli qiymati bilan to'liq tavsiflanuvchi kattalikga skalyar miqdor yoki **skalyar** deb aytiladi.*

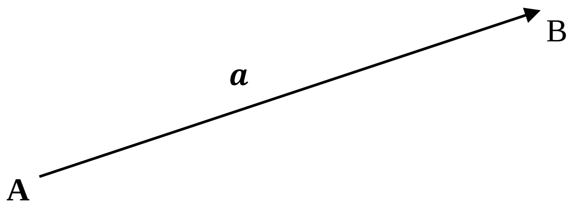
Masalan, skalyar miqdorlarga jism massasi va uning hajmi, muhit harorati, chiziq uzunligi, sirt yuzi va shu kabilar misol bo'la oladi. Skalyar miqdor musbat, manfiy, yoki nol soni bilan aniqlanadi.

2-ta'rif. *Biror tanlab olingan birliklar sistemasida o'zinig sonli qiymatidan tashqari yana yo'nalishi bilan tavsiflanuvchi kattalikga vektor miqdor yoki **vektor** deb aytiladi.*

Vektor miqdorlarga misol qilib kuch, tezlik, ko'chish kabilarni ko'rsatish mumkin**.**

Vektorlar odatda lotin alifbosining kichik harflari bilan belgila-nadi, masalan $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}$. Vektor geometrik ravishda fazodagi yo'nalgan kesma bilan tasvirlanadi va u $\boldsymbol{a} = \overline{AB}$ kabi belgilanadi, bu yerda A – kesmaning boshlang'ich nuqtasini, B esa, kesmaning oxirgi nuqtasini ifodalaydi (1-chizma). Bundan keyin mavzu bayonining ko'rgazmaligi nuqtai nazaridan vektorni yo'nalgan kesma sifatida qaraymiz.

Vektorning moduli (uzunligi) deb, unig yo'nalishi hisobga olinmagan holdagi sonli qiymati tushuniladi. $\boldsymbol{a} = \overline{AB}$ vektorning moduli $|\boldsymbol{a}| = |\overline{AB}|$ kabi belgilanadi.



1-chizma

Nol vektor deb, moduli nolga teng vektorga aytiladi va u **0** kabi belgilanadi. **0** vektorning yo'nalishi ixtiyoriy deb hisoblanishi mumkin.

Agar **a** va **b** vektorlar parallel to‘g‘ri chiziqlarda yoki aynan bir to‘g‘ri chiziqda yotib, ularning uzunliklari va yo‘nalishlari bir xil bo‘lsa, bu vektorlar teng teb hisoblanadi. Teng vektorlarni farqlamaslikka kelishamiz va shunday qilib, *ozod vektor* tushunchasini kiritamiz.

Ozod vektor – uzunligini va yo‘nalishini o‘zgartirmasdan fazonig istalgan nuqtasiga parallel ko‘chirish mumkin bo‘lgan vektor deb tushuniladi. Xususan, barcha ozod vektorlarni boshlang‘ich nuqtasi umumiy bo‘lgan nuqtaga parallel ko‘chirish mumkin bo‘ladi.

3-ta’rif. *Agar ikkita **a** va **b** vektorlar parallel to‘g‘ri chiziqlarda yoki bir to‘g‘ri chiziqda joylashgan bo‘lsa,unda bu vektorlar **kollinear** deyiladi.*

Nol vektorning yo‘nalishi ixtiyoriy bo‘lganligi sababli, nol vektorni har qanday vektorga kollinear deb hisoblash mumkin.

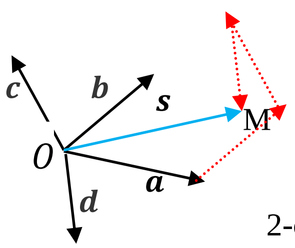
4-ta’rif. *Agar uchta **a**, **b** va **c** vektorlar parallel tekisliklarda yoki bir tekislikda joylashgan bo‘lsa, u holda bu vektorlar **komplanar** deyiladi.*

Aytish mumkinki, berilgan **a**, **b** va **c** vektorlar faqatgina ularni bitta boshlang‘ich nuqtaga keltirilganda barchasi bir tekislikda yotsagina komplanar bo‘ladilar.

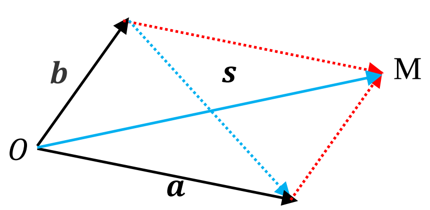
4-ta’rifga ko‘ra uchta vektordan hech bo‘lmaganda bittasi nol vektor bo‘lsa, u holda ular komplanardir.

1.2. Vektorlarnig yig‘indisi va ayirmasi. Vektorni songa ko‘paytirish

5-ta’rif. *Bir nechta, masalan **a, b, c, d** vektorlarning yig‘indisi **s=a+b+c+d** deb, yo‘nalishi $\overline{OM}$ ga va uzunligi $|\overline{OM}|$ teng bo‘lgan vektorga aytiladi (2-chizma).*



2-chizma

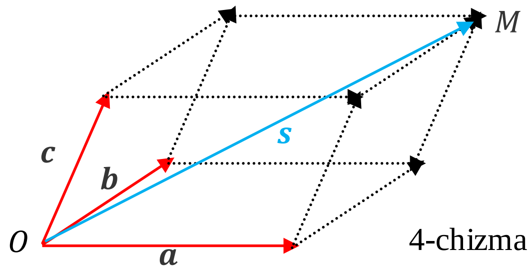


3- chizma

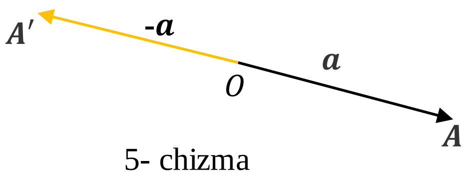
Xususiy holda, kollinear bo‘lmagan ikki **a** va **b** vektorlarning yig‘indisi $\boldsymbol{s} = \boldsymbol{a} + \boldsymbol{b}$ deb umumiy O nuqtadan chiquvchi ushbu vektorlarga qurilgan parallelogramning katta $\overline{OM}$ diagonaliga aytiladi (parallelogram qoidasi, 3-chizma).

3-chizmadan ko‘rinib turibdiki, **a** , **b** vektorlar va ularning $\boldsymbol{s} = \boldsymbol{a} + \boldsymbol{b}$ yig‘indisi uchburchak tomonlarini tashkil etadi. Ma’lumki, uchburchakda har bir tomoning uzunligi qolgan ikki tomonlar uzunliklari yig‘indisidan oshmaydi, shu sababli $|\boldsymbol{a} + \boldsymbol{b}| \leq |\boldsymbol{a}| + |\boldsymbol{b}|$, ya’ni ikki vektor yig‘indisining moduli vektorlar modullarining yig‘indisidan katta bo‘lmaydi.

Komplanar bo‘lmagan uchta **a, b, c** vektorlarning yig‘indisi bo‘lgan $\boldsymbol{s} = \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}$ vektor shu berilgan vektorlarga yasalgan parallelepipedning $\overline{OM}$ diagonalidan iborat (parallelepiped qoidasi, 4-chizma).



4-chizma



5- chizma

Vektorlarni qo‘shish amali quyidagi xossalarga ega:

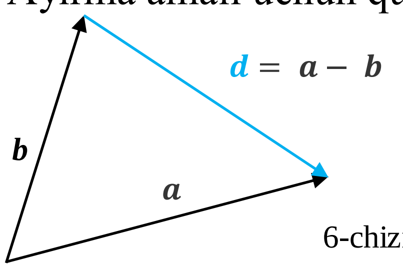
1°. $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{b} + \boldsymbol{a}$ (*o‘rin almashtirish xossasi*), ya’ni vektorlarning yig‘indisi qo‘shiluvchilar tartibiga bog‘liq emas;

2°. $\boldsymbol{a} + (\boldsymbol{b} + \boldsymbol{c}) = (\boldsymbol{a} + \boldsymbol{b}) + \boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}$ (*guruhlash xossasi*).

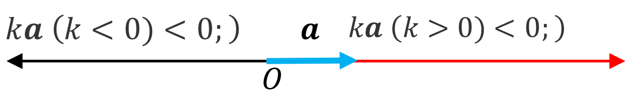
Har bir $\boldsymbol{a} = \overline{OA}$ vektor uchun yo‘nalishi unga qarama-qarshi, uzunligi shu $\boldsymbol{a}$ vektornikidek bo‘lgan $-\boldsymbol{a} = \overline{OA'}$ vektor mavjud (5-chizma). Ravshanki, $\boldsymbol{a} + (-\boldsymbol{a}) = \mathbf{0}$ bo‘ladi. Har qanday $\boldsymbol{a}$ vektor va $\mathbf{0}$ vektor uchun $\boldsymbol{a} + \mathbf{0} = \boldsymbol{a}$ o‘rinli.

6-ta’rif. Ikki $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlarning ayirmasi deb, $\boldsymbol{d} + \boldsymbol{b} = \boldsymbol{a}$ shartni qanoatlantiruvchi $\boldsymbol{d}$ vektorga aytiladi(6-chizma).

Ayirma amali uchun quyidagi qoida o‘rinli: $\boldsymbol{a} - \boldsymbol{b} = \boldsymbol{a} + (-\boldsymbol{b})$.



6-chizma



7- chizma

Qayd qilib o‘tamizki, $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar $\boldsymbol{a} - \boldsymbol{b}$ ayirmasiga shu vektorlarga yasalgan parallelogramdagi (3-chizma) ikkinchi(kichik) diagonal mos keladi.

7-ta'rif. $\boldsymbol{a}$ vektorning k-skalyarga ko'paytmasi deb $\boldsymbol{b} = k\boldsymbol{a}$ vektorga aytiladi (7-chizma).

Bu $\boldsymbol{b} = k\boldsymbol{a}$ vektor uzunligi $|\boldsymbol{b}| = |k||\boldsymbol{a}|$ | ga teng bo'lib, uning yo'nalishi esa quyidagicha aniqlanadi:

1) agar $k > 0$ bo'lsa, $\boldsymbol{b}$ vektor yo'nalishi $\boldsymbol{a}$ vektor yo'nalishi bilan ustma-ust tushadi;

2) agar $k < 0$ bo'lsa, $\boldsymbol{b}$ vektor yo'nalishi $\boldsymbol{a}$ vektor yo'nalishiga qarama-qarshi;

3) agar $k = 0$ bo'lsa, $\boldsymbol{b}$ vektor yo'nalishi ixtiyoriy bo'ladi.

Vektorni skalyarga ko'paytirish amali quyidagi xossalarga ega:

1°. $(k + l)\boldsymbol{a} = k\boldsymbol{a} + l\boldsymbol{a}$**;**

2°. $k(\boldsymbol{a} + \boldsymbol{b}) = k\boldsymbol{a} + k\boldsymbol{b}$

3°. $k(l \cdot \boldsymbol{a}) = (k \cdot l)\boldsymbol{a}$

4°. $1 \cdot \boldsymbol{a} = \boldsymbol{a}, (-\mathbf{1})\boldsymbol{a} = -\boldsymbol{a}, \mathbf{0} \cdot \boldsymbol{a} = \mathbf{0},$ (k, l-skalyarlar).

$\boldsymbol{a}$ vektorning $\frac{1}{k}$ skalyarga ko'paytmasini shu vektorni k skalyarga bo'linmasi deb tushunish mumkin, ya'ni $\frac{\boldsymbol{a}}{k} = \frac{1}{k} \cdot \boldsymbol{a}$**.**

1-misol. $(\boldsymbol{a} + \boldsymbol{b}) + (\boldsymbol{a} - \boldsymbol{b}) = \boldsymbol{a} + \boldsymbol{a} + (\boldsymbol{b} - \boldsymbol{b}) = 2\boldsymbol{a}$

2-misol. $\boldsymbol{a} + \frac{\boldsymbol{b}-\boldsymbol{a}}{2} = \boldsymbol{a} + \frac{1}{2}(\boldsymbol{b} - \boldsymbol{a}) = \boldsymbol{a} + \frac{1}{2}\boldsymbol{b} - \frac{1}{2}\boldsymbol{a} = \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b} = \frac{\boldsymbol{a}+\boldsymbol{b}}{2}$;

$$\boldsymbol{a} - \frac{\boldsymbol{b}+\boldsymbol{a}}{\mathbf{2}} = \boldsymbol{a} - \frac{1}{2}(\boldsymbol{b} + \boldsymbol{a}) = \boldsymbol{a} - \frac{1}{2}\boldsymbol{b} - \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{a} = \frac{1}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b} = \frac{\boldsymbol{a}-\boldsymbol{b}}{2}.$$

Agar noldan farqli $\boldsymbol{a}$ vektorni uning uzunligi $|\boldsymbol{a}|$ ga bo'lsak, u holda $\boldsymbol{a}$ vektor yo'nalishidagi birlik $\boldsymbol{e}$ vektorini hosil qilamiz, bu birlik vektor *ort* deyiladi: $\boldsymbol{e} = \frac{\mathbf{1}}{|\boldsymbol{a}|} \cdot \boldsymbol{a}$. Bu yerdan vektorning standart ko'rinishidagi ifodasiga ega bo'lamiz: $\boldsymbol{a} = |\boldsymbol{a}|\boldsymbol{e}$.

1.3. Vektorlarning kollinearlik va komplanarlik shartlari

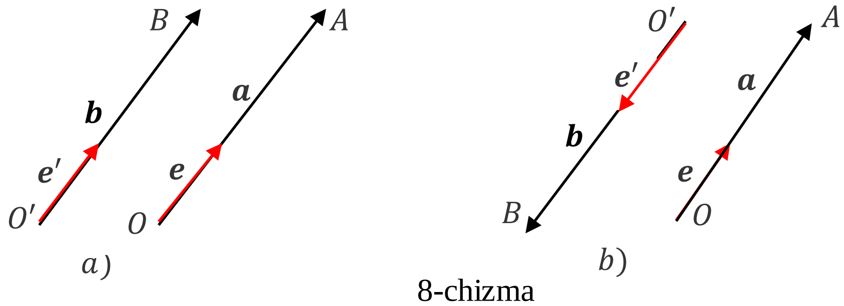
Vektorlarni qo'shish va skalyarga ko'paytirish algebraik amallari yordamida vektorlarning kollinearlik va komplanarlik shartlarini osongina ifodalash mumkin.

1-teorema. *Ikki noldan farqli* $\boldsymbol{a}$ *va* $\boldsymbol{b}$ *vektorlar faqat va faqat proporsional, ya'ni shunday* $k \neq 0$ *skalyar mavjud bo'lib,*

$$\boldsymbol{b} = k\boldsymbol{a} \tag{1}$$

tenglik bajarilsagina, kollinear bo'ladi.

Isbot. 1) Faraz qilaylik, $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar kollinear hamda mos ravishda $\boldsymbol{e}$, $\boldsymbol{e'}$ - ularning ortlari bo‘lsin (8-chizma).



8-chizma

Bu vektorlarni quyidagii standart shakllarda yozamiz: $\boldsymbol{a} = |\boldsymbol{a}|\boldsymbol{e}$, $\boldsymbol{b} = |\boldsymbol{b}|\boldsymbol{e'}$. Ko‘rinib turibdiki, $\mathrm{e}' = \pm\boldsymbol{e}$, bu yerda musbat ishora (+) $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlarning bir xil yo‘nalishida, manfiy ishora (–) esa $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlarning qarama-qarshi yo‘nalishda bo‘lgan hollarini ifodalaydi. Shunday qilib,

$$\boldsymbol{b} = \pm|\boldsymbol{b}|\boldsymbol{e}' = \pm\frac{|\boldsymbol{b}|}{|\boldsymbol{a}|}(|\boldsymbol{a}|\boldsymbol{e}) = \pm\frac{|\boldsymbol{b}|}{|\boldsymbol{a}|}\boldsymbol{a}.$$

Bu yerda $k = \pm\frac{|\boldsymbol{b}|}{|\boldsymbol{a}|}$ deb olinsa, (1) formula kelib chiqadi.

2) Agar (1) tenglik bajarilsa, u holda $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlari kollinearligi to‘g‘ridan to‘g‘ri vektorni skalyarga ko‘paytirish amali aniqlanishidan kelib chiqadi. Teorema to‘la isbotlandi.

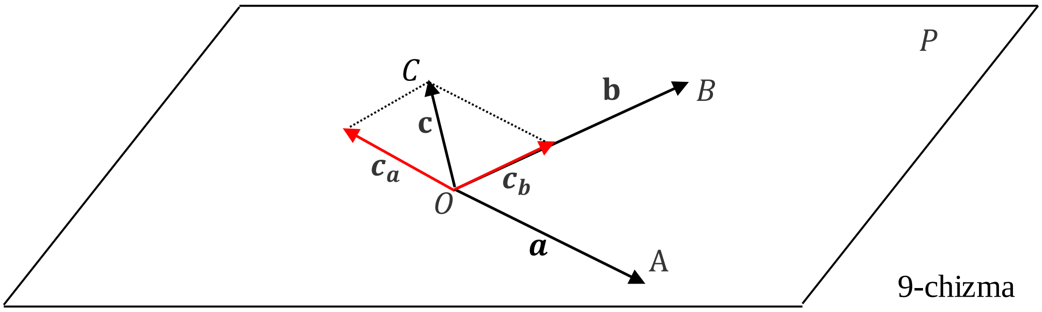
2-teorema. *Uchta noldan farqli $\boldsymbol{a}, \boldsymbol{b}$ va $\boldsymbol{c}$ vektorlari komplanar bo‘lishi uchun ulardan biri qolganlarining chiziqli kombinasiyasidan iborat bo‘lishi, ya’ni*

$$\boldsymbol{c} = k\boldsymbol{a} + l\boldsymbol{b} \tag{2}$$

tenglikning bajarilishi zarur va yetarli, bu yerda k, l - skalyarlar ($k \neq 0, l \neq 0$).

Isbot. 1) Faraz qilaylik $\boldsymbol{a}, \boldsymbol{b}$ va $\boldsymbol{c}$ vektorlar komplanar bo‘lsin. Unda ularning boshlang‘ich nuqtalarini umumiy O nuqtada keltirib, biror P tekislikda joylashtiramiz (9-chizma).

Avvalo, bu vektorlarning har qanday jufti kollinear bo‘lmasin, masalan, $\boldsymbol{a}$ va $\mathbf{b}$ vektorlari kollinear emas deb hisoblaymiz. U vaqtda $\mathbf{c}$ vektorni $\boldsymbol{a}$ va $\mathbf{b}$ vektorlarga kollinear bo‘lgan $\boldsymbol{c}_{\boldsymbol{a}}$ va $\boldsymbol{c}_{\mathbf{b}}$ vektorlarning yig‘indisi ko‘rinishda ifodalab, 1-teoremaga asosan $\mathbf{c} = \boldsymbol{c}_{\boldsymbol{a}} + \boldsymbol{c}_{\mathbf{b}} = k\boldsymbol{a} + l\boldsymbol{b}$ munosabatni olamiz, bu yerda k, l – skalyar miqdorlar. Shunday qilib, $\boldsymbol{a}$, $\boldsymbol{b}$ va $\mathbf{c}$ vektorlar komplanar bo‘lganda (2) tenglk o‘rinli bo‘ladi.



9-chizma

Endi $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ vektorlar juft-jufti bilan kollinear bo‘lsin, u holda $\mathbf{c} = k\boldsymbol{a} + 0 \cdot \boldsymbol{b}$, ya’ni bu holda ham (2) shartni olamiz.

Demak, (2) shartning bajarilishi uchta noldan farqli $\boldsymbol{a}, \boldsymbol{b}$ ва $\boldsymbol{c}$ vektorlar komplanar bo‘lishi uchun zarur ekan.

2) Endi (2) shart uchta noldan farqli a,b va c vektorlari komplanar bo‘lishi uchun yetarli ekanligini ko‘rsatamiz. 9-chizmaga ko‘ra $\boldsymbol{a} = \overline{OA}$, $\boldsymbol{b} = \overline{OB}$, $\boldsymbol{c} = \overline{OC}$ bo‘lgani uchun $\boldsymbol{a}$ ва $\mathbf{b}$ vektorlarning chiziqli kombinatsiyasidan iborat bo‘lgan $\boldsymbol{c}$ vektor ham $\boldsymbol{a}$, $\mathbf{b}$ vektorlar yotgan tekislikda yotadi. Demak, $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ vektorlar komplanar ekan. Boshqacha aytganda, (2) tenglik uchta noldan farqli $\boldsymbol{a}, \boldsymbol{b}$ ва $\boldsymbol{c}$ vektorlar komplanar bo‘lishining yetarli sharti ekan. Teorema isbotlandi.

3-misol. *$\boldsymbol{a}$, $\boldsymbol{a} + \boldsymbol{b}, \boldsymbol{a} - \boldsymbol{b}$ vektorlar komplanar bo‘ladi.*

Haqiqatan ham, $\boldsymbol{a} = \frac{1}{2}(\boldsymbol{a} + \boldsymbol{b}) + \frac{1}{2}(\boldsymbol{a} - \boldsymbol{b})$ tenglik bagarilganligi uchun 2-teoremaga ko‘ra bu $\boldsymbol{a}$, $\boldsymbol{a} + \boldsymbol{b}, \boldsymbol{a} - \boldsymbol{b}$ vektorlar komplanardir.

1.4. Vektor proyeksiyasi

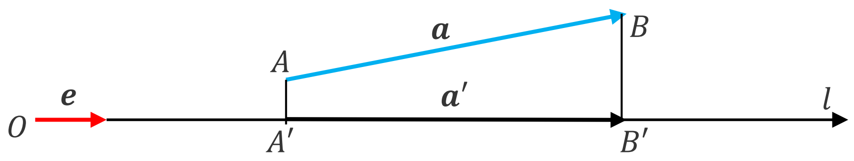
Boshlang‘ich nuqtasi va yo‘nalishi ko‘rsatilgan to‘g‘ri chiziqga *o‘q* deyiladi.

Odata o‘qning yo‘nalishi strelka orqali ko‘rsatiladi va OX kabi belgilanadi, bu yerda O boshlang‘ich nuqta, X esa to‘g‘ri chiziqning boshlang‘ich nuqtadan o‘ng tomonida joylashgan ixtiyoriy nuqtasi.

8-ta’rif. *A nuqtaning l o‘qdagi proyeksiyasi deb, A nuqtadan o‘qqa tushirilgan AA' perpendikulyarning A' asosiga aytiladi* (10-chizma).

Boshqacha qilib aytganda, A nuqtaning l o‘qdagi proyeksiyasi A nuqtadan o‘tib o‘qqa perpendikulyar bo‘lgan tekislikning kesishish nuqtasi A' ga aytiladi.

9-ta’rif. *l o‘qqa nisbatan $\boldsymbol{a} = \overline{AB}$ vektorning* ***komponentasi*** *(yasavchilari yoki tuzuvchilari) deb, l o‘qdagi $\boldsymbol{a}' = \overline{A'B'}$ vektorga aytiladi,* *bu yerda* A' *nuqta* $\boldsymbol{a} = \overline{AB}$ *vektor boshlang‘ich nuqtasi* A *ning* l *o‘qdagi proyeksiyasi,* B' *nuqta esa* $\boldsymbol{a} = \overline{AB}$ *vektor oxirgi nuqtasi* B *ning* l *o‘qdagi proyeksiyasi* (10-chizma).



10-chizma

10-ta’rif. $\boldsymbol{a}$ *vektorning* l ***o‘qdagi proyeksiyasi*** *deb,* $a_l = \pm\left|\overline{A'B'}\right|$ *skalyarga aytiladi. Bu yerda, agar komponenta yo‘nalishi* l *o‘qning yo‘nalishi bilan ustma-ust tushsa, u holda komponenta moduli(uzunligi) plyus (+) ishora bilan, agar komponenta yo‘nalishi* l *o‘qning yo‘nalishi bilan qarama-qarshi bo‘lsa, unda uning moduli minus (-) ishora bilan olinadi.*

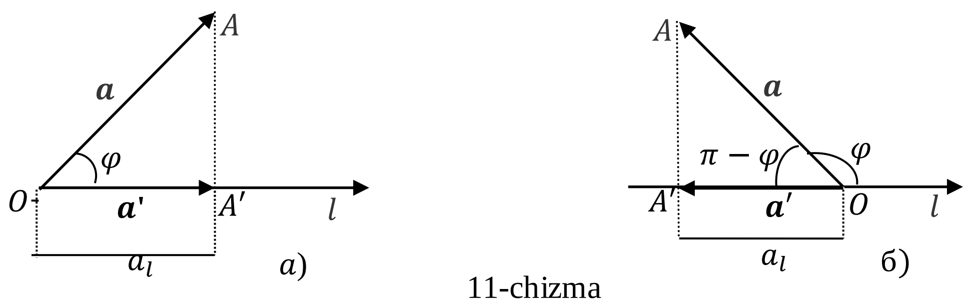
Agar $\boldsymbol{a} = \boldsymbol{0}$ bo‘lsa, u holda $a_l = 0$. Qayd etamizki, agar $\boldsymbol{e}$ – l o‘qning birlik vektori bo‘lsa, u holda $\boldsymbol{a'} = a_l\boldsymbol{e}$ tenglik o‘rinli.

$\boldsymbol{a}$ vektorning l o‘qdagi proyeksiyasi $\text{пр}_l\boldsymbol{a}$ kabi ham belgilanadi.

3-teorema. $\boldsymbol{a}$ vektorning l o‘qdagi proyeksiyasi $\boldsymbol{a}$ vektor $|\boldsymbol{a}|$ uzunligining shu vektor bilan l o‘q yo‘nalishi orasidagi burchak kosinusi ko‘paytimasiga teng, ya’ni

$$a_l = \text{пр}_l\boldsymbol{a} = |\boldsymbol{a}|\cos\varphi,\ \ \varphi = \left(\widehat{\boldsymbol{a}, l}\right) \qquad (3)$$

Isbot. $\boldsymbol{a} = \overline{OA}$ – ozod vektor bo‘lganligi uchun, uning O – boshlang‘ich nuqtasi l o‘qda yotadi deb hisoblash mumkin (11-chizma).



11-chizma

1) Agar $\boldsymbol{a}$ vektor va l o‘q orasidagi φ burchak o‘tkir $(0 \leq \varphi < \frac{\pi}{2})$ bo‘lsa, u holda $\boldsymbol{a}$ vektor $\boldsymbol{a'} = \overline{OA'}$ komponentasining yo‘nalishi l o‘q yo‘nalishi bilan ustma-ust tushadi (11a-чизма). Bu holda quyidagiga ega bo‘lamiz:

$$a_l = \text{пр}_l \boldsymbol{a} = +|OA'| = |OA'| \cos\varphi = |\boldsymbol{a}| \cos\varphi.$$

2) Agar $\boldsymbol{a}$ vektor va l o‘q orasidagi φ burchak o‘tmas ($\frac{\pi}{2} < \varphi \leq \pi$) bo‘lsa, u holda $\boldsymbol{a}$ vektor $\boldsymbol{a}' = \overline{OA'}$ komponentasining yo‘nalishi l o‘q yo‘nalishi bilan qarama-qarshi bo‘ladi (11*б*-чизма). Bu holda quyidagiga ega bo‘lamiz:

$$a_l = \text{пр}_l \boldsymbol{a} = -|OA'| = -|OA'| \cos(\pi - \varphi) = |\boldsymbol{a}| \cos\varphi.$$

3) Agar $\varphi = \frac{\pi}{2}$ bo‘lsa, u holda (3) formula o‘rinli, chunki bu holda $a_l = 0$. Teorema isbotlandi.

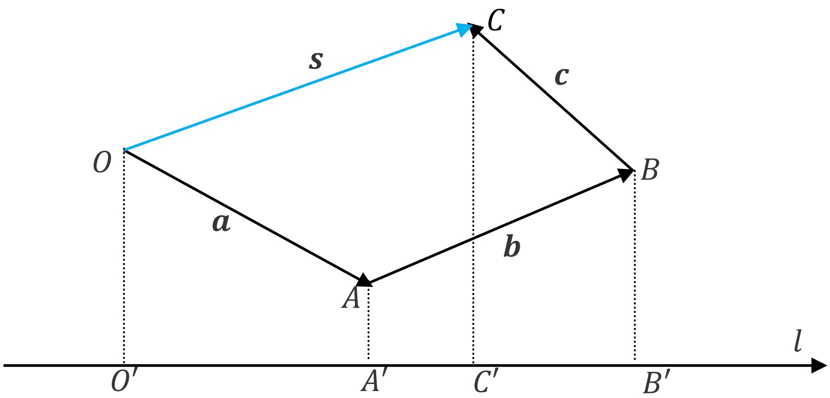
Bu teoremadan quyidagi natijalar kelib chiqadi.

1°. Vektorning o‘qdagi proyeksiyasi: a) musbat bo‘ladi, agarda vektor bilan o‘q o‘tkir burchak hosil qilsa; b) manfiy bo‘ladi, agarda bu burchak o‘tmas bo‘lsa; v) nolga teng bo‘ladi, agarda bu burchak to‘g‘ri bo‘lsa.

2°. Bitta o‘qdagi proyeksiyalari teng bo‘lgan vektorlar bir biriga teng vektorlardir.

4-teorema. *Bir nechta vektorlar yig‘indisining o‘qdagi proyeksiyasi vektorlarning shu o‘qdagi proyeksiyalari yig‘indisiga teng.*

Isbot. *Faraz qilaylik, masalan,* $\boldsymbol{s} = \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}$ *bo‘lsin, bu yerda* $\boldsymbol{a} = \overline{OA}$, $\boldsymbol{b} = \overline{AB}$, $\boldsymbol{c} = \overline{BC}$. *U holda vektorlarni qo‘shish qoidasiga asosan* $\boldsymbol{s} = \overline{OC}$ *bo‘ladi (12-chizma).*



12-chizma

O, A, B, C nuqtalarning l o‘qdagi proyeksiyalarini mos ravishda O', A', B', C' deb belgilab va $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ vektorlar komponentalarining yo‘nalishlarini hisobga olsak, (12-chizmaga q.) quyidagiga ega bo‘lamiz:

$$pr_l \boldsymbol{s} = +\left|\overline{O'C'}\right| = +\left|\overline{O'A'}\right| + \left|\overline{A'B'}\right| - \left|\overline{C'B'}\right| = pr_l \boldsymbol{a} + pr_l \boldsymbol{b} + pr_l \boldsymbol{c}.$$

Teorema isbotlandi.

Isbotlangan teoremadan kelib chiqadiki, vektorlar yopiq chizig'i-ning har qanday o'qga proyeksiyasi nolga teng.

5-teorema. $\boldsymbol{a}$ *vektorning k skalyarga ko'paytmasining* l *o'qga proyeksiyasi vektor proyeksiyasini ana shu skalyarga ko'paytirilganiga teng, ya'ni*

$$pr_l(k\boldsymbol{a}) = k \cdot pr_l\boldsymbol{a}\,. \tag{4}$$

(4) formula 3-teorema va vektorni skalyarga ko'paytirishamali ma'nosidan bevosita kelib chiqadi.

4 va 5-teoremalardan quyidagi natijani olamiz.

Natija. *Vektorlar chiziqli kombinasiyasining proyeksiysi ushu vektorlar proyeksiyalarning shunday chiziqli kombinasiyasiga teng, ya'ni*

$$pr_l(k\boldsymbol{a} + m\boldsymbol{b}) = k \cdot pr_l\boldsymbol{a} + m \cdot pr_l\boldsymbol{b} \tag{5}$$

4-misol. $\boldsymbol{a}$ *va* $\boldsymbol{b}$ *vektorlar orasidagi burchak 120° va ularning modullari* $|\boldsymbol{a}| = 3$**,** $|\boldsymbol{b}| = 4$ *bo'lsin***.** *Ularning chiziqli kombinatsiyasi bo'lgan* $\boldsymbol{c} = 2\boldsymbol{a} - \frac{3}{2}\boldsymbol{b}$ *vektorning berilgan* $\boldsymbol{a}$ *va* $\boldsymbol{b}$ *vektorlardagi proyeksiyalarini toping***.**

Yechish. (3) formulaga asosan quyidagilarga ega bo'lamiz**:**

$pr_{\boldsymbol{a}}\boldsymbol{a} = |\boldsymbol{a}|\cos 0° = |\boldsymbol{a}| = 3,$

$pr_{\boldsymbol{a}}\mathbf{b} = |\mathbf{b}|\cos 120° = |\mathbf{b}|(-\sin 30°) = -\frac{1}{2}|\mathbf{b}| = -2,$

$pr_{\mathbf{b}}\boldsymbol{a} = |\boldsymbol{a}|\cos 120° = |\boldsymbol{a}|(-\sin 30°) = -\frac{1}{2}|\boldsymbol{a}| = -\frac{3}{2},$

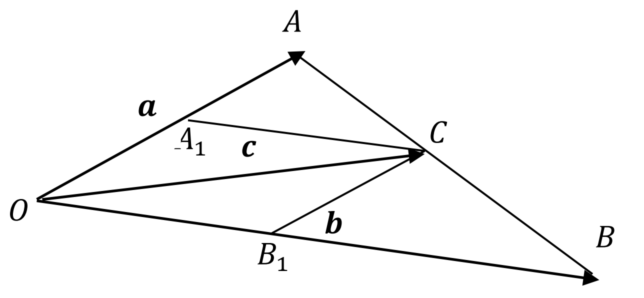
$pr_{\mathbf{b}}\mathbf{b} = |\mathbf{b}|\cos 0° = |\mathbf{b}| = 4.$

Endi (5) formuladan foydalanamiz:

$$pr_{\boldsymbol{a}}\mathbf{c} = 2pr_{\boldsymbol{a}}\boldsymbol{a} - \frac{3}{2}pr_{\boldsymbol{a}}\mathbf{b} = 2 \cdot 3 - \frac{3}{2}(-2) = 6 + 3 = 9,$$

$$pr_{\mathbf{b}}\mathbf{c} = 2\text{пр}_{\mathbf{b}}\boldsymbol{a} - \frac{3}{2}pr_{\mathbf{b}}\mathbf{b} = 2\left(-\frac{3}{2}\right) - \frac{3}{2} \cdot 4 = -3 - 6 = -9.$$

5-misol. $\boldsymbol{a} = \overline{OA}$, $\boldsymbol{b} = \overline{OB}$ *vektorlar berilgan va* $\boldsymbol{c} = \overline{OC}$ *vektor esa* OAB *uchburchakning medianasi bo'lsin (13-chizma).* $\boldsymbol{c}$ *vektorni* $\boldsymbol{a}$ *va* $\boldsymbol{b}$ *vektorlar bo'yicha yoyish talab etiladi.*



13-chizma

Yechish. *Faraz qilaylik, A_1, B_1 nuqtalar mos ravishda OAB uchburchakdagi OA va OB tomonlarining o'rtalari bo'lsin. Unda OA_1CB_1 parallelogrammdan quyidagini hosil qilamiz:*

$$\overline{OC} = OA_1 + OB_1 = \frac{1}{2}\overline{OA} + \frac{1}{2}\overline{OB} = \frac{1}{2}(\overline{OA} + \overline{OB}).$$

Shunday qilib, $\boldsymbol{c} = \frac{\mathbf{1}}{\mathbf{2}}(\boldsymbol{a} + \boldsymbol{b})$**.**

1.5. Nazorat savollari va masalalar

1. Skalyar va vektor miqdorlar deb nimaga aytiladi?
2. Vektor uzunligi deb nima?
3. Ikki vektorning tengligi qanday tushuniladi?
4. Ozod vektor deb nimaga aytiladi?
5. Kollinear va komplanar vektorlar qanday aniqlanadi?
6. Vektorlarni qo'shish amali qanday aniqlanadi (parallelogramm va parallelepiped qoidplari)?
7. Vektorlar yig'indisi uchun uchburchak tengsizligi deganda nima tushuniladi?
8. Vektorlarning ayirmasi qanday aniqlanadi (ayirish qoidasi)?
9. Vektorlarni qo'shish amalining asosiy xossalari?
10. Qarama-qarshi vektor nimaga aytiladi? Noldan farqli vectorlarni qo'shish natijasi nol-vektor bo'lishi mumkinmi?
11. Vektorni skalyarga ko'paytirish amali qanday aniqlanadi?
12. Vektorni skalyarga ko'paytirish amalining asosiy xossalari?
13. Vektorlarning kollinearlik va komplanarlik shartlari algebraik ko'rinishda qanday ifodalanadi?
14. Vektorning o'qdagi komponentasi deb nimaga aytiladi?
15. Vektorning o'qdagi proyeksiyasi deb nimaga aytiladi?
16. Vektorning o'qdagi proyeksiyasi uning uzunligi va o'q bilan vektor orasidagi burchak orqali qanday ifodalanadi?
17. Vektorning o'qdagi proyeksiyasining asosiy xossalarini ayting.
18. $\boldsymbol{a} = \boldsymbol{j} + \boldsymbol{k}, \boldsymbol{b} = \boldsymbol{k} + \boldsymbol{i}, \boldsymbol{c} = \boldsymbol{i} + \boldsymbol{j}$ vektorlari komplanarmi? Bu yerda $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ - o'zaro perpendikulyar ortlar.
19. Komplanar bo'lmagan uchta $\boldsymbol{a} = \overline{OA}$, $\boldsymbol{b} = \overline{OB}$, $\boldsymbol{c} = \overline{OC}$ vektorlar yordamida parallelepiped yasalgan. $\boldsymbol{d_1} = \boldsymbol{a} + \boldsymbol{b} - \boldsymbol{c}$, $\boldsymbol{d_2} = \boldsymbol{a} - \boldsymbol{b} + \boldsymbol{c}$, $\boldsymbol{d_3} = \boldsymbol{a} - \boldsymbol{b} - \boldsymbol{c}$, $\boldsymbol{d_4} = \boldsymbol{b} - \boldsymbol{a} - \boldsymbol{c}$ vektorlarning geometrik tasviri nimadan iborat?

20. Tomonlari $OB = AC = 4, BC = OA = 3$ bo'lgan $OACB$ to'g'ri to'rtburchakda M va N nuqtalar mos ravishda BC va AC tomonlarining o'rtalar bo'lsin. $\boldsymbol{c} = \overline{OC}$ vektorni $\boldsymbol{a} = \overline{OM}$ va $\boldsymbol{b} = \overline{ON}$ vektorlar bo'yicha yozing.

21. $\boldsymbol{c} = 3\,\boldsymbol{a} - 2\boldsymbol{b}$ vektor berilgan, bu yerda $\boldsymbol{a}$ ва **b** vektorlar bilan l o'q orasidagi burchaklar mos ravishda 180° va 30° ga teng. $\boldsymbol{c}$ vektorning l o'qdagi proyeksiyasini toping.

22. ABC uchburchakning AB tomoni M va N nuqtalar yordamida uchta kongurent bo'laklarga ajratilgan: $|AM| = |MN| = |NB|$. Agar $\overline{CA} = \boldsymbol{a}$ va $\overline{CB} = \boldsymbol{b}$ bo'lsa, $\overline{CM}$ vektorni toping.

23. Tomonlari $|OA| = 3$, $|OC| = 4$ bo'lgan $OABC$ parallelogram berilgan. M ва N nuqtalar mos ravishda AB ва BC tomonlarning o'rtalari bo'lsin. O nuqtaga $\overline{OM}$ ва $\overline{ON}$ kuchlar qo'yilgan. Bu kuchlarni OA va OC tomonlarning $\boldsymbol{i}, \boldsymbol{j}$ ortlari orqali ifodalang.

§2. TO'G'RI BURCHAKLI KOORDINATALAR SISTEMASIDA VEKTORLAR USTIDA ALGEBRAIK AMALLAR

2.1. Fazoda to'g'ri burchakli dekart koordinatalar sistemasi

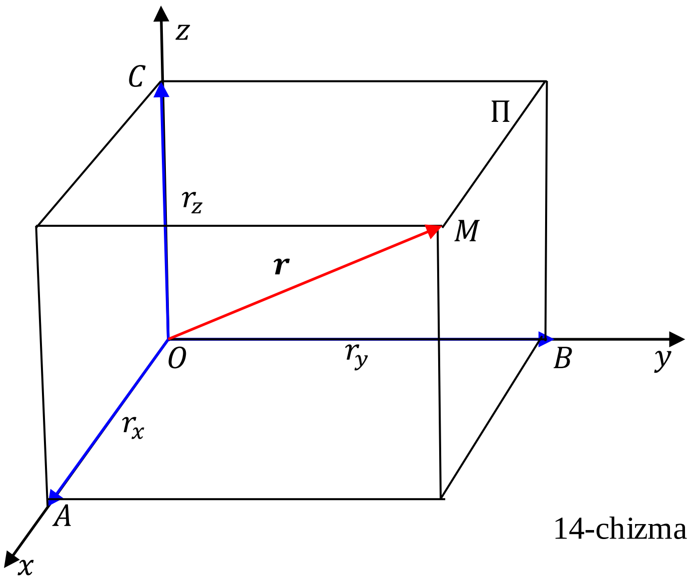
Uch o'lchamli V_3 fazoda boshlang'ich nuqtalari umumiy O nuqta bo'lgan o'zaro perpendikulyar Ox, Oy, Oz o'qlarni qaraymiz. O nuqta *koordinatalar boshi*, Ox, Oy, Oz esa – *koordinata o'qlari* deyiladi. Faraz qilaylik, koordinata o'qlari o'ng uchlikni tashkil qilsin, ya'ni kuzatuvchi Oz o'qi bo'ylab qaraganda Ox o'qidan Oy ga eng qisqa burilish soat strelkasiga qarama-qarshi yo'nalishda bo'lsin (14-chizma). Ba'zida o'ng uchlikni *o'ng sistema* ham deb ataydilar. Koordinata o'qlari orqali o'tib, o'zaro perpendikulyar bo'lgan Oxy, Oxz, Oyz tekisliklar koordinata tekisliklari deyiladi. Ular V_3 fazoni sakkizta oktantaga bo'ladi.

Fazodagi har bir M nuqta uchun boshlang'ich nuqtasi O koordinatar boshida, oxirgi nuqtasi esa M bo'lgan $\boldsymbol{r} = \overline{\mathrm{OM}}$ - *radius-vektor* mavjud. Boshqacha aytganda, boshlang'ich nuqtasi koordinatar boshida bo'lgan har qanday vektor radius-vektor deyiladi.

1-ta'rif. V_3 fazo M nuqtasining to'g'riburchakli dekart koordinatalar sistemasidagi x, y, z koordinatalari deb $\boldsymbol{r} = \overline{\mathrm{OM}}$ radius-vektorning mos ravishda Ox, Oy, Oz o'qlardagi proyeksiyalariga aytiladi, ya'ni:

$$x = r_x\,,\; y = r_y\,, z = r_z\,.$$

x, y, z koordinatalarga ega M nuqta $M(x, y, z)$ kabi belgilanadi, odatda M nuqtaning birinchi koordinatasi x – abssissa, ikkinchi koordinatasi y – *ordinata* va uchinchi koordinatasi z esa *applikata* deyiladi.



14-chizma

Bu koordinatalarni topish uchun M nuqta orqali Ox, Oy, Oz o‘qlarga perpendikulyar bo‘lgan uchta tekislik o‘tkazamiz. Bu tekisliklar bilan koordinata o‘qlarining kesishishi natijasida son qiymatlari M nuqtaning koordinatalariga teng $\overline{OA} = x, \overline{OB} = y, \overline{OC} = z$ yo‘nalgan kesmalarni hosil qilamiz.

$\boldsymbol{r} = \overline{\mathrm{OM}}$ radius-vektor MA, MB, MC tekisliklar va koordinata tekisliklari bilan hosil qilingan $|x|, |y|, |z|$ o‘lchamli Π parallelepipedning diagonalidan iborat (14-chizma). Shuning uchun

$$|\boldsymbol{r}| = \sqrt{x^2 + y^2 + z^2}. \tag{1}$$

Agar $\boldsymbol{r}$-radius-vektorning koordinata o‘qlari bilan hosil qilgan burchaklarini α, β, γ $(0 \leq \alpha, \beta, \gamma \leq \pi)$ orqali belgilasak, u holda quyidagiga ega bo‘lamiz:

$$x = |\boldsymbol{r}| \cos \alpha \,, y = |\boldsymbol{r}| \cos \beta \,, z = |\boldsymbol{r}| \cos \gamma \,. \tag{2}$$

2-ta’rif. $\boldsymbol{r}$-*radius-vektorning koordinata o‘qlari bilan hosil qilgan burchaklarning kosinuslariga uning yo‘naltiruvchi kosinuslari deyiladi.*

(1) formulani e’tiborga olib, (2) tenglikdan quyidagiga ega bo‘lamiz:

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \frac{x^2}{|\boldsymbol{r}|^2} + \frac{y^2}{|\boldsymbol{r}|^2} + \frac{z^2}{|\boldsymbol{r}|^2} = 1, \tag{3}$$

ya’ni V_3 fazoda ixtiyoriy nuqta radius-vektorining yo‘naltiruvchi kosinuslari kvadratlari yig‘indisi birga teng.

(2) formuladan quyidagi xulosani chiqaramiz: agar M nuqtaning koordinatasi musbat bo‘lsa, u holda radius-vektorning mos koordinata o‘qi bilan hosil qilgan burchagi o‘tkir, agar manfiy bo‘lsa, u holda bu burchak o‘tmas bo‘ladi.

Xususiy holda, agar $M(x, y, z)$ nuqta Oyz tekisligida yotsa, u holda $x = 0$; xuddi shunindek, $M(x, y, z)$ nuqta mos ravishda Oxz, Oxy tekisliklarda yotsa, u holda mos ravishda, $y = 0$, $z = 0$ bo‘ladi va aksincha.

O‘zaro perpendikulyar Ox, Oy, Oz koordnata o‘qlari bilan berlgan uch o‘lchamli V_3 fazoni $Oxyz$ kabi belgilaymiz.

1-misol. *V_3 fazoda $M(5, -3,4)$ nuqta berilgan. $\boldsymbol{r} = \overline{\mathrm{OM}}$ radius-vektorning uzunligi va uning yo‘naltiruvchi kosinuslarini toping.*

Yechish. (1) formulaga asosan $|\boldsymbol{r}| = \sqrt{5^2 + (-3)^2 + 4^2} = \sqrt{50} = 5\sqrt{2}$. Yo‘naltiruvchi kosinuslarn (2) formula asosida topamiz:

$$\cos\alpha = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}, \ \cos\beta = \frac{-3}{5\sqrt{2}}, \quad \cos\gamma = \frac{4}{5\sqrt{2}} = \frac{2\sqrt{2}}{5}.$$

2-misol. *M nuqtaning $\boldsymbol{r} = \overline{\mathrm{OM}}$ radius-vektori Ox o‘qi bilan $\alpha = 45°$ va Oy o‘qi bilan esa $\beta = 60°$ burchak hosil qiladi. M nuqtaning aplikatasi manfiy va $|\boldsymbol{r}| = 6$. M nuqtaning kordinatalarini toping.*

Yechish. (2) formulalarga asosan

$$x = |\boldsymbol{r}|\cos\alpha = 6\cos 45° = 3\sqrt{2}, y = |\boldsymbol{r}|\cos\beta = 6\cos 60° = 3.$$

Endi (1) formuladan foydalanib, quyidagini hosil qilamiz:

$$z^2 = |\boldsymbol{r}|^2 - x^2 - y^2 = 36 - \left(3\sqrt{2}\right)^2 - 3^2 = 9.$$

Shartga ko‘ra z (aplikata) manfiy bo‘lganligi sababli, oxirgi tenglikdan $z = -3$ ekanligini topamiz. Shunday qilib, M nuqtaning koordinatalari quyidacha: $M\left(3\sqrt{2}, 3, -3\right)$.

2.2. Vektor moduli va yo‘nalishining koordinatalarda ifodalanishi

$Oxyz$ sistema bilan aniqlangan V_3 fazoda $\boldsymbol{a}$ vektor berilgan bo‘lsin.

3-ta’rif. *$\boldsymbol{a}$ vektorning koordinata o‘qlaridagi proyeksiyalari:*

$$a_x = \text{пр}_x\boldsymbol{a}, a_y = \text{пр}_y\boldsymbol{a}, a_z = \text{пр}_z\boldsymbol{a}$$

uning koordinatalari deyiladi.

$\boldsymbol{a}$ vektorni uning koordinatalari orqali quyidagicha yozish mumkin:

$$\boldsymbol{a} = \left\{a_x, a_y, a_z\right\}.$$

$\boldsymbol{a}$ – ozod vektor bo‘lgani uchun, uni fazodagi $M(a_x, a_y, a_z)$ nuqtaning radius-vektori sifatida qarash mumkin. Shuning uchun $\boldsymbol{a}$ vektorning uzunligi quyidagi formula orqali aniqlanadi:

$$|\boldsymbol{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \tag{4}$$

ya’ni, vektorning moduli uning koordinatalari kvadratlari yig‘indisining kvadrat ildiziga teng.

$\boldsymbol{a}$ vektorning yo‘naltiruvchi kosinuslari $\cos\alpha$, $\cos\beta$, $\cos\gamma$ quyidagicha aniqlanadi:

$$\cos\alpha = \frac{a_x}{|\boldsymbol{a}|},\ \cos\beta = \frac{a_y}{|\boldsymbol{a}|},\ \cos\gamma = \frac{a_z}{|\boldsymbol{a}|}. \tag{5}$$

(4) formulani e’tiborga olsak, unda $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

Noldan farqli vektorning yo‘naltiruvchi kosinuslari uning yo‘nalishini bir qiymatli aniqlaydi. Shunday qilib, vektorning koordinatalari uni to‘la xarakterlaydi.

3-мисол. $\boldsymbol{a} = \{1, -2{,}2\}$ vektorning uzunligi va yo‘nalishini aniqlang.

Yechish. (4) va (5) formulardan foydalanib topamiz:

$$|\boldsymbol{a}| = \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{9} = 3,$$

$$\cos\alpha = \frac{a_x}{|\boldsymbol{a}|} = \frac{1}{3},\ \cos\beta = \frac{a_y}{|\boldsymbol{a}|} = -\frac{2}{3},\ \cos\gamma = \frac{a_z}{|\boldsymbol{a}|} = \frac{2}{3}.$$

Shunday qilib, berilgan vektor uzunligi 3 ga teng, bu vektor Ox, Oz koordinata o‘qlari bilan o‘tkir va Oy o‘qi bilan o‘tmas burchak tashkil qiladi.

$A(x_1, y_1, z_1)$ va $B(x_2, y_2, z_2)$ nuqtalar berilgan bo‘lsin. U holda $\boldsymbol{u} = \overline{AB}$ vektorning koordinata o‘qlaridagi proyeksiyalari quyidagicha aniqlanadi:

$$u_x = \text{пр}_x\overline{AB} = x_2 - x_1,\ u_y = \text{пр}_y\overline{AB} = y_2 - y_1,\ u_z = \text{пр}_z\overline{AB} = z_2 - z_1. \tag{7}$$

$\boldsymbol{u} = \overline{AB}$ vektorning yo‘naltiruvchi kosinuslari (5) formulaga asosan quyidagicha aniqlanadi:.

$$\cos\alpha = \frac{u_x}{|\boldsymbol{u}|},\ \cos\beta = \frac{u_y}{|\boldsymbol{u}|},\ \cos\gamma = \frac{u_z}{|\boldsymbol{u}|}. \tag{8}$$

4-misol. *$A(1{,}2{,}3)$ va $B(3, -4{,}6)$ nuqtalar berilgan. $\boldsymbol{u} = \overline{AB}$ vektorning koordinata o‘qlaridagi proyeksiyalarini, uzunligini va yo‘nalishini toping.*

Yechish. (6), (7), (8) formulalardan foydalanamiz. Quyidagilarga ega bo‘lamiz: $u_x = \text{пр}_x\overline{AB} = 3 - 1 = 2$, $u_y = \text{пр}_y\overline{AB} = -4 - 2 = -6$,

$$u_z = \text{пр}_z\overline{AB} = 6 - 3 = 3; \ |\boldsymbol{u}| = \sqrt{2^2 + (-6)^2 + 3^2} = \sqrt{49} = 7;$$
$$\cos\alpha = \frac{2}{7}, \cos\beta = -\frac{6}{7}, \cos\gamma = \frac{3}{7}.$$

$\boldsymbol{a} = \{a_x, a_y, a_z\}$ vektor berilgan bo‘lsin, bu yerda a_x, a_y, a_z – uning Ox, Oz, Oy o‘qlardagi mos proyeksiyalari. Ox, Oz, Oy o‘qlarning yo‘nalishiga mos $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ birlik vektorlar(ortlar)ni kiritamiz. Natijada, Ox, Oz, Oy koordinata o‘qlariga nisbatan $\boldsymbol{a}$ vektorning komponentalari $\boldsymbol{a_1} = a_x\boldsymbol{i}, \boldsymbol{a_2} = a_y\boldsymbol{j}$, $\boldsymbol{a_3} = a_z\boldsymbol{k}$ ko‘rinishga ega bo‘ladi. U holda $\boldsymbol{a}$ vektor o‘zining koordnata o‘qlaridagi komponentalari orqali yoyiladi, ya’ni $\boldsymbol{a} = \boldsymbol{a_1} + \boldsymbol{a_2} + \boldsymbol{a_3}$. Shunday qilib,

$$\boldsymbol{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k} \tag{9}$$

$\boldsymbol{a}$ vektorning koordinatalarini bilgan holda, hamma vaqt uni (9) formula orqali *bir qiymatli* yoyish mumkin.

5-misol. $\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$ vektorning uzunligini va uning yo‘naltiruvchi kosinuslarini toping.

Yechish. (9) formulaga asosan: $a_x = 2, a_y = 3, a_z = -4$. Endi (4) va (5) formulardan foydalansak, u holda:

$$|\boldsymbol{a}| = \sqrt{2^2 + 3^2 + (-4)^2} = \sqrt{29},$$
$$\cos\alpha = \frac{2}{\sqrt{29}}, \cos\beta = \frac{3}{\sqrt{29}}, \cos\gamma = -\frac{4}{\sqrt{29}}.$$

2.3. Koordinatalari bilan berilgan vektorlar ustida chiziqli amallar

Koordinatalari bilan berilgan vektorlarlarni qo‘shish va skalyarga ko‘paytirish amallarini quyidagicha aniqlash mumkin:

1) Агар $\boldsymbol{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}$, $\boldsymbol{b} = b_x\boldsymbol{i} + b_y\boldsymbol{j} + b_z\boldsymbol{k}$ bo‘lsa, u holda

$$\boldsymbol{a} \pm \boldsymbol{b} = (a_x \pm b_x)\boldsymbol{i} + (a_y \pm b_y)\boldsymbol{j} + (a_z \pm b_z)\boldsymbol{k}, \tag{10}$$

yoki, qisqacha

$$\boldsymbol{a} \pm \boldsymbol{b} = \{a_x \pm b_x, a_y \pm b_y, a_z \pm b_z\}. \tag{11}$$

Shunday qilib, vektorlarni qo‘shganda(yoki ayrganda) ularning bir xil koordinatalari qo‘shiladi (yoki ayriladi).

2) Agar $\boldsymbol{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}$, λ – skalyar bo‘lsa, u holda

$\lambda\boldsymbol{a} = \lambda a_x\boldsymbol{i} + \lambda a_y\boldsymbol{j} + \lambda a_z\boldsymbol{k}$ yoki, qisqacha $\lambda\boldsymbol{a} = \{\lambda a_x, \lambda a_y, \lambda a_z\}$.

Demak, vektorni skalyarga ko‘paytirilsa, unda uning barcha koordinatalari shu skalyarga ko‘paytiriladi.

6-misol. $\overline{OA} = \boldsymbol{i} + \boldsymbol{j}$, $\overline{OB} = \boldsymbol{k} - 3\boldsymbol{j}$ *vektorlari* $OACB$ *parallelogrammning tomonlari bo'lsin. Shu parallelogrammning* $\overline{OC}$ *va* $\overline{AB}$ *diagonallarini aniqlang.*

Yechish. Ma'lumki, $\overline{OC} = \overline{OA} + \overline{OB}$, $\overline{AB} = \overline{OB} - \overline{OA}$. Shu sababli, (10) formulani qo'llab, parallelogrammning diagonallarini aniqlaymiz:

$$\overline{OC} = (\boldsymbol{i} + \boldsymbol{j}) + (\boldsymbol{k} - 3\boldsymbol{j}\ ) = \boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k};$$
$$\overline{AB} = (\boldsymbol{k} - 3\boldsymbol{j}\ ) - (\boldsymbol{i} + \boldsymbol{j}) = -\boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k}.$$

7-misol. *Ikki* $\boldsymbol{F_1} = \{10, 20, 30\}$ *va* $\boldsymbol{F_2} = \{30, 20, 10\}$ *kuchlarning* $\boldsymbol{F}$ *teng ta'sir etuvchisi miqdori va yo'nalishini toping.*

Yechish. (11) formulaga asosan quyidagini hosil qilamiz:

$$\boldsymbol{F} = \boldsymbol{F_1} + \boldsymbol{F_2} = \{10 + 30, 20 + 20, 30 + 10\} = \{40, 40, 40\}.$$

(4) va (5) formulalardan foydalanib, ikki $\boldsymbol{F_1}$, $\boldsymbol{F_2}$ kuchlarning $\boldsymbol{F}$ teng ta'sir etuvchisi miqdori va uning yo'nalishini topamiz:

$$|\ \boldsymbol{F}| = 40\sqrt{3}\ , \cos\alpha = \cos\beta = \cos\gamma = \frac{1}{\sqrt{3}}.$$

8-misol. $\boldsymbol{a} = 4\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ *vektor berilgan. Agar* $||\boldsymbol{b}| = |\boldsymbol{a}|$, $b_y = a_y$ *va* $b_x = 0$ *bo'lsa,* $\boldsymbol{b}$ *vektorni toping.*

Yechish. $\boldsymbol{a}$ vektorning uzunligi $|\boldsymbol{a}| = \sqrt{4^2 + (-2)^2 + 3^2} = \sqrt{29}$ ga teng; $\boldsymbol{b}$ vektorni quyidagicha ifodalaymiz: $\boldsymbol{b} = 0 \cdot \boldsymbol{i} - 2 \cdot \boldsymbol{j} + b_z$; (4) formulaga asosan $|\boldsymbol{b}|^2 = \boldsymbol{0}^2 + (-2)^2 + (b_z)^2 = 4 + (b_z)^2$. Shartga ko'ra $|\boldsymbol{b}| = |\boldsymbol{a}| = \sqrt{29}$. Demak, $4 + (b_z)^2 = 29, (b_z)^2 = 25, b_z = \pm 5$. Shunday qilib, $\boldsymbol{b} = -2\ \boldsymbol{j} \pm 5\boldsymbol{k}$.

2.4. Nazorat savollari va masalalar

1. Fazodagi to'g'riburchakli dekart koordinatalar sistemasining asosiy elementlari.
2. Fazodagi to'g'riburchakli dekart koordinatalar sistemasidagi nuqtaning koordinatalari.
3. Fazodagi nuqtaning radius-vektori uzunligi qanday aniqlanadi?
4. Fazodagi nuqtaning radius-vektori yo'nalishini qaysi miqdorlar aniqlaydi?
5. Yo'naltiruvchi kosinuslar orasida qanday bog'lanish mavjud?
6. Nuqtaning koordinatalari berilganda, radius-vektorning yo'naltiruvchi kosinuslari qanday aniqlanadi?
7. Fazoda vektor koordinatalari deganda nima tushuniladi?
8. Vektorning uzunligi va yo'naltiruvchi kosinuslari qanday topiladi?

9. Agar A va B nuqtalarning koordinatalari berilsa, $\boldsymbol{u} = \overline{AB}$ vektorning o‘qdagi proyeksiyasi qanday aniqlanadi? |**u**|=?

10. Vektorni koordinata o‘qlariga nisbatan komponentalari orqali yoyish qaysi formula bilan ifodalanadi?

11. Vektorning koordinatalar orqali yozilish shaklidan foydalanib, vektorlarni qo‘shish (ayirish) va skalyarga ko‘paytirish amallarining ta’rifini bering..

12. $M\left(\sqrt{2}, -2, \sqrt{3}\right)$ nuqta radius-vektorining uzunligi va yo‘naltiruvchi kosinuslarini toping.

13. $M(x, y, z)$ nuqta $\boldsymbol{r}$ radius-vektori uzunligi $|\boldsymbol{r}| = 3$ ga va yo‘naltiruvchi kosinuslari $\cos\alpha = \frac{1}{\sqrt{3}}$, $\cos\beta = \frac{1}{3\sqrt{3}}$, $\cos\gamma = \frac{\sqrt{17}}{3\sqrt{3}}$ ga teng. $M(x, y, z)$ nuqtaning koordinatalarini toping.

14. Vectorning Ox va Oy o‘qlari bilan tashkil qilgan burchaklari mos ravishda $45°$ va $60°$ ga teng. Uning Oz oqi bilan tashkil qilgan burchagini toping.

15. $\boldsymbol{a} = \left\{1, -1, \sqrt{2}\right\}$ vektorning uzunligi va yo‘nalishini toping.

16. $\boldsymbol{r} = \overline{OM}$ vektor koordinata o‘qlari bilan bir xil o‘tkir burchak tashkil qiladi**.** Agar $|\boldsymbol{r}| = 2\sqrt{3}$ bo‘lsa bu burchaklarni va $\boldsymbol{r} = \overline{OM}$ vektorning koordinatalarini toping.

17. $A(2,1,-1)$ nuqtaga moduli $|\boldsymbol{F}| = 7$ ga teng $\boldsymbol{F}$ kuch qo‘yilgan. Kuch koordinatalari $F_x = 2, F_y = -3, F_z > 0$ bo‘lsa, $\boldsymbol{F}$ kuchni ifodalavchi vektorning oxirini va yo‘nalishini toping.

18. $A(1,2,3)$, $B(3,-4,6)$ nuqtalar berilgan. $\boldsymbol{u} = \overline{AB}$ vektorning uzunligi va yo‘nalishini toping.

19. ABC uchburchakning radius-vektorlari berilgan: $\boldsymbol{r}_A = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$**,** $\boldsymbol{r}_B = 3\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$**,** $\boldsymbol{r}_C = \boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}$. ABC uchburchakning teng tomonli ekanligini ko‘rsating

20. Agar $\boldsymbol{a} = \overline{AB} + \overline{CD}$ (bu yerda $(0,0,1)$, $B\ (3,2,1)$, $C(4,6,5)$, $D(1,6,3)$ bo‘lsa, $\boldsymbol{a}$ vektorning o‘qlardagi proeksiylarini toping.

21. M nuqtanind $\boldsymbol{r}$ radius-vektori Oy o‘qi bilan $60°$, Oz o‘qi bilan esa $45°$ burchaklar tashkil qiladi. Uning uzunligi $|\boldsymbol{r}| = 8$. Agar radius-vektor absissasi manfiy bo‘lsa, M nuqtaning koordinatalarini toping.

22. $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k} - \frac{\boldsymbol{1}}{\boldsymbol{5}}(4\boldsymbol{i} + 8\boldsymbol{j} + 3\boldsymbol{k})$ vektor modulini hisoblang va yo‘naltiruvch kosinuslsrini toping.

23. $\boldsymbol{F_1} = \{10, 20, 0\}$, $\boldsymbol{F_2} = \{0, -10, 20\}$, $\boldsymbol{F_3} = \{-10, 0, -20\}$ kuchlarning teng ta’sir etuvchisi bo‘lgan $\boldsymbol{F}$ kuchning miqdori va yo‘nalishini toping.

24. Oxy tekislikda $A(4,2)$,$B(2,3)$, $C(0,5)$ nuqtalar berilgan va $\boldsymbol{a} = \overline{OA}$, $\boldsymbol{b} = \overline{OB}$, $\boldsymbol{c} = \overline{OC}$ vektorlari yasalgan. $\boldsymbol{c}$ vektorning $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar bo‘yicha yoyilmasini toping.

25. $\overline{AB} = \boldsymbol{a} + 2\boldsymbol{b}, \overline{BC} = -4\boldsymbol{a} - \boldsymbol{b}, \overline{CD} = -5\boldsymbol{a} - 3\boldsymbol{b}$ berilgan**.** $ABCD$ – trapetsiya ekanligini ko‘rsating.

§3. VEKNORLARNING KO‘PAYTMALARI

3.1. Veknorlarning skalyar ko‘paytmasi

Orasidagi burchagi φ ga teng bo‘lgan $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlari berilgan bo‘lsin.

1-ta’rif. *Ikki* $\boldsymbol{a}$ *va* $\boldsymbol{b}$ *vektorlarning skalyar ko‘paytmasi* $(\boldsymbol{a}, \boldsymbol{b})$ *deb, bu vektorlar uzunliklarimi ular orasidagi burchak kosinusiga ko‘paytirishdan hosil bo‘lgan* songa aytiladi, ya’ni

$$(\boldsymbol{a}, \boldsymbol{b}) = |\boldsymbol{a}||\boldsymbol{b}| \cos \varphi. \qquad (1)$$

Ikki **a** va **b** vektorlarning skalyar ko‘paytmasi $\boldsymbol{ab}$ kabi ham belgilanadi.

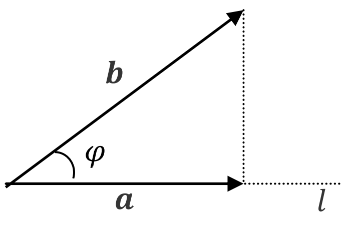
Agar vektorning o‘qdagi proyeksiyasi tushunchasidan foydalansak, u holda

$$|\boldsymbol{a}| \cos \varphi = pr_{\boldsymbol{b}}\, \boldsymbol{a}\ \boldsymbol{,}\ |\boldsymbol{b}| \cos \varphi = pr_{\boldsymbol{a}} \boldsymbol{b}.$$

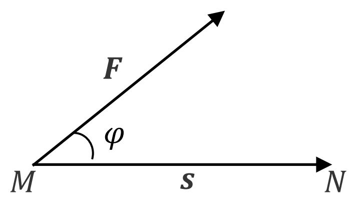
Unda (1) formulaga asosan

$$\boldsymbol{ab} = |\boldsymbol{a}| pr_{\boldsymbol{a}} \boldsymbol{b} = |\boldsymbol{b}| pr_{\boldsymbol{b}} \boldsymbol{a}, \qquad (2)$$

ya’ni, ikki vektorning skalyar ko‘paytmasi ulardan birinig uzunligini ikkinchi vektorning birinchi vektor yo‘nalishdagi o‘qdagi proyeksiysiga ko‘paytirilganiga teng (15-chizma).



15-chizma



16-chizma

Skalyar ko‘paytmaning fizik ma‘nosini tushuntramiz. $\boldsymbol{F}$ kuch moddiy nuqtani to‘g‘ri chiziq bo‘ylab $\boldsymbol{s} = \overline{MN}$ ga ko‘chsin (16-chizma).

Agar $\boldsymbol{F}$ kuch bilan ko‘chish vektori $\boldsymbol{s}$ orasidagi burchak φ bo‘lsa, u holda fizikadan ma’lumki, $\boldsymbol{F}$ ning $\boldsymbol{s}$ ko‘chishda bajargan ishi quyidagiga teng:

$$\boldsymbol{A} = |\boldsymbol{F}||\boldsymbol{s}|\cos\varphi.$$

(1) formulaga asosan bu formulani quyidagicha yozamiz:

$$\boldsymbol{A} = (\boldsymbol{F}, \boldsymbol{s}). \tag{3}$$

Shunday qilib, nuqtaga qo‘yilgan kuchning to‘gri chiziq bo‘lab ko‘chishida bajargan ishi kuch vektori va ko‘chish vektorlarning skalyar ko‘paytmasiga teng.

Vektorlar skalyar ko‘paytmasi quyidagi xossalarga ega:

1°. Ikki vektorning skalyar ko‘paytmasi ko‘paytuvchilar tartibiga bog‘liq emas (o‘rin almashtirish xossasi):

$$(\boldsymbol{a}, \boldsymbol{b}) = (\boldsymbol{b}, \boldsymbol{a}). \tag{4}$$

(4) tenglik bevosita (1) formuladan kelib chiqadi.

2°. Uch $\boldsymbol{a}, \boldsymbol{b}$, $\boldsymbol{c}$ vekrtorlar uchun quyidagi tenglik o‘rinli (taqsimot xossasi)

$$(\boldsymbol{a} + \boldsymbol{b}, \boldsymbol{c}) = (\boldsymbol{a}, \boldsymbol{c}) + (\boldsymbol{b}, \boldsymbol{c}). \tag{5}$$

Haqiqatdan ham, vektorlarning proeksiysi xossalarini e‘tiborga olib, (2) formulaga asosan quyidagini hosil qilamiz:

$$(\boldsymbol{a} + \boldsymbol{b}, \boldsymbol{c}) = \text{пр}_{\boldsymbol{c}}(\boldsymbol{a} + \boldsymbol{b}) \cdot |\boldsymbol{c}| = (\text{пр}_{\boldsymbol{c}}\boldsymbol{a} + \text{пр}_{\boldsymbol{c}}\boldsymbol{b}) \cdot |\boldsymbol{c}| =$$
$$= \text{пр}_{\boldsymbol{c}}\boldsymbol{a} \cdot |\boldsymbol{c}| + \text{пр}_{\boldsymbol{c}}\boldsymbol{b} \cdot |\boldsymbol{c}| = (\boldsymbol{a}, \boldsymbol{c}) + (\boldsymbol{b}, \boldsymbol{c}).$$

3°. $\boldsymbol{a}$ vektorning skalyar kvadrati $\boldsymbol{a}^2 = (\boldsymbol{a}, \boldsymbol{a})$ shu vektor moduli-ning kvadratiga teng, ya’ni $\boldsymbol{a}^2 = |\boldsymbol{a}|^2$.

Haqiqatdan ham, $\boldsymbol{a}^2 = (\boldsymbol{a}, \boldsymbol{a}) = |\boldsymbol{a}| \cdot |\boldsymbol{a}|\cos 0 = |\boldsymbol{a}|^2$, chunki teng vektorlaar orasidagi burchak nolga teng. Bu yerdan vektor moduli uchun

$$|\boldsymbol{a}| = \sqrt{(\boldsymbol{a}, \boldsymbol{a})}\,. \tag{6}$$

formulani hosil qilamiz.

4°. Skalyar ko‘paytuvchini skalyar ko‘paytma belgisidan tashqa-riga chqarish mumkin, ya’ni

$$(\lambda\boldsymbol{a}, \boldsymbol{b}) = (\boldsymbol{a}, \lambda\boldsymbol{b}) = \lambda(\boldsymbol{a}, \boldsymbol{b}).\ \ (\lambda - \text{skalyar}) \tag{7}$$

Bu xossa (1) dan osongina kelib chiqadi.

5°. Vektorlar chiziqli kombinatsiyasining berilgan ixtiyoriy vek-torga skalyar ko‘paytmasi shu vektorlarning berilgan vektorga skalyar ko‘paytmalarinig xuddi shunday chiziqli kombinatsiyasiga teng, ya’ni

$$(\lambda\boldsymbol{a} + \mu\boldsymbol{b}, \boldsymbol{c}) = \lambda(\boldsymbol{a}, \boldsymbol{c}) + \mu(\boldsymbol{b}, \boldsymbol{c}).\ (\lambda, \mu - skalyarlar)$$

Bu xossa 2°, 4° xossalarning natijasidir.

(1) formuladan kelib chiqadiki, noldan farqli $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar orasidagi φ burchak kosinusi quyidagiga teng:

$$\cos\varphi = \frac{(\boldsymbol{a},\boldsymbol{b})}{|\boldsymbol{a}|\cdot|\boldsymbol{b}|}\,. \qquad (8)$$

(8) formuladan $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlarning perpendikulyar (ortogonal), ya'ni ular orasdagi burchak $\varphi = \frac{\pi}{2}$ bo'lishi uchun $(\boldsymbol{a},\boldsymbol{b}) = 0$ shart bajarilishi zarur va yetarlidir.

1-**misol**. $\boldsymbol{m}$ va $\boldsymbol{n}$ birlik vektorlar orasidagi burchak $\varphi = 30°$ bo'lsin. $(\boldsymbol{m}+\boldsymbol{n})^2$ ni toping.

Yechish. $1° - 3°$ xossalardan foydalanib, quyidagini hosil qilamiz:

$$(\boldsymbol{m}+\boldsymbol{n})^2 = (\boldsymbol{m}+\boldsymbol{n},\boldsymbol{m}+\boldsymbol{n}) = (\boldsymbol{m},\boldsymbol{m}) + (\boldsymbol{m},\boldsymbol{n}) + (\boldsymbol{n},\boldsymbol{m}) + (\boldsymbol{n},\boldsymbol{n})$$
$$= \boldsymbol{m}^2 + +2(\boldsymbol{m},\boldsymbol{n}) + \boldsymbol{n}^2 = |\boldsymbol{m}|^2 + 2|\boldsymbol{m}|\cdot|\boldsymbol{n}|\cdot\cos\varphi + |\boldsymbol{n}|^2$$
$$= 1 + 2\cdot\cos 30° + 1 =$$

2-**misol**. Berilgan: $|\boldsymbol{a}| = 2\sqrt{2}, |\boldsymbol{b}| = 4,$ $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar orasidagi burchak $\varphi = 135°$. $(\boldsymbol{a}-\boldsymbol{b})^2$ni toping.

Yechish. $1° - 3°$ xossalardan foydalanib, quyidagini hosil qilamiz:

$$(\boldsymbol{a}-\boldsymbol{b})^2 = (\boldsymbol{a}-\boldsymbol{b},\boldsymbol{a}-\boldsymbol{b}) = (\boldsymbol{a},\boldsymbol{a}) - 2(\boldsymbol{a},\boldsymbol{b}) + (\boldsymbol{b},\boldsymbol{b}) = \boldsymbol{a}^2 - 2(\boldsymbol{a},\boldsymbol{b}) + +\boldsymbol{b}^2 = |\boldsymbol{a}|^2 - 2|\boldsymbol{a}|\cdot|\boldsymbol{b}|\cdot\cos\varphi + |\boldsymbol{b}|^2 = 8 - 2\cdot 2\sqrt{2}\cdot 4\cdot \cos 135° + 16 = 24 + +16\sqrt{2}\cdot\sin 45° = 24 + 16 = 40.$$

3-misol. O nuqtaga $\boldsymbol{F}_1$ va $\boldsymbol{F}_2$ kuchlar qo'yilgan. Ular orasidagi burchak $\varphi = 45°$ va modullari $|\boldsymbol{F}_1| = 10,$ $|\boldsymbol{F}_2| = 20$. Bu kuchlar teng ta'sir etuvchisinng miqdorni toping.

Yechish. Teng ta'sir etuvchi kuch $\boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2$. Unda:

$$(\boldsymbol{F}_1+\boldsymbol{F}_2)^2 = (\boldsymbol{F}_1)^2 + 2(\boldsymbol{F}_1,\boldsymbol{F}_2) + (\boldsymbol{F}_2)^2 = |\boldsymbol{F}_1|^2 + 2|\boldsymbol{F}_1||\boldsymbol{F}_2|\cos\varphi + |\boldsymbol{F}_2|^2 =$$
$$= 10^2 + 2\cdot 10\cdot 20\cdot\cos 45° + 20^2 = 500 + 400\cdot\frac{\sqrt{2}}{2} = 500 + 200\sqrt{2}.$$

Shunday qilib, $|\boldsymbol{F}| = \sqrt{(\boldsymbol{F}_1+\boldsymbol{F}_2)^2} = \sqrt{500+200\sqrt{2}} = 10\sqrt{5+2\sqrt{2}}.$

$\boldsymbol{a}$ и $\boldsymbol{b}$ vektorlar koordinatalar shaklida berilgan bo'lsin:

$$\boldsymbol{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}, \boldsymbol{b} = b_x\boldsymbol{i} + b_y\boldsymbol{j} + b_z\boldsymbol{k}, \qquad (9)$$

bu yerda $\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}$ – Ox, Oy, Oz koordinata o'qlaridagi ortlar. Ushbu vektorlarni ko'phadlarni ko'paytirgandek ko'paytiramiz(bu esa skalyar ko'paytma xossalariga ko'ra o'rinlidir) va

$$(\boldsymbol{i},\boldsymbol{j}) = (\boldsymbol{j},\boldsymbol{k}) = (\boldsymbol{k},\boldsymbol{i}) = 0, (\boldsymbol{i},\boldsymbol{i}) = (\boldsymbol{j},\boldsymbol{j}) = (\boldsymbol{k},\boldsymbol{k}) = 1,$$

munosobatlarni e'tiborga olib, quyidagini olamiz:

$$(\boldsymbol{a},\boldsymbol{b}) = a_x b_x + a_y b_y + a_z b_z. \qquad (10)$$

Shunday qilib, *vektorlarning skalyar ko'paytmasi ularning mos koordinatalari ko'paytmalari yig'indisiga teng.*

Agar (9) vektorlar orasidagi burchakni φ orqali belgilasak, unda (8) va (10) ga asosan

$$\cos\varphi = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a_x^2 + a_y^2 + a_z^2} \cdot \sqrt{b_x^2 + b_y^2 + b_z^2}}. \tag{11}$$

formulaga ega bo‘lamiz.

4-misol. $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}, \boldsymbol{b} = \boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}$ vektorlar berilgan. Ular uchun $pr_{\boldsymbol{a}}\boldsymbol{b}$ va $pr_{\boldsymbol{b}}\boldsymbol{a}$ larni toping.

Yechish. (2) formulaga ko‘ra

$$pr_{\boldsymbol{a}}\boldsymbol{b} = \frac{(\boldsymbol{a},\boldsymbol{b})}{|\boldsymbol{a}|}, pr_{\boldsymbol{b}}\boldsymbol{a} = \frac{(\boldsymbol{a},\boldsymbol{b})}{|\boldsymbol{b}|}.$$

(10) formulaga asosan

$$(\boldsymbol{a},\boldsymbol{b}) = 1 \cdot 1 + 1 \cdot (-1) + 2 \cdot 4 = 8.$$

$\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlarning uzunliklarini topamiz:

$$|\boldsymbol{a}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}, |\boldsymbol{b}| = \sqrt{1^2 + (-1)^2 + 4^2} = 3\sqrt{2}.$$

Shunday qilib, $\text{пр}_{\boldsymbol{a}}\boldsymbol{b} = \frac{8}{\sqrt{6}}, \text{пр}_{\boldsymbol{b}}\boldsymbol{a} = \frac{8}{3\sqrt{2}}..$

5-misol. Yig‘indini hisoblang : $S = (2\boldsymbol{i} - \boldsymbol{j}, \boldsymbol{j}) + (\boldsymbol{j} - 2\boldsymbol{k}, \boldsymbol{k}) + (\boldsymbol{i} - 2\boldsymbol{k})^2$.

Yechish. (10) formuladan foydalanib, topamiz:

$$(2\boldsymbol{i} - \boldsymbol{j}, \boldsymbol{j}) = (-1) \cdot 1 = -1, (\boldsymbol{j} - 2\boldsymbol{k}, \boldsymbol{k}) = (-2) \cdot 1 = -2,$$

$$(\boldsymbol{i} - 2\boldsymbol{k})^2 = (\boldsymbol{i} - 2\boldsymbol{k}, \boldsymbol{i} - 2\boldsymbol{k}) = 1 \cdot 1 + (-2) \cdot (-2) = 5.$$

Shunday qilib, $S = -1 + (-2) + 5 = 2$.

3.2. Vektorlarning kollinearlik va ortogonallik shartlari

Ma’lumki, (§1,1-teoremaga q.), $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlarning kollinearlik sharti quyidagicha ifodalanadi:

$$\boldsymbol{b} = k\boldsymbol{a}, \tag{12}$$

bu yerda k – skalyar. Koordinatalari orqali (9) shaklda berilgan vektorlar uchun (12) shart quyidagi tengliklarga ekvivalent:

$$b_x = ka_x, b_y = ka_y, b_z = ka_z, \text{ yoki } \frac{b_x}{a_x} = \frac{b_y}{a_y} = \frac{b_z}{a_z}.$$

Shunday qilib, *vektorlar faqat va faqat mos koordinatalari proporsional bo‘lsagina, kollinear bo‘ladilar.*

$\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlarning perpendikulyarligi (ortogonalligi) ular orasidagi burchak $\varphi = \frac{\pi}{2}$ ekanligini bildiradi. Bunday vektorlar uchun $\cos\varphi = 0$ va demak, $(\boldsymbol{a},\boldsymbol{b}) = 0$. Shunday qilib, (9) shakilda berilgan vektorlar uchun ortogonallik sharti quyidagicha ifodalanadi:

$$a_x b_x + a_y b_y + a_z b_z = 0.$$

Demak, *ikki vektor faqat va faqat mos koordinatalari ko'paytmalari yig'indisi nolga teng bo'lsagina, perpendikulyar bo'ladi.*

6-misol. Quyidagi vektorlar orasidagi φ burchakni aniqlang:

$$\boldsymbol{a} = -\boldsymbol{i} + \boldsymbol{j}, \boldsymbol{b} = \boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}.$$

Yechish. (10) formulaga ko'ra hisoblaymiz:

$$(\boldsymbol{a}, \boldsymbol{b}) = (-1)\cdot 1 + 1\cdot(-2) + 0\cdot 2 = -1 - 2 = -3.$$

Berilgan vektorlar modullarini topamiz:

$$|\boldsymbol{a}| = \sqrt{(-1)^2 + 1^2 + 0^2} = \sqrt{2}, |\boldsymbol{b}| = \sqrt{1^2 + (-2)^2 + 2^2} = 3.$$

Endi (11) formulaga asosan $\cos\varphi = \frac{(\boldsymbol{a},\boldsymbol{b})}{|\boldsymbol{a}|\cdot|\boldsymbol{b}|} = \frac{-3}{\sqrt{2}\cdot 3} = -\frac{1}{\sqrt{2}}$ kelib chiqadi. Demak, $\varphi = 135°$.

3.3. Vektorlarning vektor ko'paytmasi

Agar bir boshlang'ich nuqtaga keltrilgan uchta komplanar bo'lmagan $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ vektorlardan biri, masalan, $\boldsymbol{c}$ vektor uchidan qaralganda $\boldsymbol{a}$ vektordan $\boldsymbol{b}$ vektorga qarab eng kichik burilish (ya'ni π dan kichik burchakka burilishsh) soat strelkasiga teskari yo'nalishda bo'lsa, u holda ushbu vektorlar *o'ng uchlik* tashkil etadi deyiladi. Aks holda bu vektorlar *chap uchlik* tashkil etadi deb hisoblanadi.

Agar komplanar bo'lmagan $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ vektorlardan ikkitasining o'rinlari almashtirilsa, u holda bu uchlik oriyentatsiysini o'zgartiradi, ya'ni o'ng sistema chap sistemaga aylanadi va aksincha.

2-ta'rif. $\boldsymbol{a}$ *va* $\boldsymbol{b}$ *vektorlarning vektor ko'paytmasi deb shunday uchinchi* $\boldsymbol{c}$ *vektorga aytiladiki u quyidagi shartlarni qanoatlantiradi:*

1) $\boldsymbol{c}$ *vektorning moduli* $\boldsymbol{a}$ *va* $\boldsymbol{b}$ *vektorlaga yasalgan parallelogramning yuziga teng, ya'ni*

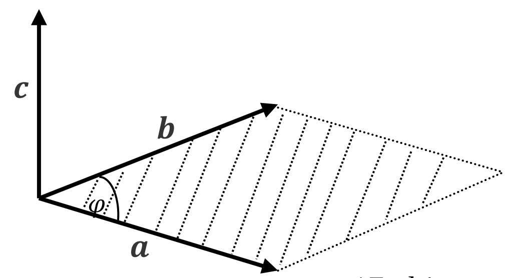
$$|\boldsymbol{c}| = |\,\boldsymbol{a}| \cdot |\,\boldsymbol{b}| \cdot \sin\varphi, \qquad (13)$$

bu yerda φ *–* $\boldsymbol{a}$ *va* $\boldsymbol{b}$ *vektorlar orasidagi burchak* $(0 \le \varphi \le \pi)$ *(17-chizma).*

2) $\boldsymbol{c}$ *vektor ko'paytirlayotgan vektorlarga perpendikulyar, ya'ni*

$$(\boldsymbol{c}, \boldsymbol{a}) = (\boldsymbol{c}, \boldsymbol{b}) = 0;$$

3) agar $\boldsymbol{a}$, $\boldsymbol{b}$ *vektorlari kollinear bo'lmasa, u holda* $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ *lar o'ng uchlik hosil qiladi.*



17-chizma

Agar $\boldsymbol{c}$ vektor $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlarning vektor ko‘paytmasidan iborat bo‘lsa**,** u holda u $\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$ kabi belgilanadi.

Vector ko‘paytmaning asosiy xossalarini keltiramiz.

1°. Agar vektor ko‘paytmada ko‘paytuvchilarning o‘rinlari almashtirilsa, u holda vector ko‘paytma moduli o‘zgarmasdan, uning ishorasi teskariga o‘zgaradi, ya’ni

$$\boldsymbol{b} \times \boldsymbol{a} = -(\boldsymbol{a} \times \boldsymbol{b}). \qquad (14)$$

Haqiqatdan ham, agar $\boldsymbol{a}, \boldsymbol{b}$-kollinear bo‘lmasalar, ularning o‘rinlari almashtirilganda, bu vektorlarga yasalgan parallelogram yuzi o‘zgarmaydi, ya’ni $|\boldsymbol{b} \times \boldsymbol{a}| = |\boldsymbol{a} \times \boldsymbol{b}|$. Ammo $\boldsymbol{b}$, $\boldsymbol{a}$, $\boldsymbol{a} \times \boldsymbol{b}$ vektorlar chap uchlikni hosil qiladi. Shu sababli $\boldsymbol{b} \times \boldsymbol{a}$ vektorning yo‘nalishi $\boldsymbol{a} \times \boldsymbol{b}$ vektorning yo‘nalishiga qarama-qarshi. Agar $\boldsymbol{a}, \boldsymbol{b}$- kollinear bo‘lsa, $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{b} \times \boldsymbol{a}$ bo‘ladi va bo holda (14) tenglikning bajarilishi ko‘rinib turibdi.

Shunday qilib, *ikki vektorning vektor ko‘paytmasi o‘rin almashtirish xossasiga ega emas*.

2°. $\boldsymbol{a}$ vektorning vektorli kvadrati nol-vektorga teng, ya’ni $\boldsymbol{a} \times \boldsymbol{a} = 0$ (bu 1° xossaning natijasidir).

3°. Skalyar ko‘paytuchini vektor ko‘paytma belgisidan tashqariga chiqarish mumkin, ya’ni agar λ – skalyar bo‘lsa, u holda

$$(\boldsymbol{\lambda a} \times \boldsymbol{b}) = (\boldsymbol{a} \times \boldsymbol{\lambda b}\ ) = \boldsymbol{\lambda}(\boldsymbol{a} \times \boldsymbol{b}\ ).$$

Bu xossa vektorni skalyarga ko‘paytirish va vektor ko‘paytmaning ta’rifidan bevosita kelib chiqadi.

4°. Har qanday $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ vektorlar uchun

$$(\boldsymbol{a} + \boldsymbol{b}) \times \boldsymbol{c} = (\boldsymbol{a} \times \boldsymbol{c}) + (\boldsymbol{b} \times \boldsymbol{c}), \qquad (15)$$

tenglik bajariladi, ya’ni taqsimot qonuni o‘rinli

7-**misol.** $\boldsymbol{a} - \boldsymbol{b}$ va $\boldsymbol{a} + \boldsymbol{b}$ *vektorlarning vektor ko‘paytmasini toping.*

Yechish. $(\boldsymbol{a} - \boldsymbol{b}) \times (\boldsymbol{a} + \boldsymbol{b}) = (\boldsymbol{a} \times \boldsymbol{a}) - (\boldsymbol{b} \times \boldsymbol{a}) + (\boldsymbol{a} \times \boldsymbol{b}) - (\boldsymbol{b} \times \boldsymbol{b}) = 0 + (\boldsymbol{a} \times \boldsymbol{b}) + (\boldsymbol{a} \times \boldsymbol{b}) + 0 = 2(\boldsymbol{a} \times \boldsymbol{b})$.

Bundan, xususiy holda $|(\boldsymbol{a} - \boldsymbol{b}) \times (\boldsymbol{a} + \boldsymbol{b})| = 2|(\boldsymbol{a} \times \boldsymbol{b})|$ ga ega bo‘lamiz, ya’ni *parallelogrammning diagonallariga yasalgan parallelogram yuzi berilgan parallelogram yuzining ikkilanganiga teng.*

Vektor ko‘paytma yordamida ikki $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlarning kollinearligining osongina tekshirib ko‘rish mumkin bo‘lgan zaruriy va yetarli shartini ifodalaymiz: $\boldsymbol{a} \times \boldsymbol{b} = 0$.

8-misol. *Agar* $\boldsymbol{a}$ *va* $\boldsymbol{b}$ *vektorlar orasidagi burchak* $\varphi = 30°$ *va* $|\boldsymbol{a}| = |\boldsymbol{b}| = 2$ *bo‘lsa,* $\boldsymbol{a} + 3\boldsymbol{b}$ *va* $3\boldsymbol{a} + \boldsymbol{b}$ *vektorlarga yasalgan parallelogram yuzini hisoblang.*

Yechish. Vektor ko‘paytma ta’rifiga ko‘ra hisoblaymiz:
$(\boldsymbol{a}+3\boldsymbol{b})\times(3\boldsymbol{a}+\boldsymbol{b}) = 3(\boldsymbol{a}\times\boldsymbol{a}) + (\boldsymbol{a}\times\boldsymbol{b}) + 9(\boldsymbol{b}\times\boldsymbol{a}) + 3(\boldsymbol{b}\times\boldsymbol{b}) = 3\cdot 0 + (\boldsymbol{a}\times\boldsymbol{b}) - 9(\boldsymbol{a}\times\boldsymbol{b}) + 3\cdot 0 = -8(\boldsymbol{a}\times\boldsymbol{b})$.

Demak, parallelogram yuzi

$$S = 8|(\boldsymbol{a}\times\boldsymbol{b})| = 8\cdot|\boldsymbol{a}|\cdot|\boldsymbol{b}|\cdot\sin\varphi = 8\cdot 2\cdot 2\cdot\sin 30^\circ = 16\ .$$

$\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar koordinatalari bilan (9) shaklda berilgan bo‘lsin. U holda, vektor ko‘paytma xossalaridan foydalanib, quyidagiga ega bo‘lamiz:

$$\boldsymbol{a}\times\boldsymbol{b} = \left[a_x b_x(\boldsymbol{i}\times\boldsymbol{i}) + a_y b_x(\boldsymbol{j}\times\boldsymbol{i}) + a_z b_x(\boldsymbol{k}\times\boldsymbol{i})\right] + \left[a_x b_y(\boldsymbol{i}\times\boldsymbol{j}) + \right.$$
$$\left. + a_y b_y(\boldsymbol{j}\times\boldsymbol{j}) + a_z b_y(\boldsymbol{k}\times\boldsymbol{j})\right] + +\left[a_x b_z(\boldsymbol{i}\times\boldsymbol{k}) + a_y b_z(\boldsymbol{j}\times\boldsymbol{k}) + a_z b_z(\boldsymbol{k}\times\boldsymbol{k})\right]. \tag{16}$$

Vektor ko‘paytma ta’rifidan, $\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}$ ortlar uchun quyidagi “ko‘paytirish jadvali” ni hosil qilamiz:

$$\boldsymbol{i}\times\boldsymbol{i} = 0,\ \boldsymbol{j}\times\boldsymbol{j} = 0,\ \boldsymbol{k}\times\boldsymbol{k} = 0,$$
$$\boldsymbol{i}\times\boldsymbol{j} = -(\boldsymbol{j}\times\boldsymbol{i}) = \boldsymbol{k},\ \boldsymbol{j}\times\boldsymbol{k} = -(\boldsymbol{k}\times\boldsymbol{j}) = \boldsymbol{i},\ \boldsymbol{k}\times\boldsymbol{i} = -(\boldsymbol{i}\times\boldsymbol{k}) = \boldsymbol{j}.$$

Bularni (16) formulada e’tiborga olib, quyidagi tenglikni hosil qilamiz:

$$\boldsymbol{a}\times\boldsymbol{b} = \boldsymbol{i}\left(a_y b_z - a_z b_y\right) + \boldsymbol{j}(a_z b_x - a_x b_z) + \boldsymbol{k}\left(a_x b_y - a_y b_x\right).$$

Chziqli algebrada keng qo‘llaniladigan ikkinchi tartibli determinant tushinchasidan foydalanib, oxirgi tenglikni quyidagicha yoza olamiz:

$$\boldsymbol{a}\times\boldsymbol{b} = \boldsymbol{i}\begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \boldsymbol{j}\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \boldsymbol{k}\begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} \tag{17}$$

(17) formulani esda saqlash oson bo‘lishi uchun uni uchinchi tartibli determinant ko‘rinishida yozamiz:

$$\boldsymbol{a}\times\boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}. \tag{18}$$

(17) formuladan

$$|\boldsymbol{a}\times\boldsymbol{b}|^2 = \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix}^2 + \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix}^2 + \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}^2 \tag{19}$$

kelib chiqadi. (19) formula $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlarga yasalgan parallelogram yuzining kvadratini ifodalaydi.

9-misol. $\boldsymbol{a} = 3\boldsymbol{k} - 2\boldsymbol{j},\ \boldsymbol{b} = 3\boldsymbol{i} - 2\boldsymbol{j}$ vektorlarga yasalgan uchburchak yuzini toping.

Yechish. Shartga ko‘ra, $a_x = 0, a_y = -2, a_z = 3,\ b_x = 3, b_y = -2, b_z = 0$. Shuning uchun (17) formulaga ko‘ra:

$$\boldsymbol{a}\times\boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & -2 & 3 \\ 3 & -2 & 0 \end{vmatrix} = \boldsymbol{i}\begin{vmatrix} -2 & 3 \\ -2 & 0 \end{vmatrix} - \boldsymbol{j}\begin{vmatrix} 0 & 3 \\ 3 & 0 \end{vmatrix} + \boldsymbol{k}\begin{vmatrix} 0 & -2 \\ 3 & -2 \end{vmatrix} = 6\boldsymbol{i} + 9\boldsymbol{j} + 6\boldsymbol{k}.$$

$$|\boldsymbol{a}\times\boldsymbol{b}| = \sqrt{6^2+9^2+6^2} = \sqrt{153} = 3\sqrt{17}.$$

Demak, uchburchak yuzi $S = \frac{1}{2}|\boldsymbol{a}\times\boldsymbol{b}| = \frac{3\sqrt{17}}{2}$.

3.4. Vektorlarning aralash ko‘paytmasi

3-ta’rif. $\boldsymbol{a},\boldsymbol{b}$ *va* $\boldsymbol{c}$ *vektorlarning aralash (yoki vector-skalyr) kopaytmasi deb* $\boldsymbol{s}=\boldsymbol{a}\times\boldsymbol{b}$ *vektorni* $\boldsymbol{c}$ *vektorga skalyar ko‘paytirish natijasida hosl bo‘lgan songa aytiladi va u quyidagicha belgilanadi:*

$$\boldsymbol{abc} = (\boldsymbol{a}\times\boldsymbol{b})\boldsymbol{c}$$

Bitta umumiy uchdan chiquvchi qirralari $\boldsymbol{a},\boldsymbol{b}$ va $\boldsymbol{c}$ vektorlardan iborat bo‘lgan parallelopipedni qaraymiz. U holda, $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlarga yasalgan parallelogramning yuzi $\boldsymbol{s}=\ \boldsymbol{a}\times\boldsymbol{b}$ vektor moduli $|\boldsymbol{a}\times\boldsymbol{b}|$ ga teng, ya’ni u parallelepiped asosining yuzidan iborat. Parallelopipedning balandligi $H=\pm\text{пр}_{\boldsymbol{s}}\boldsymbol{c}=\pm|\boldsymbol{c}|\cos\varphi$ ga teng, bu yerda agar $\boldsymbol{s}$ va $\boldsymbol{c}$ orasidagi φ burchak o‘tkir bo‘lsa , musbat ishora, φ burchak o‘tmas bo‘lsa, manfiy ishora olinadi. Birinchi holda $\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}$ vektorlari o‘ng uchlikni, ikkinch holda esa – chap uchlikni hosil qiladi.

Skalyar ko‘paytma ta‘rifiga asosan

$$(\boldsymbol{a}\times\boldsymbol{b})\boldsymbol{c} = \boldsymbol{sc} = |\boldsymbol{s}|\text{пр}_{\boldsymbol{s}}\boldsymbol{c} = \pm|\boldsymbol{s}|H = \pm V\ ,$$

bu yerda V – $\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}$ vektorlarga yasalgan parallelepiped hajmi**.**

Shunday qilib, $\boldsymbol{abc}=\pm V$, ya’ni *uchta vektorlarning aralash ko‘paytmasi, agarda vektorlar o‘ng uchlik hosil qilsa, shu vektorlarga yasalgan parallelepiped hajmi* V *ga “*$+$*” ishora bilan, agarda chap uchlik hosil qilsa, “–”ishora bilan teng bo‘ladi.*

Aralash ko‘paytma quyidagi asosiy xossalarga ega.

1°. Aralash ko‘paytmada ko‘paytuvchilarni davriy ravishda o‘rin almashtirish uning qiymatini o‘zgartirmaydi, ya’ni

$$\boldsymbol{abc} = \boldsymbol{bca} = \boldsymbol{cab}.$$

2°. Aralash ko‘paytmada yonma-yon joylashgan ko‘paytuvchilarning o‘rinlari almashtirish uning ishorasini qarama-qarshiga almashtiradi, ya’ni

$$\boldsymbol{bac} = \boldsymbol{acb} = \boldsymbol{cba} = -\boldsymbol{abc}.$$

Aralash ko‘paytma yordamida uchta vektor komplanarligining quyidagi zaruriy va yetarli shartiga ega bo‘lamiz:

$$\boldsymbol{abc} = \boldsymbol{0}.$$

Agar $\boldsymbol{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}, \boldsymbol{b} = b_x\boldsymbol{i} + b_y\boldsymbol{j} + b_z\boldsymbol{k}$, $\boldsymbol{c} = c_x\boldsymbol{i} + c_y\boldsymbol{j} + c_z\boldsymbol{k}$ bo‘lsa, u holda vektor va skalyar ko‘paytmalarning koordinatalar shaklidan foydalanib, quyidagi ifodani yoza olamiz:

$$\boldsymbol{abc} = (\boldsymbol{a} \times \boldsymbol{b})\boldsymbol{c} = \boldsymbol{a}(\boldsymbol{b} \times \boldsymbol{c}) = \left(a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}\right) \cdot \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} =$$

$$= a_x \begin{vmatrix} b_y & b_z \\ c_y & c_z \end{vmatrix} - a_y \begin{vmatrix} b_x & b_z \\ c_x & c_z \end{vmatrix} + a_z \begin{vmatrix} b_x & b_y \\ c_x & c_y \end{vmatrix},$$

yoki

$$\boldsymbol{abc} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}. \tag{20}$$

10-misol. $\boldsymbol{a} = 3\boldsymbol{i} + 4\boldsymbol{j}, \boldsymbol{b} = -3\boldsymbol{i} + \boldsymbol{k}$, $\boldsymbol{c} = 2\boldsymbol{j} + 5\boldsymbol{k}$ *vektorlargav yasalgan parallellopiped hajmini hisoblang.*

Yechish. Shartga ko‘ra $a_x = 3, a_y = 4, a_z = 0$, $b_x = 0, b_y = -3, b_z = 1$, $c_x = 0, c_y = 2, c_z = 5$. U holda (20) formulaga asosan

$$\boldsymbol{abc} = \begin{vmatrix} 3 & 4 & 0 \\ 0 & -3 & 1 \\ 0 & 2 & 5 \end{vmatrix} = 3 \cdot \begin{vmatrix} -3 & 1 \\ 2 & 5 \end{vmatrix} - 4 \cdot \begin{vmatrix} 0 & 1 \\ 0 & 5 \end{vmatrix} + 0 \cdot \begin{vmatrix} 0 & 1 \\ 0 & 5 \end{vmatrix} = -51.$$

Shunday qilib, parallelopiped hajmi $V = 51$.

3.5. Nazorat savollari va masalalar

1. Ikki vektor skalyr ko‘paytmasining ta’rifini bering.
2. Skalyar ko‘paytma qanday fizik ma’noga ega?
3. Skalyar ko‘paytmaning qanday asosiy xossalari bor?
4. Vektorning sralyar kvadrati qanday ma’noga ega va nimaga teng?
5. Vektor moduli skalyar ko‘paytma orqali qanday aniqlanadi?
6. Ikki vektor orasidagi burchak kosinusi qanday formula orqali topiladi?
7. Vektorlarning perpendikulyrlik sharti skalyar ko‘paytma orqali qanday ifodalanadi?
8. Vektorlar koordinatalari bilan berilganda skalyar ko‘paytma va ular orasidagi burchak qaysi formula orqali topiladi?
9. Vektorlar kollinearligi va ortogonalligi sharlari ularning koordinatalari orqali qanday ifodalanadi?
10. Ikki vektorning vektor ko‘paytmasi ta’rifini keltiring.

11. Vektor ko‘paytmaning asosiy xossalarini keltiring.

12. Ikki vektorning kollinearligi sharti vektor ko‘paytma orqali qanday ifodalanadi?

13. Vektor koordinatalari bilan berilganda vektor ko‘paytma qanday hisoblanadi?

14. Uch vector aralash ko‘paytmasi ta’rifini keltiring va uning geometrik ma’nosini tushuntiring.

15. Aralash ko‘paytmaning asosiy xossalarini keltiring.

16. Vektorlar koordinatalari bilan berilganda aralash ko‘paytma qaysi formula orqali hisoblanadi?

17. Uch vektorning komplanarlik sharti aralash ko‘paytma orqali qanday ifodalanadi?

18. $\boldsymbol{a} = \{1,2,-2\}$ vektorning $\boldsymbol{b} = \{1,0,-1\}$ vektordagi proeksiyasini toping.

19. Voddiy nuqtani $M(0,1,2)$ nuqtadan $N(3,-4,5)$ niqtaga to‘g‘ri chiziq bo‘ylab ko‘chiruvchii $\boldsymbol{F} = \{10,20,30\}$ kuchning bajargan ishini toping**.**

20. Hisoblang: 1) $(\boldsymbol{a}+\boldsymbol{b})^2$; 2) $(\boldsymbol{a}+\boldsymbol{b})^2 + (\boldsymbol{a}-\boldsymbol{b})^2$. Hosil bo‘lgan formulalarning geometrik ma’nosini bering.

21. $\boldsymbol{a} = 2\boldsymbol{i} + \boldsymbol{j}$ va $\boldsymbol{b} = -2\boldsymbol{j} + \boldsymbol{k}$ vektorlarga yasalgan parallelogram diagonallari orasidagi burchakni toping.

22. $A(a,0,0), B(0,0,2a)$ va $C(a,0,a)$ nuqtalar berilgan. $\overline{OC}$ va $\overline{AB}$ vektorlar orasidagi burchakni toping**.**

23. $\boldsymbol{a} = 3\boldsymbol{m} + \boldsymbol{n}$ va $\boldsymbol{b} = \boldsymbol{m} - 2\boldsymbol{n}$ vektorlarga yasalgan parallelogram diagonallari uzunliklarini toping. Bu yerda $\boldsymbol{m}$ va $\boldsymbol{n}$ lar birlik vektorlar bo‘lib ular orasidagi burchak $60°$.

24. $\boldsymbol{a} = \{1,-2,3\}$ va $\boldsymbol{b} = \{3,2,1\}$ vektorlarga yasalgan parallelogram yuzini hamda o‘tkir burchagini toping.

25. Harakatdagi nuqtaning $\boldsymbol{s}$ ko‘chishi va ta’sir etuvchi $\boldsymbol{F}$ kuchning koordinata o‘qlaridagi proeksiyalari quyidagucha: $s_x = 2m, s_y = 1m, s_z = -2m$; $F_x = 5H, F_y = 4H$, $F_z = 3H$. Bajarilgan A ishni va $\boldsymbol{F}$ kuch bilan $\boldsymbol{s}$ ko‘chish orasidagi burchakni toping**.**

26. $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar orasidagi burchak $30°$. Agar $|\boldsymbol{a}| = |\boldsymbol{b}| = 4$ bo‘lsa, $\boldsymbol{c} = \boldsymbol{a} - 2\boldsymbol{b}$ va $\boldsymbol{d} = 3\boldsymbol{a} + 2\boldsymbol{b}$ vektorlarga yasalgan uchburchak yuzini toping.

27. $\boldsymbol{a} = \boldsymbol{m} + 2\boldsymbol{n}$ va $\boldsymbol{b} = 2\boldsymbol{m} + \boldsymbol{n}$ vektorlarga yasalgan parallelogram yuzini toping, bu yerda $\boldsymbol{m}$ va $\boldsymbol{n}$ – birlik vektorlar bo‘lib, ular orasidagi burchak $30°$.

28. Ixtiyoriy $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ vektorlar berilgan. $(\boldsymbol{a}-\boldsymbol{b})(\boldsymbol{b}-\boldsymbol{c})(\boldsymbol{c}-\boldsymbol{a})$ aralash ko‘paytmani hisoblang.

29. $\boldsymbol{a}=\boldsymbol{j}+\boldsymbol{k}, \boldsymbol{b}=\boldsymbol{i}+\boldsymbol{k}, \boldsymbol{c}=\boldsymbol{i}+\boldsymbol{j}$ vektorlar komplanarmi?

30. $A(5,7,-2)$, $B(3,1,-1), C(9,4,-4)$ va $D(1,5,0)$ nuqtalarning bir tekslikda yotishini ko‘rsating.

31. Qavuslarni oching va ifodani soddalashtiring:

1) $\boldsymbol{i}\times(\boldsymbol{j}+\boldsymbol{k})-\boldsymbol{j}\times(\boldsymbol{i}+\boldsymbol{k})++\boldsymbol{k}\times(\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k})$;

2) $2\boldsymbol{i}\cdot(\boldsymbol{j}\times\boldsymbol{k})+3\boldsymbol{j}\cdot(\boldsymbol{i}\times\boldsymbol{k})+4\boldsymbol{k}\cdot(\boldsymbol{i}\times\boldsymbol{j})$.

32. $\boldsymbol{a}=\boldsymbol{i}+\boldsymbol{j}+4\boldsymbol{k}, \boldsymbol{b}=\boldsymbol{i}-2\boldsymbol{j}$ va $\boldsymbol{c}=3\boldsymbol{i}-3\boldsymbol{j}+4\boldsymbol{k}$ vektorlarning komplanarligini va ularning chiziqli bog‘liqligini ko‘rsating.

I-BOB UCHUN TEST SAVOLLARI (1-TEST)

1. $\boldsymbol{c}=\boldsymbol{a}+\boldsymbol{b}$, $\boldsymbol{d}=\boldsymbol{a}-\boldsymbol{b}$ bo‘lsin. Quyidagilar topilsin: $\frac{1}{2}(\boldsymbol{c}+\boldsymbol{d}), \frac{1}{2}(\boldsymbol{c}-\boldsymbol{d})$.

A) $\boldsymbol{a}$; $\boldsymbol{b}$ B) $\boldsymbol{b}$; $\boldsymbol{a}$ C) $2\boldsymbol{a}$; $2\boldsymbol{b}$ D) $\boldsymbol{a}$; $2\boldsymbol{b}$ E) $\boldsymbol{b}$; $2\boldsymbol{a}$

2. $\boldsymbol{a}=\overline{OA}$ va $\boldsymbol{b}=\overline{OB}$ vektorlar OAB uchburchakning tomonlari. C nuqta AB tomonda yotadi va bu tomonni 3:1 nisbatta bo‘ladi (A uchidan hisoblaganda). $\overline{OC}$ vektorni $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar bo‘yicha yozing. A) $\frac{1}{4}(\boldsymbol{a}+\boldsymbol{b})$ B) $\frac{3}{4}(\boldsymbol{a}+\boldsymbol{b})$ C) $\frac{1}{4}(\boldsymbol{a}+3\boldsymbol{b})$ D) $\frac{1}{4}(3\boldsymbol{a}+\boldsymbol{b})$ E) $\frac{1}{4}(\boldsymbol{a}-3\boldsymbol{b})$

3. $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlari berilgan, ular orasidagi burchak 60° va $|\boldsymbol{a}|=5$, $|\boldsymbol{b}|=6$. $\boldsymbol{c}=2\boldsymbol{a}-3\boldsymbol{b}$ vektorning $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlardagi proyeksiyalarini toping.

A)$\text{пр}_{\boldsymbol{a}}\boldsymbol{c}=1, \text{пр}_{\boldsymbol{b}}\boldsymbol{c}=-1$; B)$\text{пр}_{\boldsymbol{a}}\boldsymbol{c}=-1, \text{пр}_{\boldsymbol{b}}\boldsymbol{c}=1$;

C) $\text{пр}_{\boldsymbol{a}}\boldsymbol{c}=2, \text{пр}_{\boldsymbol{b}}\boldsymbol{c}=-2$; D) $\text{пр}_{\boldsymbol{a}}\boldsymbol{c}=-2, \text{пр}_{\boldsymbol{b}}\boldsymbol{c}=2$;

E) $\text{пр}_{\boldsymbol{a}}\boldsymbol{c}=1, \text{пр}_{\boldsymbol{b}}\boldsymbol{c}=1$.

4. $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar l o‘qi bilan mos ravishda 30° va 120° burchaklar tashkil qiladi. Agar $|\boldsymbol{a}|=2\sqrt{3}$, $|\boldsymbol{b}|=2$ bo‘lsa, $\boldsymbol{c}=\boldsymbol{a}+\boldsymbol{b}$ vektorning l o‘qdagi proyeksiyasini toping.

A) 2; B) $\frac{1}{2}$; C) 1; D) -1; E) -2.

5. $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlari orasidagi burchak 120° ga teng. Agar $|\boldsymbol{a}|=3$, $|\boldsymbol{b}|=4$ bo‘lsa, ular orasidagi burchak bissektrisasi l yo‘nalish bo‘yicha proyeksiyasini toping.

A) $\text{пр}_l\boldsymbol{a}=\frac{3}{2}, \text{пр}_l\boldsymbol{b}=2$; B) $\text{пр}_l\boldsymbol{a}=2, \text{пр}_l\boldsymbol{b}=\frac{3}{2}$; C) $\text{пр}_l\boldsymbol{a}=-\frac{3}{2}, \text{пр}_l\boldsymbol{b}=2$;

D) $\text{пр}_l\boldsymbol{a}=-2, \text{пр}_l\boldsymbol{b}=\frac{3}{2}$; E) $\text{пр}_l\boldsymbol{a}=-2, \text{пр}_l\boldsymbol{b}=-\frac{3}{2}$.

6. $\boldsymbol{a} = \overline{OA}$, $\boldsymbol{b} = \overline{OB}$ vektorlari berilgan. Agar ular orasidagi burchak $90°$, $|\boldsymbol{a}| = 3\sqrt{2}$, $|\boldsymbol{b}| = 4\sqrt{2}$ bo‘lsa, u holda $\boldsymbol{c} = \frac{1}{2}(\boldsymbol{a} + \boldsymbol{b})$ vektorning $\boldsymbol{b}$ vektordagi proyeksiyasini toping.

A) $5\sqrt{2}$; B) $2\sqrt{2}$; C) $\frac{5\sqrt{2}}{3}$; D) $\frac{2\sqrt{2}}{3}$; E) $\frac{3\sqrt{2}}{4}$.

7. $M(-\sqrt{2}, 2, \sqrt{3})$ nuqta $\boldsymbol{r}$ radius-vektorining uzunligi $|\boldsymbol{r}|$ ni va yonaltiruvch kosinuslarini toping.

A) $|\boldsymbol{r}| = 3$, $\cos\propto = -\frac{\sqrt{2}}{3}$, $\cos\beta = -\frac{2}{3}$, $\cos\gamma = \frac{\sqrt{3}}{3}$;

B) $|\boldsymbol{r}| = 3$, $\cos\propto = \frac{\sqrt{2}}{3}$, $\cos\beta = -\frac{2}{3}$, $\cos\gamma = -\frac{\sqrt{3}}{3}$;

C) $|\boldsymbol{r}| = 3, \cos\propto = \frac{\sqrt{2}}{3}, \cos\beta = \frac{2}{3}, \cos\gamma = -\frac{\sqrt{3}}{3}$;

D) $|\boldsymbol{r}| = 3$, $\cos\propto = -\frac{\sqrt{2}}{3}$, $\cos\beta = \frac{1}{3}$, $\cos\gamma = \frac{1}{\sqrt{3}}$;

E) $|\boldsymbol{r}| = 3$, $\cos\propto = \frac{1}{\sqrt{3}}$, $\cos\beta = \frac{2}{3}$, $\cos\gamma = -\frac{1}{\sqrt{3}}$.

8. $M(x, y, z)$ nuqta radius-vektori $\boldsymbol{r}$ ning uzunligi 5 ga, yonaltiruvchi kosinuslari $\cos\propto = \frac{1}{\sqrt{5}}$, $\cos\beta = \frac{1}{2\sqrt{5}}$, $\cos\gamma = \frac{\sqrt{3}}{2}$ ga teng. M nuqtaning koordinatalarini toping.

A) $x = -\sqrt{5}, y = \frac{\sqrt{5}}{2}, z = \frac{5\sqrt{3}}{2}$; B) $x = \sqrt{5}, y = \frac{\sqrt{5}}{2}, z = \frac{5\sqrt{3}}{2}$;

C) $x = \sqrt{3}, y = \frac{\sqrt{3}}{2}, z = 5\sqrt{3}$; D) $x = -\sqrt{3}, y = \frac{\sqrt{3}}{2}, z = 5\sqrt{3}$;

E) $x = \sqrt{5}, y = \frac{\sqrt{3}}{2}, z = \frac{5\sqrt{3}}{2}$

9. $M(x, y, z)$ nuqtaning radius-vektori $\boldsymbol{r}$ bilan Ox o‘qi orasidagi burchak $\propto = 60°$, Oy o‘qi orasidagi burchak $\beta = 45°$ va $|\boldsymbol{r}| = 4$ bo‘lsin. $M(x, y, z)$ nuqtaning koordinatalarini toping.

$A)$ $x = 2$, $y = 2\sqrt{2}$, $z = 4$; $B)$ $x = -2$, $y = 2\sqrt{2}$, $z = 2$;

$C)$ $x = 2$, $y = 2\sqrt{2}$, $z = -2$; $D)$ $x = -2$, $y = 2\sqrt{2}$, $z = -4$;

$E)$ $x = 2$, $y = 2\sqrt{2}$, $z = \pm 2$.

10. $A(2,1,3)$ va $B(4,-3,6)$ nuqtalar berilgan. $\boldsymbol{u} = \overline{AB}$ vektorning uzunligini va yo‘nalishini aniqlang.

$A)$ $|\boldsymbol{u}| = \sqrt{30}$, $\cos\propto = \frac{2}{\sqrt{30}}$, $\cos\beta = -\frac{4}{\sqrt{30}}$, $\cos\gamma = \frac{3}{\sqrt{30}}$;

$B)$ $|\boldsymbol{u}| = \sqrt{21}$, $\cos\propto = \frac{2}{\sqrt{21}}$, $\cos\beta = -\frac{4}{\sqrt{21}}$, $\cos\gamma = \frac{3}{\sqrt{21}}$;

$C)$ $|\boldsymbol{u}| = \sqrt{29}$, $\cos\propto = \frac{2}{\sqrt{29}}$, $\cos\beta = -\frac{4}{\sqrt{29}}$, $\cos\gamma = \frac{3}{\sqrt{29}}$;

$D)$ $|\boldsymbol{u}| = 6$, $\cos\propto = \frac{1}{3}$, $\cos\beta = -\frac{2}{3}$, $\cos\gamma = \frac{1}{2}$;

$E)\ |\boldsymbol{u}| = 7,\ cos \propto = \frac{2}{7},\ cos\beta = -\frac{4}{7},\ cos\gamma = \frac{3}{7}.$

11. $A(2,1,4)$, $B(3,2,-1)$ nuqtalar berilgan. $\boldsymbol{a} = \overline{AB}$ vektorning a_x, a_y, a_z koordinatalarini toping.

$A)\ a_x = -1,\quad a_y = -1,\quad a_z = 5$; $B)\ a_x = 1,\quad a_y = 1,\quad a_z = -5$;

$C)\ a_x = -1,\quad a_y = 1,\quad a_z = 5$; $D)\ a_x = 5,\quad a_y = 3,\quad a_z = 3$;

$E)\ a_x = \frac{5}{2},\quad a_y = \frac{3}{2},\quad a_z = \frac{3}{2}.$

12. $A(2,1,2)$, $B(1,2,3)$ nuqtalar berilgan. $\boldsymbol{a} = \overline{AB}$ vektorni $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ – ortlar bo‘yicha yozing.

$A)\ \boldsymbol{a} = -\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$; $B)\ \boldsymbol{a} = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$; $C)\ \boldsymbol{a} = -\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$;

$D)\ \boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$; $E)\ \boldsymbol{a} = \boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}.$

13. $A(2,0,1), B(-2,1,2), C(1,2,2)$ nuqtalar berilgan. $\boldsymbol{a} = \overline{AB}$, $\boldsymbol{b} = \overline{BC}$, $\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b}$ bo‘lsa, $\boldsymbol{c} = c_x\boldsymbol{i} + c_y\boldsymbol{j} + c_z\boldsymbol{k}$, vektorni $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ –ortlar bo‘yicha yozing .

$A)\ \boldsymbol{c} = \boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$; $B)\ \boldsymbol{c} = -\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$; $C)\ \boldsymbol{c} = 2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$;

$D)\ \boldsymbol{c} = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$; $E)\ \boldsymbol{c} = \boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}.$

14. $\boldsymbol{a}(3,2,-4), \boldsymbol{b}(2,-1,3)$ vektorlari berilgan. $\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b}$ bo‘lsa, $\boldsymbol{c}$ vektorning koordinatalarini toping.

$A)\ \boldsymbol{c}(4,1,-1)$; $B)\ \boldsymbol{c}(5,3,-1)$; $C)\ \boldsymbol{c}(5,1,-1)$; $D)\ \boldsymbol{c}(5,-1,1)$; $E)\ \boldsymbol{c}(4,-1,1)$.

15. $\boldsymbol{a} = \overline{AB}$, $A(1,0,2)$, $B(2,1,3)$, $\mathbf{b} = \overline{\mathrm{BA}}$, $\boldsymbol{c} = \boldsymbol{2a} - \boldsymbol{b}$ bo‘lsa, $\boldsymbol{c}$ vektorning koordinatalarini toping**.**

$A)\ \boldsymbol{c}(2,2,2)$; $B)\ \boldsymbol{c}(2,3,3)$; $C)\ \boldsymbol{c}(-3,3,3)$; $D)\ \boldsymbol{c}(3,3,3)$; $E)\ \boldsymbol{c}(-3,-2,2)$.

16. $\boldsymbol{a} = 2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ vektorlar berilgan**.** $\boldsymbol{c} = \boldsymbol{a} - 2\boldsymbol{b}$ vektorni $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ - ortlar bo‘yicha yoyilmasini toping.

$A)\ \boldsymbol{c} = 7\boldsymbol{j} + \boldsymbol{k}$; $B)\ \boldsymbol{c} = -7\boldsymbol{j} - \boldsymbol{k}$; $C)\ \boldsymbol{c} = 7\boldsymbol{i} - \boldsymbol{j}$; $D)\ \boldsymbol{c} = \boldsymbol{5j} + \boldsymbol{k}$;

$E)\ \boldsymbol{c} = 5\boldsymbol{i} - \boldsymbol{j}.$

17. $\overline{OA} = \boldsymbol{i} - 2\boldsymbol{j}$, $\overline{OB} = \boldsymbol{k} - 2\boldsymbol{j}$ vektorlar $OACB$ parallellogramning tomonlarini tashkil qilsin. Bu parallellogramning OC va AB diagonallarini toping.

$A)\quad \overline{OC} = \overline{OA} + \overline{OB} = \boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k},\quad \overline{AB} = \overline{OB} - \overline{OA} = -\boldsymbol{i} - \boldsymbol{k}$;

$B)\quad \overline{OC} = \overline{OA} + \overline{OB} = \boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k},\quad \overline{AB} = \overline{OB} - \overline{OA} = \boldsymbol{i} + \boldsymbol{k}$;

$C)\quad \overline{OC} = \overline{OA} - \overline{OB} = \boldsymbol{i} - \boldsymbol{k},\quad \overline{AB} = \overline{OB} + \overline{OA} = \boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k}$;

$D)\quad \overline{OC} = \overline{OA} - \overline{OB} = \boldsymbol{i} - \boldsymbol{k},\quad \overline{AB} = \overline{OB} + \overline{OA} = \boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k}$;

$E)\quad \overline{OC} = \overline{OA} + \overline{OB} = 2\boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k},\quad \overline{AB} = \overline{OB} - \overline{OA} = \boldsymbol{i} - \boldsymbol{k}$;

18. $\mathbf{r}$ vektor Oy va Oz o‘qlari bilan mos ravishda $60°$ va $120°$ burchaklar tashkil qiladi. Bu vector Ox o‘qi bilan qanday α burchak tashkil qiladi?

A) $\propto = 30°$ B) $\propto = 60°$ C) $\propto = 45°$ yoki $\propto = 135°$;

D) $\propto = 75°$ E) $\propto = 50°$

19. Oxy tekislikda $A(4{,}2), B(2{,}3)$, $C(0{,}5)$ nuqtalar yordamida $\boldsymbol{a} = \overline{OA}$, $\boldsymbol{b} = \overline{OB}$, $\boldsymbol{c} = \overline{OC}$ vektorlari yasalgan. $\boldsymbol{c}$ vektorni $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar bo‘yicha yoying.

A) $\boldsymbol{c} = \frac{5}{2}\boldsymbol{a} - \frac{5}{4}\boldsymbol{b}$ B) $\boldsymbol{c} = \frac{3}{2}\boldsymbol{a} - \frac{3}{4}\boldsymbol{b}$ C) $\boldsymbol{c} = \frac{7}{2}\boldsymbol{a} - \frac{7}{4}\boldsymbol{b}$

Д) $\boldsymbol{c} = \frac{3}{4}\boldsymbol{a} + \frac{3}{2}\boldsymbol{b}$ E) $\boldsymbol{c} = -\frac{5}{4}\boldsymbol{a} + \frac{5}{2}\boldsymbol{b}$

20. $A(1{,}2{,}0)$, $B(2{,}3{,}1)$, $C(-1{,}0{,}4)$, $D(1{,}3{,}2)$ nuqtalar berilgan. D nuqtaning $\boldsymbol{r_D}$ radius-vektorini $\boldsymbol{r}_A, \boldsymbol{r}_B, \boldsymbol{r}_C$ radius-vektorlari orqali yoying.

$A)\ \boldsymbol{r}_D = \frac{3}{2}\boldsymbol{r}_A + \frac{1}{2}\boldsymbol{r}_B$; $B)\ \boldsymbol{r}_D = \frac{3}{2}\boldsymbol{r}_A + \frac{1}{2}\boldsymbol{r}_C$; $C)\ \boldsymbol{r}_D = \frac{3}{2}\boldsymbol{r}_A - \frac{1}{2}\boldsymbol{r}_B$;

$D)\ \boldsymbol{r}_D = \frac{1}{2}\boldsymbol{r}_A + \frac{3}{2}\boldsymbol{r}_B$; $E)\ \boldsymbol{r}_D = \frac{3}{2}\boldsymbol{r}_A - \frac{1}{2}\boldsymbol{r}_B + \frac{1}{2}\boldsymbol{r}_C$.

21. Agar $|\boldsymbol{a}| = 2, |\boldsymbol{b}| = 3$ hamda $\boldsymbol{a}$ va $\boldsymbol{b}$ orasidagi burchak $\propto = 60°$ bo‘lsa, bu vektorlarning skalyar ko‘paytmasini toping

$A)\ (\boldsymbol{a}, \boldsymbol{b}) = 6$; $B)\ (\boldsymbol{a}, \boldsymbol{b}) = 2$; $C)\ (\boldsymbol{a}, \boldsymbol{b}) = 3$; $D)\ (\boldsymbol{a}, \boldsymbol{b}) = -3$; $E)\ (\boldsymbol{a}, \boldsymbol{b}) = -6$.

22. $|\boldsymbol{a}| = \sqrt{3}, |\boldsymbol{b}| = 3$, $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar orasidagi burchak $\propto = 30°$ bo‘lsa, $\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b}$ va $\boldsymbol{b}$ vektorlarning skalyar ko‘paytmasini toping.

A) $(\boldsymbol{c}, \boldsymbol{b}) = \frac{15}{2}$; B) $(\boldsymbol{c}, \boldsymbol{b}) = 8$; C) $(\boldsymbol{c}, \boldsymbol{b}) = 7$;

D) $(\boldsymbol{c}, \boldsymbol{b}) = 9$; E) $(\boldsymbol{c}, \boldsymbol{b}) = \frac{17}{2}$.

23. $|\boldsymbol{a}| = \sqrt{5}$, $|\boldsymbol{b}| = \sqrt{3}$. $\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b}$ va $\boldsymbol{d} = \boldsymbol{a} - \boldsymbol{b}$ vektorlarning skalyar ko‘paytmasini toping.

$A)\ (\boldsymbol{c}, \boldsymbol{d}) = 4$; $B)\ (\boldsymbol{c}, \boldsymbol{d}) = 2$; $C)\ (\boldsymbol{c}, \boldsymbol{d}) = 3$; $D)\ (\boldsymbol{c}, \boldsymbol{d}) = -2$; $E)\ (\boldsymbol{c}, \boldsymbol{d}) = -3$.

24. $|\boldsymbol{a}| = |\boldsymbol{b}|$ bo‘lsin. $\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b}$ va $\boldsymbol{d} = \boldsymbol{a} - \boldsymbol{b}$ vektorlarning skalyar ko‘paytmasini toping.

$A)\ (\boldsymbol{c}, \boldsymbol{d}) = 0$;$B)$ $(\boldsymbol{c}, \boldsymbol{d})$=1; $C)$ $(\boldsymbol{c}, \boldsymbol{d})$= -1; $D)$ $(\boldsymbol{c}, \boldsymbol{d})$=$\frac{1}{2}$; $E)$ $(\boldsymbol{c}, \boldsymbol{d})$=$\frac{3}{2}$;

25. $|\boldsymbol{a}| = \sqrt{2}, |\boldsymbol{b}| = \sqrt{3}$. $\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b}$ va $\boldsymbol{d} = \boldsymbol{a} - \boldsymbol{b}$ vektorlar orasidagi α burchakng toping.

$A)\ \propto = \arccos\frac{1}{\sqrt{7}}$; $B)\ \propto = \arccos\frac{2}{\sqrt{7}}$; $C)\ \propto = \arccos\frac{3}{\sqrt{7}}$;

$D)\ \propto = \pi - \arccos\frac{1}{\sqrt{7}}$; $E)\ \propto = \pi - \arccos\frac{2}{\sqrt{7}}$.

26. Agar $\boldsymbol{a}(2,4,1), \boldsymbol{b}(3,-5,7)$ bo‘lsa, $\boldsymbol{p}=2\boldsymbol{a}$ va $\boldsymbol{q}=3\boldsymbol{b}$ vektorlarning skalyar ko‘paytmasini toping.

$A)\ (\boldsymbol{p},\boldsymbol{q})=43$; $B)\ (\boldsymbol{p},\boldsymbol{q})=-43$; $C)\ (\boldsymbol{p},\boldsymbol{q})=42$; $D)\ (\boldsymbol{p},\boldsymbol{q})=44$; $E)\ (\boldsymbol{p},\boldsymbol{q})=-42$.

27. $\boldsymbol{c}=\boldsymbol{a}+\boldsymbol{b}$ vektorning uzunligini toping, bu yerda $\boldsymbol{a}(1,2,3)$, $\boldsymbol{b}(4,-2,9)$.

$A)\ |\boldsymbol{c}|=14$; $B)|\boldsymbol{c}|=13$; $C)\ |\boldsymbol{c}|=12$; $D)\ |\boldsymbol{c}|=15$; $E)\ |\boldsymbol{c}|=11$.

28. $\boldsymbol{a}(6,2,1)$ va $\boldsymbol{b}(0,-1,2)$ vektorlari berilgan. $\boldsymbol{c}=2\boldsymbol{a}-\boldsymbol{b}$ vektor uzunligini toping.

$A)\ |\boldsymbol{c}|=15$; $B)|\boldsymbol{c}|=12$; $C)\ |\boldsymbol{c}|=13$; $D)\ |\boldsymbol{c}|=11$; $E)\ |\boldsymbol{c}|=14$.

29. x va y ning qanday qiymatlarida $\boldsymbol{a}(3,-2,x)$ va $\boldsymbol{b}(y,4,2)$ vektorlar kollinear bo‘ladi?

$A)\ x=-1,\ y=-6$; $B)\ x=1,\ y=-6$; $C)\ x=-2,\ y=6$;

$D)\ x=-1,\ y=-3$; $E) x=2,\ y=6$.

30. x ning qanday qiymatlarida $\boldsymbol{a}(-1,1,2)$ va $\boldsymbol{b}(x^2,x-2,x^2-12)$ vektorlar kollinear bo‘ladi?

$A)\ x=\pm2$; $B)\ x=\pm3$; $C)\ x=3$; $D)\ x=-3$; $E)\ x=-2$.

31. $\boldsymbol{a}=\boldsymbol{2i}-\boldsymbol{3j}+\boldsymbol{k}$, $\boldsymbol{b}=3\boldsymbol{i}+2\boldsymbol{j}$ vektorlar orasidagi burchakni toping.

$A)\ 0°$; $B)\ 30°$; $C)\ 45°$; $D)\ 90°$; $E)\ 60°$.

32. x ning qanday qiymatlarida $\boldsymbol{a}(x,3,4)$ va $\boldsymbol{b}(5,6,3)$ vektorlar perpendikulyar bo‘ladi?

$A)\ x=5$; $B)\ x=4$; $C)\ x=-6$; $D)\ x=-5$; $E)\ x=6$.

33. Agar $\boldsymbol{a}(x,2,z)$ vektor $\boldsymbol{b}(2,3,-2)$ vektorga va Ox o‘qiga perpendikulyar bo‘lsa, x va z koordinatalarning ko‘paytmasini toping.

$A)\ 1$; $B)\ -1$; $C)\ -2$; $D)\ 0$; $E)\ 2$.

34. z ning qanday qiymatlarida $\boldsymbol{c}=2\boldsymbol{i}-9\boldsymbol{j}+z\boldsymbol{k}$ vektor uzunligi 11 ga teng?

$A)\ z=6$; $B)\ z=5$; $C)\ z=\pm6$; $D)\ z=\pm5$; $E)\ z=6$.

35. $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar 120° burchak tashkil qiladi va $|\boldsymbol{a}|=3, |\boldsymbol{b}|=5$. $|\boldsymbol{a}-\boldsymbol{b}|$ ni toping.

A) 8; $B)$ 9; $C)$ 6; $D)$ 5; $E)$ 7.

36. $A(1,2,3)$ va $B(-3,3,2)$ nuqtalardan baravar uzoqlikda va Ox o‘qida yotyvchi $M(x,0,0)$ nuqtaning x koordinatasini toping.

A) $x=-1$; $B)\ x=-2$; $C)\ x=1$; $D)\ x=2$; $E)\ x=-3$.

37. Agar $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar 45° burchak tashkil qilib, $(\boldsymbol{a},\boldsymbol{b})=4$ bo ‘lsa, u holda $\boldsymbol{a}$ va **b** vektorlarga yasalgan uchburchakning S yuzini toping.

A) $S = 4$; $B)\ S = 2$; $C)\ S = 3$; $D)\ S = 6$; $E)\ S = 5$.

38. Agar $\boldsymbol{a}(-4,2,4)$ va $\boldsymbol{b}(2,-2,0)$ bo‘lsa, $2\boldsymbol{a}$ va $\frac{\mathbf{b}}{\mathbf{2}}$ vektorlar orasidagi burchakni toping.

A) $\frac{\pi}{4}$; $B)\ \frac{\pi}{2}$; $C)\ \frac{\pi}{3}$; $D)\ \frac{3\pi}{4}$; $E)\ \frac{\pi}{6}$.

39. $\boldsymbol{c}$ vektor $\boldsymbol{a}(2,2,-1)$ va $\boldsymbol{b}(3,-1,1)$ vektorlarga perpendikulyar, Oz o‘qi bilan o‘tmas burchak tashkil qiladi va $|\boldsymbol{c}| = \sqrt{30}$. $yz - x^2$ ifodaning qiymatini toping, bu yerda x, y, z – $\boldsymbol{c}(x, y, z)$ vektorning koordinatalari.

A) 15; $B)$ 18; $C)$ 20; $D)$ 12; $E)$ 13.

40. Nolga teng bo‘lmagan $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar uzunliklari teng. Agar $\boldsymbol{p} = \boldsymbol{a} + 2\boldsymbol{b}$ va $\boldsymbol{q} = 5\boldsymbol{a} - 4\boldsymbol{b}$ vektorlar perpendikulyar ekanligi ma’lum bo‘lsa, ular orasidagi burchakni toping.

A) 45°; $B)$ 30°; $C)$ 60°; $D)$ 90°; $E)$ 75°.

41. Uchlari $A(3,-2,1), B(3,0,2), C(1,2,5)$ nuqtalarda bo‘lgan uchburchak berilgan. Uchburchakning $\overline{BD}$ mediana va $\overline{AC}$ asosi tashkil qilgan burchakni toping.

A) 30°; $B)$ 60°; $C)$ 150°; $D)$ 90°; $E)$ 45°.

42. $ABCD$ parallelogramning tomonlari $|\overline{AB}| = 7$, $|\overline{AD}| = 8$ hamda $\overline{AC}$ va $\overline{AD}$ tomonlar orasidagi burchak 60° teng bo‘lsa, $|\overline{AC}|$ dioganalni toping.

A) 13; $B)$ 14; $C)$ 15; $D)$ 16; $E)$ 12.

43. Agar $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlar orasidagi burchak 30° va $(\boldsymbol{a}, \boldsymbol{b}) = \sqrt{3}$ bo‘lsa, unda $\boldsymbol{a}$ va $\boldsymbol{b}$ vektorlarga yasalgan parallellogramning yuzi S ni toping.

A) $S = 2$; $B)\ S = 1$; $C)\ S = \dfrac{1}{2}$; $D)\ S = 3$; $E)\ S = \dfrac{3}{2}$.

44. $\boldsymbol{a}(-1,-2,1)$ va $\boldsymbol{b}(-1,1,2)$ vektorlarning $\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$ vektor ko‘paytmasini toping.

$A)\ \boldsymbol{c} = 6\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$; $B)\ \boldsymbol{c} = 3\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$; $C)\ \boldsymbol{c} = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$;

$D)\ \boldsymbol{c} = -6\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$; $E)\ \boldsymbol{c} = 4\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k}$.

45. $\boldsymbol{a} = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ va $\boldsymbol{b} = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ vektorlarga yasalgan parallellogrammning S yuzini toping.

A) $S = 8$; $B)\ S = 8\sqrt{2}$; $C)\ S = 8\sqrt{3}$;

$D)\ S = 8\sqrt{5}$; $E)\ S = 16$.

46. $\boldsymbol{a} = \boldsymbol{i} - 2\boldsymbol{j}$ va $\boldsymbol{b} = \boldsymbol{j} - 3\boldsymbol{k}$ vektorlarga yasalgan uchburchak S yuzini toping.

A) $S=\frac{\sqrt{45}}{2}$; $B)$ $S=\frac{\sqrt{23}}{2}$; $C)$ $S=\frac{\sqrt{26}}{2}$;

$D)$ $S=\frac{\sqrt{28}}{2}$; $E)$ $S=\frac{\sqrt{46}}{2}$.

47. $\boldsymbol{a}$ va **b** vektorlar orasidagi burchak $60°$, $|\boldsymbol{a}|=|\boldsymbol{b}|=\sqrt{3}$ sharlar bajarilganda $\boldsymbol{a}+2\boldsymbol{b}$ va $2\boldsymbol{a}+\boldsymbol{b}$ vektorlarga yasalgan parallellogrammning S yuzini toping.

A) $S=\frac{9\sqrt{3}}{2}$; $B)$ $S=9\sqrt{3}$; $C)$ $S=9\sqrt{2}$; $D)$ $S=3\sqrt{3}$;

$E)$ $S=\frac{3\sqrt{3}}{2}$.

48. x ning qanday qiymatlarida $\boldsymbol{a}=x\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k}$, $\boldsymbol{b}=\boldsymbol{i}+x\boldsymbol{j}-\boldsymbol{k}$, $\boldsymbol{c}=2\boldsymbol{i}+\boldsymbol{j}+x\boldsymbol{k}$ vektorlar komplanar bo‘ladi?

$A)$ $x=-1, x=2$; $B)$ $x=-1, x=\frac{1\pm\sqrt{5}}{2}$; $C)$ $x=1, x=-2$;

$D)$ $x=\frac{1+\sqrt{3}}{2}, x=-1$; $E)$ $x=\frac{1\pm\sqrt{3}}{2}, x=1$.

49. $\boldsymbol{a}(1,2,-1)$, $\boldsymbol{b}(2,-1,1)$, $\boldsymbol{c}(-1,2,1)$ vektorlarning aralash ko‘-paytmasini toping.

A) 12; $B)$ 10; $C)-12$; $D)$ 14; $E)$ 6.

50. $\boldsymbol{a}=\boldsymbol{i}-\boldsymbol{j}$, $\boldsymbol{b}=\boldsymbol{j}+\boldsymbol{k}$, $\boldsymbol{c}=\boldsymbol{i}-2\boldsymbol{j}$ vektorlarga yasalgan parallelopipedning V hajmini toping**.**

A) $V=2$; $B)$ $V=\frac{1}{2}$; $C)$ $V=\frac{3}{2}$; $D)$ $V=\frac{5}{2}$; $E)$ $V=3$.

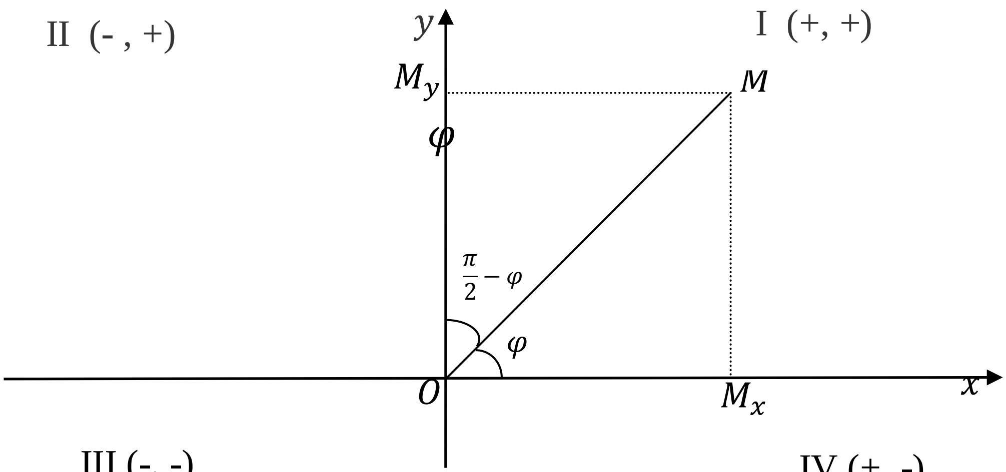
II BOB. TEKISLIKDAGI ANALITIK GEOMETRIYANING ASOSIY TUSHUNCHALARI

§ 1. TEKISLIKDAGI TO'G'RI BURCHAKLI KOORDINATALAR SISTEMASI VA UNING SODDA MASALALARGA TATBIQLARI

1.1. Tekislikdagi to‘g‘ri burchakli koordinatalar sistemasi

Tekislikda to‘g‘ri burchakli dekart (fransuz matematigi Dekart nomi bilan atalgan) koordinatalari sitemasi quyidagicha kiritiladi.

Tekislikda ixtiyoriy O (*koordinata boshi*) nuqtani tanlaymiz va undan o‘zaro perpendikulyar bo‘lgan to‘g‘ri chiziqlar – Ox va Oy *koordinata o‘qlarini* o‘tkazamiz (1-chizma). Qulaylik uchun Ox – *abstsisa o‘qini* gorizantal va chapdan o‘nga yonalgan, Oy *ordinata o‘qini* esa vertikal va pastdan yuqoriga yonalgan deb hisoblaymiz. Bundan tashqari masofani o‘lchsh uchun masshtab (olcham birligi) tanlangan bo‘lsin.



1-chizma

Tekislikdagi nuqtaning koordinalari deb uning tekislikdagi holatini aniqlovchi sonlarga aytiladi. Tekislikdagi har bir M nuqtaga (x, y) sonlar juftligini, bu yerda x *abstsisa* va y *ordinatalar*, quyidagicha mos qo‘yamiz. M nuqtadan koordinata o‘qlariga parallel bo‘lgan to‘g‘ri chiziqlar oʻtkazamiz. Ular Ox – abstsisa oʻqini M_x nuqtada, Oy – ordinata oʻqini esa M_y nuqtada kesib oʻtadi.

M nuqtaning *abstsisasi* deb absolyut qiymati koordinata boshi O nuqtadan M_x nuqtagacha boʻlgan masofaga teng boʻlgan x songa aytiladi; bunda agar M_x nuqta koordinata boshidan oʻngda yotsa, abstsisa musbat, aks holda esa – manfiy boʻladi.

M nuqtaning *ordinatasi* deb absolyut qiymati koordinata boshi O nuqtadan M_y nuqtagacha boʻlgan masofaga teng boʻlgai y songa aytiladi; bunda agar M_y nuqta koordinata boshidan yuqorida yotsa, ordinata musbat, aks holda esa –manfiy boʻladi.

Tekislikda x abstsisa va y ordinatalar M nuqtaning holatini toʻliq aniqlaganligi uchun uning *dekart koordinatalari* deb qabul qilinad. *Har bir* (x, y) *sonlar juftiga tekislikda dekart koordinatalari shu sonlardan iborat yagona nuqta mos keladi; aksincha, har qanday tekislik nuqtasi aniq* (x, y) *dekart koordinatalarga ega.* Abstsisasi x va ordinatasi y boʻlgan M nuqta quyidagicha belgilanadi: $M(x, y)$. x va y dekart kordinatalari kiritilgan tekislikni Oxy deb belgilaymiz.

Ox va Oy oʻqlari tekislikni kvadrantlar deb ataluvchi toʻrta (I, II, III, IV) qisimlarga ajratadi. Kvadrantlardagi nuqtalarning koordinatalarining ishoralari:

I - kvadrantda $(+,+)$; II- kvadrantda $(-,+)$; III - kvadrantda $(-,-)$; IV - kvadrantda $(+,-)$ (1-chizma).

Abstsisa oʻqidagi nuqtaning ordinatasi nolga teng, ordinata oʻqidagi nuqtaning esa abstsisasi nolga teng. Agar nuqta koordinata boshi bilan ustma-ust tushsa, u holda uning ikkala koordinatasi ham nolga tengdir.

Koordinata boshini M nuqta bilan birlashtiruchi kesmaga M nuqtaning *radius-vektori* deyiladi va $\boldsymbol{r} = \overline{OM}$ kabi belgilanadi. Ox oʻqi bilan $\boldsymbol{r} = \overline{OM}$ radius-vektori orasidagi musbat burchakni φ bilan, $r = |\boldsymbol{r}|$ bilan esa radius-vektorining uzunligini belgilaymiz. I kvadrantda yotgan M nuqta koordinatalari uchun quyidagi formulalar oʻrinli:

$$x = r\cos\varphi, y = r\cos\left(\frac{\pi}{2} - \varphi\right) = r\sin\varphi\ . \qquad (1)$$

Osonlik bilan koʻrish mumkinki, (1) formulalar barch kvadrantlar uchun ham oʻrinli. Sunday qilib, M nuqta abstsisasi x ning ishorasi mos kvadrantlarda *kosinus* ishorasi bilan, y ordinatasi esa *sinus* ishorasi bilan ustma-ust tushar ekan.

1-misol. $M_1(2,3), M_2(-4,2), M_3(-2,-4), M_4(5,-2)$ *nuqtalar berilgan*. Bu nuqtalar mos ravishda I, II, III, IV kvadrantlarda yotadi.

2-**misol**. Oxy tekislikda x abstsisasi $|x| = a$ $(a > 0)$ shartni qanoatlantiruvchi to‘g‘ri chiziqlarni aniqlang.

Yechish. Misol shartini qanoatlantiruvch nuqtalar $M(a, y)$ *va* $M(-a, y)$ koordinatalarga ega. Shu sababli, bu nuqtalar Oy o‘qga parallel ravishda $M_1(a, 0)$ *va* $M_2(-a, 0)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqlardan iborat.

1.2. To‘g‘ri burchakli koordinatalar sistemasini almashtirish

Ba’zi masalalarni yechishda berilgan Oxy to‘g‘riburchakli koordinatalar sistemasi o‘rniga unga nisbatan ma’lum ma’noda oriyentasiyalangan boshqa bir $O'x'y'$ to‘g‘ri burchakli koordinatalar sistemasini qarash qulay bo‘ladi. Masalan, osmon mexanikasi va astronomiyadagi tadqiqotlarda Yer markazi bilan bog‘liq geotsentrik sistemadan foydalanish mumkin. Ammo, markazi Quyoh bo‘lgan geliosentrik sistemadan foydalanish qulayroq hisoblanadi.

Bir dekart koordinatalar sistemasidan shunday tipdagi boshqa ikkinch sistemaga qanday o‘tiladi? – degan savol paydo bo‘ladi. Bu savolga koordinatalar sistemasini o‘zgartish qanday bajarilisini ko‘rsatish bilan javob beramiz.

Koodinatalar sistemasini o‘zgartirishning uchta holini qaraymiz:

1) koordinatalar sistemasini parallel ko‘chirush;

2) koordinatalar sistemasini burish;

3) koordinatalar sistemasini bir vaqtning o‘zida parallel ko‘chirish va burish.

A) *Koordinatalar sistemasini parallel ko‘chirush*. Bu holda koordinatalar boshi tekislikning boshqa nuqtasiga ko‘chirilgan va koordinata o‘qlarining yo‘nalishi o‘zgarmagan bo‘ladi (2-chizma).

Faraz qilaylik, “yangi” koordinatalar sistemasining boshlang‘ich O' nuqtasi “eski” koordinatalar sistemasida (a, b) koordinatalarga ega bo‘lsin. Unda tekislikning “eski” koordinatalar sistemasida (x, y) kordinatalarga ega bo‘lgann M niqtasi “yangi” koordinatalar sistemasida qandaydir (x', y') kordinatalarga ega bo‘ladi. Bunda asosiy masala – (x, y) *va* (x', y') *koordinatalar orasidagi bog‘lanishni topishdan iborat.*

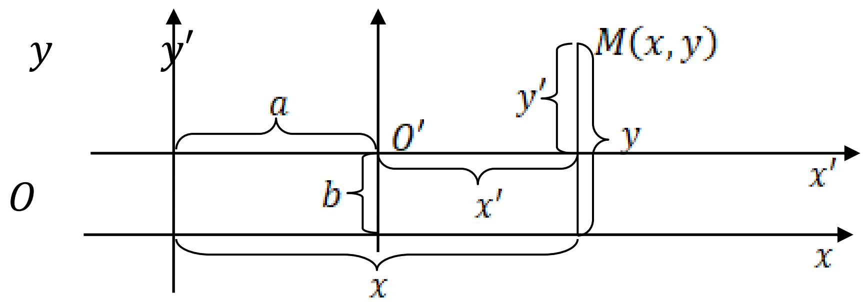
2-chizmadan ko‘rinib turibdiki,

$$x' = x - a,\ \ y' = y - b. \tag{2}$$

Ushbu (2) tengliklardan esa

$$x = x' + a, \;\; y = y' + b \tag{3}$$

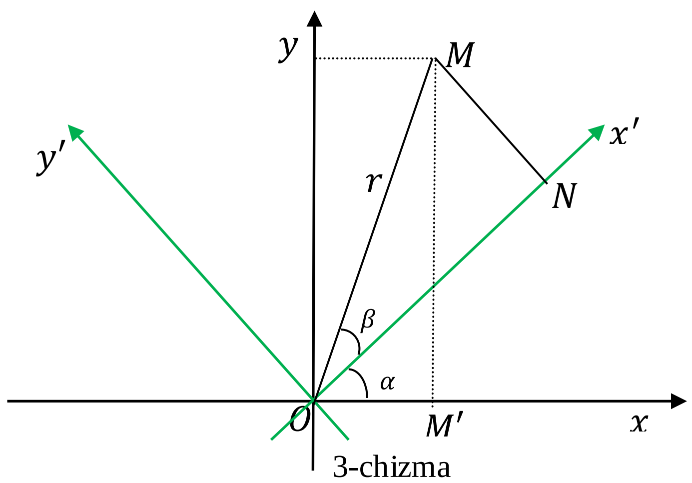
ekanlgini topamiz.



2-chizma

Keltirilgan (2) va (3) formulalar koordinatalar sistemasini parallel ko‘chirishda M nuqtaning eski (x, y) va yangi (x', y') koordinatalari orasidagi bog‘lanishni ifodalaydi.

b) *Koordinatalar sistemasini burish*. Endi “yangi” koordinatalar sistemasi deb, O koordinata boshini o‘zgartirmasdan, “eski” Oxy koordinatalar sistemasini α burchakka burish natijasida hosil bo‘lgan $Ox'y'$ sistemaga aytiladi (3-chizma). Bu yerda, agar Oxy sistema soat strelkasiga teskari yo‘nalishda burilsa, α burchak musbat, aks holda esa – manfiy deb hisoblanadi.



3-chizma

M nuqtaning radius-vektori $\overline{OM}$ ning Ox' o‘qi bilan hosil qilgan musbat burchakni β belgilaymiz. U vaqtda OM kesma Ox o‘qi bilan $\alpha + \beta$ musbat burchakni hosil qiladi. Shuni e’tiborga olib, (1) formulaga asosan M nuqtaning koordinatalarini hisoblaymiz:

$$x = r\cos(\alpha + \beta) = r\cos\alpha\cos\beta - r\sin\alpha\sin\beta, \tag{4}$$

$$y = r\sin(\alpha + \beta) = r\sin\alpha\cos\beta + r\cos\alpha\sin\beta, \tag{5}$$

bu yerda $r = |\overline{OM}|$. M nuqtaning yangi koordinatalari

$$x' = r\cos\beta, y' = r\sin\beta,$$

bo‘lganligi uchun (4) va (5) formulalarni quyidagicha yozish mumkin:

$$\left.\begin{aligned} x &= x'\cos\alpha - y'\sin\alpha, \\ y &= x'\sin\alpha + y'\cos\alpha. \end{aligned}\right\} \tag{6}$$

(6) formula M nuqtaning eski koordinatalari x va y ni uning yangi x' va y' koordinatalari orqali ifodalaydi. Yangi x' va y' koordinatalarni uning eski koordinatalari x va y orqali ifodalash uchun (6) sistemani x' va y' larga nisbatan yechish yetarli. Biroq buni soddaroq usulda bajarish mumkin. Buning uchun $Ox'y'$ sistemani “eski” deb, Oxy sistemani esa “yangi” deb qabul qilamiz. U holda, ikkinchi sistema birinchisiga nisbatan $-\alpha$ burchakka burulganligini hisobga olamiz, (6) formulada mos ravishda x' va y'larni x va y almashtiramiz hamda

$$\cos(-\alpha) = \cos\alpha, \sin(-\alpha) = -\sin\alpha$$

tengliklarni e’tiborga olib, quyidagini hosil qilamiz:

$$\left.\begin{aligned} x' &= x\cos\alpha + y\sin\alpha, \\ y' &= -x\sin\alpha + y\cos\alpha \end{aligned}\right\} \tag{7}$$

c) *Boshqa koordinatalar sistemasiga o‘tishning umumiy holi*. Umumiy holda, yangi koordinatalar boshi $O'(a, b)$ nuqtada va $O'x'$ o‘q Ox o‘qi bilan α tashkil etganda, (3) va (6) formulalarini birlashtirib quyidagini hosil qilaniz:

$$\left.\begin{aligned} x &= a + x'\cos\alpha - y'\sin\alpha, \\ y &= b + x'\sin\alpha + y'\cos\alpha \end{aligned}\right\} \tag{8}$$

Xuddi shuningdek, (2) va (7) formulalaridan

$$\left.\begin{aligned} x' &= (x - a)\cos\alpha + (y - b)\sin\alpha, \\ y' &= -(x - a)\sin\alpha + (y - b)\cos\alpha. \end{aligned}\right\} \tag{9}$$

hosil bo‘ladi.

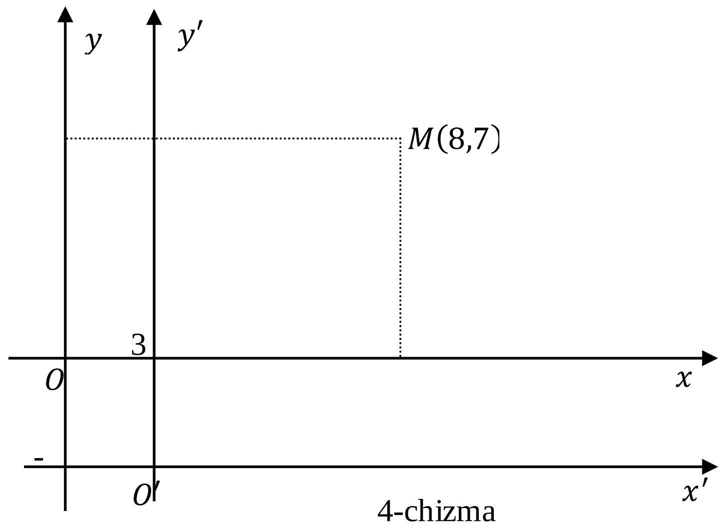
(8) va (9) formulalaridan quyidagicha xulosa qilamiz: *bir to‘g‘ri burchakli koordinatalar sistemasidan ikkinchi to‘g‘ri burchakli koordinatalar sistemasiga o‘tish formulalari yangi va eski kordinatalarni bog‘lovchi chiziqli funksiyalar bo‘lar ekan*.

3-misol. *To‘g‘ri burchakli koordinatalar sistemasi parallel ko‘chirilgan, yangi kordinatalar boshi* $O'(3, -4)$ *nuqtada. M nuqtaning eski koordinatalari* $(8,7)$. *Shu nuqtaning yangi sistemadagi koordinatalarini toping.*

Yechish. Misol shartiga ko‘ra: $a = 3, b = -4;\ x = 8, y = 7$. Demak, (2) formuladan foydalanib, quyidagilarni topamiz:

$$x' = x - a = 8 - 3 = 5,\ y' = y - b = 7 - (-4) = 11.$$

Shunday qilib, berilgan nuqtaning yangi sistemadagi koordinatasi $M(5,11)$ (4-chizma).



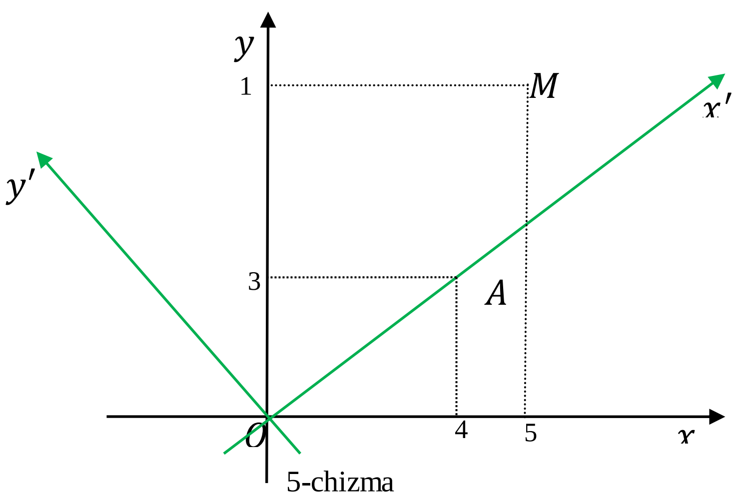
4-chizma

4-misol. Oxy tekislikda $A(4,3)$ nuqta berilgan. Koordinatalar sistemasi koordinata boshiga nisbatan shunday burulganki, yangi $O'x'$ o‘qi A nuqtadan o‘tadi. Agar M nuqtaning yangi $x' = 5, y' = 10$ koordinatalari berilgan bo‘lsa, uning eski koordinatalarini toping.

Yechish. Avvalo eski Ox abssissa o‘qining burulbsh burchagini aniqlaymiz. $|OA| = \sqrt{4^2+3^2} = 5, \sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}$. Endi (6) formulaga asosan

$$x = \frac{4}{5}x' - \frac{3}{5}y', y = \frac{3}{5}x' + \frac{4}{5}y'.$$

Misol shartiga ko‘ra $x' = 5, y' = 10$ bo‘lgani uchun $x = 5, y = 11$ (5-chizma).



5-chizma

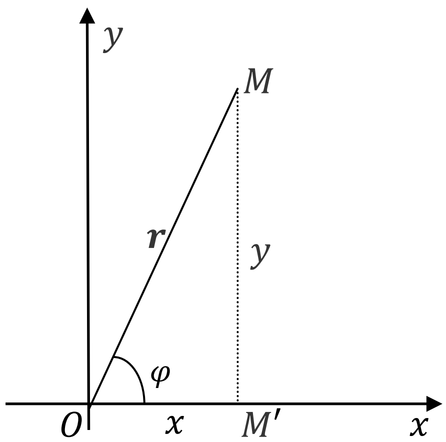
1.3. Tekislikdagi ikki nuqta orasidagi masofa

A) Dastlab $O(0{,}0)$ koordinata boshidan tekislikning ixtiyoriy $M(x, y)$ nuqtasigacha bo‘lgan r masofani topish masalasini qaraymiz.

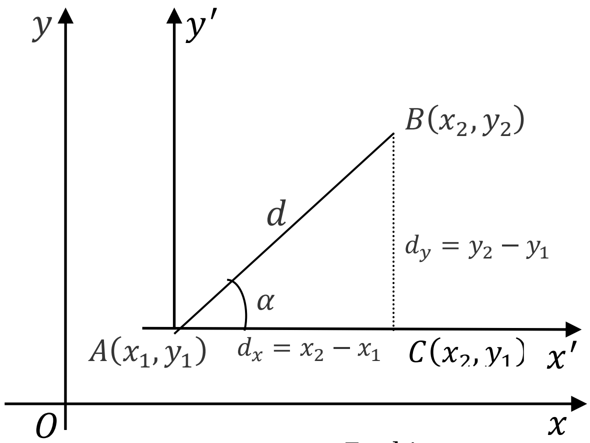
6-chizmadan ko‘rinib turibdiki, $r = |OM|$ masofa katetlari $[OM']$ va $[MM']$ lardan iborat to‘g‘ri burchakli OMM' uchburchakning $[OM]$ gipotenuzasi uzunliga teng. $|OM'| = |x|, |MM'| = |y|$ bo‘lgani uchun Pifagor teoremasiga asosan

$$r = \sqrt{x^2 + y^2}. \qquad (10)$$

Shunday qilib, koordinata boshidan tekislikning biror nuqtasigacha bo‘lgan masofa shu nuqtaning koordinatalari kvadratlari yig‘indisidan chiqarilgan kvadrat ildizga teng.



6-chizma



7- chizma

B) Endi umumiy holni, ya’ni berilgan $A(x_1, y_1)$ va $B(x_2, y_2)$ (7-chizma) nuqtalar orasidagi $d = |AB|$ masofani topishni qaraymiz.

O$x'y'$ yangi koordinatalar sistemasini shunday tanlaymizki O koordinatalar boshi A nuqtada va o‘qlar oldingi o‘qlarga parallel va ular bilan bir xil yo‘nalishda. U vaqtdaa B va A nuqtalar yangi sistemada $B(x_2 - x_1, y_2 - y_1)$ va $A(0{,}0)$ koordinatalarga ega bo‘ladi. Bularni e’tiborga olsak, (10) formulaga asosan

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, \qquad (11)$$

ya’ni tekislikda berilgan ikki nuqta orasidagi masofa ularning mos koordinatalari ayirmalari kvadratlari yig‘indisidan chiqarilgan kvadrat ildizga teng.

1-eslatma. (11) formula $[AB]$ kesma uzunligini ifodalaydi.

5-misol. *Berilgan* $A(2{,}6)$ *va* $B(-6{,}12)$ *nuqtalar orasidagi masofa-ni toping* ***Yechish.*** (11) formuladan foydalanib, quyidagini olamiz:

$$d = \sqrt{(-6-2)^2 + (12-6)^2} = \sqrt{64+36} = 10.$$

6-misol. *$A(0, a)$ va $B(b, 0)$ nuqtalardan baravar uzoqlikda yotuvch Ox o'qidagi M nuqtaning koordinatalarini toping.*

Yechish. M nuqta Ox o'qida yotganligi uchun uning ordinatasi $y = 0$. (11) formuladan foydalansak, unda

$$|AM| = \sqrt{x^2 + a^2}, \ |MB| = \sqrt{(x-b)^2}.$$

Shartga ko'ra $|AM| = |MB|$. Shunday qilib,

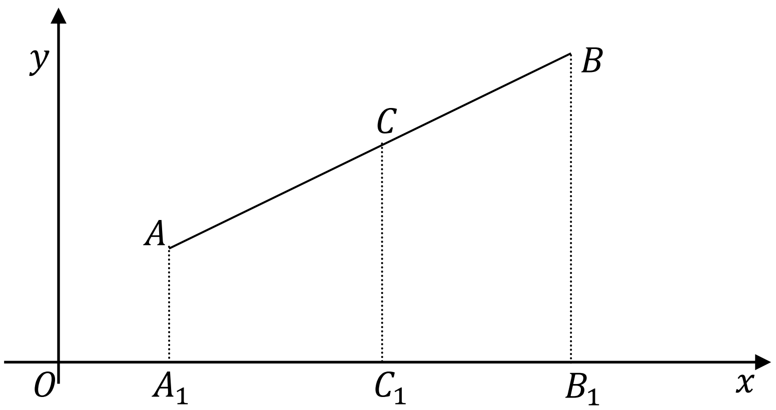
$$x^2 + a^2 = (x-b)^2 \text{ yoki } x^2 + a^2 = x^2 - 2bx + b^2.$$

Bundan $2bx = b^2 - a^2$, demak, $x = \frac{b^2 - a^2}{2b}$ – M nuqtaning abstsisasi.

1.4. Kesmani berilgan nisbatda bo'lish. Massalar sistemasining markazi.

A) Kesmani berilgan nisbatda bo'lish. Oxy tekisligida uchlari $A(x_1, y_1)$ va $B(x_2, y_2)$ nuqtalarda bo'lgan $[AB]$ kesmani berilgan nisbatda ikki qismga bo'lish masalasini qaraymiz. Bu masala $[AB]$ kesmani $\lambda_1 : \lambda_2$ nisbatda bo'luvchi, ya'ni $\frac{[AC]}{[CB]} = \frac{\lambda_1}{\lambda_2}$ shartni qanoatlantiruvchi C nuqtaning x va y koordinatalarini topishdan iborat.

Faraz qilaylik $[AB]$ kesma Oy o'qiga parallel bo'lmasin. A_1, B_1, C_1 nuqtalar mos ravishda A, B, C nuqtalarning Ox o'qidagi proyeksiylari bo'lsin (8 - chizma).



8-chizma

Masala shartini hisobga olsak, $\frac{|AC|}{|CB|} = \frac{|A_1C_1|}{|C_1B_1|} = \frac{\lambda_1}{\lambda_2}$. A_1, B_1, C_1 nuqtalarning abssissalari A, B, C nuqtalarning abssissalariga тeng bo'lganligidan

$$|A_1C_1| = |x_1 - x|, |C_1B_1| = |x - x_2|.$$

Shuning uchun,

$$\frac{|x_1-x|}{|x-x_2|}=\frac{\lambda_1}{\lambda_2}.$$

C_1 nuqta A_1 va B_1 nuqtalar orasida yotganligi uchun x_1-x va $x-x_2$ ayirmalar bir xil ishoraga ega. Shuning uchun

$$\frac{|x_1-x|}{|x-x_2|}=\frac{x_1-x}{x-x_2}=\frac{\lambda_1}{\lambda_2}.$$

Bundan keib chiqadiki,

$$x=\frac{\lambda_2x_1+\lambda_1x_2}{\lambda_1+\lambda_2}. \qquad (12)$$

Agar $[AB]$ kesma Oy o'qiga parallel bo'lsa, u holda $x_1=x_2=x$. Shu natijani (12) formuladan ham olamiz. Demak, bu formula A va B nuqtalarning qanday joylashishdan qat'iy nazar o'rinlidir.

C nuqtaning ordinatasi ham shunga o'xshash topiladi, ya'ni quyidagi formula o 'rinli:

$$y=\frac{\lambda_2y_1+\lambda_1y_2}{\lambda_1+\lambda_2}. \qquad (13)$$

$\lambda_1=\lambda_2$ bo'lganda (12) va (13) formulalardan $[AB]$ kesmani teng ikkiga bo'luvchi nuqtaning koordinatalarini ifodalovchi formulani hosil qilamiz:

$$x=\frac{x_1+x_2}{2}, y=\frac{y_1+y_2}{2}. \qquad (14)$$

7-misol. *$[AB]$ kesmaning chetki nuqtalari $A(-2{,}5)$ va $B(4{,}17)$. Bu kesmada C nuqta shunday joylashganki, uning A nuqtagacha bo'lgan masofasi B nuqtagacha bo'lgan masofadan ikki baravar ortiq. C nuqtaning koordinatasini toping.*

Yechish. Berilganlarga ko'ra $x_1=-2, y_1=5, x_2=4, y_2=17$. Masala shartiga ko'ra $|AC|=2|CB|$, ya'ni $|AC|:|CB|=2$. Demak, (12), (13) formulalarda $\lambda_1=2, \lambda_2=1$ ekanligini hisobga olib, C nuqtaning koordinatalarinui topamiz:

$$x=\frac{-2+2\cdot 4}{1+2}=2, y=\frac{5+2\cdot 17}{1+2}=13.$$

B) Berilgan massalar sistemasining markazini topish masalasi. $A_1(x_1,y_1)$ va $A_2(x_2,y_2)$ nuqtalarga mos ravishda m_1 va m_2 massalar qo'yilgan. Berilgan massalar sistemasining markazi (yoki m_1, m_2 massalarning og'irlik markazi) deb, $[A_1A_2]$ kesmani $m_2{:}m_1$ nisbatda bo'luvchi A nuqtaga aytiladi. Shunday qilib, A_1, A_2 nuqtalarga joylashtirilgan massalar sistemasi markazining koordinatalari quyidagiga teng:

$$x=\frac{m_1x_1+m_2x_2}{m_1+m_2}, \qquad y=\frac{m_1y_1+m_2y_2}{m_1+m_2}.$$

Agar berilgan $A_i(x_i, y_i), i = \overline{1,n}$ nuqtalarga mos ravishda $m_1, m_2, m_3, \dots, m_n$ massalar qo‘yilgan bo‘lsa, ushbu massalar sistemasi markazining koordinatalari induksiya metodi orqali aniqlanadi. Masalan, agar A'_{n-1} nuqta dastlabki $(n-1)$ ta massaning markazi bo‘lsa, u holda barcha n ta массaning markazi ikki massaning, ya’ni A_n nuqtaga qo‘yilgan m_n massa va A'_{n-1} nuqtaga qo‘yilgan $m_1 + m_2 + m_3 + \dots + m_{n-1}$ massalarning og‘irlik markazi sifatida aniqlanadi.

$m_1, m_2, m_3, \dots, m_n$ massalar mos ravishda $A_i(x_i, y_i), i = \overline{1,n}$ nuqtalarga qo‘yilgan bo‘lsa, bu massalarning og‘irlik markazining koordinatasi uchun quyidagi formula o‘rinli:

$$x = \frac{m_1x_1 + m_2x_2 + \dots + m_nx_n}{m_1 + m_2 + \dots + m_n}, \quad y = \frac{m_1y_1 + m_2y_2 + \dots + m_ny_n}{m_1 + m_2 + \dots + m_n}. \qquad (15)$$

8-misol. $A_1(-2,4), A_2(3,-1), A_3(2,3)$ *nuqtalarga mos ravishda* $m_1 = 60\ g, m_2 = 40\ g, m_3 = 100\ g$ *massalar qo‘yilgan. Bu massalar sistemasi markazinining koordinatalarini toping.*

Yechish. (15) formuladan $n = 3$ bo‘lgan hol uchun foydalanamiz. Masala shartiga ko‘ra $x_1 = -2,\ x_2 = 3, x_3 = 2,\ y_1 = 4, y_2 = -1, y_3 = 3.$ Demak,

$$x = \frac{-2 \cdot 60 + 3 \cdot 40 + 2 \cdot 100}{60 + 40 + 100} = \frac{200}{200} = 1\,,$$

$$y = \frac{4 \cdot 60 + (-1) \cdot 40 + 3 \cdot 100}{60 + 40 + 100} = \frac{500}{200} = \frac{5}{2}.$$

1.5. Uchburchak va ko‘pburchak yuzini topish

Uchlari $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ nuqtalarda bo‘lgan ABC uchburchakning S yuzasini topish masalasini qaraymiz. $|AB| = c$, $|AC| = b$, hamda $[AB]$, $[AC]$ tomonlarning Ox o‘qi bilan tashkil qilgan musbat burchaklari mos ravishda α и β ga teng bo‘lsin (9-chizma). U holda

$$|A'B'| = c \cdot \cos\alpha = x_2 - x_1,\ |A''B''| = c \cdot \sin\alpha = y_2 - y_1, \qquad (16)$$

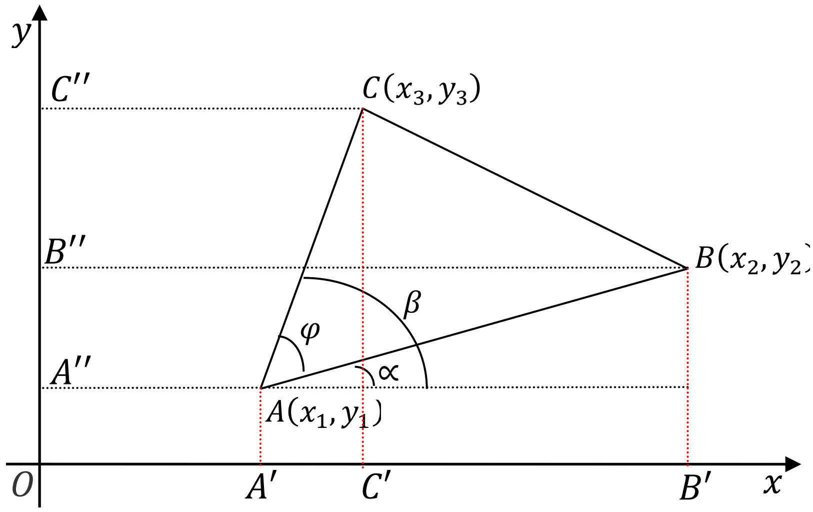
$$|A'C'| = b \cdot \cos\beta = x_3 - x_1,\ |A''C''| = b \cdot \sin\beta = y_3 - y_1. \qquad (17)$$

Agar $\varphi = \angle(CAB)$ deb belgilasak, u holda 9-chizmadan $\varphi = \beta - \alpha$. Trigonometriyadan bizga ma’lum bo‘lgan ABC uchburchakning S yuzini topish formulasini yozamiz :

$$S = \frac{1}{2}bc\sin\varphi = \frac{1}{2}bc\sin(\beta - \alpha) = \frac{1}{2}bc(\sin\beta\cos\alpha - \cos\beta\sin\alpha).$$

Bu formuladan foydalansak, (16) va (17) tengliklarga asosan uchburchak yuzi uchun quyidagi formulaga kelamiz:

$$S = \frac{1}{2}[(y_3 - y_1)(x_2 - x_1) - (x_3 - x_1)(y_2 - y_1)]. \qquad (18)$$



9-chizma

Uchburchak uchlari boshqacha joylashganda uchburchak yuzi uchun (18) formula manfiy ishorali qiymat berishi mumkin. Shuning uchun uchburchak yuzini topish formulasi odatda quyidagi shaklda yoziladi:

$$S = \frac{1}{2}[(y_3 - y_1)(x_2 - x_1) - (x_3 - x_1)(y_2 - y_1)], \qquad (19)$$

bu yerda $(\pm)$ ishora shunday tanlanadiki, yuza uchun musbat son hosil bo‘lsin.

Ikkinchi tartibli determinant tushunchasidan foydalanib, (19) formulani eslab qolishga qulay shaklda yozish mumkin:

$$S = \pm\frac{1}{2}\begin{vmatrix} x_2 - x_1 & x_3 - x_1 \\ y_2 - y_1 & y_3 - y_1 \end{vmatrix} = \pm\frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}\right]. \qquad (20)$$

Agar $A(x_1, y_1)$ nuqta koordinata boshida joylashsa, u holda (19) formula soddalashadi. Bu holda $x_1 = 0, y_1 = 0$ bo‘lgani uchun

$$S = \pm\frac{1}{2}(x_2 y_3 - x_3 y_2).$$

Shuni taakidlaymizki, agar A, B, C nuqtalar bir to‘g‘ri chiziqda yotsa, u holda uchburchak yuzi $S = 0$; aksincha, agar $S = 0$, bo‘lsa, u holda uchburchak uchlari A, B, C bir to‘g‘ri chiziqda yotadi.

2-eslatma. Kopburchak yuzini hisoblashni uchburchak yuzini hisoblashga keltirish mumkin. Buning uchun ko‘pburchakni yuzalari (20) formula bilan hisoblanadigan uchburchaklarga ajratish yetarli. Uchlari

$A_1(x_1,y_1)$, $A_2(x_2,y_2)$,…, $A_n(x_n,y_n)$ nuqtalarda bo‘lgan ko‘pburchak yuzi quyidagi formula bilan hisoblanadi:

$$S=\pm\frac{1}{2}\left[\begin{vmatrix}x_1 & y_1\\ x_2 & y_2\end{vmatrix}+\begin{vmatrix}x_2 & y_2\\ x_3 & y_3\end{vmatrix}+\cdots+\begin{vmatrix}x_n & y_n\\ x_1 & y_1\end{vmatrix}\right], \qquad (21)$$

(21) formula (20) formulaning umumlashmasidir. Bu yerda shuni taakidlaymizki, (21) formuladan foydalana olish uchun $A_i(x_i,y_i)$ $i=\overline{1,n}$ nuqtalar ketma-ketligi $A_1A_2\ldots A_nA_1$ ko‘pburchakni hosil qilishi kerak.

9-misol. *Uchlari* $A(-2,-4)$, $B(2{,}8)$, $C(10{,}2)$ *nuqtalarda bo‘lgan uchburchak yuzi topilsin.*

Yechish. Misol shartiga ko‘ra

$$x_1=-2, x_2=2, x_3=10, y_1=-4, y_2=8, y_3=2.$$

(19) formuladan foydalanamiz:

$$S=\frac{1}{2}|(x_2-x_1)(y_3-y_1)-(x_3-x_1)(y_2-y_1)|=$$

$$=\frac{1}{2}|(2+2)(2+4)-(10+2)(8+4)|=\frac{1}{2}|24-144|=60.$$

10-misol. *Uchlari* $A(3,2), B(10,3), C(10,8), D(2,12), E(14,7)$ *nuqtalarda bo‘lgan ko‘pburchak yuzi topilsin.*

Yehish. Berilgan nuqtalarni $ABCDEA$ ketma-ketlikda to‘g‘ri chiziq kesmalari bilan tutashtirib ko‘pburchak yasab bo‘lmaydi. Bunda beshburchak hosil bo‘lishi uchun nuqtalarni $ABECDA$ tartibda olishmiz kerak. Shu tartibda olingan nuqtalarning koordinatalarini ustunlarga yozib chiqamiz:

$x_1=3$	$y_1=2$
$x_2=10$	$y_2=3$
$x_3=14$	$y_3=7$
$x_4=10$	$y_4=8$
$x_5=2$	$y_5=12$
$x_1=3$	$y_1=2$

Endi $n=5$ deb olib, (21) formuladan foydalanamiz. $ABECDA$ beshburchak uchlari koordinatalarining jadvaldagi qiymatlarini hisobga olib, quyidagini hosil qilamiz:

$$S=\pm\frac{1}{2}\left[\begin{vmatrix}x_1 & y_1\\ x_2 & y_2\end{vmatrix}+\begin{vmatrix}x_2 & y_2\\ x_3 & y_3\end{vmatrix}+\begin{vmatrix}x_3 & y_3\\ x_4 & y_4\end{vmatrix}+\begin{vmatrix}x_4 & y_4\\ x_5 & y_5\end{vmatrix}+\begin{vmatrix}x_5 & y_5\\ x_1 & y_1\end{vmatrix}\right]=$$

$$=\pm\frac{1}{2}\left[\begin{vmatrix}3 & 2\\ 10 & 3\end{vmatrix}+\begin{vmatrix}10 & 3\\ 14 & 7\end{vmatrix}+\begin{vmatrix}14 & 7\\ 10 & 8\end{vmatrix}+\begin{vmatrix}10 & 8\\ 2 & 12\end{vmatrix}+\begin{vmatrix}2 & 12\\ 3 & 2\end{vmatrix}\right]=$$

$$=\frac{1}{2}[(9-20)+(70-42)+(112-70)+(120-16)+(4-36)]=$$

$$=65{,}5 \text{ kv.b.}$$

1.6. Nazorat savollari va masalalar

1. Tekislikda to‘g‘ri burchakli dekart koordinatalari qanday kiritiladi?

2. Tekislikni to‘g‘ri burchakli koordinatalar o‘qlari qanday qismlarga ajratadi?

3. Tekislikdagi nuqtaning koordinatalari to‘g‘ri burchakli koordinatalar sistemasida qanday formula orqali ifodalanadi?

4. To‘g‘ri burchakli koordinatalar sistemasini koordinata o‘qlarini parallel ko‘chirish yordamida almashtirish.

5. To‘g‘ri burchakli koordinatalar sistemasini koordinata o‘qlarini burush yordamida almashtirish.

6. Umumiy holda koordinatalar sistemasini boshqa sistemaga o‘tkazilganda “eski” va “yangi” koordinatalar orasidagi bog‘lanish qanday formula orqali ifodalanadi?

7. Tekislikdagi ikki nuqta orasidagi masofani hisoblash formulasi.

8. Kesmani berilgan nisbatta bo‘ladigan nuqtaning koordinatalari qanday aniqlanadi?

9. Kesma o‘rtasining koordinatalari qanday formula orqali hisoblanadi?

10. Sistemaning massalar markazi qanday aniqlanadi?

11. Uchlari berilgan uchburchak yuzini hisoblash va uning ko‘pburchaklar uchun umumlashmasi.

12. Nuqtalarni yasang:

$$A(2,3), B(-4,1), C(2,-3), D(2,-5), E(-6,0).$$

13. I kvadrantda yotuvchi, tomoni 12 bo‘lgan tengtomonli uchburchak uchlaridan biri koordinatalar boshi O bilan ustma-ust tushgan, asosi esa Ox o‘qida yotadi. Uning uchlarining koordinatalarini aniqlang.

14. $A(1,2)$ nuqtaga: a) Ox o ‘qiga; b) Oy o‘qiga; c) I va III koordinata burchaklari bissektrisasiga nisbatan simmetrik joylashgan $M(x, y)$ nuqta koordinatalarini aniqlang.

15. Ordinata o‘qiga parallel bo‘lgan MN to‘g‘ri chiziq undan o‘ng tomonda 5 birlik masofadan o‘tgan. MN to‘g‘ri chiziqqa nisbatan $A(2,4)$ nuqtaga simmetrik bo‘lgan A_1 nuqtaning koordinatlarini, xuddi shunindek $B(-1,3)$ nuqtaga simmetrik bo‘lgan B_1 nuqtaning koordinatlarini toping.

16. Koordinatalar sistemasi $\alpha = \frac{\pi}{6}$ burchakga burilgan. $M(\sqrt{3}, 3)$ nuqtaning koordinatalarini aniqlang.

17. Nuqta berilgan: $M(4\sqrt{5}, 2\sqrt{5})$. Yangi abstsisalar o‘qi sifatida $y = 2x$ to‘g‘ri chiziq, yangi ordinatalar o‘qi esa – $y = -0{,}5x$ to‘g‘ri chiziq olingan, yangi koordinata o‘qlari eski o‘qlar bilan o‘tkir burchak tashkil qiladi. Yangi sistemada M nuqtaning koordinatalarini toping.

18. $C(2, 3)$ nuqta AB kesmani 1:2 nisbatda bo‘ladi. Agar A nuqtaning koordinatalari $x = 1, y = 2$ bo‘lsa, B nuqtaning koordinatalarini toping.

19. $A(-2{,}0), B(6{,}6)$ va $C(1, -4)$ nuqtalar uchburchakning uchlari bo‘lsin. A uchdan o‘tkazilgan bissektrisa uzunligini toping.

20. $A(-2{,}1)$, $B(7{,}4)$ niqtalarga mos ravishda $m_1 = 10\ g$ va $m_2 = 20\ g$ massalar qo‘yilgan. Bu massalar sistemasining markazi koordinatalarini toping.

21. ABC uchburchak uchlari berilgan: $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$. Uchburchak medianalarining kesishish nuqtasi koordinatasi topilsin.

22. Uchlari bilan berilgan ABC uchburchak massasi N markazining koordinatasi topilsin: $A(-2{,}1), B(2, -1), C(4{,}3)$ (uchburchak massasi markazi uning medianalari kesishish nuqtasi bilan ustma-ust tushadi).

23. Uchlari $A(2, -1), B(4{,}2)$, $C(5{,}1)$ nuqtalarda bo‘lgan uchburchak teng yonli ekanliligini ko‘rsting.

24. Uchlari $O(0{,}0)$, $A(8{,}0)$, $B(0{,}6)$ nuqtalarda bo‘lgan uchburchakning OC medianasi va OD bissektrisasi uzunliklarini toping.

25. Uchburchakning ikki $A(3{,}8)$, $B(10{,}2)$ uchlari va medianalari kesishish nuqtasi $M(1{,}1)$ berilgan. Uchburchak uchinchi uchining koordinatalarini toping.

26. Uchlari $A(2{,}0), B(6{,}3)$, $C(2{,}7)$ niqtalarda bo‘lgan uchburchak yuzi hisoblansin.

27. $A(1{,}1), B(-1{,}7)$, $C(0{,}4)$ nuqtalar bir to‘g‘ri chiziqda yotishini ko‘rsating.

28. $L(0{,}0), M(3{,}0)$ va $N(0{,}4)$ nuqtalar uchburchak o‘rtalari bo‘lsin. Shu uchburchak yuzini hisoblang.

29. Uchlari $A(-2{,}1), B(2{,}2)$, $C(4, y)$ nuqtalarda bo‘lgan ABC uchburchak yuzi 15 ga teng. C nuqtaning ordinatasini toping.

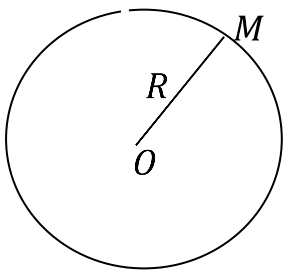
30. O‘rmon uchlari $A(0{,}200)$, $B(200{,}300)$, $C(500{,}100)$, $D(100{,}100)$ niqtalarda bo‘lgan to‘tburchakdan shaklida (koordinatalar metrlarda berilgan). O‘rmon yuzi topilsin.

§2. TEKISLIKDAGI CHIZIQ TENGLAMASI

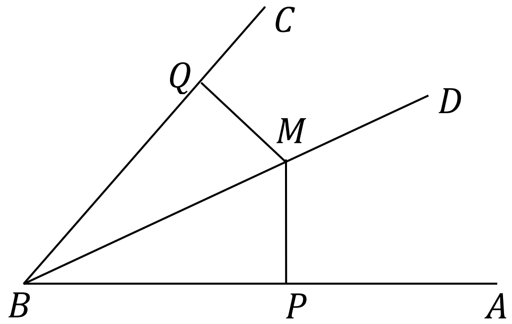
2.1. Chiziq tenglamasi tushunchasi

Tekislikdagi analitik geometriya chiziqlarning shakllari, joylashishi va xossalarini algebra va matematik tahlil usullaridan foydalanib tadqiq etishdan iborat umumiy masalani hal tish bilan shug‘ullanadi. Tekislikdagi chiziq odatda faqat chiziqning o‘zigagna ta’luqli xossalarga ega bo‘lgan nuqtalar to‘plami sifatida beriladi.

1-misol. *R radiusli aylana tekislikning biror O(aylana markazi) nuqtadan R ga teng masofada chetlangan barcha nuqtalari to‘plamidan iborat (1-chizma).*



1-chizma



2-chizma

Boshqacha aytganda, markazdan faqat va faqat radiusga teng uzoqlikda yotuvch nuqtalar aylanaga tegishli bo‘ladi.

2-misol. *ABC burchak bissektrisasi tekislikning shu burchak ichida yotovchi va burchak tomonlaridan baravar uzoqlikda joylashgan nuqtalar to‘plamidir(2-chizma).*

Bu misol quyidagini tasdiqlaydi: BD bissektrisada joylashgan har qanday M nuqtadan burchakning BA va BC tomonlariga mos ravishda tushurilgan MP va MQ perpendilulyarlar uzunliklari o‘zaro teng, ya’ni $|MP| = |MQ|$ va ABC burchak ichidagi bissertrisada yotmagan har qanday nuqta tomonlarning biriga boshqasga nisbatan yaqinroq joylashadi.

Keltirilgan misollarda: aylana – to‘g‘ri chiziq bo‘lmagan yopiq chiziq (*egri chiziq*); burchak bissektrisasi – to‘g‘ri chiziq (*yarim to‘g‘r chiziq*).

Faraz qilaylik, Oxy tekisligida biror chiziq berilgan. Bu chiziqda yotuvchi nuqaning x va y koordinatalari ixtiyoriy bo‘la olmaydi: ular berilgan chiziqning geometrik xossalari bilan aniqlanuvchi qandaydir cheklashlar, shartlarga bo‘ysungan bo‘ladilar. Koordinatalari x va y

bo‘lgan nuqtaning berilgan chiziqda yotishi analitik shaklda biror tenglama ko‘rinishida yoziladi.

Agar $M(x,y)$ nuqta K chiziq ustida harakatlansa, u holda uning x va y koordinatalari o‘zgarib borib, hamma vaqt chiziq tenglamasini qanoatlantiradi. Shuning uchun $M(x,y)$ nuqta koordinatalariga K chiziqning *joriy koordinatalari* deyiladi.

Agar chiziqning har qanday nuqtasining (x,y) *koordinatalari* $f(x,y)=0$ *tenglamani qanoatlantirsa va aksincha, bu tenglamani qanoatlantiruvchi* $(x,\ y)$ *juftliklar chiziqning joriy koordinatalari bo‘lsa, u holda* $f(x,y)=0$ *tenglama chiziqning oshkormas shakldagi tenglamasi deyiladi.*

Tekislikdagi har qanday chiziqqa uning joriy koordinatalarini bog‘lovchi biror tenglama mos keladi. Aksincha, berilgan x va y o‘zgaruvchilar – tekislikdagi nuqta koordinatalarini bog‘lovchi tenglamaga, umuman olganda, xossalari shu tenglama bilan to‘la aniqlanuvchi biror chiziq mos keladi.

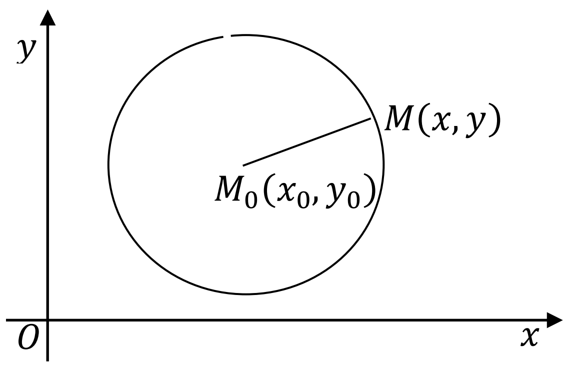
Bu yerdan, tabiiy ravishda, tekislikdagi analitik geometriyaning ikkita asosiy masalasi kelib chiqadi.

1) Nuqtalar to‘plami sifatida qaraluvchi chiziq berilgan. Chiziqning ma’lum geometrik xossalari asosida uning tenglamasi tuzish.

2) Biror chiziq tenglamasi berilgan. Shu tenglama asosida uning geometrik xossalarini o‘rganish (shakli, joylashishi va h.k.).

Chiziq tenglamasini tuzishga doir misollar qaraymiz.

3-misol. *Markazi* $M_0(x_0,y_0)$ *nuqtada va radiusi* R *bo‘lgan aylana tenglamasini tuzing (3-chizma).*



3-chizma

Yechish. Faraz qilaylik, $M(x,y)$ – markazi $M_0(x_0,y_0)$ nuqtada va radiusi R bo‘lgan aylananing ixtiyoriy nuqtasi bo‘lsin. U holda $M(x,y)$ va $M_0(x_0,y_0)$ nuqtalar orasidagi masofa R ga teng, ya’ni

$\sqrt{(x-x_0)^2+(y-y_0)^2}=R$. Bundan berilgan aylananing tenglamasini hosil qilamiz:

$$(x-x_0)^2+(y-y_0)^2-R^2=0. \qquad (1)$$

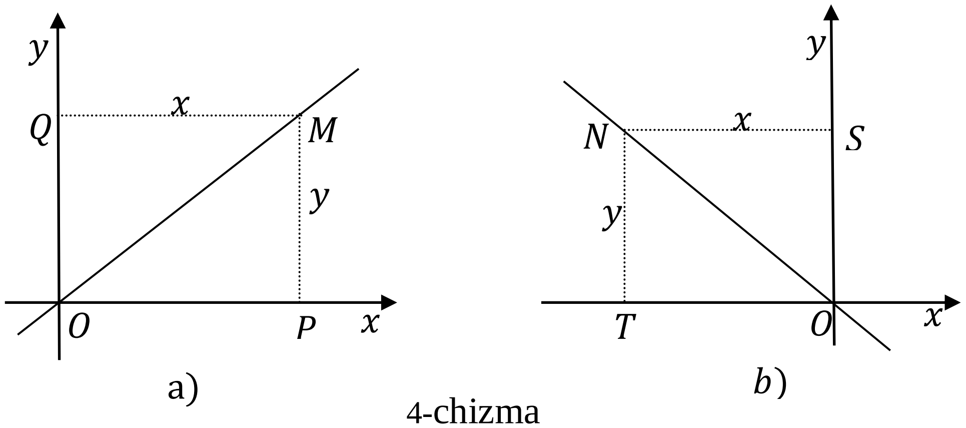
Shunday qilib, aylananing oshkormas shaklidagi tenglamasini hosil qildik:

$$f(x,y)=0,$$

bu yerga $f(x,y)=(x-x_0)^2+(y-y_0)^2-R^2$.

4-misol. *Koordinata burchaklari bissektrisalarining tenglamalarini tuzing.*

Yechish. Avvalo I va III koordinata burchaklarning bissektrisalarini qaraymiz (4a-chizma).



4-chizma

Bissektrisa ustida ixtiyoriy $M(x,y)$ nuqtani tanlaymiz. Agar $M(x,y)$ nuqta birinchi kvadrantda joylashgan bo'lsa, u holda ikkala abtsissa va ordinatalar musbat, hamda bir-biriga teng. Agar $M(x,y)$ nuqta III kvadrantda joylashsa, u holda ikkala abstsisa va ordinatalar manfiy, hamda ularning modullari teng; suning uchun bu nuqtalarning x va y koordinatalari o'zaro teng. Demak, ikkala holda ham $x=y$. Bu esa I va III koordinata bissektrisasining tenglamasidir. Bu tenglamani oshkormas shakilda ham uozish mumkin $x-y=0$.

Endi II va IV koordinata burchaklarning bissektrisalarini qaraymiz (4б-chzma). Bissektrisadagi ixtiyoriy $N(x,y)$ nuqtani tanlaymiz. Bu nuqta II yoki IV kvadrantlarning qaysi birida joylashishidan qa'tiy nazar, uning x va y koordinatalari modullari teng va ishoralari bilan farq qiladi. Shuning uchun, ikkala holda ham $y=-x$ ga ega bo'lamiz. Bu esa biz izlayotgan bissektrisa tenglamasidir. Uni oshkormas shakilda ham yozish mumkin $x+y=0$.

5-misol. *Nuqtalaridan* $B(12{,}16)$ *nuqtagacha masofai* $A(3{,}4)$ *nuqtagacha masofadan ikki marta katta bo'lgan chiziq tenglamasini nuzing.*

Yechish. Agar $M(x,y)$ – izlanayotgan chiziqning ixtiyoriy nuqtasi bo‘lsa, u holda masala shartiga ko‘ra

$$2|AM| = |BM|. \qquad (2)$$

Ushbu chiziq tenglamasini tuzish uchun $|AM|$ va $|BM|$ masofalarni M nuqtaning x va y koordinatalari orqali ifodalab olishimiz kerak. Ikki nuqta orasidagi masofani toppish formulalariga ko‘ra

$$|AM| = \sqrt{(x-3)^2 + (y-4)^2},$$

$$|BM| = \sqrt{(x-12)^2 + (y-16)^2}.$$

Unda, (2) tenglikka asosan quyidagini hosil qilamiz:

$$2\sqrt{(x-3)^2 + (y-4)^2} = \sqrt{(x-12)^2 + (y-16)^2}.$$

Bu esa izlanayotgan chiziq tenglamasidir. Ammo, tenglamaning bu ko‘rinishi uning qanday chiziqni ifodalashini bilish qiyinchilik tug‘diradi. Shuning uchun uni soddalashtiramiz. Tenglikning har ikkala tomonini kvadratga ko‘tarib va qavslarni ochib chiqib, quyidagini hosil qilamiz:

$$4x^2 - 24x + 36 + 4y^2 - 32y + 64 = x^2 - 24x + 144 + y^2 - 32y + 256,$$

yoki uncha qiyin bo‘lmagan almashtirishlardan so‘ng yuqoridagi tenglamaga teng kuchli $x^2 + y^2 = 100$ tenglamani hosil qilamiz. Hosil qilingan tenglamani (1) tenglama bilan solishtirsak, izlanayotgan chiziq – markazi $O(0,0)$ koodinata boshida, radiusi $R = 10$ ga teng aylanadan iborat ekanligini ko‘ramiz.

6-misol. *$A(1,1)$ va $B(3,3)$ nuqtalardan baravar uzoqlikda yotuvch nuqtalar to‘plamini ifodalovchi chiziq tenglamasini tuzing.*

Yechish. Faraz qilaylik $M(x,y)$ nuqta izlanayotgan to‘plamga tegishli bo‘lsin. Ikki nuqta orasidagi masofani topish formulasiga asosan

$$|MA| = \sqrt{(x-1)^2 + (y-1)^2}, \quad |MB| = \sqrt{(x-3)^2 + (y-3)^2}$$

Sharga ko‘ra $|MA| = |MB|$. Demak, izlanayotgan chiziq tenglamasini quyidagi ko‘rinishda yozish mumkin:

$$\sqrt{(x-1)^2 + (y-1)^2} = \sqrt{(x-3)^2 + (y-3)^2}.$$

Bu tenglikning har ikkala tomonini kvadratga ko‘tarib,

$$x^2 - 2x + 1 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 6y + 9,$$

tenglikni hosil qilamiz. Endi o‘xshash hadlarni ixchamlashtirib, quyidagi tenglamani hosil qilamiz:

$$x + y - 4 = 0.$$

Shunday qilib, izlanayotgan chiziq $[AB]$ kesmaning o‘rtasidan unga perpendikulyar tarzda o‘tuvch to‘g‘ri chiziqdan iborat ekan.

7-misol. *Har bir nuqtasi* $F\left(0,\frac{1}{2}\right)$ *nuqta va* $y=-\frac{1}{2}$ *to'g'ri chiziq-dan baravar uzoqlikda yotuvchi chiziq tenglamasinu tuzing.*

Yechish. Izlanayotgan chiziqdan ixtiyoriy $M(x,y)$ nuqtani tanlaymiz. M nuqtadan F nuqtagacha masofani topamiz:

$$|MF|=\sqrt{(x-0)^2+\left(y-\frac{1}{2}\right)^2}.$$

M nuqtadan $y=-\frac{1}{2}$ to'g'ri chiziqqacha masofa $|MN|=|y+\frac{1}{2}|$ ga teng. Masala shartiga ko'ra, $|MF|=|MN|$ tenglik izlanayotgan chiziqda yotuvchi ixtiyoriy M nuqta uchun o'rinli. Demak, bu chiziq tenglamasini quyidagicha yozish mumkin:

$$\sqrt{x^2+\left(y-\frac{1}{2}\right)^2}=|y+\frac{1}{2}|,$$

yoki

$$x^2+y^2-y+\frac{1}{4}=y^2+y+\frac{1}{4}.$$

Bundan $2y-x^2=0$ tenglamani olamiz. Bu tenglama bilan aniqlangan chiziq – *parabola* deb ataladi.

2.2. Chiziqning parametrik tenglamalari

Ba'zida chiziqning x va y to'g'ri burchakli koordinatalarini bog'-lovchi tenglama o'rniga uning *parametrik tenglamalarini*, ya'ni joriy koordinatalarni biror o'zgaruvchi miqdor – t parametrning funksiyalari ko'rinishda ifodalavchi tenglamalarni qarash qulay bo'ladi.

Chiziqdagi ixtiyoriy nuqta koordinatalarini t parametrning funk-siyalari shakilda ifodalovchi ushbu

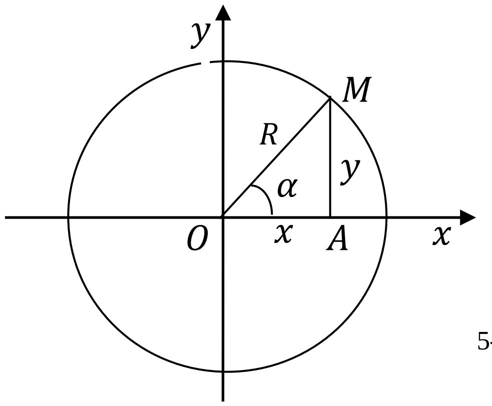
$$x=\varphi(t), y=\psi(t), \tag{3}$$

tenglamalar sistemasiga chiziqning *parametrik tenglamalari* deyiladi.

Parametrik tenglamalar obyekt va jarajonlarlar holatlarining o'zga-rish qonuniyatlarini o'rganishda muhum ahamiyatga ega. Masalan, me-xanikada harakatdagi $M(x,y)$ moddiy nuqtaning x va y koordinatalari vaqtning funksiylari sifatida ifodalanadi hamda bu hol nuqta *harakat tenglamasi* deb qaraladi.

8-misol. *Aylananing parametrik shakldagi tenglamasini tuzing.*

Yechish. Faraz qilaylik, aylana markazi koordinata boshida, radiusi esa R ga teng bo'lsin (5-chizma).



5-chzma

Aylanadagi $M(x, y)$ nuqtaning holatini OM radius bilan Ox musbat yarim o‘q orasidagi α burchak orqali ifodalaymiz. OAM uchburchakdan quyidagilarga ega bo‘lamiz: $|OA| = |OM|\cos\alpha\,, |AM| = |OM|\sin\alpha$. Ravshanki, $|OA| = x, |AM| = y, |OM| = R$. Shularni e’tiborga olib, aylananing parametrik shakldagi tenglamasini

$$x = R\cos\alpha\,, y = R\sin\alpha\,. \qquad (4)$$

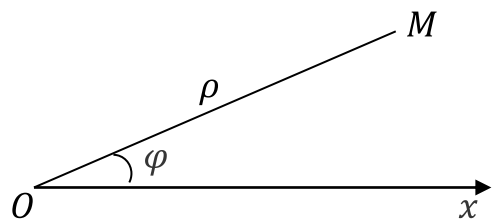
ko‘rinishda yozamiz.

Agar chiziqning (3) parametrik shakldagi tenglamasi berilsa, undan uning oshkormas shakldagi $f(x, y) = 0$ tenglamasini hosil qilish mumkin. Buning uchun (3) tenglamalardan t parametrni yo‘qotish, ya’ni tenglamalarda biridan t ni topib, keyin uni ikkinchisiga qo‘yish yoki boshqa biror usulni qo‘llash yetarli. Masalan, aylananing (4) shakldagi parametrik tenglamasidan oshkormas tenglamasini hosil qilish uchun, undagi α ni yo‘qotamiz. Buning uchun (4) tenglamalarni kvadratga ko‘tarib va ularni qo‘shamiz: $x^2 + y^2 = R^2(\cos^2\alpha + \sin^2\alpha) = R^2$.

2.3. Tekislikdagi qutb koordinatalar sistemasi

To‘g‘ri burchakli koordinatalar sistemasidan keyin muhim ahamiyatli sistema qutb koordinatalar sistemasidir. Tekislikdagi nuqtaning qutb koordinatalari quyidagicha kiritiladi.

Tekislikda *qutb* deb ataluvchi O nuqtani tanlaymiz. So‘ngra, O qutbdan *qutb o‘qi* deb ataluvchi Ox yo‘nalgan yarim to‘g‘ri chiziqni o‘tkazamiz (6-chizma).



6-chizma

M – tekislikdagi ixtiyoriy nuqta bo‘lsin. M nuqtani O qutrb bilan OM kesma orqali tutashtiramiz. Kesma uzunligi $|OM| = \rho$ ga M nuqtaning *qutb radiusi* deb, qutb o‘qidan OM kesmagacha soat strelkasiga teskari yonalishidagi hisoblangan $\varphi = \angle(xOM)$ burchak esa – *qutb burchagi* deb ataladi. ρ qutb radiusi va φ qutb burchagi M nuqtaning qutb koordinatalarini tashkil qiladi. $M(\rho, \varphi)$ yozuv M nuqtaning qutb radiusi ρ va qutb burchagi φ ekanligini bildiradi.

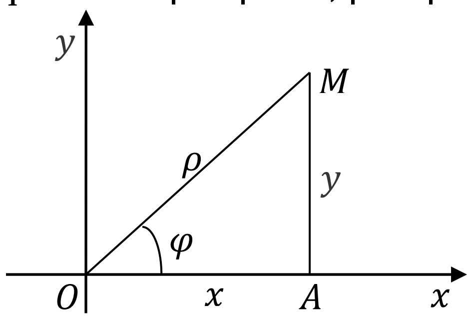
Nuqtaning ρ va φ qutb koordinatalari quyidagi hartlarni qanoatlantiradi:

$$0 \leq \rho < +\infty, 0 \leq \varphi < 2\pi.$$

Shuni takidlab o‘tamizki, agar M nuqta ρ, φ koordinatalariga ega bo‘lsa, u holda $(\rho, \varphi + 2k\pi)$ juftlikdan iborat qutb koordinatalar to‘plami ham shu nuqtaning koordinatalari bo‘ladi, bu yerda $k \in Z$ (Z – butun sonlar to‘plami).

Endi qutb koordinatalar sistemasidan to‘g‘riburchakli koordinatalar sistemasiga o‘tishni va aksincha holni qaraymiz.

Faraz qilaylik, qutb koordinatalar sistemasining qutb Oxy to‘g‘ri burchakli koordinatalar sistemasining boshlang‘ich nuqtasi bilan, qutb o‘qi esa Ox musbat yarim o‘q bilan mos tushsin. U holda 7-chizmadan ravshanki, M nuqta uchun $|OA| = x, |AM| = y, |OM| = \rho, \angle(xOM) = \varphi$.



7-chizma

φ ni o‘tkir burchak deb hisoblab, AOM uchburchakdan

$$|OA| = |OM| \cos \varphi,\ \ |AM| = |OM| \sin \varphi,$$

tengliklarni hosil qilamiz, yoki

$$x = \rho \cos \varphi,\ \ y = \rho \sin \varphi. \tag{5}$$

Hosil qilingan formula ixtiyoriy $\varphi, (0 \leq \varphi < 2\pi)$ burchak uchun ham o‘rinli.

(5) formulalar M nuqtaning to‘g‘ri burchakli koordinatalarini uning qutb koordinatalari orqali ifodalaydi.

AOM to‘g‘ri burchakli uchburchakdan quyidagi tengliklarni olamiz:

$$|OM| = \sqrt{|AO|^2 + |AM|^2}, \qquad tg\varphi = \frac{|AM|}{|AO|},$$

yoki

$$\rho = \sqrt{x^2 + y^2}, tg\varphi = \frac{y}{x}. \tag{6}$$

(6) formulalar M nuqtaning qutb koordinatalarini uning to‘g‘ri burchakli koordinatalari orqali ifodalaydi. Ta’kidlab o‘tamizki, $tg\varphi$ bo‘yicha φ burchakni aniqlaganimizda x va y koordinatalarning ishoralarini hisobga olish kerak bo‘ladi.

9-misol. *Qutb koordinatalar boshi bilan ustma-ust tushsin, qutb o‘qi esa abstsisa o‘qining musbat yonalishi bilan mos tushsin. $M\left(1, -\sqrt{3}\right)$ nuqtaning qutb koordinatalarini toping.*

Yechish. (6) tengliklarga asosan

$$\rho = \sqrt{1^2 + \left(-\sqrt{3}\right)^2} = 2, tg\varphi = -\sqrt{3}.$$

Ko‘rinib turibdiki, M niqta IV kvadrantda joylashgan, shuning uchun, $\varphi = \frac{5\pi}{3}$. Demak, $M\left(2, \frac{5\pi}{3}\right)$.

10-misol. *Agar qutb koordinatalar boshi bilan ustma-ust tushgan, qutb o‘qi esa abstsisa o‘qining musbat yonalishi bilan mos tushgan bo‘lsa, u holda $A\left(2\sqrt{2}, \frac{3\pi}{4}\right)$ nuqtaning to‘g‘r iburchakli koordinatalarini toping.*

Yechish. (5) formulalardan foydalanib, hisoblaymis.Unda

$$x = 2\sqrt{2}\cos\frac{3\pi}{4} = -2, \qquad y = 2\sqrt{2}\sin\frac{3\pi}{4} = 2.$$

Shunday qilib, qutb koordinatalar sistemasida berilgan $A\left(2\sqrt{2}, \frac{3\pi}{4}\right)$ nuqta to‘g‘ri burchakli koordinatalar sistemasida quyidagicha yoziladi: $A(-2,2)$.

Yuqorida chiziq o‘z nuqtasining to‘g‘ri burchakli joriy koordinatalarini bog‘lovchi tenglamalar orqali berilishini ko‘rdik. Chiziq nuqtaning qutb koordinatalari orqali aniqlanadigan tenglama bilan ham berilishi mumkin.

11-misol. *Qutb koordinatalari $\rho = a\varphi$ tenglamani qanoatlantiruvchi nuqtalarni qaraymiz, bu yerda a – qandaydir musbat son. Bu tenglama* ***Arximed spirali*** *deb ataluvchi chizqni ifodalaydi.*

2.4. Algebraik chiziqlar

Joriy nuqtalari to‘g‘ri burchakli koordinatalarga nisbatan n-darajali tenglama ($n = 1,2,3, ...$) bilan aniqlangan chiziqlar n-tartibli yoki *algebraik* chiziqlar deyiladi.

Misol uchun tenglamalari quyidagicha berilgan

$$x + y - 1 = 0, x^2 + y^2 = 1, x^3 + y^3 - 3xy = 0,$$

chiziqlar mos ravishda birinchi, ikkinchi va uchunch tartibli chiziqlardir.

Birinchi tartibli ciziqlar quyidagi umumiy ko‘rinishi ega:

$$Ax + By + C = 0,$$

bu yerda A va B koeffitsiyentlar bir vaqtda nolga teng emas, ya’ni $A^2 + B^2 \neq 0$. Barcha birinch tartibli chiziqlar – to‘g‘ri chiziqlardir.

Ikkinchi tartibli ciziqlarning umumiy ko‘rinishi quyidagicha:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0,$$

bu yerda A,B va C koeffitsiyentlar bir vaqtda nolga teng emas, ya’ni $A^2 + B^2 + C^2 \neq 0$.

n – tartibli chiziq tenglamasini quyidagicha yozish mumkin:

$$\sum_{\substack{p,q=0 \\ p+q\leq n}}^{n} a_{pq}x^p y^q = 0,$$

bu yerda yuqori tartibli o‘zgaruvchlarning a_{pq} koeffitsiyentlaridan hech bo‘lmaganda bittasi, ya’ni, $p + q = n$ bo‘lganda, noldan farqli. Xususiy holda algebraik chiziqlar bitta yoki bir nechta nuqtalardan iborat bo‘lishi mumkin.

Masalan, $(x - 1)^2 + (y - 2)^2 = 0$ tenglamaga yagona $(1,2)$ nuqta mos keladi, chunki bu tenglamani yagona $x = 1$ va $y = 2$ sonlar juftligi qanoatlantiradi.

$x^2 + 2xy + y^2 + 1 = 0$ tenglamaga Oxy tekisligida mos keluvchi birorta ham nuqta mavjud emas, chunki $x^2 + 2xy + y^2 + 1 = (x + y)^2$ +1>0.

Keyingi mavzularda birinchi tartibli chiziq (to‘g‘ri chiziq) va ikkinch tartibli chiziqlarning ba’zilarini (aylana, ellips, giperbola, parobola) o‘rganamiz.

2.5. Chiziq tenglamalaridan foydalishga oid sodda masalalar

Agar chiziq tenglamasi ma’lum bo‘lsa, u holda tekislikdagi chiziq joylashuvi bilan bog‘liq sodda masalalarni osongina yechish mumkin. Shunga doir misollar qaraymiz.

1-masala. *K chiziq tenglamasi va $M(a, b)$ nuqta koordinatalari berilgan. M nuqta K chiziqga tegishli yoki yo‘qligini aniqlang.*

Boshqacha aytganda, K chiziq berilgan M nuqtadan o‘tadimi yoki yo‘qmi, shuni aniqlash talab etiladi.

Chziq tenglamasi tushunchasiga asosan ***1-masalani yechish uchun quyidagi qoidani olamiz:*** *M nuqtaning K chiziqda yotishi yoki yotmasligini aniqlash uchun, chiziq tenglamasiga shu nuqtaning koordinatalarini olib borib qo‘yish kerak; agar koordinatalar tenglamani qanoatlantirsa, u holda nuqta chiziqda yotadi; aks holda, ya’ni agar koordinatalar tenglamani qanoatlantirmasa, u holda nuqta chiziqda yotmayadi.*

12-misol. *Aylana tenglamasi berilgan: $x^2 + y^2 = 25$. Shu aylanada $M(3, -4)$ va $N(-2,4)$ nuqtalar yotadimi?*

Yechish. M nuqtaning koordinatalarini aylana tenglamasiga olib borib qo‘ysak, $3^2 + (-4)^2 = 25$ ayniyatni hosil qilamiz. Demak, M nuqta berilgan aylanaga tegishli. Shunga o‘xshash, N nuqtaning koordinatalarini aylana tenglamasiga olib borib qo‘ysamiz: $(-2)^2 + 4^2 \neq 25$. Bundan, N nuqta aylanada yotmasligi kelib chiqadi.

2-masala. *Tenglamalari bilan berilgan ikki chiziqning kesishish nuqtasini toping.*

Kesishish nuqtasi bir vaqtning o‘zida ham birinch chiziqda, ham ikkinchi chiziqda yotadi. Shuning uchun, bu nuqtaning koordinatalari ikkala chiziqning tenglamasini qanoatlantiradi. Bu yerdan ***2-masalani yechish uchun quyidagi qoidaga ega bo‘lamnz:*** *ikki chiziqning kesishish nuqtasi koordinatalarini topish uchun, ularning tenglamalarini birgalikda yechish etarli. Agar bu sistema haqiqiy yechimlarga ega bo‘lmasa, u holda chiziqlar kesishmaydi.*

13-misol. *$y = x^2$ parabola va $y = 9$ chiziqning kesishish nuqtasini toping.*

Yechish. 2-masala yechimini topish qoidasaga ko‘ra quyidagi sistemani qaraymiz:

$$\begin{cases} y = x^2, \\ y = 9. \end{cases}$$

Bu sistema ikkita yechimga ega: $x_1 = -3$, $y_1 = 9$; $x_2 = +3$, $y_2 = 9$. Demak, berilgan parabola va to‘g‘ri chiziq ikkita $A(-3{,}9)$ va $B(3{,}9)$ nuqtalarda kesishadi.

3-masala. *Berilgan chiziqning koordinata o‘qlari bilan kesishish nuqtalarini toping.*

Bu masala 2-masalaning xususiy holidir.

Ox o‘qining tenglamasi $y = 0$, Oy o‘qining tenglamasi esa $x = 0$ ekanligini hisobga olsak, ***3-masala yechimi uchun quyidagi qoidagaga ega bo‘lamiz:*** *berilgan chiziqning Ox abstsisa o‘qi bilan kesishish nuqtasini topish uchun uning tenglamasiga $y = 0$ deb olamiz va tenglamani x ga nisbatan yechamiz; shunga o‘xshash, chiziqning Oy ordinata o‘qi bilan kesishish nuqtasini topish uchun uning tenglamasiga $x = 0$ deb olamizz va tenglamani y ga nisbatan yechamiz.*

14-misol. $x^2 + y^2 = 4$ *aylananing koordinata o‘qlari bilan kesishish niqtalarini topng.*

Yechish. Aylana tenglamasida $y = 0$ deb, $x^2 = 4$ tenglamaga kelamiz. Uning yechimlari $x_1 = -2, x_2 = 2$ ga teng. Bu yerdan, Ox o‘qi bilan aylananing ikkita kesishish nuqtasini topamiz: $A(-2{,}0)$ va $B(2{,}0)$. Shunga o‘xshash, aylana tenglamasida $x = 0$ deb, $y^2 = 4$ tenglamaga kelamiz, bundan $y_1 = -2, y_2 = 2$.

Shunday qilib, aylana Oy o‘qi bilan ham ikki nuqtada kesishadi: $C(0,-2)$ va $D(0,2)$.

2.6. Nazorat savollari va masalalar

1. Tekislikdagi chiziq ta’rifini nuqtalar to‘plami sifatida bering. Misollar keltiring.
2. Chizq tenglamasi tushunchasi. Chiziq tenglamasining oshkormas shakli.
3. Tekislikdagi analitik geometriyaning ikki asosiy masalasi.
4. Chiziq tenglamasini tuzishga doir misollar keltiring.
5. Chiziqning parametrik tenglamalari.
6. Qutb koordinatalari. Tekislikdagi nuqtanig to‘g‘ri burchakli va qutb koordinatalari orasidagi bog‘lanish.
7. Qutb koordinatalar sistemasidagi chiziq tenglamasi. Misollar keltiring.
8. Algebraik chiziqlar. Birinchi va ikkinch tartibli chiziqlar tenglamalarining umumiy ko‘rinishi.

9. Nuqtalari Oy o‘qiga nisbatan Ox o‘qidan ikki baravar uzoqlikda joylashgan chiziq tenglamasini tuzing.

10. Berilgan ikki niqtadan bir xil masofada yotuvch chiziq tenglamasini tuzing: $A(3,2)$ va $B(-3,-2)$.

11. Qo‘zg‘almas uchlari $A(5,0)$ va $B(-5,0)$ nuqtalarda, uchinchi $C(x_3, y_3)$ uchi esa $(x_3)^2 + (y_3)^2 = 81$ aylani chizuvchi ABC uchburchak og‘irlik markazi qanday chiziqni hosil qiladi?

12. Berilgan tenglamalarga qanday geometrik obrazlar(chiziqlar) mos keladi: a) $xy = 0$; b) $x^2 + y^2 = 0$; c) $x^2 - 1 = 0$; d) $y^2 - 3y + 2 = 0$; e) $y^2 - xy = 0$.

13. Berilgan $A(0,0), B(1,1), C(-1,1), D(-2,2), E(2,1)$ nuqtalardan qaysilari $y = x^2$ chiziqda yotadi, qaysilari esa yotmaydi?

14. $x^2 + y^2 = 0$ aylana va $x + y = 0$ to‘g‘ri chiziqlarning kesishish nuqtalarini toping.

15. $y = x^2 - x - 2$ chiziqning koordinata o‘qlari bilan kesishish nuqtalarini toping.

16. Quyidagi parametrik tenglamalar bilan qanday chiziqlar ifodalangan:

а) $x = t^2, y = t^2$; б) $x = \cos t,\ y = cos^2 t$;

в) $x = \sin t,\ y = cosect$; г) $x = 2\,t,\ y = 4t$.

17. Chziq parametrik tenglamalar bilan berilgan: $x = a\cos t,\ y = b\sin t$. Uning to‘g‘ri burchakli koordinatalar sistemasidagi tenglamasini toping.

18. O‘zining qutb koordinatalari bilan berilgan $A(4,0)$, $B\left(5, \frac{\pi}{4}\right)$, $C\left(3, \frac{\pi}{2}\right), D\left(6, \frac{5\pi}{4}\right)$ nuqtalarning to‘g‘ri burchakli koordinatalini toping.

19. O‘zining to‘g‘ri burchakli koordinatalari bilan berilgan $A\left(\frac{9}{2}, \frac{9}{2}\right), B(3, \sqrt{3}), C(\sqrt{3}, 3), D(-2,2)$ nuqtalarning qutb koordinatalarini toping.

20. Quyidagi chiziq tenglamalarini qutb koordinatalar sistemasida yozing:

a) $x = 1$; b) $y = -3$; c) $y = x$; d) $x + y = 4$; e) $x^2 + y^2 = 49$.

21. Chiziq tenglamasi qutb koordinatalarda $\rho = a\cos\varphi$ ko‘rinishga ega. Bu tenglamani to‘g‘ri burchakli koordinatalar sistemasida yozing.

22.Chiziq parametrik tenglamasi bilan berilgan: $x = \frac{a}{2}\left(t + \frac{1}{t}\right)$, $y = \frac{b}{2}\left(t - \frac{1}{t}\right)$. Bu tenglamani to‘g‘ri burchakli koordinatalar sistemasida yozing.

23. Oxy tekisligidagi harakatlanaytgan nuqta, boshlang‘ich $t = 0$ vaqt lahzasidan hisoblanganda, t vaqt lahzasida $M(20 - t, 10 - t)$ holatni olgan bo‘lsin. Nuqta qaysi vaqtda $8x - 2y + 10 = 0$ chiziqga kelib tushadi va bu vaqtdagi uning koordinatalari qanday bo‘ladi?

§3. TO‘G‘RI CHIZIQ

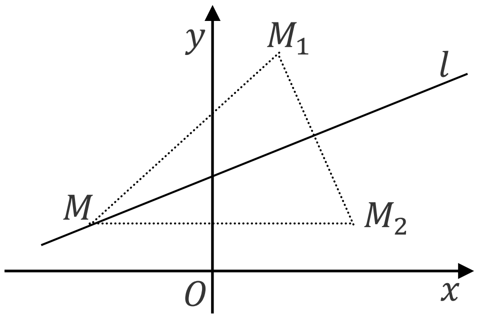
3.1. To‘g‘ri chiziqning umumiy tenglamasi

Chiziqlar ichida eng soddasi va ko‘proq qo‘llaniladigani to‘g‘ri chiziqdir.**1-теорема**. *Tekislikdagi har qanday to‘g‘ri chiziq tenglamasi*

$$Ax + By + C = 0 \tag{1}$$

ko‘rinishga ega, bu yerda A, B, C – o‘zgarmaslar. Va aksincha, agar A va B bir vaqtda nolga teng bo‘lmasa ($A^2 + B^2 \neq 0$), *u holda shunday to‘g‘ri chiziq mavjudki,* (1) – *uning tenglamasidir*.

Isboti. Faraz qilaylik $M_1(a_1, b_1), M_2(a_2, b_2)$ lar l to‘g‘ri chiziqqa nisbatan simmetrik joylashgan qandaydir ikki nuqta bo‘lsin(1-chizma). U holda l to‘g‘ri chiziqdagi ixtiyoriy $M(x, y)$ nuqta M_1 va M_2 nuqtalardan baravar uzoqlikda bo‘ladi. Aksincha, M_1 va M_2 nuqtalardan baravar uzoqlikdagi nuqta l to‘g‘ri chiziqga tegishli bo‘ladi.



1-chizma

Shunig uchun $(x - a_1)^2 + (x - b_1)^2 = (x - a_2)^2 + (x - b_2)^2$. Bu tenglamada barcha hadlarni chap tovonga o‘tkazib, kvadratlarni ochib chiqib, soddalashtirsak, quyidagini hosil qilamiz:

$$2(a_2 - a_1)x + 2(b_2 - b_1)y + (a_1^2 + b_1^2 - a_2^2 - b_2^2) = 0.$$

Bu esa, $A = 2(a_2 - a_1), B = \ 2(b_2 - b_1), C = a_1^2 + b_1^2 - a_2^2 - b_2^2$

bo‘lgan (1) ko‘rinishdagi tenglamadan iborat. Teoremaning birinchi qismi isbotlandi.

Teoremaning ikkinchi qismini sbotlaymiz. Oxy tekislikning koordinatalari (1) tenglamani qanoatlantiruvchi turli D_1 va D_2 nuqtalarini qaraymiz.

Faraz qilaylik, $A_1x + B_1y + C_1 = 0$ tenglama D_1 va D_2 nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasi bo‘lsin. U vaqtda

$$\begin{cases} Ax + By + C = 0, \\ A_1x + B_1y + C_1 = 0 \end{cases} \quad (2)$$

tenglamalar sistemasi birgalikda bo‘ladi, chunki D_1 va D_2 niqtalarning koordinatalari bu sistemani qanoatlantiradi. Algebra kursidan ma’lumki, birgalikdagi ikkita chiziqli tenglamalar sistemasi yoki yagona yechimga ega, yoki (agar yechim yagona bo‘lmasa) biri ikkinchisi natijasidir, ya’ni uni biror songa ko‘paytirishdan hosil bo‘ladi.

Yuqorida aytilganlarga ko‘ra (2) tenglamalar sistemasi kamida ikkita yechimga ega. Bundan kelib chiqadiki, $Ax + By + C = 0$ tenglama $A_1x + B_1y + C_1 = 0$ tenglamadan kelib chiqadi, demak, D_1 va D_2 nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasi $Ax + By + C = 0$ tenglamadan iborat deyish mumkin. Teoremaning ikkinchi qismi ham isbotlandi.

1-misol. Ushbu $A^2x^2 + 2ABxy + B^2y^2 - C^2 = 0$ *tenglama bilan bir juft to‘g‘ri chiziqlar berilganligini ko‘rsating.*

Yechish. Berilgan tenglamani quyidagicha yozib olamiz:

$$A^2x^2 + 2ABxy + B^2y^2 - C^2 = (Ax + By)^2 - C^2 = (Ax + By + C)(Ax + By - C) = 0.$$

Bundan ko‘rinadiki, berilgan tenglama ikkita tenglamaga ajraladi:

$$Ax + By + C = 0, \quad Ax + By - C = 0.$$

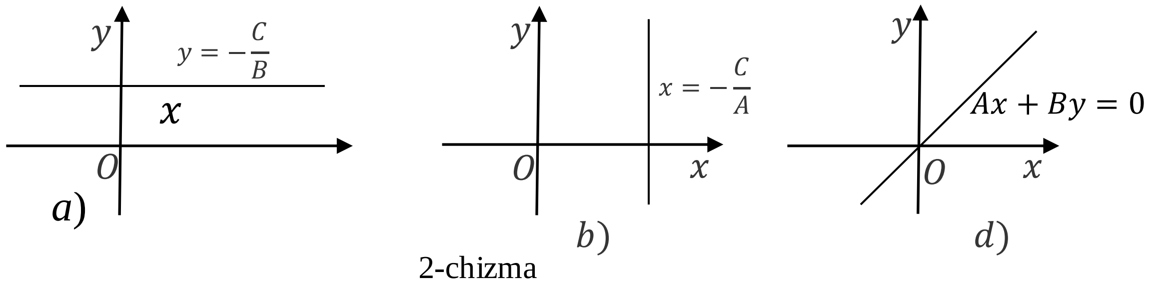
Bularning har biri to‘g‘ri chiziq tenglamasidir.

3.2. To‘g‘ri chiziqning koordinatalar sistemasiga nisbatan joylashishi

To‘g‘ri chiziqning koordinatalar sistemasiga nisbatan joylashuvi uning $Ax + By + C = 0$ tenglamasining u yoki bu xususiy ko‘rinishlarida qanday o‘ziga xoslikga ega ekanligini aniqlashtiramiz. Quyidagi hollarni qaraymiz.

a) $A = 0$ va $B, C \neq 0$. Bu holda to‘g‘ri chiziq tenglamasini quyidagicha yozish mumkin: $y = -\frac{C}{B}$. Shunday qilib, bu holda to‘g‘ri chiziqning barcha niqtalari bir xil $(-\frac{C}{B})$ ordinataga ega va demak, to‘g‘ri chiziq Ox o‘qiga parallel (2a-chizma). Agar $C = 0$ bo‘lsa u Ox bilan ustma-ust tushadi.

b) $B = 0$ va $A, C \neq 0$. Bu hol yuqoridagiga o‘xshash. Hosil bo‘lgan $x = -\frac{C}{A}$ to‘g‘ri chiziq Oy o‘qiga parallel (2b- chizma). Agar $C = 0$ bo‘lsa u Oy o‘qi bilan ustma-ust tushadi.



2-chizma

$d)$ $C = 0$ va $A, B \neq 0$. Bu holda $(0,0)$ koordinata boshi (1) to‘g‘ri chiziq tenglamasini qanoatlantirganligi uchun to‘g‘ri chiziq koordinata boshidan o‘tadi (2c-chizma).

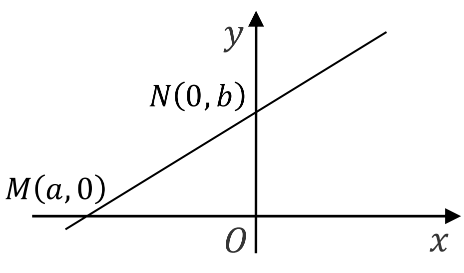
e) Endi (1) tenglamaning barcha koeffitsiyentlari noldan farqli bo‘lsin. Bu holda to‘g‘ri chiziq koordinata boshidan ham o‘tmaydi, u Ox o‘qiga ham, Oy o‘qiga ham parallel emas. Tenglamani $(-\frac{1}{C})$ ga ko‘paytiramiz va

$$a = -\frac{C}{A}, b = -\frac{C}{B}$$

belgilashlarni kiritib, quyidagi tenglamani hosil qilamiz:

$$\frac{x}{a} + \frac{y}{b} = 1. \qquad (3)$$

Bu shaklda yozilgan to‘g‘ri chiziq tenglamasidagi koeffitsiyentlar sodda geometrik ma’noga ega: a va b sonlar – ishoralar aniqligida, to‘g‘ri chiziqning koordinata o‘qlaridan kesib olgan kesmalari uzunliklariga teng (3-chizma).



3-chizma

Haqiqatdan ham, to‘g‘ri chiziq $Ox(y=0)$ o‘qini $M(a,0)$ nuqtada, $Oy(x=0)$ o‘qini esa $N(0,b)$ nuqtada kesib o‘tadi. Suning uchun (3) tenglamaga to‘g‘ri chiziqning “*kesmalardagi*” tenglamasi deyiladi.

2-misol. *Ox o‘qini $OM=-5$, Oy o‘qini esa $ON=4$ kesmalarda kesib otuvchi to‘g‘ri chiziq tenglamasini tuzing.*

Ye*chish*. (3) tenglamada $a=-5, b=4$ deb olsak, unda quyidagiga ega bo‘lamiz:

$$\frac{x}{-5}+\frac{y}{4}=1 \text{ yoki } \frac{x}{5}-\frac{y}{4}=-1.$$

Eslatma. Koordinata boshidan o‘tuvch yoki koordinata o‘qlariga parallel bo‘lgan to‘g‘ri chiziqlarning tenglamalarini “*kesmalardagi*” tenglamasi shaklda yozib bo‘lmaydi.

3-misol. *Qanday shartlarda $Ax+By+C=0$ to‘g‘ri chiziq Ox musbat yarim o‘qni, qanday shartlarda Ox manfiyt yarim o‘qni kesib o‘tadi?*

Yechish. Berilgan tenglamani (3) ko‘rinishga keltiramiz, bunda $a=-\frac{C}{A}, b=-\frac{C}{B}$. Demak, agar $\frac{C}{A}>0$ bo‘lsa, u holda $a<0$, ya’ni to‘g‘ri chiziq Ox manfiy yarim o‘qni kesadi; agar $\frac{C}{A}<0$ bo‘lsa, u holda $a>0$, ya’ni to‘g‘ri chiziq Ox musbat yarim o‘qni kesib o‘tadi.

3.3. To‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi. To‘g‘ri chiziqlar orasidagi burchak

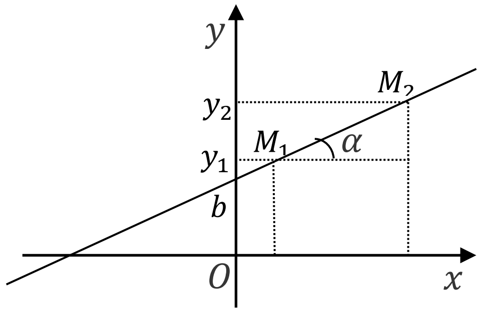
Faraz qilaylik, $Ax+By+C=0$ to‘g‘ri chiziq Oy o‘qiga parallel emas. U holda, to‘g‘ri chiziq tenglamasini $\frac{1}{B}$ ga ko‘paytirb va $k=-\frac{A}{B}, b=-\frac{C}{B}$ deb belgilash kiritib, quyidagini hosil qilamiz:

$$y=kx+b. \tag{4}$$

(4) to‘g‘ri chiziq tenglamasining koeffitsiyentlari sodda geometrik ma’noga ega: k – to‘g‘ri chiziqning Ox o‘qi bilan hosil qilgan α burchagining tangensi; b – to‘g‘ri chiziqning Oy o‘qidan kesib olgan kesmasi uzunligi (ishora aniqligida).

Haqiqatdan ham, $M_1(x_1,y_1), M_2(x_2,y_2)$ – to‘g‘ri chiziqdagi ikki nuqta bo‘lsin (4-chizma). U vaqtda:

$$\operatorname{tg}\alpha=\frac{y_2-y_1}{x_2-x_1}=\frac{(kx_2+b)-(kx_1+b)}{x_2-x_1}=k.$$



4-chzma

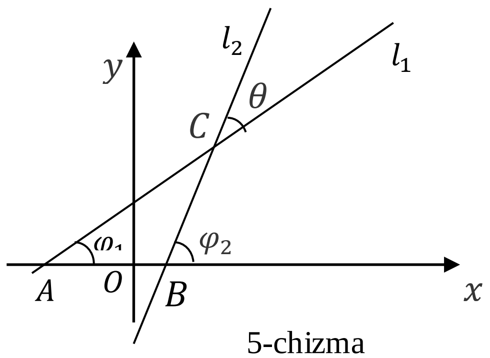
Ko‘rinib turibdiki (4-chizma), to‘g‘ri chiziq Oy $(x = 0)$ o‘qini $(0, b)$ nuqtada kesib o‘tadi.

(4) tenglamaga to‘g‘ri chiziqning *burchak koeffitsiyentli tenglamasi* deyiladi

Oy o‘qiga parallel bo‘lmagan ikkita to‘g‘ri chiziq burchak koeffitsiyentli tenglamalari bilan berilgan bo‘lsin(5-chizma):

$$\begin{cases} y = k_1x + b_1\,, \operatorname{tg}\varphi_1 = k_1, & (5) \\ y = k_2x + b_2\,, \operatorname{tg}\varphi_2 = k_2. & (6) \end{cases}$$

Ular orasidagi θ burchakni topish talab qilinadi. Aniqroq qilib aytganda, ikkinchi to‘g‘ri chiziq birinch to‘g‘ri chiziqqa nisbatan soat strelkasiga teskari yo‘nalishda burilgan eng kichik burchakni θ deb velgilaymiz. Bu burchak, ABC uchburchakdagi $\theta = \left(\widehat{ACB}\right)$ burchakka teng (5-chizma).



5-chizma

Elementar geometriya kursidan ma’lumki, ucburchakning tashqi burchagi unga qo‘shni bo‘lmagan ichki burchaklar yig‘indisiga teng. Shuning uchun, $\varphi_2 = \varphi_1 + \theta$ yoki $\theta = \varphi_2 - \varphi_1$. Bundan

$$\operatorname{tg}\theta = \operatorname{tg}(\varphi_2 - \varphi_1) = \frac{\operatorname{tg}\varphi_2 - \operatorname{tg}\varphi_1}{1 + \operatorname{tg}\varphi_2 \cdot \operatorname{tg}\varphi_1}.$$

kelib chiqadi. Bu yerda, agar $\operatorname{tg}\varphi_1$ va $\operatorname{tg}\varphi_2$ miqdorlarni mos ravishda k_1 va k_2 deb almashtirsak, u holda quyidagini hosil qilamiz:

$$\operatorname{tg}\theta = \frac{k_2 - k_1}{1 + k_1k_2}. \qquad (7)$$

(7) formula (5) va (6) to‘g‘ri chiziqlar orasidagi burchak tangensining ularning burchak koeffitsiyentlari orqali ifodalanishidir.

Endi, ikki to‘g‘ri chiziq tenglamalari umumiy shaklda berilgan bo‘lsin:

$$A_1x + B_1y + C_1 = 0, \tag{8}$$

$$A_2x + B_2y + C_2 = 0. \tag{9}$$

Agar, $B_1 \neq 0, B_2 \neq 0$ deb olsak, unda quyidagiga ega bo‘lamiz

$$y = -\frac{A_1}{B_1}x - \frac{C_1}{B_1},$$

$$y = -\frac{A_2}{B_2}x - \frac{C_2}{B_2}.$$

Demak, (8) va (9) to‘g‘ri chiziqlarning burchak koeffitsiyentlari mos ravishda $k_1 = -\frac{A_1}{B_1}$ va $k_2 = -\frac{A_2}{B_2}$ ga teng. Shularni hisobga olib (7) formuladan foydalanamiz va uncha qiyin bo‘lmagan almashtirishlardan so‘ng (8) va (9) to‘g‘ri chiziqlar orasidagi burchak tangensini topamiz:

$$\operatorname{tg}\theta = \frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2} \tag{10}$$

4-misol. $y = 2x + 1$ *va* $y = -3x + 7$ *to‘g‘ri chiziqlar orasidagi o‘tkir burchakni toping.*

Yechish. Masala shartiga ko‘ra $k_1 = 2, k_2 = -3$. Shunig uchun (7) formulaga asosan

$$\operatorname{tg}\theta = \frac{k_2 - k_1}{1 + k_1k_2} = \frac{-3-2}{1+2(-3)} = 1\ .$$

Demak $\theta = \frac{\pi}{4}$.

3.4. To‘g‘ri chiziqlarning parallellik va perpendikulyarlik shartlari

Endi ikki to‘g‘ri chiziqning parallellik va perpendikulyarlik shartlarini keltirib chiqaramiz.

Avvalo, to‘g‘ri chiziqlar burchak koeffitsiyentli tenglamalari bilan berilgan holni qaraymiz.

Agar (5) va (6) tenglamalar bilan berilgan to‘g‘ri chiziqlar parallel bo‘lsalar, u holda $\varphi_1 = \varphi_2$ va shu sababli, $tg\varphi_1 = tg\varphi_2$, ya’ni

$$k_1 = k_2. \tag{11}$$

Aksincha, agar (11) shart bajarilsa, u holda, (7) dan $tg\theta = 0$ va demak, φ_1 va φ_2 burchaklar 0 dan π gacha o‘zgarishimi inobatga olsak, dan $\theta = 0$, ya’ni $\varphi_1 = \varphi_2$ kelib chiqadi. Bu holda qaralayotgan to‘g‘ri chiziqlar yoki parallel, yoki ustma-ust tushadi (*keng ma‘nodagi parallellik*). Demak, quyidagi o‘rinli.

2-teorema. *Tekislikdagi to‘g‘ri chiziqlar faqat va faqat ularning burchak koeffitsiyentlari o‘zaro teng ($k_1 = k_2$) bo‘lsagina parallel* (*keng ma’nodagi parallel*) *bo‘ladilar.*

Agar (5) va (6) to‘g‘ri chiziqlar perpendikulyr bo‘lsalar, u holda ular orasidagi burchak $\theta = \frac{\pi}{2}$ va shuning uchun

$$ctg\theta = \frac{1}{tg\theta} = \frac{1+k_1k_2}{k_2-k_1} = 0.$$

Bu yerdan $1 + k_1k_2 = 0$, yoki

$$k_2 = -\frac{1}{k_1}. \tag{12}$$

Teskari mulohaza ham to‘g‘ri, ya’ni agar (12) shart bajarilsa, u holda (5) va (6) to‘g‘ri chiziqlar perpendikulyar bo‘ladi. Shunday qilib, quyidagi o‘rinli.

3-teorema. *Tekislikdagi ikki to‘g‘ri chiziq perpendikulyar bo‘lishi uchun ularning burchak koeffitsiyentlari o‘zaro teskari va qarana-qarshi ishorali bo‘lishi ($k_2 = -\frac{1}{k_1}$) zarur va yetarlidir.*

Endi ikki to‘g‘ri chiziq (8), (9) – umumiy ko‘rinishdagi tenglama-lari bilan berilgan holni qaraymiz. Bu holda ular orasidagi burchak tan-gensi uchun (10) formuladan foydalanib, yuqoridagiga o‘xshash mulo-hazalar asosida ularning quyidagi parallellik va perpendikulyarlik shart-lariga ega bo‘lamiz.

a) *To‘g‘ri chiziqlarning parallellik sharti* ($\theta = 0$):

$$\frac{A_1}{A_2} = \frac{B_1}{B_2}. \tag{13}$$

b) *To‘g‘ri chiziqlarning perpendikulyarlik sharti* ($\theta = \frac{\pi}{2}$):

$$A_1A_2 + B_1B_2 = 0. \tag{14}$$

Bu alomatlardan xususiy holda $Ax + By + C = 0$ va $Bx - Ay + C_1 = 0$ to‘g‘ri chiziqlarning o‘zaro perpendikulyar ekanligi kelib chi-qadi.

5-misol. Quyidagi *to‘g‘ri chiziqlar juftlarini qaraymiz:*

$$4x - 6y + 7 = 0 \text{ va } 20x - 30y - 11 = 0;$$

$$3x - 5y + 7 = 0 \text{ va } 10x + 6y - 3 = 0.$$

Bu to‘g‘ri chiziqlar uchun parallellik va perpendikulyarlik shart-larini tekshiring.

Yechish. Birinchi juftlik uchun (13) shart bajariladi: $\frac{4}{20} = \frac{-6}{-30}$. Demak, bu to‘g‘ri chiziqlar parallel. Ikkinchi juftlik uchun (14) perpen-dikulyarlik sharti bajariladi: $3 \cdot 10 + (-5) \cdot 6 = 0$.

3.5. Berilgan nuqtadan berilgan yo‘nalish bo‘yicha o‘tuvchi to‘g‘ri chiziq tenglamasi

Faraz qilaylik, l to‘g‘ri chiziq Ox o‘qi bilan musbat φ burchak tashkil qilib $M_1(x_1, y_1)$ nuqtadan o‘tsin (6-chizma). Avvalo, to‘g‘ri chiziq Ox yoki Oy o‘qlariga parallel emas deb hisoblab, uning tenglamasini keltirib chiqaramiz.

Bizga ma’luvki, burchak koeffitsiyenti $k = tg\varphi$ bo‘lgan to‘g‘ri chiziq tenglamasi quyidagi

$$y = kx + b, \tag{15}$$

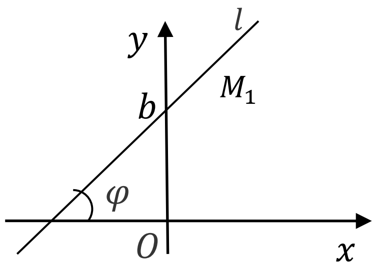
ko‘rinishga ega, bu yerda b – to‘g‘ri chiziqning Oy o‘qidan ajratgan kesmasi. $M_1(x_1, y_1)$ nuqta l to‘g‘ri chiziqda yotganligi uchun uning koordinatalari (15) tenglamani qanoatlantiradi, ya’ni

$$y_1 = kx_1 + b \quad . \tag{16}$$

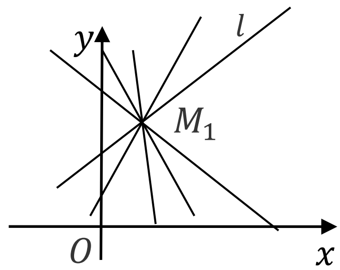
Endi (15) tenglikdan (16) tenglikni ayirib, quyidagi tenglamani hosil qilamiz

$$y - y_1 = k(x - x_1). \tag{17}$$

Bu esa biz izlayotgan to‘g‘ri chiziq tenglamasidir.



6-chizma



7-chizma

Agar to‘g‘ri chiziq $M_1(x_1, y_1)$ nuqtadan o‘tib, Oy o‘qqa parallel bo‘lsa, u holda uning tenglamasi $x = x_1$ ko‘rinishda bo‘ladi.

Agar to‘g‘ri chiziq $M_1(x_1, y_1)$ nuqtadan o‘tib, Ox o‘qqa parallel bo‘lsa, u holda uning tenglamasi $y = y_1$ ko‘rinishda bo‘ladi.

k parametr o‘zgaruvchi bo‘lgan holda (17) tenglama $M_1(x_1, y_1)$ nuqtadan o‘tuvchi to‘g‘ri chiziqlar dastasini aniqlaydi; bunda k ga dastaning parametri deyiladi (7-chizma).

6-misol. *$M(5,2)$ nuqtadan o‘tib, $y = \frac{4}{5}x - 3$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.*

Yechish. Izlanayotgan to‘g‘ri chiziq berilgan to‘g‘ri chiziqqa parallel bo‘lgani uchun, uning burchak koeffitsiyenti $k = \frac{4}{5}$ ga teng. Shuning uchun, (17) formulaga asosan izlanayotgan to‘g‘ri chiziq tenglamasi $y-2=\frac{4}{5}(x-5)$, yoki $y=\frac{4}{5}x-3$ bo‘ladi.

7-misol. *$M(5,2)$ nuqtadan o‘tib, $y=-\frac{2}{3}x+4$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.*

Yechish. Shartga ko‘ra, izlanayotgan to‘g‘ri chiziq burchak koeffitsiyenti $k_1=-\frac{2}{3}$ ga teng to‘g‘ri chiziqqa perpendikulyar bo‘lgani uchun uning burchak koeffitsiyenti

$$k_2=-\frac{1}{k_1}=\frac{3}{2}$$

bo‘ladi. Shuning uchun (17) formulaga asosan, topish kerak bo‘lgan to‘g‘ri chiziq tenglamasini tuzamiz: $y-5=\frac{3}{2}(x-2)$, yoki $y=\frac{3}{2}x+2$.

3.6. Berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi

Ma’lumki, o‘zaro ustma-ust tushmaydigan ikki nuqtadan bitta va faqat bitta to‘g‘ri chiziq o‘tkazish mumkin.

$M_1(x_1,y_1)$ va $M_2(x_2,y_2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzamiz. Faraz qilaylik, $x_1\neq x_2$ bo‘lsin, ya’ni izlanayotgan to‘g‘ri chiziq Oy o‘qiga parallel bo‘lmasin. To‘g‘ri chiziq $M_1(x_1,y_1)$ nuqtadan o‘tganligi uchun uning tenglamasi quyidagi ko‘rinishga ega:

$$y-y_1=k(x-x_1), \qquad (18)$$

bu yerda k – to‘g‘ri chiziqning bizga noma’lum bo‘lgan burchak koeffitsiyenti. Ammo, izlanayotgan to‘g‘ri chiziq $M_2(x_2,y_2)$ nuqtadan o‘tganligi uchun bu nuqtaning x_2 va y_2 koordinatalari (18) tenglamani qanoatlantiradi. Bundan

$$y_2-y_1=k(x_2-x_1),$$

va $x_2\neq x_1$ bo‘lganligidan, quyidagiga ega bo‘lamiz:

$$k=\frac{y_2-y_1}{x_2-x_1}. \qquad (19)$$

(19) ifodani (18) tenglamadagi k koeffitsiyentning o‘rniga olib borib qo‘ysak, $M_1(x_1,y_1)$ va $M_2(x_2,y_2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini hosil qilamiz:

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1).$$

Bu tenglamani, $y_2\neq y_1$ bo‘lganda, quyidagi shaklda ham yozish mumkin:

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}. \tag{20}$$

Agar $x_2=x_1$ bo‘lsa, ya’ni $M_1(x_1,y_1)$ va $M_2(x_2,y_2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq Oy o‘qiga parallel bo‘lsa, u holda bu to‘g‘ri chiziq tenglamasi $x=x_1$ ko‘rinishda bo‘lishi ravshan.

8-misol. $M_1(4,-3)$ *va* $M_2(2,1)$ *nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.*

Yechish. Masala shartiga ko‘ra berilgan nuqtalar koordinatalari $x_1=4, y_1=-3, x_2=2, y_2=1$. U holda (20) formulaga asosan

$$\frac{x-4}{2-4}=\frac{y+3}{1+3},$$

yoki $y=-2x+5$ izlanayotgan to‘g‘ri chiziq tenglamasini hosil qilamiz.

3.7. Ikki to‘g‘ri chiziqning kesishish nuqtasi

Faraz qilaylik, ikki to‘g‘ri chiziq berilgan:

$$A_1x+B_1y+C_1=0, \tag{21}$$

$$A_2x+B_2y+C_2=0. \tag{22}$$

Ravshanki, ularning kesishish nuqtasi bu to‘g‘ri chiziqlarning har birida yotadi. Shuning uchun kesishish nuqtasining koordinatalari birinchi va ikkinchi to‘g‘ri chiziq tenglamalarini qanoatlantiradi. Demak, bu chiziqlarning kesishish nuqtasi koordinatalarini topish uchun (21) va (22) tenglamalar sistemasini yechish yetarli.

(21) va (22) tenglamalardan mos ravishda y va x noma’lumlarni yo‘qotgandan so‘ng quyidagini hosil qilamiz:

$$(A_1B_2-A_2B_1)x+(C_1B_2-C_2B_1)=0, \tag{23}$$

$$(A_1B_2-A_2B_1)y+(A_1C_2-A_2C_1)=0. \tag{23}$$

Agar $A_1B_2-A_2B_1\neq 0$ bo‘lsa, u holda oxirgi tengliklardan ikki to‘g‘ri chiziq kesishish nuqtasining koordinatalari uchun quyidagi ifodalarni olamiz:

$$x=-\frac{C_1B_2-C_2B_1}{A_1B_2-A_2B_1},\ y=-\frac{A_1C_2-A_2C_1}{A_1B_2-A_2B_1}, \tag{25}$$

(25) formulani ikkinchi tartibli determinantlardan foydalanib

$$x=-\frac{\begin{vmatrix} C_1 & B_1 \\ C_2 & B_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}},\ y=-\frac{\begin{vmatrix} A_1 & C_1 \\ A_2 & C_2 \end{vmatrix}}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}} \tag{26}$$

ko‘rinishda ham yozish mumkin.

(21), (22) to‘g‘ri chiziqlar uchun quyidagi uchta hol mavjud.

a) $A_1B_2 - A_2B_1 \neq 0$, ya'ni $\frac{A_1}{A_2} \neq \frac{B_1}{B_2}$. Bu holda to‘g‘ri chiziqlar parallel emas.

Bunda yagona kesishish nuqtasining koordinatalari (25) (yoki (26)) formulalar orqali aniqlanadi.

b) $A_1B_2 - A_2B_1 = 0 \; va \; C_1B_2 - C_2B_1 \neq 0$ yoki $A_1C_2 - A_2C_1 \neq 0$, ya'ni

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} \neq \frac{C_1}{C_2}.$$

Bu holda (21), (22) to‘g‘ri chiziqlar parallel va kesishish nuqtasi yo‘q.

c) $A_1B_2 - A_2B_1 = 0, C_1B_2 - C_2B_1 = 0\,, A_1C_2 - A_2C_1 = 0$, ya'ni

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}.$$

Bu holda (21), (22) to‘g‘ri chiziqlar ustma-ust tushadi, va demak, kesishish nuqtalari cheksiz ko‘p bo‘ladi.

9-misol. $3x - 2y + 1 = 0$ *va* $2x + 5y - 12 = 0$ *to‘g‘ri chiziqlar berilgan. Bu to‘g‘ri chiziqlarning kesishish nuqtasini toping.*

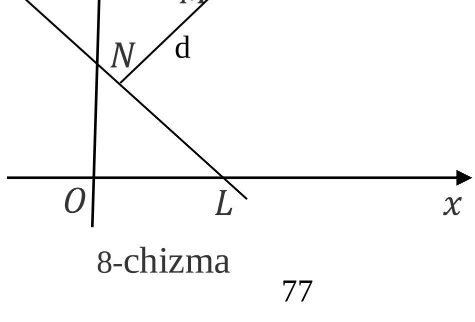
Yechish. $\frac{3}{2} \neq -\frac{2}{5}$, ya'ni $\frac{A_1}{A_2} \neq \frac{B_1}{B_2}$ shart bagarilganligi uchun, berilgan to‘g‘ri chiziqlar yagona nuqtada kesishadi. (26) formulalardan foydalanib, kesishish nuqtasining koordinatalarini topamiz:

$$x = -\frac{\begin{vmatrix} 1 & -2 \\ -12 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 2 & 5 \end{vmatrix}} = -\frac{-19}{19} = 1\,,\; y = -\frac{\begin{vmatrix} 3 & 1 \\ 2 & -12 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 2 & 5 \end{vmatrix}} = -\frac{-38}{19} = 2\,.$$

Shunday qilib, berilgan to‘g‘ri chiziqlar $M(1,2)$ nuqtada kesishadi.

3.8. Nuqtadan to‘g‘ri chiziqgacha bo‘lgan masofa. To‘g‘ri chiziqning normal tenglamasi

Umumiy tenglamasi $Ax + By + C = 0$ bolgan KL to‘g‘ri chiziqni va shu chiziqda yotuvchi $M(x_1, y_1)$ nuqtani qaraymiz. M nuqtadan KL to‘g‘ri chiziqgacha bo‘lgan d masofa deb, M nuqtadan KL to‘g‘ri chiziqga tushirilgan perpendikulyar $[MN]$ ning uzunligiga aytiladi (8-chizma).



8-chizma

$[MN]$ perpendikylyarning tenglamasini quyidagi ko‘rinishda yozish mumkin:

$$B(x-x_1)-A(y-y_1)=0.$$

Bu yerdan, perpendikulyarning asosi bo‘lgan $N(x_2,y_2)$ nuqta uchun quyidagi tenglikga ega bo‘lamiz: $B(x_2-x_1)-A(y_2-y_1)=0$, va bundan,

$$\frac{x_2-x_1}{A}=\frac{y_2-y_1}{B}=t, \qquad (27)$$

bu yerda t – proporsionallik koeffitsiyenti. Shuning uchun $[MN]$ perpendikulyarning uzunligi quyidagiga teng:

$$d=|MN\,|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{A^2+B^2}\cdot|t|. \qquad (28)$$

Ikkinchi tomondan, $N(x_2,y_2)$ nuqtaning KL to‘g‘ri chiziqda yotishini va (27) dan kelib chiquvchi $x_2=x_1+At$, $y_2=y_1+Bt$ tengliklarni e’tiborga olsak, u holda:

$$Ax_2+By_2+C=A(x_1+At)+B(y_1+Bt)+C=$$
$$=(Ax_1+By_1+C)+t(A^2+B^2)=0.$$

Bundan,

$$t=-\frac{Ax_1+By_1+C}{A^2+B^2}.$$

Shunday qilib, (28) ga asosan

$$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}. \qquad (29)$$

Xususiy holda, agar (29) formulada $x_1=0$, $y_1=0$ deb olinsa, koordinata boshidan to‘g‘ri chiziqgacha bo‘lgan masofa formulasini hosil qilamiz:

$$d_0=\frac{|C|}{\sqrt{A^2+B^2}}\,.$$

Agar $Ax+By+C=0$ to‘g‘ri chiziqning umumiy tenglamasini normallovchi deb ataluvchi

$$\mu=\pm\frac{1}{\sqrt{A^2+B^2}}$$

songa ko‘paytirsak (ildiz oldidagi ishora $\mu\cdot C<0$ tengsizlikni qanoatlantiradigan qilib tanlahadi), u holda quyidagi tenglamani hoail qilamiz:

$$x\cos\varphi+y\sin\varphi-p=0.$$

Bu tenglamaga to‘g‘ri *chiziqning normal tenglamasi* deyiladi. Bu yerda p – koordinata boshidan to‘g‘ri chiziqga tushirilgan perpendikulyar uzunligi, φ esa perpendikulyarning Ox o‘qi bilan tashkil qilgan musbat burchagi.

10-misol. $M(1,2)$ *nuqtadan* $3x-4y-5=0$ *to‘g‘ri chiziqgacha bo‘lgan masofani toping.*

Yechish. (29) formuladan foydalanib, izlanayotgan masofani topamiz:

$$d=\frac{|3\cdot 1-4\cdot 2-5|}{\sqrt{3^2+(-4)^2}}=2.$$

11-misol. $x+\sqrt{3}y-6=0$ *to‘g‘ri chiziq tenglamasini normal shaklga keltiring.*

Yechish. $C=-6<0$ ekanligini hisobga olib, berilgan tenglamaning normallovchi ko‘paytuvchisini topamiz:

$$\mu=\frac{1}{\sqrt{A^2+B^2}}=\frac{1}{\sqrt{1^2+\left(\sqrt{3}\right)^2}}=\frac{1}{2}.$$

Berilgan tenglama normallovchi ko‘paytiruvchiga ko‘paytirilgandan so‘ng quyidagi ko‘rinishni oladi:

$$\frac{1}{2}x+\frac{\sqrt{3}}{2}y-3=0,$$

yoki $x\cos\varphi+y\sin\varphi-p=0$, bu yerda $\cos\varphi=\frac{1}{2}$, $\sin\varphi=\frac{\sqrt{3}}{2}$, $p=3$. Demak, koordinata boshidan to‘g‘ri chiziqqa tushirilgan perpendikulyar uzunligi $d_0=3$, uning Ox o‘qi bilan tashkil qilgan musbat burchagi $\varphi=60°$ ga teng ekan.

3.9. To‘g‘ri chiziqlar dastasining tenglamasi

Agar kesishuvch $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ – to‘g‘ri chiziqlarning tenglamalari bo‘lsa, u holda ularning kesishish nuqtasidan o‘tuvch to‘g‘ri chiziqlar tenglamasi quyidagi ko‘rinishga ega bo‘ladi:

$$A_1x+B_1y+C_1+\lambda(A_2x+B_2y+C_2)=0,$$

bu yerda λ – sonli ko‘paytiriluvchi bo‘lib, berilgan to‘g‘ri chiziqlarning kesishish nuqtasidan o‘tuvchi to‘g‘ri chiziqlarni aniqlovchi parametrdir. Oxirgi tenglamadagi λ parametrga har xil qiymatlarni berib, markazi berilgan to‘g‘ri chiziqlar kesishish nuqtasida bo‘lgan to‘g‘ri chiziqlar dastasiga tegishli bo‘lgan turli to‘g‘ri chiziqlarni hosil qilamiz.

12-misol. $3x-4y+7=0$ va $5x+2y+3=0$ *to‘g‘ri chiziqlarning kesishish nuqtasidan o‘tib, ordinata o‘qiga parallel bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.*

Yechish. Izlanayotgan to‘g‘ri chiziq quyidagi

$$3x-4y+7+\lambda(5x+2y+3)=0,$$

yoki $(3+5\lambda)x+(-4+2\lambda)y+(7+3\lambda)=0$ to g‘ri chiziqlar dastasiga tegishli.

Masala shartiga ko‘ra izlanayotgan to‘g‘ri chiziq ordinata o‘qiga parallel bo‘lganligi uchun unda y oldidagi koeffitsiyent nolga teng bo‘lishi kerak, ya’ni $-4+2\lambda=0$, $\lambda=2$. Topilgan $\lambda=2$ qiymatini dasta tenglamasiga olib borib qo‘yib, izlanayotgan tenglamani hosil qilamiz: $x+1=0$.

3.10. Nazorat savollari va masalalar

1. To‘g‘ri chiziqning umumiy tenglamasi va uning xususiy ko‘rinishlari.
2. To‘g‘ri chiziqning “kesmalar” bo‘yicha tenglamasi.
3. To‘g‘ri chiziqning burchak koeffitsiyentli tenglamasi.
4. To‘g‘ri chiziqlar orasidagi burchak.
5. To‘g‘ri chiziqlarning parallelik va perpendikulyarlik shartlari.
6. Berilgan nuqtadan berilgan yo‘nalishda o‘tuvchi to‘g‘ri chiziq tenglamasi.
7. Berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi.
8. Ikki to‘g‘ri chiziqning kesishish nuqtasini aniqlash formulalari.
9. Nuqtadan to‘g‘ri chiziqgacha bo‘lgan masofani topish formulasi.
10. To‘g‘ri chiziqning normal tenglamasi.
11. To‘g‘ri chiziqlar dastasining tenglamasi.
12. $M(3,4)$ nuqtadan o‘tib, $y=2x+1$ to‘g‘ri chiziq bilan $\varphi=45°$ burchak hosil qiluvchi to‘g‘ri chiziq tenglamasini tuzing.
13. Uchlari $A(-3,2)$ va $B(1,-1)$ nuqtalarda bo‘lgan AB kesma berilgan. Bu kesmaning o‘rtasi va koordinata boshidan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.
14. Uchlari $A(4,2), B(-2,4)$ va $C(-1,-4)$ nuqtalarda bo‘lgan ABC uchburchak berilgan. C uchdan o‘tuvch mediana tenglamasni tuzing va mediana uzunligini toping.
15. $M(5,2)$ nuqtadan shunday to‘gri chiziq o‘tkazingki, u koordinata o‘qlaridan bir xil kesmalar ajratsin.
16. $3x+2y-6=0$, $6x+4y-3=0$ parallel to‘g‘ri chiziqlardan baravar uzoqlikda yotuvchi to‘g‘ri chiziq tenglamasini tuzing.

17. Uchlari $A(5,3), B(-3,4)$ va $C(-2,-5)$ nuqtalarda bo‘lgan ABC uchburchak berilgan. B uchidan tushirilgan balandlik tenglamasni tuzing va balandlik uzunligini toping.

18. To‘gri chiziqlarning kesishish nuqtalarini toping:

a) $5x - 7y - 20 = 0$ va $7x - 10y + 15 = 0$;

b) $2x + 3y - 7 = 0$ va $4x + 6y + 11 = 0$.

19. $3x + 4y - 7 = 0, 5x + 3y - 8 = 0$ chiziqlarning kesishish nuqtasi va koordinata boshidan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

20. $M(3,4)$ nuqtaning $5x + 2y + 20 = 0$ to‘g‘ri chiziqdagi proyeksiyasini toping.

21. $O(0,0), B(1,2), C(4,3)$ nuqtalardan $3x - 4y + 10 = 0$ chiziqgacha bo‘lgan masofalarni toping.

22. Uchlari $A(-2,1), B(2,-1), C(4,3)$ nuqtalarda bo‘lgan ABC uchburchak berilgan. Shu uchburchakning medianalari kesishish nuqtasining koordinatalarini toping.

23. $3x + 4y - 10 = 0$ va $3x + 4y - 45 = 0$ to‘gri chiziqlar orasida yotib, ularga perpendikulyar bo‘lgan kesma uzunligini toping.

24. $8x - 6y + 5 = 0$ to‘g‘ri chiziqqa parallel va undan 2 birlik uzoqlikdan o‘tuvchi to‘g‘ri chiziqlar tenglamalarini tuzing.

25. $3x + 4y - 20 = 0$ va $8x + 6y - 5 = 0$ to‘g‘ri chiziqlar orasidagi burchak bissektrisasi tenglamalarini tuzing.

26. m parametrning qanday qiymatlarida $7x - 2y - 5 = 0, x + 7y - 8 = 0$ va $mx + my - 8 = 0$ to‘g‘ri chiziqlar bir nuqtada kesishadi?

27. Abssissa o‘qidan shunday nuqta topingki, undan $8x + 15y + 10 = 0$ to‘g‘ri chiziqgacha bo‘lgan masofa 1 ga teng bo‘lsin.

28. $x + 2y + 3 = 0$, $2x + 3y + 4 = 0$ va $5x + 8y = 0$ to‘g‘ri chiziqlarning kesishish nuqtasidan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

29. Koordinata o‘qlaridan $a = 8, b = 6$ kesmalar ajratuvchi to‘g‘ri chiziqdan $M(2,-1)$ nuqtagacha bo‘lgan masofani toping.

30. Tomonlari $x + \sqrt{3}y + 1 = 0$, $\sqrt{3}x + y + 1 = 0, x - y - 10 = 0$ to‘g‘ri chiziqlarda yotuvchi uchburchak teng yonli ekanliligini ko‘rsating. Uning uchidagi burchagini toping.

§4. IKKINCHI TARTIBLI CHIZIQLAR

4.1. Aylana

Eng sodda ikkinchi tartivbli chiziq aylana bo‘lib, biz bu chiziq haqida yuqorida chiziqlar haqida umumiy tushunchalar mavzusida ba’zi ma’lumotlar keltirdik. Keltirilganlar asosida ushbu ta’rifni eslatamiz.

1-ta’rif. *Berilgan* $C(x_0, y_0)$ *nuqtadan baravar* R *uzoqlikda yotuvchi nuqtalarning geometrik o‘rniga* ***aylana*** *deyiladi* (1-chizma).

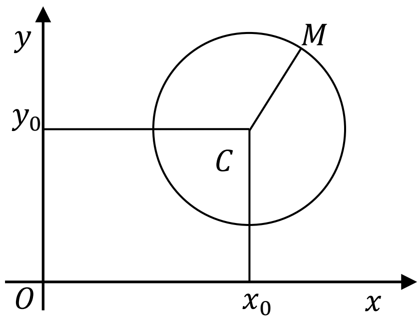
Bunda $C(x_0, y_0)$ – aylana markazi, R – aylana radiusi deyiladi.

Ta’rifga ko‘ra, aylanada yotgan ixtiyoriy $M(x, y)$ nuqta $|MC| = R$ tenglikni qanoatlantiradi. $M(x, y)$ va $C(x_0, y_0)$ nuqtalar orasidagi masofa quyidagi formula orqali aniqlanadi: $|MC| = \sqrt{(x - x_0)^2 + (y - y_0)^2}$. Shuning uchun,

$$\sqrt{(x - x_0)^2 + (y - y_0)^2} = R. \qquad (1)$$

(1) tenglamaning ikkala tomoni ham musbat bo‘lgani uchun uni kvadratga ko‘tarib, teng kuchli tenglamani hosil qilamiz:

$$(x - x_0)^2 + (y - y_0)^2 = R^2. \qquad (2)$$



1-chizma

Demak, aylanada berilgan har qanday $M(x, y)$ nuqta koordinatalari (2) tenglamani qanoatlantiradi va aksincha, koordinatalari (2) tenglamani qanoatlantiruvchi har qanday $M(x, y)$ nuqta aylanaga tegishli bo‘ladi. Xususiy holda, agar $x_0 = 0$ va $y_0 = 0$ bo‘lsa, u holda aylananing markazi koordinatalar boshida bo‘ladi:

$$x^2 + y^2 = R^2.$$

(2) aylana tenglamasini quyidagi

$$x^2 + y^2 + \alpha x + \beta y + \gamma = 0, \qquad (3)$$

ko‘rinishga keltirish mumkin, bu yerda

$$\alpha = -2x_0, \beta = -2y_0, \gamma = {x_0}^2 + {y_0}^2 - R^2.$$

Aylananing (3) ko‘rinishdagi tenglamasi umumiy ko‘rinishi

$$Ax^2 + Cy^2 + Bxy + Dx + Ey + F = 0 \qquad (4)$$

bo‘lgan ikkinchi tartibli chiziqlarning $A = C \neq 0, B = 0$ xususiy holidir.

Haqiqatdan ham, bu holda

$$Ax^2 + Ay^2 + Dx + Ey + F = 0. \qquad (5)$$

(5) tenglamani hadma-had $A \neq 0$ ga bo‘lib va $\frac{D}{A} = \alpha, \frac{E}{A} = \beta, \frac{F}{A} = \gamma$ belgilashlarni kiritsak, u holda (3) ko‘rinishdagi tenglamaga kelamiz.

Qayd etamizki, (5) tenglama faqat $\frac{D^2+E^2}{4A^2} - \frac{F}{A} > 0$ bo‘lgandagina aylana ko‘rinishidagi haqiqiy chiziqni aniqlaydi. Chunki faqat shu holdagina uni

$$(x + \frac{D}{2A})^2 + (y + \frac{E}{2A})^2 = R^2, R = \frac{D^2+E^2}{4A^2} - \frac{F}{A}$$

ko‘rinishda aylana tenglamasi kabi yozish mumkin.

1-misol. $2x^2 + 2y^2 - 8x + 5y - 4 = 0$ *tenglama bilan berilgan aylananing markazi koordinatalarini va radiusini toping*.

Ye*chish.* Berilgan tenglamani 2 ga bo‘lib va tenglama hadlarini guruhlagandan so‘ng, quyidagi tenglamaga kelamiz:

$$x^2 - 4x + y^2 + \frac{5}{2}y = 2.$$

$x^2 - 4x$ va $y^2 + \frac{5}{2}y$ ifodalarninig birinchisiga 2^2=4 ni va ikkinchisiga $\left(\frac{5}{4}\right)^2$ni qo‘shamiz va ayiramiz (yoki, bu sonlarni tenglikning ikkala tomoniga ham qo‘shamiz). Natijada to‘la kvadratlarni ajratib, quyidagini hosil qilamiz:

$$(x^2 - 4x + 4) + \left(y^2 + \frac{5}{2}y + \frac{25}{16}\right) = 2 + 4 + \frac{25}{16}$$

yoki $(x-2)^2 + \left(y + \frac{5}{4}\right)^2 = \frac{121}{16}$.

Shunday qilib, aylana markazining koordinatalari $x_0 = 2\ y_0 = -\frac{5}{4}$ va radiusi $R = \frac{11}{4}$ ekan.

2-misol. *Markazi* $x + y - 3 = 0$ *to‘g‘ri chiziqda yotuvchi va berilgan* $A(5{,}0)$, $B(1{,}4)$ *nuqtalardan o‘tuvchi aylana tenglamasini tuzing.*

Yechish. Avvalo, izlanayotgan aylana $[AB]$ vatarining o‘rtasidagi M nuqtaning koordinatalarini topamiz: $x_M = \frac{5+1}{2} = 3, y_M = \frac{4+0}{2} = 2$, ya’ni $M(3{,}2)$. Aylana markazi esa $[AB]$ ga perpendikulyar bo‘lgan kesmada yotadi. (AB) to‘g‘ri chiziqning tenglamasini tuzamiz:

$$\frac{y-0}{4-0}=\frac{x-5}{1-5}, \text{ ya'ni } x+y-5=0.$$

Bu to‘g‘ri chiziqning burchak koeffitsiyenti -1 ga teng bo‘lgani uchun, unga perpendikulyar to‘g‘ri chiziqning burchak koeffitsiyenti 1 ga teng bo‘ladi. $M(3,2)$ nuqtadan o‘tuvchi shu perpendikulyar to‘g‘ri chiziqning burchak koeffitsiyentli tenglamasini yozamiz:

$$y-2=1\cdot(x-3), \text{ ya'ni } x-y-1=0.$$

Osonlik bilan ko‘rish mumkinki, izlanayotgan aylana markazi C – berilgan $x+y-3=0$ va tuzilgan $x-y-1=0$ to‘g‘ri chiziqlarning kesishish nuqtasidan iborat, ya’ni markaz koordinatasi (x_0, y_0) quyidagi tenglamalar sistemasining yechimidir:

$$\begin{cases} x+y-3=0, \\ x-y-1=0. \end{cases}$$

Demak, $x_0=2$, $y_0=1$, ya’ni $C(2;1)$. Aylana radiusi $[CA]$ kesma uzunligiga teng, ya’ni $R=\sqrt{(5-2)^2+(1-0)^2}=\sqrt{10}$. Shunday qilib, izlanayotgan aylana tenglamasi quyidagi ko‘rinishga ega:

$$(x-2)^2+(y-1)^2=10.$$

3-misol. $x^2+y^2=4(y+1)$ *aylananing koordinata boshidan o‘tuvchi barcha vatarlari o‘rtalarining geometric o‘rnini toping.*

Ye*chish*. Masala shartida aytilayotgan vatarlar tenglamalari $y=kx$ ko‘rinishga ega. Shu vatarlarning aylana bilan kesishish nuqtalarining koordinatalarini k orqali ifodalaymiz; buning uchun $y=kx$ va $x^2+y^2-4y-4=0$ tenglamalarni birgalikda yechamiz. U holda $(k^2+1)x^2-4kx-4=0$ kvadrat tenglamani hosil qilamiz. Kvadrat tenglama ildizlari haqidagi Viet teoremaidan foydalanib, oxirgi tenglamaning x_1, x_2 yechimlari uchun $x_1+x_2=\frac{4k}{1+k^2}$ tenglamani hosil qilamiz. Bundan esa vatar o‘rtasining abstsisasini aniqlaymiz:

$$\bar{x}=\frac{x_1+x_2}{2}=\frac{2k}{1+k^2}.$$

Vatar o‘rtasining ordinatasi esa $\bar{y}=k\bar{x}=\frac{2k^2}{1+k^2}$ tenglikni qanoatlantiradi. Shunday qilib, izlanayotgan nuqtalar geometrik o‘rnini anglatuvchi chiziqning parametrik tenglamalarin hosil qildik:

$$\bar{x}=\frac{2k}{1+k^2}, \ \bar{y}=\frac{2k^2}{1+k^2}.$$

Bu tengliklardan k parametrni yoqotib,

$$\bar{x}^2+\bar{y}^2-2\bar{y} \quad =0$$

tenglamani hosil qilamiz. Demak, izlanayotgan chiziq ham aylanadan iborat ekan.

4.2. Markazli ikkinchi tartibli chiziqlar. Ellips va giperbola.

x va y o‘zgaruvchilar ko‘paytmasi qatnashmagan ikkinchi tartibli egri chiziq tenglamasini qaraymiz (ya’ni (4) tenglamada $A \neq 0, C \neq 0$, $B = 0$ bo‘lgan hol):

$$Ax^2 + Cy^2 + Dx + Ey + F = 0. \tag{6}$$

(6) tenglamani quyidagicha yozish mumkin:

$$A\left(x + \frac{D}{2A}\right)^2 + C\left(y + \frac{E}{2C}\right)^2 = \frac{D^2}{4A} + \frac{E^2}{4C} - F.$$

Agar bu yerda $x_0 = -\frac{D}{2A}, y_0 = -\frac{E}{2C}, H = \frac{D^2}{4A} + \frac{E^2}{4C} - F$ deb belgilasak, unda

$$A(x - x_0)^2 + C(y - y_0)^2 = H. \tag{7}$$

$O'(x_0, y_0)$ nuqta (7) *chiziqning simmetriya markazi* (*chiziq markazi*) *ni* ifodalaydi. Haqiqatdan ham, agar $M_1(x_1, y_1)$ nuqta (7) chiziqda yotsa, u holda $x_2 = 2x_0 - x_1$, $y_2 = 2y_0 - y_1$ koordinatali $M_2(x_2, y_2)$ nuqta O' nuqtaga nisbatan simmetrik bo‘lib, u ham (7) chiziqqa tegishli bo‘ladi. Ox va Oy koordinata o‘qlariga parallel bo‘lgan $x = x_0$ va $y = y_0$ to‘g‘ri chiziqlar (7) *chiziqning simmetriya o‘qlari* (*chiziq o‘qlari*) deyiladi.

Bundan keyin, soddalik uchun chiziq markazini koordinata boshida deb faraz qilamiz, yani $x_0 = 0$, $y_0 = 0$. U holda chiziq tenglamasi quyidagi ko‘rinishga ega bo‘ladi:

$$Ax^2 + Cy^2 = H. \tag{8}$$

2-ta’rif. A*gar (8) tenglamada A va C koeffitsiyentlar bir xil ishoraga ega bo‘lsalar, ya’ni $AC > 0$* bo‘lsa, i*kkinchi tartibli chiziqqa **ellips** (aniqrog‘i **elliptik tipga** tegishli) deyiladi.*

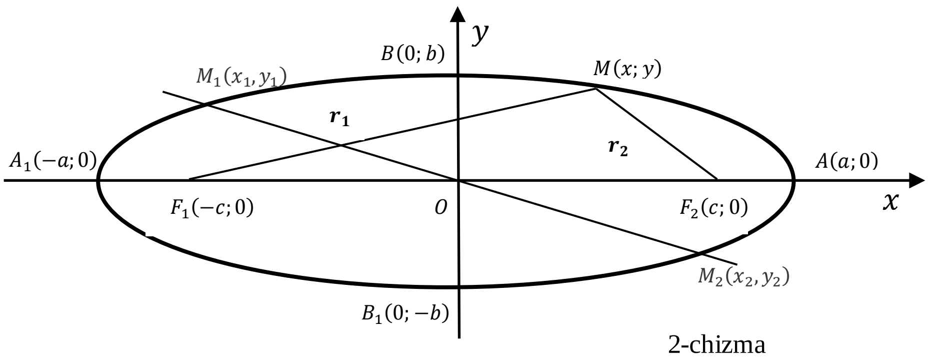
Aniqlik uchun $A > 0$ va $C > 0$ deb hisoblaymiz (aks holda, (8) tenglamadagi hadlarning ishoralari teskarisiga almashtiriladi). Bunda mumkin bo‘lgan uchta hol mavjud:

1) $H > 0$; 2) $H = 0$; 3) $H < 0$.

Birinchi $H > 0$ holda haqiqiy ellipsni hosil qilamiz:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\,, \tag{9}$$

bu yerda $a=\sqrt{\frac{H}{A}}, b=\sqrt{\frac{H}{C}}$ sonlarga ellipsning *yarim o'qlari* deyiladi. (9) tenglama esa a va b yarim o'qlarga ega bo'lgan ellipsning *kanonik tenglamasi* deyiladi.



2-chizma

$A(a;0), B(0;b), A_1(-a;0), B_1(0;-b)$ nuqtalar *ellips uchlari* va $|A_1A|=2a$, $|B_1B|=2b$ kesmalar – *uning o'qlari* deyiladi. Ellips grafigi Ox va Oy o'qlariga hamda koordinata boshi O nuqtaga nisbatan simmetrik joylashgan yopiq chegaralangan egri chiziqdan iborat (2-chizma). Ta'kidlaymizki, (9) tenglamadan $|x|\le a\,, |y|\le b$ kelib chiqadi. Xususiy holda, $a=b$ bo'lganda (9) dan aylana tenglamasini hosil qilamiz: $x^2+y^2=a^2$.

Ikkinchi $H=0$ holda (8) chiziq yagona $O(0{,}0)$ nuqtani ifodalaydi (*maxsus ellips*).

Nihoyat, uchunchi $H<0$ holda (8) chiziqning haqiqiy nuqtalari yo'q; bu holda uni shartli ravishda *mavhum ellips* deb ataymiz.

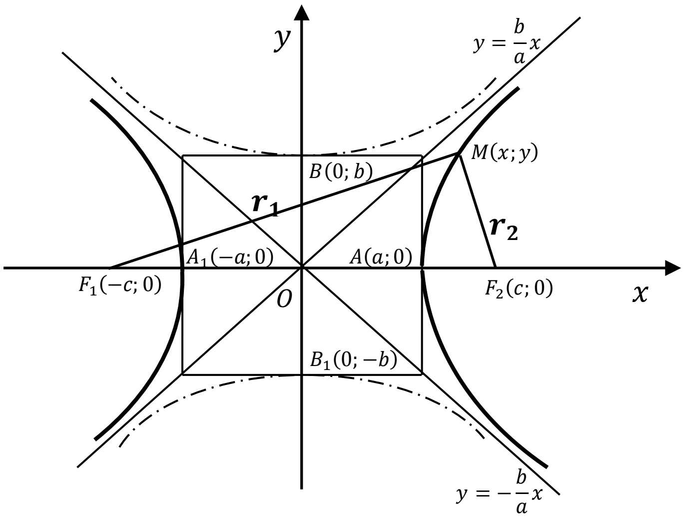
3-ta'rif. A*gar (8) tenglamada A va C koeffitsiyentlar har xil ishoraga ega, ya'ni $AC<0$ bo'lsa*, *ikkinchi tartibli chiziqqa* ***giperbola*** *(aniqrog'i* ***giperbolik tipga*** *tegishli) deyilad.*

Aniqlik uchun $A>0$, $C<0$ bo'lsin. Bu yerda ham mumkin bo'lgan hollar uchta: 1) $H>0$; 2) $H=0$; 3) $H<0$.

Birinchi $H>0$ holda giperbolaning *kanonik tenglamasini* olamiz:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\,, \tag{10}$$

bu yerda $a=\sqrt{\frac{H}{A}}$ son – *haqiqiy yarim* o'q, $b=\sqrt{\frac{H}{C}}$ son esa – *mavhum yarim* o'q deyiladi (3-chizma). $A(a;0), A_1(-a;0)$ nuqtalarga giperbola uchlari deyiladi. $|AA_1|=2a$ kesma (10) giperbolaning *haqiyqiy o'qi*, $|BB_1|=2b$ kesma esa – giperbolaning *mavhum o'qi* deyiladi. Ta'kidlaymizki, (10) tenglamadan $|x|\ge a$ kelib chiqadi.



3- chizma

Ikkinchi holda, ya’ni $H = 0$ bo‘lganda, kesishuvchi to‘g‘ri chichiq jufti hosil bo‘ladi (*maxsus giperbola*):

$$\left(\sqrt{A}\cdot x - \sqrt{-C}\cdot y\right)\cdot\left(\sqrt{A}\cdot x + \sqrt{-C}\cdot y\right) = 0.$$

Nihoyat, uchunchi holda, ya’ni $H < 0$ bo‘lganda, yarim o‘qlari

$$a_1 = \sqrt{\frac{-H}{A}},\ b_1 = \sqrt{\frac{H}{C}}$$

bo‘lgan giperbolani hosil qilamiz:

$$\frac{x^2}{(a_1)^2} - \frac{y^2}{(b_1)^2} = -1\ . \qquad (11)$$

Agar $a_1 = a,\ b_1 = b$ bo‘lsa, u holda (11) giperbola (10) giperbolaning *qo‘shmasi* deyiladi; uning uchlari $B(0, b)$ va $B_1(0, -b)$ nuqtalardadir.

4-misol. *Quyidagi chiziqning qanday tipdagi chiziq ekanligini aniqlang:*

$$3x^2 + 2y^2 - 9x + 6y - 1 = 0.$$

Yechish. Berilgan tenglama chap tarafida hadlarni to‘la kvadratga keltirgandan so‘ng quyidagini hosil qilamiz

$$3\left(x - \frac{3}{2}\right)^2 + 2\left(y - \frac{3}{2}\right)^2 = \frac{49}{4}$$

Bu yerdan

$$\frac{\left(x - \frac{3}{2}\right)^2}{\frac{49}{12}} + \frac{\left(y + \frac{3}{2}\right)^2}{\frac{49}{8}} = 1.$$

Demak, berilgan chiziq markazi $O'\left(-\frac{3}{2};\frac{3}{2}\right)$ nuqtada, yarim o‘qlari esa $a = \frac{7}{2\sqrt{3}}$ va $b = \frac{7}{2\sqrt{2}}$ bo‘lgan ellipsdan iborat ekan.

4.3. Ellips va giperbolaning fokal xossalari

$F_1(-c,0)$ va $F_2(c,0)$ nuqtalar (9) va (10) kanonik tenglamalar bilan berilgan ellips va giperbolaning *fokuslari* deyiladi, bu yerda

$$c = \sqrt{a^2 \mp b^2} \tag{12}$$

("-" ishora ellipsga, "+" ishora esa giperbolaga mos keladi.

$\varepsilon = \frac{c}{a}$ son markazga ega ikkinchi tartibli chiziqning *ekssentrisiteti* deyiladi. (12) formulaga asosan, ellips uchun $0 \le \varepsilon < 1$, giperbola uchun esa $1 < \varepsilon < +\infty$. Ta'kidlaymizki, aylana uchun $\varepsilon = 0$.

Markazga ega ikkinchi tartibli chiziqning ixtiyoriy M nuqtasidan fokuslarigacha bo'lgan $r_1 = MF_1$ va $r_2 = MF_2$ masofalarga ularning *fokal radiuslari* deyiladi (2-chizma, 3-chizma). Fokal radiuslar uchun

$$r_1 = \sqrt{(x-c)^2 + y^2}\,, r_2 = \sqrt{(x+c)^2 + y^2}$$

formulalarni yoza olamiz.

Ellips va giperbolaning kanonik tenglamalari

$$\frac{x^2}{a^2} \pm \frac{y^2}{b^2} = 1$$

bo'lgani uchun (bu yerda "+" ishora ellipsga, "–" ishora giperbolaga mos keladi), bundan $y^2 = \pm b^2\left(1 - \frac{x^2}{a^2}\right)$, va demak, (12) ni hisobga olsak quyidagini yoza olamiz:

$$r_1 = \sqrt{x^2 - 2cx + c^2 \pm b^2\left(1 - \frac{x^2}{a^2}\right)} = \sqrt{\left(1 \mp \frac{b^2}{a^2}\right)x^2 - 2cx + (c^2 \pm b^2)} =$$

$$= \sqrt{\frac{c^2}{a^2}x^2 - 2cx + a^2} = \sqrt{\left(\frac{c}{a}x - a\right)^2} = \left|\frac{c}{a}x - a\right| = |\varepsilon x - a|.$$

Xuddi shunga o'xshash

$$r_2 = \sqrt{x^2 + 2cx + c^2 \pm b^2\left(1 - \frac{x^2}{a^2}\right)} = |\varepsilon x + a|.$$

Agar chiziq ellips bo'lsa, u holda $0 \le \varepsilon < 1, |x| \le a$ va shuning uchun $r_1 = a - \varepsilon x, r_2 = a + \varepsilon x$. Bundan esa,

$$r_1 + r_2 = 2a\,.$$

Shunday qilib, *ellips ixtiyoriy nuqtasining fokal radiuslari yig'indisi o'zgarmas miqdordir*. Ko'pincha bu xarakteristik xossani ellipsning ta'rifi sifatida qabul qilinadi.

Giperbola uchun $\varepsilon > 1, |x| \ge a$. Shuning uchun

$$r_1 = \pm(\varepsilon x - a), r_2 = \pm(\varepsilon x + a),$$

bu yerda “+” ishora giperbolaning o‘ng tarmog‘iga ($x > 0$), “–“ ishora esa giperbolaning chap tarmog‘iga ($x < 0$ mos keladi. Bundan

$$r_1 - r_2 = \pm 2a.$$

Shunday qilib, *giperbola ixtiyoriy nuqtasining fokal radiuslari ayirmasining absolyt qiymati o‘zgarmas miqdordir*. Bu xarakteristik xossa orqali giperbolani ta’riflash mumkin.

5-misol. $\frac{x^2}{25} + \frac{y^2}{9} = 1$ *ellipsda shunday nuqtani topingki, uning fokal radiuslari ayirmasi mos ravishda 6,4 teng bo‘lsin.*

Yechish. Berilgan ellipsda $a = 5, b = 3$ bolgani uchun $c = \sqrt{25 - 9} = 4$ va ekssentrisiteti $\varepsilon = \frac{c}{a} = \frac{4}{5}$ ga teng. Fokal radiuslari $r_1 = a - \varepsilon x,\ r_2 = a + \varepsilon x$ formulalarga asosan aniqlangani uchun, $r_1 - r_2 = -2\varepsilon x = -\frac{8}{5}x$. shartga ko‘ra $-\frac{8}{5}x = 6{,}4$. Bundan

$$x = -4; y = \pm b\sqrt{1 - \frac{x^2}{a^2}} = \pm 3\sqrt{1 - \frac{(-4)^2}{25}} = \pm 3\frac{\sqrt{25-16}}{5} = \pm\frac{9}{5}.$$

Demak, masala shartini qanoatlantiruvch nuqtalar

$$M_1\left(-4; \frac{9}{5}\right), M_2\left(-4; -\frac{9}{5}\right).$$

6-misol. $\frac{x^2}{16} - \frac{y^2}{9} = 1$ *giperbolaning o‘ng tarmog‘ida shunday nuqta topingki, undan o‘ng fokusigacha bo‘lgan masofa chap fokusigacha bo‘lgan masofadan ikki marta qisqa bo‘lsin.*

Yechish. Giperbolaning o‘ng tarmog‘i uchun fokal radiuslar $r_1 = \varepsilon x - a$ va $r_2 = \varepsilon x + a$ formulalar orqali aniqanadi. Shunday qilib, masala shartiga ko‘ra $\varepsilon x + a = 2(\varepsilon x - a)$ tenglamaga ega bo‘lamiz. Bundan

$$x = 3\frac{a}{\varepsilon}.$$

Endi $a = 4, \varepsilon = \frac{c}{a} = \frac{\sqrt{a^2+b^2}}{a} = \frac{\sqrt{16+9}}{4} = \frac{5}{4}$ ekanligini hisobga olsak, izlanayotgan nuqtaning abssisasi $x = 9{,}6$ ga teng. Izlanayotgan nuqtaning ordinatasini giperbola tenglamasidan topamiz:

$$y = \pm\frac{3}{4}\sqrt{x^2 - 16} = \pm\frac{3}{4}\sqrt{\left(\frac{48}{5}\right)^2 - 16} = \pm\frac{3}{5}\sqrt{119}.$$

Shunday qilib, masala shartini ikkita

$M_1(9{,}6; 0{,}6\sqrt{119}), M_2(9{,}6; -0{,}6\sqrt{119})$ nuqta qanoatlantiradi.

7-misol. Giperbola ekssentrisiteti $\sqrt{2}$ *ga teng.* $M(\sqrt{3}; \sqrt{2})$ *nuqtadan o‘tuvchi giperbolaning kanonik tenglamasini tuzing .*

Yechish. Ekssentrisitetning ta’rifi va masala shartiga asosan $\varepsilon = \frac{c}{a} = \sqrt{2}$ yoki $c^2 = 2a^2$, ya’ni giperbola teng yonli ekan. $M\left(\sqrt{3};\sqrt{2}\right)$ nuqta giperbola nuqtasi bo‘lgani uchun,

$$\frac{\left(\sqrt{3}\right)^2}{a^2} - \frac{\left(\sqrt{2}\right)^2}{b^2} = 1 \text{ , yoki } \frac{3}{a^2} - \frac{2}{b^2} = 1.$$

$a^2 = b^2$ bo‘lganligidan, $\frac{3}{a^2} - \frac{2}{a^2} = 1$, ya’ni $a^2 = 1$. Shunday qilib, izlanayotgan giperbola tenglamasi $x^2 - y^2 = 1$ ko‘rinishga ega.

4.4. Markazsiz ikkinchi tartibli chiziqlar. Parabolalar

Agar ikkinchi tartibli chiziqlar simmetriya markaziga ega bo‘lmasa yoki cheksiz ko‘p markazga ega bo‘lsa (ya’ni markazi yagona bo‘lmasa), bunday holda ular markazsiz chiziqlar deyiladi.

Quyidagi ikkinchi tartibli chiziqni qaraymiz:

$$Ax^2 + Cy^2 + Dx + Ey + F = 0 \text{ ,}$$

bu yerda $AC = 0$ va $A^2 + C^2 \neq 0$ deb hisoblaymiz. Aniqlik uchun $A = 0, C \neq 0$ deb olamiz. Bundan tashqari $D \neq 0$ deb faraz qilamiz, aks holda juft parallel chiziqlarga ega bo‘lar edik. Shunday qilib, ikkinchi tartibli chiziq

$$Cy^2 + Dx + Ey + F = 0 \qquad (13)$$

tenglama bilan berilgan bo‘lsin.

(13) tenglamadagi y o‘zgaruvchi qatnashgan hadlarni to‘la kvadratgacha to‘ldirib, quyidagi tenglikni olamiz:

$$C\left(y + \frac{E}{2C}\right)^2 = -Dx - F + \frac{E^2}{4C} \text{ ,}$$

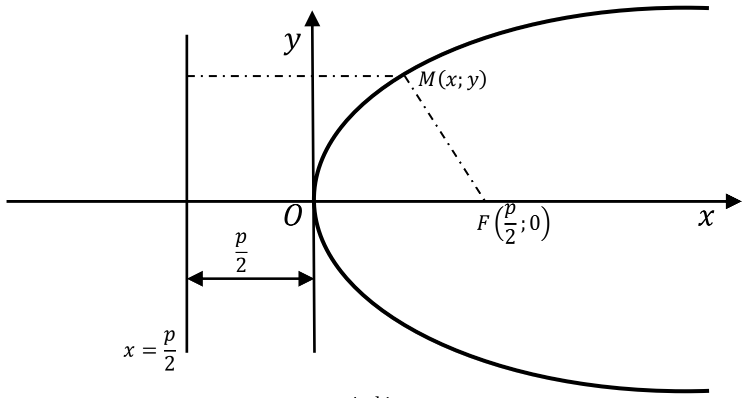
yoki

$$x_0 = -\frac{F}{D} + \frac{E^2}{4DC} \text{ ,} y_0 = -\frac{E}{2C} \text{ ,} 2p = -\frac{D}{C} \text{ ,}$$

belgilashlardan so‘ng, quyidagi tenglamaga kelamiz:

$$(y - y_0)^2 = 2p(x - x_0). \qquad (14)$$

(14) tenglama bilan berilgan chiziqga *parabola* deyiladi (4-chizma); $O'(x_0; y_0)$ nuqta parabolaning *uchi*, p son esa parabola *parametri* deyiladi. Osonlik bilan ko‘rish mumkinki, $y = y_0$ to‘g‘ri chichiq parabolaning *simmetriya o‘qi (parabola o‘qi)* bo‘ladi; (14) parabola simmetriya markaziga ega emas.



4-chizma

Agar parabola uchi koordinatalar boshida bo‘lsa, Ox o‘qi parabola o‘qi bo‘ladi. Bu holda parabolaning *kanonik tenglamasi* deb ataluvchi ushbu

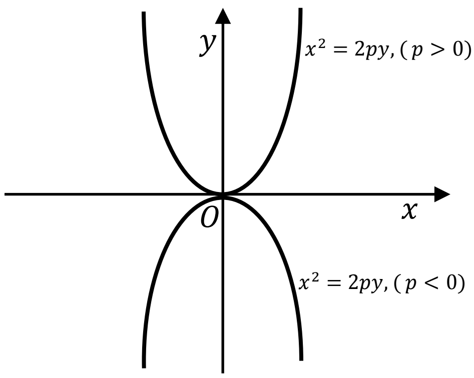
$$y^2 = 2px$$

tenglamani hosil qilamiz, bunda p parametrni odatda musbat deb hisoblaymiz (bunga Ox o‘qining kerakli yo‘nalishini tanlab olish bilan erishish mumkin) (4-chizma).

Ta’kidlaymizki, agar Ox va Oy o‘qlar rollarini almashirsak, u holda parabolaning kanonik tenglamasi quyidagi ko rinishni oladi:

$$x^2 = 2py.$$

Bu – vertikal o‘qqa ega paraboladir (5-chizma).



5-chizma

8-misol. *Ox o‘qqa nisbatan simmetrik va uchi koordinatalar boshida bo‘lgan shunday parabola tenglamasi tuzilsinki, shu parabolaning Ox o‘qqa perpendikulyar bo‘lgan biror vatarining uzunligi 16 ga teng va bu vatardan parabola uchigacha bo‘lgan masofa 6 ga teng bo‘lsin.*

Yechish. Vatar Ox o‘qqa perpendikulyar, uning uzunligi 16 va undan parabola uchigacha bo‘lgan masofa 6 ga teng. Bu ma’lumotlardan parabola vatarining chetki nuqtalari koordinatalari osongina topiladi, ya'ni $M_1(6;8), M_2(6;-8)$. Parabolaning umumiy tenglamasi $y^2 = 2px$. Bunda $x = 6, y = \pm 8$ deb olinsa, $8^2 = 2p \cdot 6$ tenglik o'rinli bo'ladi, yoki $2p = \frac{32}{3}$. Shunday qilib, izlanayotgan parabola tenglamasi $y^2 = \frac{32}{3}x$ bo'ladi.

9-misol. *Uchi koordinatalar boshida, Oy o'qqa simmetrik va I va III koodinata burchaklari bissektrisalaridan $8\sqrt{2}$ uzunlikda kesma ajratuvchi parabola tenglamasini tuzing.*

Yechish. Izlanayotgan parabola tenglamasi $x^2 = 2py$ va ko'rsatilgan koordinatalar burchaklari bissektrisasi tenglamasi esa $y = x$ ko'rinishda bo'ladi.

Parabola va bissektrisaning kesishish nuqtalari: $O(0;0)$ и $M(2p;2p)$. Vatar uzunligi ikki nuqta orasidagi masofa kabi aniqlanadi: $\sqrt{4p^2 + 4p^2} = |OM|$. Masala shartiga ko'ra $|OM| = 8\sqrt{2}$, bo'lgani uchun, $8\sqrt{2} = \sqrt{8p^2}$, bundan esa $2p = 8$. Demak, izlanayotgan parabola tenglamasi $x^2 = 8y$ ko'rinishga ega.

Parabola tenglamasini qaraymiz

$$y^2 = 2px, (p > 0) \tag{14}$$

$F\left(\frac{p}{2};0\right)$ nuqta uning *fokusi*, $x = -\frac{p}{2}$ to'g'ri chiziq esa – *directrisasi* deyiladi.

Parabola $M(x;y)$ nuqtasining $r = |MF|$ fokal radiusi quyidagicha aniqlanadi

$$r = \sqrt{\left(x - \frac{p}{2}\right)^2 + y^2} = \sqrt{x^2 - px + \frac{p^2}{4} + 2px} = \sqrt{x^2 + px + \frac{p^2}{4}} = x + \frac{p}{2}.$$

M nuqtadan direktrisasigacha bo'lgan masofani esa, quyidagi formula orqali topamiz

$$|MN| = x + \frac{p}{2} = r.$$

Shunday qilib, *berilgan nuqta (fokusi) va berilgan to'g'ri chiziqdan (direktrisa) baravar uzoqlikda yotuvch barcha nuqtalar to'plami, parabolani ifodalayd*.

Bu xossa – parabolaning xarakteristik xossasidir.

10-misol. *$y^2 = 8x$ parabolaning fokusi koordinatalarini va directrisa tenglamasini aniqlang.*

Yechish. $y^2 = 8x$ bilan (14) tenglamani taqqoslab, $2p = 8$ teglikni hosil qilamiz; bundan esa $\frac{p}{2} = 2$. Demak, parabola fokusi koordinatalari $(2,0)$ ga teng, direktrisasi tenglamasi esa $x = -2$ bo‘lar ekan.

4.5. Nazorat savollari va masalalar

1. Aylana tenglamasi.
2. Qanday shartlarda $x^2 + y^2 + \alpha x + \beta y + \gamma = 0$ tenglama bilan berilgan chiziq aylanani aniqlaydi?
3. $Ax^2 + Cy^2 + Dx + Ey + F = 0, A \neq 0, C \neq 0$ tenglama bilan berilgan ikkinch tartibli chiziq markazi qanday aniqlanadi?
4. Markazli ikkinchi tartibli chiziqning simmetriya o‘qi deb nimaga aytiladi?
5. Qanday shartlarda $Ax^2 + Cy^2 = H$ ikkinchi tartibli chiziq tenglamasi ellipsni aniqlaydi (maxsus ellips, mavhum ellips tushunchalari)?
6. Qanday shartlarda $Ax^2 + Cy^2 = H$ ikkinchi tartibli chiziq tenglamasi giperbolani aniqlaydi?
7. Ellips va giperbolaning kanonik tenglamalari. Qo‘shma giperbola tenglamasi.
8. Ellipsning o‘qlari, yarim o‘qlari va uchlari tushunchalari.
9. Giperbolaning haqiyqiy va mavhum yarim o‘qlari. Giperbola uchlari.
10. Chiziq qachon markazsiz deyiladi?
11. $Cy^2 + Dx + Ey + F = 0$ parabolaning uchi va simmetriya o‘qlari qanday aniqlanadi?
12. Parabolaning kanonik tenglamasi.
13. Ellips, giperbola va parabolalarning fokal xossalari.
14. $M\left(\frac{5}{2}; \frac{\sqrt{6}}{4}\right)$ va $N\left(-2; \frac{\sqrt{15}}{5}\right)$ nuqtalardan o‘tuvchi ellipsning kanonik tenglamasini tuzing.
15. $\frac{x^2}{25} + \frac{y^2}{16} = 1$ ellipsning chap fokusi va quyi uchidan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.
16. Markazi koordinata boshida bo‘lgan ellips $M(1;1)$ nuqtadan o‘tadi va ekssentrisiteti $\varepsilon = \frac{3}{5}$. Shu ellipsning tenglamasini tuzing.
17. Agar ellipsning yuqori uchidan fokal kesmasi α ($\alpha = 90°$, $\alpha = 120°$) burchak ostida ko‘rinsa, uning ekssentrisitetini toping?

18. $\frac{x^2}{20}+\frac{y^2}{4}=1$ ellipsning yqori uchidan va chap fokusidan baravar uzoqlikda yotuvchi $x+5=0$ to‘g‘ri chiziq nuqtasini toping.

19. Tekislikning $A(0;1)$ nuqtagacha masofasi $y-4=0$ to‘g‘ri chiziqqacha masofadan ikki marta kichik bo‘lgan nuqtalar to‘plamining tenglamasini tuzing.

20. Agar ellipsning fokuslari $F_1(0;0)$, $F_2(1;1)$ nuqtalar va katta o‘qi 2 ga teng bo‘lsa, u holda uning tenglamasini tuzing.

21. Uchi va fokuslari mos ravishda $\frac{x^2}{8}+\frac{y^2}{5}=1$ ellipsning uchi va fokuslarida joylashgan giperbolaning tenglamasini tuzing.

22. $M(0;-2)$ nuqta va $3x^2-4y^2=12$ giperbolaning o‘ng uchidan to‘g‘ri chiziq o‘tkazilgan. Shu to‘g‘ri chiziqnng giperbola bilan ikkinchi kesishish nuqtasining koordinatalarini toping.

23. $\frac{x^2}{64}-\frac{y^2}{36}=1$ giperbolaning chap tarmog‘ida shunday nuqtani topingki, uning o ‘ng fokal radiusi 18 ga teng bo‘lsin.

24. Ekssentrisiteti $\varepsilon=2$ va fokuslari $\frac{x^2}{25}+\frac{y^2}{16}=1$ ellipsning focuslariga teng bo‘lgan giperbola tenglamasini tuzing.

25. $\frac{x^2}{16}-\frac{y^2}{9}=1$ giperbola va $x^2+y^2=81$ aylananing kesishish nuqtalarining fokal radiuslarini toping.

26. Agar parabolaning fokusi $4x-3y-4=0$ to‘g‘ri chiziq bilan Ox o‘qi kesishish nuqtasida yotishi ma’lum bo‘lsa, shu parabolaning sodda tenglamasini tuzing.

27. $y^2=8x$ parabolada shunday nuqtani topingki, undan directrisagacha bo‘lgan masofa 4 ga teng bo‘lsin.

28. Uchi koordinatalar boshida bo‘lgan, Ox o‘qiga nisbatan simmetrik va $y=x$ to‘g‘ri chiziqdan uzunligi $4\sqrt{2}$ ga teng kesma ajratuvchi parabola tenglamasini tuzing.

29. Agar parabola o‘z simmetriya o‘qiga perpendikulyar va uchi bilan fokusi orasidagi masofani teng ikkiga bo‘luvchi to‘g‘ri chiziqdan uzunligi 1 ga teng kesma ajratsa, shu parabolaning sodda tenglamasini tuzing.

30. Uchi koordinatalar boshida bo‘lgan, Ox o‘qiga nisbatan simmetrik va $M(4;2)$ nuqtadan o‘tuvchi parabola tenglamasini tuzing; shu berilgan nuqtaning radius-vektori bilan Ox o‘qi orasidagi α burchakni aniqlang.

II BOB UCHUN TEST SAVOLLARI (2-TEST)

1. $x = 2, y = 3$ – Oxy tekisligidagi to‘g‘ri burchakli koordinatalar sistemasidagi $M(x, y)$ nuqtaning koordinatalari. Mos ravishda abssisa o‘qi, ordinata o‘qi, koordinata boshiga nisbatan M nuqtaga simmetrik bo‘lgan $M_1(x_1, y_1), M_2(x_2, y_2), M_3(x_3, y_3)$ nuqtalarni toping.

$A)$ $M_1(2, -3), M_2(-2,3), M_3(-2, -3)$;

$B)$ $M_1(-3,2), M_2(3, -2), M_3(-3, -2)$;

$C)$ $M_1(-2,3), M_2(2, -3), M_3(-2, -3)$;

$D)$ $M_1(3,2), M_2(3, -2), M_3(-3, -2)$;

$E)$ $M_1(3, -2), M_2(2, -3), M_3(-3, -2)$.

2. Parallel ko‘chirish yordamida O koordinata boshi $O'(-3,4)$ nuqtaga ko‘chirilgan. Yangi $O'x'y'$ koordinatalar sistemasida eski koordinatal sistemasidagi $M(x, y)$ nuqtaning koordinatalari qanday ko‘rinishga ega?

$A)$ $x' = x - 3, y' = y + 4$; $B)$ $x' = x + 3, y' = y - 4$;

$C)$ $x' = x - 3, y' = y - 4$; $D)$ $x' = x + 3, y' = y + 4$;

$E)$ $x' = x - 3, y' = 4 - y$.

3. Koordinata o‘qlarini $\alpha = 30°$ burchakka burulgandan keyin $M(x, y)$ nuqtanig yangi x', y' koordinatalari qanday ko‘rinishga ega?

$A)$ $x' = \frac{\sqrt{3}}{2}x + \frac{1}{2}y,\ y' = -\frac{1}{2}x + \frac{\sqrt{3}}{2}y$;

$B)$ $x' = \frac{\sqrt{3}}{2}x - \frac{1}{2}y, y' = \frac{1}{2}x - \frac{\sqrt{3}}{2}y$;

$C)$ $x' = \frac{\sqrt{2}}{2}x + \frac{1}{2}y, y' = -\frac{1}{2}x + \frac{\sqrt{2}}{2}y$;

$D)$ $x' = \frac{\sqrt{2}}{2}x - \frac{1}{2}y, y' = \frac{1}{2}x - \frac{\sqrt{2}}{2}y$;

$E)$ $x' = -\frac{\sqrt{3}}{2}x + \frac{1}{2}y, y' = \frac{1}{2}x - \frac{\sqrt{3}}{2}y$.

4. Koordinatalar boshi $O'(1,1)$ nuqtaga ko‘chirilgan va o‘qlar $\alpha = 60°$ burchakka burulgandan so‘ng M nuqtaning yangi koordinatalari $x' = 2,\ \ y' = -2$ ga teng. M nuqtaning eski x va y koordinatalarini toping.

$A)$ $x = 1 + \sqrt{3},\ \ y = 2 + \sqrt{3}$; $B)$ $x = \sqrt{3}, y = 1 + \sqrt{3}$;

$C)$ $x = 2 + \sqrt{3}, y = \sqrt{3}$; $D)$ $x = 2 - \sqrt{3}, y = \sqrt{3}$;

$E)$ $x = 1 - \sqrt{3}, y = 2 + \sqrt{3}$.

5. Koordinatalar boshi $O'(1,1)$ nuqtaga ko‘chirilgan va o‘qlar α burchakka burulgandan so‘ng M nuqtaning yangi x', y' koordinatalarini toping?

$A)\ x' = (x+1)\cos\alpha + (y-1)\sin\alpha, y' = (x-1)\sin\alpha + (y-1)\cos\alpha$
$B)\ x' = (x+1)\sin\alpha + (y+1)\cos\alpha, y' = (x-1)\cos\alpha + (y-1)\sin\alpha$
$C)\ x' = (x-1)\sin\alpha + (y-1)\cos\alpha, y' = (x+1)\sin\alpha + (y-1)\cos\alpha$
$D)\ x' = (x-1)\cos\alpha + (y+1)\sin\alpha, y' = -(x-1)\sin\alpha + (y+1)\cos\alpha$
$E)\ x' = (1-x)\cos\alpha + (y-1)\sin\alpha, y' = (1-x)\sin\alpha + (y-1)\cos\alpha.$

6. $M(x,y)$ nuqta koordinatalar boshini $O'(2,-2)$ nuqtaga ko‘chirilgandan bso‘ng x', y' koordinatalarga ega. O' koordinata boshiga nisbatan $M(x',y')$ nuqtaga nisbatan simmetrik bo‘lgan $M_1(x_1,y_1)$ nuqtani toping.

$A)\ x_1 = x-2, y_1 = -y-2;$ $B)\ x_1 = 2-x, y_1 = -y-2;$
$C)\ x_1 = 2-x, y_1 = y+2;$ $D)\ x_1 = x-3, y_1 = -y-3;$
$E)\ x_1 = 3-x, y_1 = -y+3.$

7. Chetki nuqtalari $A(-2,5), B(4,17)$ bo‘lgan $[AB]$ kesma $M(x,y)$ nuqta yordamida $\lambda = \frac{|AM|}{|MB|} = 2$ nisbatda $[AM], [MB]$ kesmalarga ajratilgan. M nuqtaning koordinatalarini toping.

$A)\ x = 3, y = 12;$ $B)\ x = 4, y = 13;$ $C)\ x = 2, y = 12;$
$D)\ x = 5, y = 13;$ $E)\ x = 2, y = 13.$

8. $A(5,-3), B(6,4)$ bolsin, $[AB]$ kesma $M(x,y)$nuqta yordamida teng bo‘lingan. M nuqtaning koordinatalarini toping.

$A)\ x = \frac{13}{2}, y = \frac{7}{2};$ $B)\ x = \frac{11}{2}, y = -\frac{3}{2};$
$C)\ x = 6, y = \frac{5}{2};$ $D)\ x = \frac{11}{2}, y = \frac{1}{2};$ $E)\ x = \frac{9}{2}, y = \frac{3}{2}.$

9. $A(x_1,y_1), B(x_2,y_2), C(x_3,y_3)$ – nuqtalar ABC uchburchakning uchlari bo‘lsin. Uchburchak medianalari kesishish nuqtasi D ning koordinatalarini toping.

$A)\ x = \frac{x_1+x_2+x_3}{2}, y = \frac{y_1+y_2+y_3}{2};$
$B)\ x = \frac{x_1+x_2+x_3}{3}, y = \frac{y_1+y_2+y_3}{3};$
$C)\ x = \frac{x_1+x_2+x_3}{4}, y = \frac{y_1+y_2+y_3}{4};$
$D)\ x = \frac{x_1+2x_2+x_3}{3}, y = \frac{y_1+2y_2+y_3}{3};$
$E)\ x = \frac{x_1+2x_2+x_3}{4}, y = \frac{y_1+2y_2+y_3}{4}.$

10. $M_1(1,2), M_2(2,-1), M_3(-1,2)$ nuqtalarga mos ravishda $m_1 = 1$, $m_2 = 2$, $m_3 = 3$ massalar qo‘yilgan. Bu sistema markazining koordinatalarini toping.

$A)\ x = \frac{1}{3}, y = 1;\ B)\ x = \frac{1}{4}, y = 2;$

$C)\ x = \frac{1}{2}, y = 1\ D)\ x = 2, y = 1;\quad E)\ x = 3, y = 2.$

11. $C(2,3)$ nuqta $[AB]$ kesmani $1:2$ nisbatda bo‘ladi. Agar A nuqtaning koordinatalari $x = 1, y = 2$ bo‘lsa, u holda B nuqtaning koordinatalarini toping.

$A)\ A(3,5);\ \ B)\ A(5,3);\ \ C)\ A(4,5)\ \ D)\ A(5,4);\ \ E)\ A(3,6).$

12. $A(2,3)$ va $B(-10,-2)$; $C(\sqrt{2},-\sqrt{7})$ va $D(2\sqrt{2},0)$ nuqtalar orasidagi masofalarni toping.

$A)\ |AB| = 12,\ |CD| = 2;\qquad B)\ |AB| = \sqrt{166},\ |CD| = \sqrt{10};$

$C)\ |AB| = 11,\ |CD| = 4;\qquad D)\ |AB| = 13,\ |CD| = 3;$

$E)\ |AB| = \sqrt{47},\ |CD| = \sqrt{17}.$

13. Ox o‘qida $A(3,4)$ nuqtadan masofasi 5 ga teng bo‘lgan $M(x,0)$ nuqtani toping.

$A)\ x = 1, x = 6;\ B)\ x = -1, x = 2; C)\ x = 0, x = 6;$

$D)\ x = 2, x = 6;\quad E)\ x = -1, x = 4.$

14. Uchburchak uchlari berilgan: $A(-2,0), B(6,6), C(1,-4)$. A uchidan o‘tkazilgan bissektrisa uzunligi l ni toping.

$A)\ l = \frac{10\sqrt{2}}{3};\ B)\ l = \frac{7\sqrt{2}}{3}; C)\ l = \frac{10\sqrt{3}}{3};\ D)\ l = \frac{10\sqrt{2}}{5}; E)\ l = \frac{7\sqrt{3}}{3}.$

15. Uchburchak uchlari berilgan: $A(-1,0), B(3,3), C(2,-2)$. A uchidan o‘tkazilgan mediana uzunligi m ni toping.

$A)\ m = \frac{3\sqrt{2}}{2};\ B)\ m = \frac{5\sqrt{2}}{2}; C)\ m = 4\sqrt{2};\ D)\ m = 5\sqrt{2};$

$E)\ m = 3\sqrt{2}.$

16. Uchlari $A(-2,-2), B(-1,3)\ , C(3,-1)$ nuqtalarda bo‘lgan uchburchakning S yuzini hisoblang.

$A)\ S = 15;\ B)\ S = 14; C)\ S = 13;\ D)\ S = 12; E)\ S = 16.$

17. Uchlari $A(-2,1), B(2,2)$ и $C(4,y)$ nuqtalarda bo‘lgan ABC uchburchak yuzi 15 ga teng. C uchining y ordinatasini toping.

$A)\ y = 10;\ B)\ y = 8; C)\ y = -8;\ D)\ y = 9; E)\ y = 6.$

18. Ox o‘qigacha bo‘gan masofasi Oy o‘qigach bo‘lgan masofadan ikki baravar uzoqlikda yotuvchi nuqtalari tenglamasini tuzing.

$A) y^2 = 2x^2; B)\ y^2 = 4x^2; C)\ x^2 = 2y^2;$

$D)\ x^2 = 4y^2; E)\ x^2 + 4y^2 = 0.$

19. $A(2,1)$ va $B(-3,0)$ nuqtalardan teng uzoqlikda yotuvchi chiziqning tenglamasini tuzing.

$A)\ y-5x+2=0;\ B)\ 2y+5x-2=0;\ C)\ y+5x+2=0;$
$D)\ y+4x+2=0; E)\ y-4x-2=0.$

20. Qutb koordinatalari bilan berilgan $A(5,0), B\left(6,\frac{\pi}{4}\right)$ nuqtalarning to‘g‘ri burchakli koordinatalari qanday ko‘rinishga ega?

$A)\ A(5,0), B(3\sqrt{2},3\sqrt{2});\ B)\ A(5,1), B(4\sqrt{2},4\sqrt{2});$
$C)\ A(4,2), B(5\sqrt{2},5\sqrt{2});\ D)\ A(0,5), B(3\sqrt{2},3\sqrt{2});$
$E) A(5,2), B(4\sqrt{2},5\sqrt{2}).$

21. To‘g‘ri burchakli koordinatalar sistemasida berilgan $M(3,\sqrt{3})$ nuqtaning qutb koordinatalar sistemasidagi koordinatalari qanday ko‘rinishga ega?

$A)\ \rho=2\sqrt{3}, \varphi=60°;\quad B)\ \rho=2\sqrt{3}, \varphi=30°;$
$C)\ \rho=3\sqrt{3}, \varphi=60°;\quad D)\ \rho=3\sqrt{3}, \varphi=35°;$
$E)\ \rho=3\sqrt{3}, \varphi=45°.$

22. $x^2+y^2=8$ aylana va $x-y=0$ to‘g‘ri chiziqning kesishish nuqtasini toping.

$A) M_1(3,3), M_2(-3,-3);\ B)\ M_1(2,3), M_2(-2,-3);$
$C)\ M_1(3,2), M_2(-3,-2);\ D)\ M_1(2,4), M_2(-2,-4);$
$E) M_1(2,2), M_2(-2,-2).$

23. Chziq parametrik tenglamasi bilan berilgan: $x=acost, y=bsint$. Uning to‘g ri burchakli koordinatalar sistemasidagi tenglamasini toping.

$A)\ x^2+y^2=a^2+b^2;\ B)\ x^2+y^2=\sqrt{a^2+b^2};$
$C)\ x^2+y^2=\sqrt{ab};\quad D)\ \frac{x^2}{a^2}+\frac{y^2}{b^2}=1; E)\ \frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$

24. Quyidagi $x=cos^2t, y=sin^2t$ parametrik ko‘rinishdagi tenglama qanday chiziqni aniqlaydi?

$A)\ y=1-x, 0\le x\le 1;\ B)\ y=1-x;\ C)\ y=1+x;$
$D)\ y=1-x, |x|\le 1; E)\ y=1+x, |x|\le 1.$

25. Chiziqlar berilgan:

1) $x+y-1=0;$ 2) $x^2+y^2=4;$ 3) $x^2+y-3x^2y=0;$
4) $x^2-2xy+y^2+1.$

Ularning qaysi birlari ikkinchi tartibli chiziq?

$A)$ 1,2,4; $B)$ 2,4; $C)$ 2; $D)$ 3,4; $E)$ 2,3,4.

26. To‘g‘ri chiziq tenglamasi berilgan $\frac{x+2\sqrt{5}}{4}+\frac{y-2\sqrt{5}}{2}=0$.Bu tenglamani umumiy shaklga keltiring.

$A)\ x+y+2\sqrt{5}=0;\ B)\ x+2y-2\sqrt{5}=0;$

$C)\ x-y-2\sqrt{5}=0;\ D)\ 2x+y+\sqrt{5}=0;$

$E)\ 2x+y-2\sqrt{5}=0.$

27. Ordinata o‘qini $b=1$ birlik kesmada kesib o‘tuvchi va abssisalar o‘qi bilan musbat yo‘nalishda $\alpha=\frac{\pi}{3}$ burchak tashkil qiluvchi to‘g‘ri chiziq tenglamasini tuzing.

$A)\ y=\sqrt{3}x+1;\ B)\ y=-\sqrt{3}x+1;\ \ C)\ y=\frac{\sqrt{3}}{3}x+1;$

$D)\ y=-\frac{\sqrt{3}}{3}x+1; E)\ y=\sqrt{3}x-1.$

28. $y=-3x+7$ va $y=2x+1$ to‘g‘ri chiziqlar orasidagi φ o‘tkir burchakni toping.

$A)\ \varphi=\frac{\pi}{3};\ B)\ \varphi=\frac{\pi}{6}; C)\ \varphi=\frac{\pi}{4};\ D)\ \varphi=\frac{5\pi}{12}; E)\ \varphi=\frac{\pi}{12}.$

29. $M(1,-1)$ nuqtadan o‘tuvch va $3x-5y+7=0$ chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

$A)\ 5x+3y+2=0;\ B)\ 5x+3y-2=0;\ \ C)\ 10x+6y-6=0;$

$D)\ 10x+6y+6=0; E)\ 6x+3y-3=0.$

30. $M(-1,2)$ nuqtadan o‘tuvchi va $3x-2y+1=0$ chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

$A)\ 3x-2y+8=0;\ B)\ 3x+2y-1=0;\ \ C)\ 3x-2y+7=0;$

$D)\ 6x-3y=0; E)\ 8x+4y=0.$

31. m ning qanday qiymatlarida $mx+5y+6=0, 3x+2y-8=0$ to‘g‘ri chiziqlar parallel bo‘ladi?

$A)\ m=\frac{5}{2};\ B)\ m=\frac{7}{2}; C)\ m=\frac{9}{2};\ D)\ m=\frac{15}{2}; E)\ m=8.$

32. n ning qanday qiymatlarida $3x+ny+7=0\ , 4x+6y-5=0$ to‘g‘ri chiziqlar perpendikulyar bo‘ladi?

$A)\ n=2;\ B)\ n=3; C)\ n=4;\ D)\ n=-3; E)\ n=-2.$

33. $M_1(-1,3)$ va $M_2(2,5)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

$A)\ 2x+3y-7=0;\ B)\ 3x-4y+14=0;$

$C)\ 2x-3y+11=0;\ D)\ 2x+3y-19=0;$

$E)\ 3x-4y+15=0.$

34. $3x-2y+1=0,\ 2x+5y-12=0$ to‘g‘ri chiziqlarning kesishish nuqtasining koordinatalarini toping.

$A)\ x=2, y=1;\ B)\ x=1, y=2;\ \ C)\ x=-1, y=-2;$
$D)\ x=-2, y=-1; E)\ x=\frac{1}{3}, y=\frac{1}{2}.$

35. Uchburchak uchlari berilgan: $A(0,1), B(6,5)$ и $C(12,-1)$. C uchidan tushirilgan balandlik tenglamasini tuzing .

$A)\ 2x+3y-32=0;\ B)\ 2x-3y+32=0;$
$C)\ 3x-2y+32=0;\ D)\ 3x+2y-34=0;$
$E)\ 3x-2y+36=0.$

36. Uchburchak uchlari berilgan: $A(1,1), B(10,13)$ и $C(13,6)$. A burchak bissektrisasi tenglamasini tuzing.

$A)\ 7x-9y+2=0;\ B)\ 9x-7y+3=0;\ \ C)\ 9x+7y+6=0;$
$D)\ 7x+9y-2=0; E)\ 7x-9y+4=0.$

37. $M(5,1)$ nuqtadan o‘tuvchi va и $2x+y-4=0$ to‘g‘ri chiziq bilan $\varphi=\frac{\pi}{4}$ burchak tashkil qiluvchi to‘g‘ri chiziq tenglamasini tuzing.

$A)\ x+3y+8=0, 3x-y+14=0;$
$B)\ x+5y-8=0, 3x+y+14=0;$
C) $x+3y-8=0, 3x-y-14=0;$
$D)\ 3x-4y+5=0, 2x+5y-8=0;$
$E)\ 5x+y-6=0, 3x+4y-7=0.$

38. $3x+y-3\sqrt{10}=0$ va $6x+2y+5\sqrt{10}=0$ parallel to‘g‘ri chiziqlar orasidagi d masofani aniqlang.

$A)\ d=6;\ B)\ d=5; C)\ d=7;\ D)\ d=\frac{11}{2}; E)\ d=\frac{13}{2}.$

39. To‘g‘ri chiziqning umumiy tenglamasi berilgan: $12x-5y-60=0$. Bu tenglamani “kesmalar” ko‘rinishiga keltiring.

$A)\ \dfrac{x}{6}+\dfrac{y}{-5}=1;\ B)\ \dfrac{x}{5}+\dfrac{y}{-12}=1;\ C)\ \dfrac{x}{6}+\dfrac{y}{12}=1;$
$D)\ \frac{x}{5}+\frac{y}{12}=1;\ E)\ \frac{x}{6}+\frac{y}{5}=1.$

40. $2x+3y+5+\lambda(x+8y+6)=0$ to‘g‘ri ciziqlar dastasiga tegishli va $M(1,1)$ nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini toping.

$A)\ 4x-7y+3=0;\ B)\ 5x-6y+2=0;\ \ C)\ 6x-5y+2=0;$
$D)\ 7x-4y-5=0; E)\ 6x-5y+3=0.$

41. $x^2+y^2-2x-4y+1=0$ aylananing $O(a,b)$ markazi koordinatalarini va R radiusini toping.

$A)\ O(1,-2),\ R=2;\ B)\ O(-1,2),\ R=1;\ \ C)\ O(1,2),\ R=2;$
$D)\ O(2,1),\ R=1;\ \ E)\ O(2,1),\ R=2.$

42. $A(1,2)$ nuqtada teng bo‘linadigan $x^2+y^2=49$ aylana vatarining tenglamasini tuzing.

$A)\ x-3y-5=0;\ B)\ x+2y-5=0;\ \ C)\ 2x-y-4=0;$
$D)\ 2x-3y+4=0; E)\ x-2y-4=0.$

43. $x-y-3=0$ to‘g‘ri chiziqqa nisbatan $x^2+y^2-2x-4y+4=0$ aylana simmetrik bo‘lgan aylana tenglamasini tuzing.

$A)\ (x-3)^2+(y-3)^2=1;\ B)\ (x-2)^2+(y-3)^2=1;$
C) $(x-5)^2+(y+2)^2=1;\ D)\ (x+3)^2+(y-2)^2=1;$
$E)\ (x-5)^2+(y-2)^2=1.$

44. $M\left(\frac{5}{2},\frac{\sqrt{6}}{4}\right)$ va $N\left(-2,\frac{\sqrt{15}}{5}\right)$ nuqtalardan o‘tuvchi ellipsning kanonik tenglamasini tuzing.

$A)\ \dfrac{x^2}{10}+\dfrac{y^2}{4}=1;\ B)\ \dfrac{x^2}{10}+\dfrac{y^2}{5}=1;\ \ C)\ \dfrac{x^2}{12}+\dfrac{y^2}{6}=1;$
$D)\ \frac{x^2}{12}+\frac{y^2}{10}=1;\ E)\ \frac{x^2}{10}+y^2=1.$

45. $M(1,1)$ nuqtadan o‘tuvchi eksentrisiteti $\varepsilon=\frac{3}{5}$ bo‘lgan ellipsning kanonik tenglamasini tuzing.

$A)\ \frac{x^2}{\frac{41}{16}}+\frac{y^2}{\frac{41}{25}}=1;\ B)\ \frac{x^2}{\frac{42}{16}}+\frac{y^2}{\frac{42}{25}}=1;\ \ C)\ \frac{x^2}{16}+\frac{y^2}{25}=1;$
$D)\ \frac{x^2}{25}+\frac{y^2}{16}=1;\ E)\ \frac{x^2}{\frac{49}{16}}+\frac{y^2}{\frac{49}{25}}=1.$

46. Giperbola eksentrisitenti $\sqrt{2}$ ga teng bo‘lsin. Bu holda $M(\sqrt{3},\sqrt{2})$ nuqtadan o‘tuvch giperbolaning kanonik tenglamasini tuzing.

$A)\ \frac{x^2}{4}-\frac{y^2}{9}=1;\ B)\ \frac{x^2}{9}-\frac{y^2}{4}=1;\ \ C)\ x^2-y^2=1;$
$D)\ \frac{x^2}{5}-\frac{y^2}{16}=1;\ E)\ \frac{x^2}{16}-\frac{y^2}{5}=1.$

47. Agar giporbola asimptotalari tenglamasi $y=\pm\frac{2\sqrt{2}}{3}x$ ko‘rinishga ega bo‘lsa, u holda $M(9,8)$ nuqtadan o‘tuvchi giperbolaning kanonik tenglamasini tuzing.

$A)\ \frac{x^2}{16}-\frac{y^2}{25}=1;\ B)\ \frac{x^2}{9}-\frac{y^2}{8}=1;\ \ C)\frac{x^2}{25}-\frac{y^2}{16}=1;$
$D)\ \frac{x^2}{12}-\frac{y^2}{8}=1;\ E)\ \frac{x^2}{10}-\frac{y^2}{16}=1.$

48. Agar giperbolaning ekssentrisiteti $\varepsilon=2$ va fokuslari $\frac{x^2}{25}+\frac{y^2}{9}=1$ ellips fokuslari bilan ustma-ust tushsa, u holda giperbola tenglamasini tuzing.

$A)\ \frac{x^2}{12}-\frac{y^2}{4}=1;\ B)\ \frac{x^2}{14}-\frac{y^2}{6}=1;\ \ C)\frac{x^2}{6}-\frac{y^2}{12}=1;$

$D)\ \frac{x^2}{4}-\frac{y^2}{12}=1;\ E)\ \frac{x^2}{6}-\frac{y^2}{14}=1.$

49. Uchi koordinatalar boshida, Oy o‘qqa nisbatan simmetrik va I va III koordinatalar burchaklari bissektrisasidan uzunligi $8\sqrt{2}$ ga teng kesma ajratadigan parabola tenglamasini tuzing.

$A)\ x^2=4y;\ B)\ x^2=6y; C)\ y^2=8x;\ D)\ y^2=4x; E)\ x^2=8y.$

50. Agar parabolaning fokusi Ox o‘qi bilan $4x-3y-4=0$ to‘g‘ri chiziq kesishish nuqtasida bo‘sa, u holda bu parabolaning sodda tenglamasini tuzing.

$A)\ y^2=4x;\ B)\ y^2=8x; C)\ y^2=10x;\ D)\ x^2=4y; E)\ x^2=8y.$

III BOB. FAZODAGI ANALITIK GEOMETRIYADAN MA'LUMOTLAR

§1. FAZODAGI TEKISLIK VA CHIZIQ TEGLAMALARI. TEKISLIK VA TO'G'RI CHIZIQ

1.1. Sirt tenglamasi

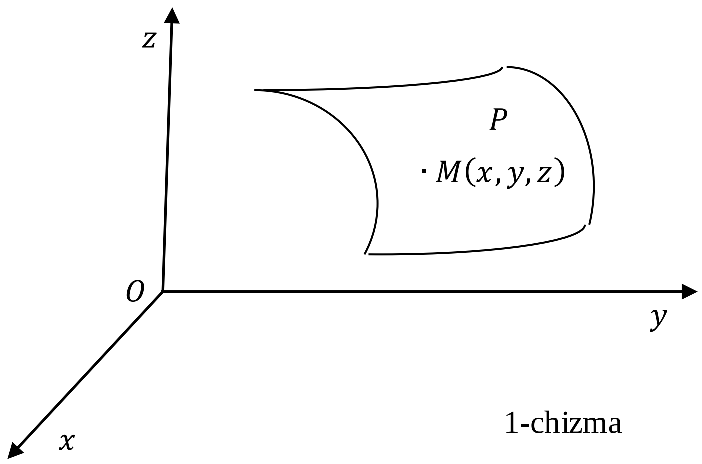
$Oxyz$ to'g'ri burchakli dekart koordinatalar sistemasi kiritilgan uch o'lchamli fazoda P sirtni qaraymiz (1-chizma).

1-ta'rif. *P sirt tenglamasi deb shunday tenglamaga aytiladiki, uni P sirtga tegishli bo'lgan barcha nuqtalarning x, y, z koordinatalari qanoatlantiradi va sirtga tegishli bo'lmaganlarining koordinatalari esa qanoatlantirmaydi.*

Sirtning oshkormas shakldagi tenglamasi quyidagicha yoziladi:

$$F(x, y, z) = 0 \quad (1)$$

Sirtning ixtiyoriy nuqtasining koordinatalari sirt nuqtasining *joriy koordinatalari* deyiladi.



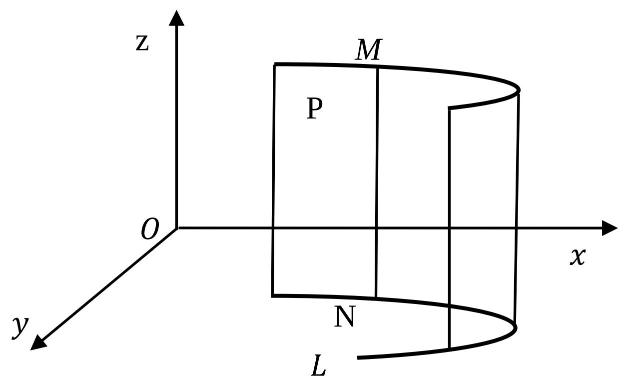
1-chizma

(1) tenglamani P sirtning (1-chizma) faqat joriy koordinatalari qanoatlantiradi.

1-misol. *(Koordinata tekisliklarining tenglamalari).* Oyz koordinatalar tekisligida yotuvchi har qanday $M(x, y, z)$ nuqta $x = 0$ abssisaga ega; aksincha, agar qandaydir $M(x, y, z)$ nuqtaning abssisasi $x = 0$ bo'lsa, u holda bu nuqta Oyz koordinatalar tekisligida yotadi. Shuning uchun, $x = 0$ tenglama Oyz koordinatalar tekisligining tenglamasidir. Xuddi shu kabi, $y = 0$ va $z = 0$, mos ravishda Oxz va Oxy koordinatalar tekisliklarining tenglamalaridir.

Теорема. *Jasovchilari koordinata o‘qlariga parallel bo‘lgan silindrik sirt tenglamalarida shu koordinata o‘qlari bilan bir xil nomli joriy koordinatalar qatnashmaydi va aksincha.*

Isbot. Faraz qilaylik, masalan, P silindrik sirt MN to‘g‘ri chiziqni (*yasovchi*) Oxy tekislikda yotuvchi L chiziq bo‘ylab Oz o‘qqa parallel ($MN||Oz$) tarzda siljitish natijasida hosil qilingan bo‘lsin (2-chizma). $M(x, y, z)$ deb P sirtnig joriy koordinatalari x, y va z bo‘lgan nuqtasini belgilaymiz. M nuqtadan o‘tuvchi MN yasovchi L yo‘naltiruvchini $N(x, y, 0)$ nuqtada kesadi.



2- chizma

Faraz qilaylik,

$$F(x, y) = 0 \tag{2}$$

tenglama Oxy koordinatlar tekisligidagi L yo‘naltiruvchi chiziqning tenglamasi bo‘lsin. Bu tenglamani N nuqtaning koordinatalari qanoatlantiradi. P sirtdagi M nuqta koordinatalari N nuqtaning x abssissasi va y ordinatasi bilan br xil hamda z o‘zgaruvch esa (2) tenglamada qatnashmaganligi uchun M nuqtaning koordinatalari (2) tenglamani qanoatlantiradi. Demak, P silindrik sirtdagi ixtiyoriy $M(x, y, z)$ nuqtaning koordinatalari (2) tenglamani qanoatlantiradi.

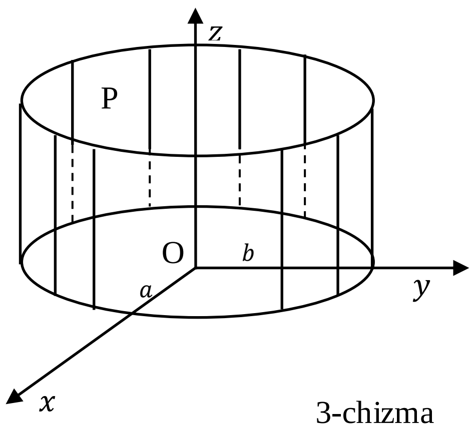
Teskarisi, agar qandaydir $M(x, y, z)$ nuqta (2) tenglamani qanoatlantirsa, u holda bu nuqta $MN||Oz$ to‘g‘ri chiziqda yotadi, chunki uning Oxy tekisligidagi izi bo‘lgan $N(x, y, 0)$ nuqta L chiziqda yotadi. Bu esa M nuqta P silindirik sirtga tegishli ekanligini bildiradi.

Shunday qilib, $F(x, y) = 0$ tenglama $Oxyz$ fazodagi yazovchilari Oz koordinata o‘qiga parallel bo‘lgan *silindrik sirt tenglamasi* bo‘lib, unda z koordinatasi qatnashmaydi.

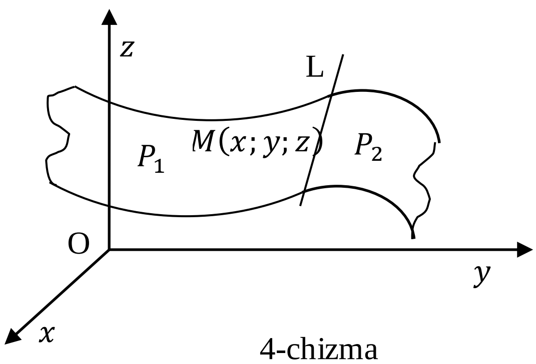
2-misol. (*elliptik silindr tenglamasi).* Yuqoridagi teoremaga asosan, o‘qi Oz bo‘lib, asosida yarim o‘qlari a va b dan iborat ellips yotadigan *elliptik silindrning* (3-chizma) tenglamasi quyidagi ko‘rinishga ega:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Xususiy holda, $a = b$ bo‘lganda, doiraviy silindrning $x^2 + y^2 = a^2$ tenglamasini hosil qilamiz.



3-chizma



4-chizma

1.2. Fazodagi chiziq tenglamasi

Uch o‘lchamli fazodagi L chiziq ikkita P_1 va P_2 sirtlarning kesishishidan hosil bo‘lgan chiziq sifatida aniqlanadi(4-chima).

L chiziqda yotuvchi $M(x, y, z)$ nuqta ham P_1 sirtga, ham P_2 sirtga tegishli, shuning uchun, bu nuqtaning koordinatalari ikkala sirt tenglamalarini ham qanoatlantiradi.

2-ta’rif. *$Oxyz$ fazodagi chiziq tenglamasi deb x, y, z o‘garuvchilarga bog‘liq bo‘lgan shunday bir juft tenglamalarga aytiladiki, chiziqqa tegishli bo‘lgan ixtiyoriy nuqta koordinatalari bu tenglamalar sistemasini qanoatlantiradi, tegishli bo‘lmagan nuqta koordinatalari esa qanoatlantirmaydi.*

Ta’rifga asosan, fazodagi chiziqning *oshkormas tenglamasi* deb shu chiziqni aniqlovchi ikkita sirtning ushbu oshkormas tenglamalari sistemasi tushuniladi:

$$\begin{cases} F_1(x,y,z) = 0, \\ F_2(x,y,z) = 0. \end{cases} \tag{3}$$

Qayd etamizki, berilgan chiziqni har xil sirtlarning kesishishidan hosil qilish mumkin. Shuning uchun, fazodagi chiziqqa (3) tenglamalar sistemasiga teng kuchli bo‘lgan cheksiz ko‘p sirt tenglamalari mos keladi.

3-misol *(kordinata o‘qlarining tenglamalari).* Ox *o‘qini* Oxy *va* Oxz *koordihatalar tekisliklarining kesishishmasi deb qarash mumkin.*

Oxy tekislikda $z = 0$ bo‘ladi, xuddi shunday Oxz tekislikda $y = 0$ bo‘ladi, shuning uchun Ox o‘qning tenglamasi quyidagi ko‘rinishga ega:

$$\begin{cases} y = 0, \\ z = 0 \end{cases}$$

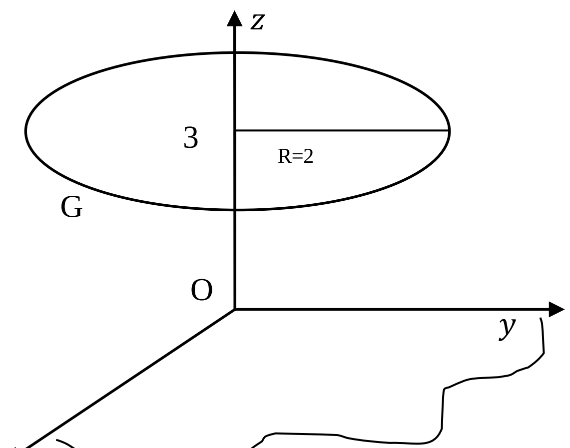
Xuddi shu kabi, Oy va Oz o‘qlarning tenglamalari mos ravishda quyidagi ko‘rinishga ega:

$$\begin{cases} x = 0, \\ z = 0 \end{cases} \quad \text{va} \quad \begin{cases} x = 0, \\ y = 0 \end{cases}$$

4-misol. *Radiusi* $R = 2$, *markazi* $C(0,0,3)$ *nuqtada va* Oxy *koordinata tekisligiga parallel bo‘lgan tekislikda yotuvchi* G *aylananing tenglamasini tuzing.*

Yechish. Masala shartiga ko‘ra, G aylana radiusi 2 ga teng doiraviy silindr bilan Oxy koordinata tekisligiga parallel va Oz o‘qidan 3 birlik masofadan o‘tuvchi tekisliklarning kesishishidan hosil bo‘ladi (5-chizma). Shuning uchun, berilgan G aylananing tenglamasi quyidagi ko‘rinishga ega:

$$\begin{cases} x^2 + y^2 = 4, \\ z = 3 \end{cases}$$



5-chizma

Mexanikada L chiziqni ko‘pincha harakatlanayotgan nuqtaning izi sifatida qaraladi (6-chizma). Faraz qilaylik, L chiziqdagi M nuqtaning joriy koordinatalari x, y, z bo‘lsin. M nuqta vaqt bo‘yicha qo‘zg‘alganda uning koordinatalari ham o‘zgarib boradi va ular t vaqtning funksiyalari bo‘ladi. Shuning uchun, quyidagiga ega bo‘lamiz:

$$x = \varphi(t), y = \psi(t), z = \chi(t)\,, \qquad (4)$$

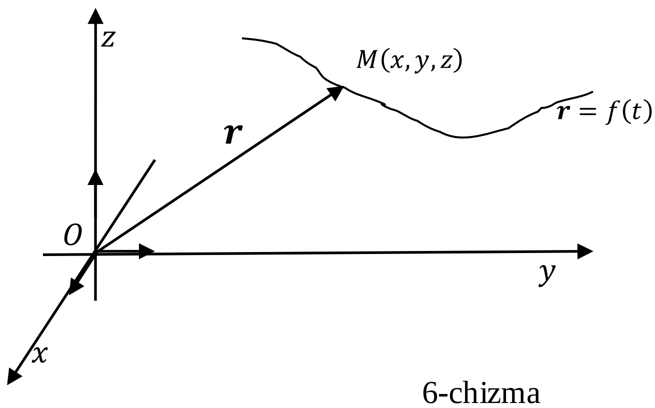
bu yerda φ, ψ, χ – qandaydir brilgan funksiyalar.

(4) tenglamlarda yordamchi t o‘zgaruvchini (parametr) vaqt deb tushunish shart emas. Shuning uchun (4) tenglamalar *chiziqning fazodagi parametrik tenglamalari* deyiladi.

L chiziqning $M(x, y, z)$ joriy nuqtasini $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ ($\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ - ortlar) radius-vektor orqali tavsiflsh mumkin (6-chizma). U holda (4) dan *chiziqning vektor tenglamasini* hosil qilamiz:

$$\boldsymbol{r} = f(t), \qquad (5)$$

bu yerda $f(t) = \varphi(t)\boldsymbol{i} + \psi(t)\boldsymbol{j} + \chi(t)\boldsymbol{k}$ – skalyar t argumentning *vektor-funksiyasi.*



6-chizma

Mexanikada odatda t parametr sifatida vaqt olinadi. Bu holda (5) chiziq tenglamasi harakatlanayotgan M nuqtaning *trayektoriyasini* ifodalaydi.

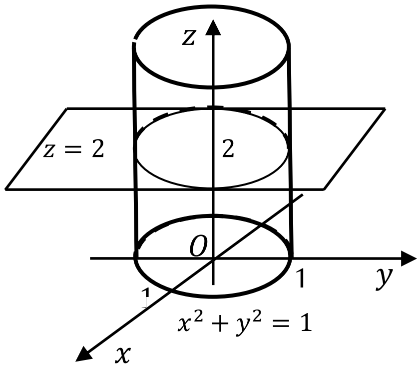
Berilgan (5) tenglamani qanoatlantiruvchi barcha $M(x, y, z)$ nuqtalar to‘plamiga fazodagi chiziq tenglamasining *geometrik obrazi* (yoki *grafigi*) deyiladi.

5-misol. *Quyidagi tenglamaga qanday geometrik obraz mos keladi?*

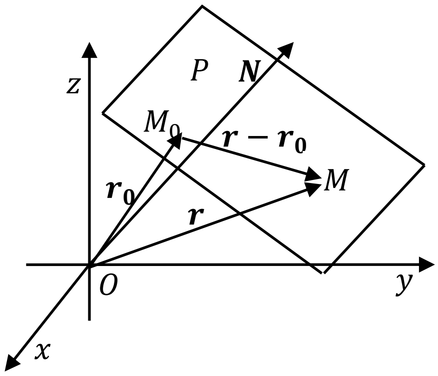
$$(x^2 + y^2 - 1)(z - 2) = 0. \qquad (6)$$

Yechish. (6) tenglamadan $x^2 + y^2 = 1$ yoki $z = 2$ tenglamalarni hosil qilamiz. Bundan (6) tenglamaning grafigi yo‘naltiruvchisi $x^2 +$

$y^2 = 1$ bo‘lgan silindrik sirt va Oxy tekisligiga parallel ravishda Oz o‘qidan musbat ikki birlik masofadan o‘tuvchi $z = 2$ sirtdan iborat (7-chizma).



7-chizma



8-chizma

1.3. Tekislikning umumiy tenglamasi

Fazodagi P tekislik uning $M_0(x_0, y_0, z_0)$ nuqtasi va bu tekislikka perpendikulyar bo‘lgan noldan farqli $\boldsymbol{N} = \{A, B, C\}$ $(A^2 + B^2 + C^2 \neq 0)$ (tekislik *normali* yoki *yo‘naltiruvchi* vector) vektor orqali berilishi mumkin.

Faraz qilaylik, tekislikdagi berilgan M_0 nuqtaning radius-vektori $\boldsymbol{r}_0 = \{x_0, y_0, z_0\}$ va tekislik ixtiyoriy M nuqtasining radius-vektori esa $\boldsymbol{r} = \{x, y, z\}$ bo‘lsin (8-chizma). U holda $\boldsymbol{r} - \boldsymbol{r}_0 = \{x - x_0, y - y_0, z - z_0\}$ vektor berilgan tekislikda yotadi, shuning uchun, bu vector $\boldsymbol{N}$ vektorga ortogonal. Vektorlarning ortogonallik shartidan foydalanib, quyidagiga ega bo‘lamiz:

$$\boldsymbol{N} \cdot (\boldsymbol{r} - \boldsymbol{r}_0) = 0. \quad (7)$$

Bu tenglama *tekislikning vektor shaklidagi* tenglamasidir.

(7) tenglama koordinatalar shaklida quyidagi ko‘rinishda bo‘ladi:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0, \quad (8)$$

yoki

$$Ax + By + Cz + D = 0, \quad (9)$$

bu yerda $D = -Ax_0 - By_0 - Cz_0$.

Har qandy (9) ko‘rinishdagi tenglama biror tekislik tenglamasi ekanliligini ko‘rsatamiz $(A^2 + B^2 + C^2 \neq 0)$. (9) sirtda yotuvchi biror $M_0(x_0, y_0, z_0)$ nuqtani tanlaymiz. Unda

$$Ax_0 + By_0 + Cz_0 + D = 0. \quad (10)$$

(9) tenglikdan (10) tenglikni ayiramiz, natijada quyidagini hosil qilamiz:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0.$$

Bu yerda $\boldsymbol{N} = \{A, B, C\}$, $\boldsymbol{r}_0 = \{x_0, y_0, z_0\}$, $\boldsymbol{r} = \{x, y, z\}$ vektorlarni kiritsak, $\boldsymbol{N} \cdot (\boldsymbol{r} - \boldsymbol{r}_0) = 0$ tenglamaga kelamiz. Demak, (9) tenglama bilan berilgan sirt $M_0(x_0, y_0, z_0)$ nuqtadan o‘tib, $\boldsymbol{N} = \{A, B, C\}$ vektorga perpendikulyar bo‘lgan tekislik ekan.

2-ta’rif. *(9) tenglamaga tekislikning* ***umumiy tenglamasi*** *deyiladi va u x, y, z joriy koordinatalariga nisbatan birinchi tartibli tenglamani ifodalaydi.*

Shunday qilib, tekislik birinchi tartibli sirt ekan.

Agar (7) tenglamada yo‘naltiruvchi vektor o‘rniga $\boldsymbol{n} = \frac{\boldsymbol{N}}{|\boldsymbol{N}|}$ $\left(|\boldsymbol{N}| = \sqrt{A^2 + B^2 + C^2} \neq 0\right)$ birlik vektorini olsak, u holda *normallangan tekislik tenglamasini* hosil qilamiz:

$$\boldsymbol{n} \cdot (\boldsymbol{r} - \boldsymbol{r}_0) = 0,$$

yoki koordinatalarda,

$$\frac{Ax + By + Cz + D}{\sqrt{A^2 + B^2 + C^2}} = 0,$$

bunda $D = -Ax_0 - By_0 - Cz_0$.

6-misol. $M(2{,}3{,}5)$ *nuqtadan o‘tib,* $\boldsymbol{N} = 4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$ *vektorga perpendikulyar bo‘lgan tekislik tenglamasini tuzing.*

Yechish. Masala shartiga ko‘ra $A = 4, B = 3, C = 2$, $x_0 = 2, y_0 = 3, z_0 = 5$ bo‘lgani uchun, (8) ga asosan quyidagini hosil qilamiz:

$4(x - 2) + 3(y - 3) + 2(z - 5) = 0$, yoki $4x + 3y + 2z - 27 = 0$.

1.4. Koordinatalar sistemasiga nisbatan tekisliklarning joylashuvi

(9) umumiy tenglama bilan aniqlangan tekislik joylashuvining xususiy hollarini qaraymiz.

1. $A = 0; By + Cz + D = 0$; Ox o‘qiga parallel bo‘lgan tekislik tenglamasi;

2. $B = 0; Ax + Cz + D = 0$; Oy o‘qiga parallel bo‘lgan tekislik tenglamasi;

3. $C = 0; Ax + By + D = 0$; Oz o‘qiga parallel bo‘lgan tekislik tenglamasi;

4. $D = 0; Ax + By + Cz = 0$; koordinata boshidan o‘tuvch tekislik tenglamasi;

5. $A = B = 0, Cz + D = 0$; Oz o‘qiga perpendikulyar yoki Oxy tekisligiga parallel tekislik tenglamasi;

6. $A = C = 0, By + D = 0$; Oy o‘qiga perpendikulyar yoki Oxz tekisligiga parallel tekislik tenglamasi;

7. $B = C = 0, Ax + D = 0$; Ox o'qiga perpendikulyar yoki Oyz tekisligiga parallel tekislik tenglamasi;

8. $A = D = 0, By + Cz = 0$; Ox o'qidan o'tuvchi tekislik tenglamasi;

9. $B = D = 0, Ax + Cz = 0$; Oy o'qidan o'tuvchi tekislik tenglamasi;

10. $C = D = 0, Ax + By = 0$; Oz o'qidan o'tuvchi tekislik tenglamasi;

11. $A = B = D = 0, C \neq 0$; Oxy $(z = 0)$ tekisligi bilan ustma-ust tushuvchi tekislik tenglamasi;

12. $A = C = D = 0, B \neq 0$; Oxz $(y = 0)$ tekisligi bilan ustma-ust tushuvchi tekislik tenglamasi;

13. $B = D = C = 0, A \neq 0$; Ozy $(x = 0)$ tekisligi bilan ustma-ust tushuvchi tekislik tenglamasi.

Faraz qilaylik $A \neq 0, B \neq 0, C \neq 0, D \neq 0$. Bu holda, (9) tenglamaning har ikkala tomoninidagi barcha hadlarni $-D$ bo'lib, tekislik tenglamasini quyidagi ko'rinishga keltirish mumkin:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, \tag{11}$$

bu yerda $a = -\frac{D}{A}, b = -\frac{D}{B}, c = -\frac{D}{C}$. (11) tenglamaga tekislikning *kesmalarga nisbatan tenglamasi* deyiladi, bu yerda a, b va c – tekislikning Ox, Oy va Oz o'qlari bilan kesishish nuqtasining mos abssissa, ordinata va aplikatalari.

7-misol. $M(2{,}3{,}5)$ *nuqtadan o'tib, koordinata o'qlaridan kongruent kesmalar ajratadigan tekislik tenglamasini tuzing.*

Ye*chish.* (11) tekislikning kesmalarga nisbatan tenglamasida $a = b = c$ deb olinsa, u holda

$$\frac{x}{a} + \frac{y}{a} + \frac{z}{a} = 1.$$

M nuqtaning $x = 5, y = 4, z = 3$ koordinatalari izlanayotgan tekislik tenglamasini qanoatlantirganligi uchun $\frac{5}{a} + \frac{4}{a} + \frac{3}{a} = 1$ tenglik o'rinli. Bundan esa $a = 12$.

Shunday qilib, izlanayotgan tekislik tenglamasi $x + y + z - 12 = 0$ ko'rinishga ega bo'ladi.

1.5. Fazodagi to‘g‘ri chiziq tenglamasi

Fazodagi L to‘g‘ri chiziqni unda yotuvchi $M_0(x_0, y_0, z_0)$ nuqta va yo‘naltiruvchi $\boldsymbol{s} = \{l, m, n\}$ $(l^2 + m^2 + n^2 \neq 0)$, $\boldsymbol{s}\|\|L$, vektor bir qiymatli aniqlaydi.

Faraz qilaylik, L to‘g‘ri chiziqdagi $M_0(x_0, y_0, z_0)$ nuqtaning radius-vektori $\boldsymbol{r_0} = \{x_0, y_0, z_0\}$, shu chiziq ixtiyoriy $M(x, y, z)$ nuqtasining radius-vektori esa $\boldsymbol{r} = \{x, y, z\}$ bo‘lsin. Unda quyidagi tenglik o‘rinli (9-chizma)

$$\boldsymbol{r} = \boldsymbol{r_0} + \overline{M_0M}. \tag{12}$$

$\overline{M_0M}$ va $\boldsymbol{s}$ vektorlari kollinear bo‘lgani uchun $\overline{M_0M} = t\boldsymbol{s}$ tenglik o‘rinli, bunda t–skalyar $(-\infty < t < +\infty)$. Bu ifodani (12) ga olib borib qo‘ysak, *fazodagi to‘g‘ri chiziq tenglamasini* hosil qilamiz

$$\boldsymbol{r} = \boldsymbol{r_0} + t\boldsymbol{s}, \tag{13}$$

bu yerda parametr t skalyar.

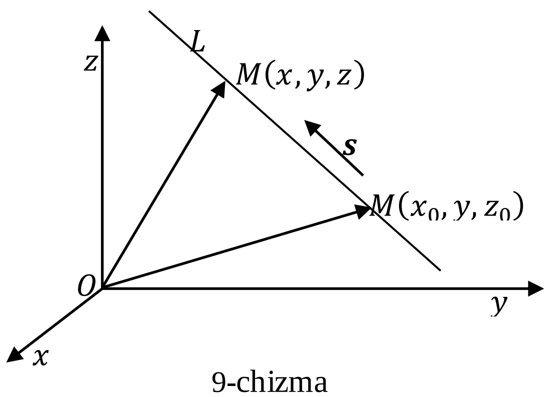
(13) tenglikni koordinata o‘qlariga nisbatan proyeksiyalab, *fazodagi to‘g‘ri chiziqning parametrik tenglamalarini* hosil qilamiz:

$$x = x_0 + lt, y = y_0 + mt, z = z_0 + nt\ . \tag{14}$$

Agar (14) tengliklardan t parametrni yo‘qotsak, u holda *fazodagi to‘g‘ri chiziqning kanonik tenglamasi* deb ataluvchi ushbu tenglamani hosil qilamiz:

$$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}, \tag{15}$$

bu yerda l, m, n sonlar to‘g‘ri chiziq yo‘naltiruvchisining koeffitsiyentlaridir.



9-chizma

L to‘g‘ri chiziqning koordinata o‘qlari bilan hosil qilgan musbat burchaklarni α, β, γ bilan belgilasak hamda $\cos\alpha, \cos\beta, \cos\gamma$ lar $\boldsymbol{s}$ vektorning yo‘naltiruvchi kosinuslari ekanligini hisobga olsak, quyidagi tengliklarni hosil qilamiz:

$$l = |\boldsymbol{s}|\cos\alpha\,, m = |\boldsymbol{s}|\cos\beta\,, n = |\boldsymbol{s}|\cos\gamma, \tag{16}$$

bu yerda $|\boldsymbol{s}| = \sqrt{l^2 + m^2 + n^2}$ – $\boldsymbol{s}$ vektorning uzunligi.

(16) tengliklarni e'tiborga olsak, (15) tenglamani quyidagicha yozish mumkin:

$$\frac{x-x_0}{\cos\alpha} = \frac{y-y_0}{\cos\beta} = \frac{z-z_0}{\cos\gamma}\,. \tag{17}$$

bu yerda $\cos\alpha\,, \cos\beta\,, \cos\gamma$ – to'g'ri chiziqning yo'naltiruvchi kosinuslari.

Fazodagi L to'g'ri chiziq P_1 va P_2 tekisliklarning kesishmasi sifatida ham berilishi mumkin:

$$\begin{cases} A_1x + B_1x + C_1x + D_1 = 0, \\ A_2x + B_2x + C_2x + D_2 = 0 \end{cases}$$

bu yerda P_1 va P_2 tekisliklarni parallel emas va ustma-ust tushmaydi deb faraz qilinadi. $\boldsymbol{N}_1 = \{A_1, B_1, C_1\}$ va $\boldsymbol{N}_2 = \{A_2, B_2, C_2\}$ vektorlar bu tekisliklarining mos normallari bo'ladi. L chiziqning $\boldsymbol{s}$ yo'naltiruvchsi quyidagi shartlarni qanoatlantiradi: $\boldsymbol{s} \perp \boldsymbol{N}_1$ va $\boldsymbol{s} \perp \boldsymbol{N}_2$. Unda $\boldsymbol{s} = \boldsymbol{N}_1 \times \boldsymbol{N}_2$ deb olish mumkin (ya'ni, $\boldsymbol{s}$ vektor, $\boldsymbol{N}_1$ va $\boldsymbol{N}_2$ vektorlarning vektor ko'paytmasiga teng).

8-misol. Berilgan $x - 2y + 3z - 4 = 0, 3x - 2y + z = 0$ *tekisliklarning kesishishidan hosil bo'lgan to'g'ri chiziq tenglamasini yozing va uning yo'naltiruvchi kosinuslarini aniqlang.*

Yechish. Masala shartiga ko'ra, $\boldsymbol{N}_1 = \{1, -2{,}3\}$ va $\boldsymbol{N}_2 = \{3, -2{,}1\}$. U holda

$$\boldsymbol{s} = \boldsymbol{N}_1 \times \boldsymbol{N}_2 = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1 & -2 & 3 \\ 3 & -2 & 1 \end{vmatrix} = 4\boldsymbol{i} + 8\boldsymbol{j} + 4\boldsymbol{k} = 4(\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}).$$

To'g'ri chiziqning yo'naltiruvchi vektori sifatida $\boldsymbol{s}_0 = \frac{1}{4}\boldsymbol{s} = \{1{,}2{,}1\}$ vektorni olamiz. $M_0(-2, -3{,}0)$ nuqta chiziqda yotadi. Demak, berilgan to'g'ri chiziqning kanonik tenglamasi quyidagi ko'rinishga ega:

$$\frac{x+2}{1} = \frac{y+3}{2} = \frac{z}{1}.$$

Yo'naltiruvchi vektor uzunligi $|\boldsymbol{s}_0| = \sqrt{6}$ ga teng. Shunday qilib, berilgan to'g'ri chiziqning yo'naltiruvchi kosinuslari

$\cos\alpha = \frac{1}{\sqrt{6}}, \cos\beta = \frac{2}{\sqrt{6}}, \cos\gamma = \frac{1}{\sqrt{6}}$ ga teng ekan.

1.6. Nazorat savollari va masalalar

1. Fazodagi sirt tenglamasining ta'rifini bering.

2. Sirtning oshkormas shakldagi tenglamasi qanday ko‘rinishga ega?

3. Yasovchisi koordinata o‘qiga parallel bo‘lgan silindirik sirtning oshkormas tenglamasi qanday ko‘rinishga ega?

4. Fazodagi chiziq tenglamasi ta’rifini bering.

5. Fazodagi chiziqnung oshkormas tenglamasi qanday ko‘rinishga ega?

6. Ox, Oy va Oz koordinata o‘qlarining tenglamalari qanday aniqlanadi?

7. Oxy, Oxz va Oyz koordinata tekisliklarining tenglamalari qanday yoziladi?

8. Fazodagi chiziqnung vektor tenglamasi ta’rifini bering.

9. Tekislikning umumiy tenglamasi. Tekislik tenglamasining vektor shakli.

10. Tekislikning normal tenglamasi.

11. $Oxy(Oxz,Oyz)$ koordinata tekisligiga parallel bo‘lgan tekislik tenglamasi qanday ko‘rinishga ega?

12. $Ox(Oy, Oz)$ o‘qqa parallel bo‘lgan tekislik tenglamasi qanday ko‘rinishga ega?

13. $Ox(Oy, Oz)$ o‘qdan o‘tuvchi tekislik tenglamasi qanday ko‘rinishga ega?

14. Kordinata boshidan o‘tuvchi tekislik tenglamasi qanday ko‘rinishga ega?

15. Tekislikning kesmalarga nisbatan tenglamasi ta’rifini bering.

16. Fazodagi to‘g‘ri chiziqning vektor va parametrik ko‘rinishdagi tenglamalari.

17. Fazodagi to‘g‘ri chiziqning kanonik tenglamasi. Chiziqning yo‘naltiruvchi kosinuslari.

18. Ikki tekislikning kesishishidan hosil bo‘lgan to‘g‘ri chiziq tenglamasi ta’rifi.

19. Quyida berilgan tenglamalarga fazoda qanday geometrik obrazlar mos keladi: a) $xy = 0$; b) $xz = yz$; c) $z^2 = 2x$; d) $y^2 + y - 2 = 0$; e) $x^2 + y^2 = 0$; f) $x^2 + y^2 + z^2 = 0$.

20. Oz o‘qiga parallel bo‘lib Ox va Oy oqlardan mos ravishda 2 va 3 uzunlik birligida kesma ajratuvchi tekislik tenglamasini yozing.

21. $M(1,3,4)$ nuqtadan o‘tib Oz o‘qqa perpendikulyar bo‘lgan tekislik tenglamasini yozing.

22. Koordinatalari $(A_1x + B_1y + C_1z + D_1)^2 + (A_2x + B_2y + C_2z + D_2)^2 = 0$ tenglamani qanoatlantiruvchi nuqtalarning geometrik o‘rni nimani ifodalaydi?

23. Quyidagi tekislik tenglamalarini normal shakilga keltiring:

1) $x + y - z - 2 = 0$; 2) $3x + 5y - 4z + 7 = 0$.

24. Koordinata boshidan tushirilgan perpendikulyarning asosi $M(4, -3,12)$ nuqtada bo‘lgan tekislik tenglamasini toping.

25. Agar tekislik $M(2, -1,4)$ nuqtadan o‘tib, Oz o‘qni Ox va Oy o‘qlarga nisbatan ikki baravar ko‘p uzunlik birligida kesib o‘tgan bolsa, u holda bu tekislik tenglamasini toping.

26. To‘g‘ri chiziq quyidagi tenglamalari bilan berilgan: $2x - y + 3z - 1 = 0, 5x + 4y - z - 7 = 0$. Shu chiziq tenglamalarini kanonik shakldа yozing va yo‘naltiruvchi kosinuslarini toping.

27. $M(5,3,4)$ nuqtadan o‘tib, $\boldsymbol{s} = 2\boldsymbol{i} + 5\boldsymbol{j} - 8\boldsymbol{k}$ vektorga parallel bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

28. $M(1,1,1)$ nuqtadan o‘tib, $\boldsymbol{s}_1 = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ va $\boldsymbol{s}_2 = 3\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ vektorlarga perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

29. $\begin{cases} 2x + 3y - 16z - 7 = 0, \\ 3x + y - 17z = 0 \end{cases}$ tenglamalar bilan berilgan to‘g‘ri chiziq tenglamasini kanonik shakilga keltiring.

30. $M(5, -1, -3)$ nuqtadan otib, $\begin{cases} 2x + 3y + z - 6 = 0, \\ 4x - 5y - z + 2 = 0 \end{cases}$ to‘g‘ri chiziqqa parallel bo‘lgan to‘g‘ri chiziq tenglamasini toping.

§2. TO‘G‘RI CHIZIQ VA TEKISLIKLARGA DOIR ASOSIY MASALALAR

2.1. Tekisliklar orasidagi burchak. Tekisliklarning parallellik va perpendikulyarlik shartlari.

Ikki tekislik tenglamalari berilgan bo‘lsin:

$$A_1x + B_1y + C_1z + D_1 = 0, \qquad (1)$$

$$A_2x + B_2y + C_2z + D_2 = 0. \qquad (2)$$

Bu tekisliklarning normal (yo‘naltiruvchi) vektorlari: $\boldsymbol{N}_1 = \{A_1, B_1, C_1\}$, $\boldsymbol{N}_2 = \{A_2, B_2, C_2\}$.

(1) va (2) tekisliklar orasidagi ikki yoqli burchak deb $\boldsymbol{N_1}$va $\boldsymbol{N_2}$ vektorlar tashkil qilgan φ burchakka aytiladi. Shunday qilib, quyidagiga ega bo‘lamiz:

$$\cos\varphi = \frac{\boldsymbol{N}_1 \cdot \boldsymbol{N}_2}{|\boldsymbol{N}_1| \cdot |\boldsymbol{N}_2|}.$$

Ma'lumki, $\boldsymbol{N}_1 \cdot \boldsymbol{N}_2 = A_1A_2 + B_1B_2 + C_1C_2, |\boldsymbol{N}_1| = \sqrt{A_1^2 + B_1^2 + C_1^2}$, $|\boldsymbol{N}_2| = \sqrt{A_2^2 + B_2^2 + C_2^2}$. Shuning uchun quyidagi tenglikni hosil qilamiz:

$$\cos\varphi = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}}. \qquad (3)$$

1-misol. $x - z = 0, y - z = 0$ *tekisliklar orasidagi* φ *burchakni aniqlang.*

Yechish. Masala shartiga ko'ra $\boldsymbol{N}_1 = \{1{,}0,-1\}$, $\boldsymbol{N}_2 = \{0{,}1,-1\}$. (3) formulaga asosan

$$\cos\varphi = \frac{1\cdot0 + 1\cdot0 + (-1)\cdot(-1)}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2}.$$

Bu yerdan esa, $\varphi = 60°$.

Agar (1) va (2) tekisliklarning $\boldsymbol{N}_1 = \{A_1, B_1, C_1\}$ $\boldsymbol{N}_2 = \{A_2, B_2, C_2\}$ normal (yo'naltiruvchi) vektorlari parallel bo'lsalar, ya'ni ularning koordinatalari proporsional bo'lsa:

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}, \qquad (4)$$

bu tekisliklar parallel bo'ladilar.

(1) va (2) tekisliklar perpendikulyar bo'lishi uchun ularning normal vektorlari perpendikulyar bo'lishi zarur va yetarli, ya'ni $\boldsymbol{N}_1 \cdot \boldsymbol{N}_2 = 0$. Bu shart koordinatalar shaklida quyidagi ko'rinishda yoziladi:

$$A_1A_2 + B_1B_2 + C_1C_2 = 0. \qquad (5)$$

Takidlaymizki, agar (1) va (2) tekisliklar uchun (4) shart bajarilmasa, u holda ular parallel ham emas, ustma-ust ham tushmaydi, ya'ni kesishadi.

2-misol. $P(2{,}0,-1)$, $Q(1,-1{,}3)$ *nuqtalardan o'tuvch va* $3x + 2y - z + 5 = 0$ *tekislikga perpendikulyar tekislik tenglamasini toping.*

Yechish. Faraz qilaylik, izlanayotgan tekislikning yonaltiruvchi vektori $\boldsymbol{N} = \{A, B, C\}$ bo'lsin. U holda masala shartiga ko'ra, $\boldsymbol{N} \perp \boldsymbol{N}_1$, $\boldsymbol{N} \perp \boldsymbol{N}_2$, bu yerda $\boldsymbol{N}_1 = \overline{QP} = \{-1,-1{,}4\}$, $\boldsymbol{N}_2 = \{3{,}2,-1\}$. Demak,

$$\begin{cases} A\cdot(-1) + B\cdot(-1) + C\cdot 4 = 0 \\ A\cdot 3 + B\cdot 2 + C\cdot(-1) = 0 \end{cases}.$$

Bundan, $A = -7C$, $B = 11C$. Izlanayotgan tekislik $P(2{,}0,-1)$ nuqtadan o'tganligi uchun uning tenglamasi

$$A(x-2) + B(y-0) + C(z+1) = 0,$$

ko'rinishga ega, ya'ni $-7C(x-2) + 11C(y-0) + C(z+1) = 0$. Bu yerdan esa $-7(x-2) + 11(y-0) + z + 1 = 0$, yoki $7x - 11y - z - 15 = 0$ izlanayotgan tekislik tenglamasi hosil bo'ladi.

2.2. Uch tekislikning o‘zaro joylashuvi

Faraz qilaylik uchta har xil tekisliklar berilgan:

$$\begin{cases} A_1x + B_1y + C_1z + D_1 = 0, \\ A_2x + B_2y + C_2z + D_2 = 0, \\ A_3x + B_3y + C_3z + D_3 = 0. \end{cases} \tag{6}$$

Agar bu tekisliklar bir nuqtada kesishsa, u holda (6) sistema yagona yechimga ega. Algebra kursidan ma’lumki, (6) sistema yagona yechimga ega bo‘lishi uchun uning asosiy determinant noldan farqli bo‘lish kerak, ya’ni

$$\Delta = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_2 & B_2 & C_2 \end{vmatrix} \neq 0. \tag{7}$$

Δ determinant $\boldsymbol{N}_1 = \{A_1, B_1, C_1\}, \boldsymbol{N}_2 = \{A_2, B_2, C_2\}$, $\boldsymbol{N}_3 = \{A_3, B_3, C_3\}$ vektorlarning aralash ko‘paytmasiga teng. Shuning uchun, (7) shart tekisliklarning $\boldsymbol{N}_1$, $\boldsymbol{N}_2$, $\boldsymbol{N}_3$ yo‘naltiruvchi vektorlari bu tekisliklarning birortasiga ham parallel emasligini bildiradi.

Agar (6) $\Delta = 0$ bo‘lsa, u holda tekisliklar biror to‘g‘ri chiziqqa parallel bo‘ladi, ya’ni bu shart $\boldsymbol{N}_1$, $\boldsymbol{N}_2$, $\boldsymbol{N}_3$ vektorlar biror tekislikka parallel ekanligini bildiradi. Agar bu holda sistema birgalikda(yechimga ega) bo‘lsa, u holda tekisliklar to‘g‘ri chiziq bo‘ylab kesishadilar.

3-misol. Berilgan tekisliklarning o‘zaro joylashishini tekshiring:

$$\begin{cases} x - y + z - 1 = 0, \\ 2x + y - z + 2 = 0, \\ x - y + 2z - 5 = 0. \end{cases}$$

Yechish. Sistemaning determinantini hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 3 \neq 0\ .$$

Demak, tekisliklar bir nuqtada kesishadi.

4-misol. Brilgan tekisliklarning o‘zaro joylashishini aniqlang:

$$\left.\begin{aligned} x + y + z - 3 = 0, \\ y + z - 3 = 0, \\ y + z - 1 = 0. \end{aligned}\right\}$$

Yechish. Sistemaning determinantini hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0.$$

Demak, tekisliklar bitta chiziqqa parallel. $x+y+z-3=0$, $y+z-3=0$ tekisliklari $x=0$, $y+z=3$ to‘g‘ri chiziq bo‘ylab kesishganligi uchun, barcha berilgan tekisliklar, $x=0$, $y+z=3$ to‘g‘ri chiziqqa parallel bo‘ladi.

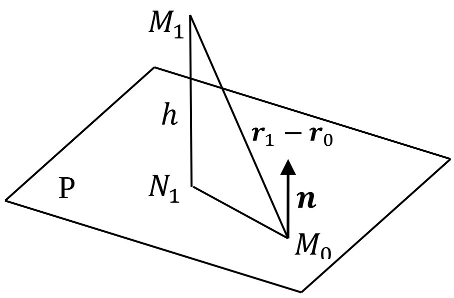
2.3. Nuqtadan tekislikgacha bo‘lgan masofa

Berilgan nuqtadan berilgan tekislikgacha bo‘lgan masofani topish masalasin qaraymiz.

Biror $\mathrm{M}_1(\mathrm{x}_1,\mathrm{y}_1,\mathrm{z}_1)$ nuqta va

$$Ax+By+Cz+D=0 \tag{8}$$

tenglama bilan berilgan P tekislikni qaraymiz. $h=|M_1N_1|$ bo‘lsin, bu yerda $N_1\in P, \boldsymbol{M_1N_1}\perp P$. $\boldsymbol{M_0M_1}=\boldsymbol{r}_1-\boldsymbol{r}_0$ vektorni qaraymiz, bu yerda $\boldsymbol{r}_0$ va $\boldsymbol{r}_1$ – mos ravishda $M_0\in P$ va M_1 nuqtalarning radius-vektorlari (1-chizma).



1-chizma

$\boldsymbol{M_1N_1}\|\boldsymbol{n}$ ekanliligini hisobga olib, $M_0M_1N_1$ uchburchakdan quyidagi tenglikni olamiz:

$$h=|\text{пр}_{\boldsymbol{n}}(\boldsymbol{r}_1-\boldsymbol{r}_0)|=|\boldsymbol{n}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_0)|.$$

Endi, $\boldsymbol{n}=\frac{\boldsymbol{N}}{|\boldsymbol{N}|}, \boldsymbol{N}=\{A,B,C\}$, $|\boldsymbol{N}|=\sqrt{A^2+B^2+C^2}$,

$\boldsymbol{r}_1-\boldsymbol{r}_0=\{x_1-x_0,y_1-y_0,z_1-z_0,\}$, $Ax_0+By_0+Cz_0+D=0$ formulalarni e‘tiborga olsak, u holda

$$h=\frac{|A(x_1-x_0)+B(y_1-y_0)+C(z_1-z_0)|}{\sqrt{A^2+B^2+C^2}}=\frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}. \tag{9}$$

Xususiy holda, $x_1=0, y_1=0, z_1=0$ deb olinsa, koordinata boshidan (8) tekislikgacha bo lgan masofani topish formulasini hosil qilamiz:

$$h_0=\frac{|D|}{\sqrt{A^2+B^2+C^2}}.$$

$d=Ax_1+By_1+Cz_1+D$ ifodaning ishorasiga qarab $M_1(x_1,y_1,z_1)$ nuqta va tekislikning koordinata boshiga nisbatan joylashishni aniqlash mumkin. Agar $d>0$ bo‘lsa, u holda M_1 nuqta va koordinata boshi tekislikning turli tomonlarida joylashadi; agar $d<0$ bo‘lsa, u holda M_1 nuqta va koordinata boshi tekislikdan bir tomonida joylashadi.

5-misol. *$M(1,3,-2)$ nuqtadan $x-3y-2z+12=0$ tekislikgacha bo‘lgan masofani toping. M nuqta tekislikga nisbatan qanday joylashgan?*

Yechish. (9) formulaga asosan

$$h=\frac{|1\cdot 1-3\cdot 3-2\cdot(-2)+12|}{\sqrt{1^2+(-3)^2+(-2)^2}}=\frac{8}{\sqrt{14}}.$$

$d=1\cdot 1-3\cdot 3-2\cdot(-2)+12=8>0$ bo‘lgani uchun $M(1,3,-2)$ nuqta va koordinata boshi $x-3y-2z+12=0$ tekislikning turli tomonida joylashgan.

2.4. To‘g‘ri chiziq va tekislikning o‘zaro joylashuvi

Tekislik va to‘g‘ri chiziq tenglamalari berilgan:

$$Ax+By+Cz+D=0, \tag{10}$$

$$\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}. \tag{11}$$

(11) to‘g‘ri chiziq va (10) tekislik orasidagi φ burchak quyidagi formula bilan aniqlanadi

$$\sin\varphi=\frac{|Al+Bm+Cn|}{\sqrt{A^2+B^2+C^2}\cdot\sqrt{l^2+m^2+n^2}}\,. \tag{12}$$

Bu yerdan to‘g‘ri chiziq va tekislikning parallellik shartini:

$$Al+Bm+Cn=0, \tag{13}$$

va to‘g‘ri chiziq va tekislikning perpendikulyarlik shartini hosil qilamiz:

$$\frac{A}{l}=\frac{B}{m}=\frac{C}{n}. \tag{14}$$

Agar (13) bilan birgalikda $Ax_0+By_0+Cz_0+D=0$ tenglik bajarilsa, u holda (11) to‘g‘ri chiziq (10) tekislikda yotadi.

Agar $Al+Bm+Cn\neq 0$ bo‘sa, u holda (11) to‘g‘ri ciziq (10) tekislik bilan kesishadi. To‘g‘ri chiziq va tekislikning kesishish nuqtasini topish uchun ularning tenglamalarini birgalikda yechish kerak. Bunda to‘g‘ri chiziqning parametrik tenglamalaridan foydalanamiz:

$$x=x_0+lt, y=y_0+mt, z=z_0+nt, -\infty<t<+\infty.$$

(11) to‘g‘ri ciziq va (10) tekislikning $M_1(x_1,y_1,z_1)$ kesishish nuqtasining koordinatalari quyidagicha ifodalanadi:

$$x_1=x_0-l\,\frac{Ax_0+By_0+Cz_0+D}{Al+Bm+Cn},$$

$$y_1 = y_0 - m\frac{Ax_0 + By_0 + Cz_0 + D}{Al + Bm + Cn},$$
$$z_1 = z_0 - n\frac{Ax_0 + By_0 + Cz_0 + D}{Al + Bm + Cn}.$$

To‘g‘ri chiziq ikki tekislikning kesishishidan hosil bo‘lgan holda, ya’ni

$$A_1x + B_1y + C_1z + D_1 = 0,$$
$$A_2x + B_2y + C_2z + D_2 = 0,$$

to‘g‘ri ciziq va (10) tekislikning o‘zaro joylashishini aniqlash uchun avvalo to‘g‘ri chiziq tenglamasini (11) ko‘rinishda kanonik shaklda yozib olish kerak. Bu holda:

$$l = \begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix}, m = \begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix}, n = \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}.$$

Ta’kidlaymizki, bu yerda ham (12) formula va (13), (14) shartlardan foydalansh mumkin.

6-misol. $\frac{x-1}{1} = \frac{y+2}{1} = \frac{z}{-\sqrt{2}}$ *to‘g‘ri ciziq koordinata tekisliklari bilan qanday burchaklarni tashkil qiladi?*

Yechish. Berilgan to‘g‘ri chiziqning $\boldsymbol{s} = \{l, m, n\}$ yo‘naltiruvchi vektorining koordinatalari $l = 1, m = 1, n = -\sqrt{2}$ ga teng. Faraz qilaylik, φ, ψ, θ burchaklar to‘g‘ri chiziqning Oxy, Oyz, Oxz koordinata tekisliklari bilan hosil qilgan mos burchaklar bo‘lsin. $z = 0$ – Oxy tekislikning tenglamasi, $A = B = D = 0$; (12) formuladan foydalansak, unda:

$$\sin\varphi = \frac{|1\cdot(-\sqrt{2})|}{\sqrt{1^2+1^2+(-\sqrt{2})^2}} = \frac{\sqrt{2}}{2}, \varphi = \frac{\pi}{4}.$$

Xuddi shunga o‘xshash:

$x = 0$ – Oyz tekislikning tenglamasi, $B = D = C = 0$; $\sin\psi = \frac{1}{2}, \psi = \frac{\pi}{6}$.

$y = 0$ – Oxz tekislikning tenglamasi, $A = D = C = 0$; $\sin\theta = \frac{1}{2}, \theta = \frac{\pi}{6}$.

2.5. Ikki to‘g‘ri chiziqning o‘zaro joylashuvi

Agar ikki to‘g‘ri chiziq berilsa, u holda ular orasidagi burchakni topish qiyin emas. Buning uchun ularning yo‘naltiruvch vektorlari orasidagi burchakni topish yetarli.

To‘g‘ri chiziqlar kanonik tenglamalari bilan berilgan:

$$\begin{cases} \frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}, \\ \frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}. \end{cases} \qquad (13)$$

Ular orasidagi φ burchak quyidagi formula bilan aniqlanadi

$$\cos\varphi = \frac{l_1 l_2 + m_1 m_2 + n_1 n_2}{\sqrt{l_1^2 + m_1^2 + n_1^2} \cdot \sqrt{l_2^2 + m_2^2 + n_2^2}}. \qquad (14)$$

Bundan esa quyidagi shartlarni hosil qilamiz. Ikki to‘g‘ri chiziqning parallelik sharti:

$$\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2} \qquad (15)$$

va ikki to‘g‘ri chiziqning perpendikulyarlik sharti:

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0. \qquad (16)$$

Xususiy holda, agar (15) shart bajarilsa va bir to‘g‘ri chiziqning nuqtalari ikkinchisini qanoatlantirsa, ya’ni

$$\frac{x_2-x_1}{l_1} = \frac{y_2-y_1}{m_1} = \frac{z_2-z_1}{n_1}.$$

bo‘lsa, (13) chiziqlar ustma-ust tushadi.

(13) to‘g‘ri chiziqlarning bir tekislikda yotishining zaruriy va yetarli sharti (*ikki to‘g‘ri chiziqning komplanarlik sharti*) quyidagi bilan ifodalanadi:

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0. \qquad (17)$$

Agar l_1, m_1, n_1 miqdorlar l_2, m_2, n_2 miqdorlarga proporsional bo‘lmasa, ya’ni (15) munosobat bajarilmasa, u holda (17) shart fazodagi ikki to‘g‘ri chiziqning kesishishining zarur va yetarli sharti bo‘ladi.

7-misol. $\frac{x}{2} = \frac{y}{-3} = \frac{z}{n}$ *to‘g‘ri chiziqdagi* n *parametrni shunday aniqlangki, bu to‘g‘ri chiziq* $\frac{x+1}{3} = \frac{y+5}{2} = \frac{z}{1}$ *to‘gri chiziq bilan kesishsin. Shu kesishish nuqtasini toping.*

Yechish. n ning har qanday qiymatida bu to‘g‘ri chiziqlar parallel emas $\left(\frac{2}{3} \neq \frac{-3}{2}\right)$. Shuning uchun (17) shart bajarilganda ular kesishadi. (17) tenglamada

$x_1 = -1, y_1 = -5, z_1 = 0, x_2 = 0, y_2 = 0, z_2 = 0, l_1 = 3, m_1 = 2, n_1 = 1,$
$l_2 = 2, m_2 = -3, n_2 = n$ ekanligini e’tiborga olsak, unda:

$$\begin{vmatrix} 1 & 5 & 0 \\ 3 & 2 & 1 \\ 2 & -3 & n \end{vmatrix} = 0, \text{ yoki } 2n + 10 + 3 - 15n = 0, \text{ ya'ni } n = 1.$$

$\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ va $\frac{x+1}{3} = \frac{y+5}{2} = \frac{z}{1}$ to‘g‘ri chiziqlarning kesishish nuqtasi koordinatalarini topish uchun, birinchi tenglamadagi x va y larni z orqali ifodalaymiz: $x = 2z$, $y = -3z$. Bu qiymatlarni $\frac{x+1}{3} = \frac{y+5}{2}$ tenglikka qo‘yamiz va $\frac{2z+1}{3} = \frac{-3z+5}{2}$ tenglikni hosil qilamiz; bundan esa $z = 1$. Endi z ni bilgan holda, $x = 2z = 2$, $y = -3z = -3$ ni topamiz. Demak, to‘g‘ri chiziqlarning kesishish nuqtasi $M(2, -3, 1)$ nuqta ekan.

2.6. To‘gri chiziq va tekislik tenglamalarini tuzishga doir masalalar

To‘gri chiziq va tekislik tenglamalarini tuzishga doir bir nechta masalalarni qaraymiz.

1-masala. $M_0(x_0, y_0, z_0)$ *nuqtadan o‘tuvchi ixtiyoriy tekislik tenglamasini tuzing.*

Yechish. Har qanday tekislik tenglamasi quyidagi umumiy ko‘rinishga ega :

$$Ax + By + Cz + D = 0.$$

$M_0(x_0, y_0, z_0)$ nuqta tekislikda yotgani uchun $Ax_0 + By_0 + Cz_0 + D = 0$ tenglik o‘rinli. Unda izlanayotgan tekislik tenglamasi quyidagi ko‘rinishga ega bo‘ladi:

$$Ax + By + Cz - (Ax_0 + By_0 + Cz_0) = 0,$$

yoki $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$.

2-masala. $M_0(x_0, y_0, z_0)$ *nuqtadan o‘tib,* $A_0x + B_0y + C_0z + D_0 = 0$ *tekislikga parallel bo‘lgan tekislik tenglamasini tuzing.*

Yechish. Izlanayotgan tekislik tenglamasi $A_0x + B_0y + C_0z + D_1 = 0$ ko‘rinishga ega. Shartga ko‘ra, $M_0(x_0, y_0, z_0)$ nuqta berilgan tekislikda yotganligi uchun $A_0x_0 + B_0y_0 + C_0z_0 + D_1 = 0$, ya’ni $D_1 = -(A_0x_0 + B_0y_0 + C_0z_0)$. Demak, izlanayotgan tekislik tenglamasi quyidagi ko‘rinishga ega:

$$A_0(x - x_0) + B_0(y - y_0) + C_0(z - z_0) = 0.$$

3-masala. $M_0(x_0, y_0, z_0)$ *nuqtadan o‘tib,* $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ to‘g‘ri *chiziqqa perpendikulyar bo‘lgan tekislik tenglamasini ntuzing.*

Yechish. Tekislikning yo‘naltiruvchi (normali) vektori sifatida berilgan to‘g‘ri chiziqning $\boldsymbol{s} = \{l, m, n\}$ yo‘naltiruvchi vektorini olish mumkin. Shuning uchun izlanayotgan tekislik tenglamasi quyidagi ko‘rinishga ega:

$$l(x-x_0)+m(y-y_0)+n(z-z_0)=0.$$

4-masala. *$M_0(x_0,y_0,z_0)$ nuqtadan o‘tuvchi va quyidagi*

$$\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1},$$

$$\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$$

to‘g‘ri chiziqlarga parallel bo‘lgan tekislik tenglamasini tuzing.

Yechish. Masala shartiga ko‘ra $\boldsymbol{s_1}=\{l_1,m_1,n_1\}$ va $\boldsymbol{s_2}=\{l_2,m_2,n_2\}$ vektorlar izlanayotgan tekislikka parallel bo‘lgani uchun, bu vektorlarning $\boldsymbol{s}=\boldsymbol{s_1}\times\boldsymbol{s_2}$ vektor ko‘paytmasi shu tekislikga perpendikulyar bo‘ladi. Shu vektor ko‘paytmani hisoblaymiz:

$$\boldsymbol{s_1}\times\boldsymbol{s_2}=\begin{vmatrix}\boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k}\\ l_1 & m_1 & n_1\\ l_2 & m_2 & n_2\end{vmatrix}=\begin{vmatrix}m_1 & n_1\\ m_2 & n_2\end{vmatrix}\boldsymbol{i}+\begin{vmatrix}n_1 & l_1\\ n_2 & l_2\end{vmatrix}\boldsymbol{j}+\begin{vmatrix}l_1 & m_1\\ l_2 & m_2\end{vmatrix}\boldsymbol{k}.$$

Shuni hisobga olib, yo‘naltiruvchi vektori $\boldsymbol{s}=\boldsymbol{s_1}\times\boldsymbol{s_2}$ bo‘lgan va $M_0(x_0,y_0,z_0)$ nuqtadan o‘tuvchi tekislik tenglamasini yosamiz:

$$\begin{vmatrix}m_1 & n_1\\ m_2 & n_2\end{vmatrix}\cdot(x-x_0)+\begin{vmatrix}n_1 & l_1\\ n_2 & l_2\end{vmatrix}\cdot(y-y_0)+\begin{vmatrix}l_1 & m_1\\ l_2 & m_2\end{vmatrix}\cdot(z-z_0)=0,$$

yoki uni ixcham shaklda yozsak,

$$\begin{vmatrix}x-x_0 & y-y_0 & z-z_0\\ l_1 & m_1 & n_1\\ l_2 & m_2 & n_2\end{vmatrix}=0.$$

5-masala. *Bir to‘g‘ri chiziqda yotmaydigan uchta $M_1(x_1,y_1,z_1)$, $M_2(x_2,y_2,z_2)$, $M_3(x_3,y_3,z_3)$ nuqtalardan o‘tuvchi tekislik tenglamasini tuzing.*

Yechish. $M(x,y,z)$ – izlanayotgan tekislikning ixtiyoriy nuqtasi bo‘lsin. Uchta $\overline{M_1\mathrm{M}}=(x-x_1,y-y_1,z-z_1)$, $\overline{M_1M_2}=(x_2-x_1,y_2-y_1,z_2-z_1)$, $\overline{M_1M_3}=(x_3-x_1,y_3-y_1,z_3-z_1)$ vektorlar bir tekislikda yotadi. Demak, ularning aralash ko‘paytmasi nolga teng:

$(\overline{M_1\mathrm{M}}\times\overline{M_1M_2},\overline{M_1M_3})=0$, ya’ni

$$\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ x_3-x_1 & y_3-y_1 & z_3-z_1\end{vmatrix}=0.$$

Bu esa izlanayotgan tekislik tenglamasidir.

6-masala. *Quyidagi*

$$A_1x+B_1y+C_1z+D_1=0,\quad A_2x+B_2y+C_2z+D_2=0$$

tekisliklarning kesishish to‘g‘ri chizig‘i orqali o‘tuvchi ixtiyoriy tekislik tenglamasini tuzing.

Yechish. Izlanayotgan tekislik tenglamasi

$$A_1x + B_1y + C_1z + D_1 + \lambda(A_2x + B_2y + C_2z + D_2) = 0$$

ko‘rinishga ega, bu berda λ – parametr. Bu tenglama berilgan tekisliklarning kesishish chizigidan o‘tuvchi *tekisliklar dastasining* tenglamasidir.

7-masala. *Berilgan $M_0(x_0, y_0, z_0)$ nuqtadan o‘tuvchi ixtiyoriy to‘g‘ri chiziq tenglamasini tuzing.*

Yechish. Izlanayotgan to‘g‘ri chiziq tenglamasi quyidagi ko‘rinishga ega:

$$\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}.$$

Haqiqatdan ham, bu tenglama $M_0(x_0, y_0, z_0)$ nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi bo‘lib, ravshanki, berilgan nuqtaning koordinatalari ushbu tenglamani qanoatlantiradi. l, m, n o‘zgaruvchlarga ixtiyoriy, ammo barchasi bir vaqtda nolga teng bo‘lmagan qiymatlar berib, ixtiyoriy yo‘nalishdagi to‘g‘ri chiziq tenglamasini hosil qilamiz.

8-masala. *Berilgan ikki $M_1(x_1, y_1, z_1)$ va $M_2(x_2, y_2, z_2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.*

Yechish. Izlanayotgan tenglamani quyidagi ko‘rinishda yozish mumkin:

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}.$$

$M_2(x_2, y_2, z_2)$ nuqta shu to‘g‘ri chiziqda yotgani uchun quyidagi

$$\frac{x_2-x_1}{l} = \frac{y_2-y_1}{m} = \frac{z_2-z_1}{n}.$$

tenglik o‘rinli. Bu munosabatlardan foydalanib l, m, n larni yo‘qotamiz va izlanayotgan to‘g‘ri chiziq tenglamasini hosil qilamiz:

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}.$$

9-masala. *Berilgan $M_0(x_0, y_0, z_0)$ nuqtadan o‘tuvchi va berilgan*

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

to‘g‘ri chiziqga parallel bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

Yechish. Izlanayotgan to‘g‘ri chiziq $\boldsymbol{s} = \{l, m, n\}$ yo‘naltiruvchi vektorga ega, va shuning uchun uning tenglamasi quyidagi ko‘rinishga ega bo‘ladi:

$$\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}.$$

10-masala. $M_0(x_0, y_0, z_0)$ *nuqtadan o'tuvchi va* $Ax + By + Cz + D = 0$ *tekislikga perpendikulyar to'g'ri chiziq tenglamasini tuzing.*

Yechish. Izlanayotgan to'g'ri chiziqning yo'naltiruvchi vektori sifatida berilgan tekislikning $\boldsymbol{s} = \{A, B, C\}$ normal vektorini olish mumkin. Shuning uchun izlanayotgan to'g'ri chiziq tenglamasi quyidagi ko'rinishga ega bo'ladi:

$$\frac{x - x_0}{A} = \frac{y - y_0}{B} = \frac{z - z_0}{C}.$$

Endi qaralgan masalalarga oid bir nechta misollar ko'ramiz.

8-misol. $M(3,4,-2)$ *nuqtadan o'tib,* $3x - 5y + 2z - 8 = 0$ *tekislikga parallel bo'lgan tekislik tenglamasini toping*.

Yechish. $M(3,4,-2)$ nuqtadan o'tuvchi ixtiyoriy tekislik tenglamasini yozamiz:

$$A(x - 3) + B(y - 4) + C(z + 2) = 0.$$

Izlanayotgan tekislikning normal vektori berilgan tekislikning $\boldsymbol{N} = \{3, -5, 2\}$ normali bilan ustma-ust tushadi. Shuning uchun, $A = 3$, $B = -5$, $C = 2$, va demak, izlanayotgan tekislik tenglamasi quyidagi ko'rinishda bo'ladi:

$$3(x - 3) - 5(y - 4) + 2(z + 2) = 0, \text{ yoki } 3x - 5y + 2z + 15 = 0\ .$$

9-misol. $M(2,3,-4)$ *nuqtadan koordinatalar tekisliklariga perpendikulyarlar tushirilgan. Bu perpendikulyarlar asoslaridan o'tuvchi tekislik tenglamasini tuzing.*

Yechish. Koordinata tekisliklariga tushirilgan perpendikulyarlarning asoslari quyidagi nuqtalardan iborat: $M_1(2,3,0)$, $M_2(2,0,-4)$, $M_3(0,3,-4)$. 5-masalaning yechimidan foydalanib, M_1, M_2, M_3 nuqtalardan o'tuvchi tekislik tenglamasini yozamiz:

$$\begin{vmatrix} x - 2 & y - 3 & z \\ 0 & -3 & -4 \\ -2 & 0 & -4 \end{vmatrix} = 0,$$

yoki $12x + 8y - 6z - 48 = 0.$

10-misol. $x + y + 5z - 1 = 0$, $2x + 3y - z + 2 = 0$ *tekisliklarning kesishish chizig'i va* $M(3,4,2)$ *nuqtadan o'tuvchi tekislik tenglamasini tuzing.*

Yechish. 6-masalaning yechimidan foydalanamiz. Berilgan tekisliklarning kesishish chizig'idan o'tuvchi tekisliklar dastasining tenglamasini yozamz:

$$x + y + 5z - 1 + \lambda(2x + 3y - z + 2) = 0.$$

Bu yerdagi λ ning qiymatini $M(3,4,2)$ nuqta shu tenglamani qanoatlantirish shartidan foydalanib topamiz:

$$3+4+10-1+\lambda(6+12-2+2)=0,$$

bundan $\lambda=-\frac{8}{9}$. Shunday qilib, izlanayotgan tekislik tenglamasi quyidagi

$$x+y+5z-1-\frac{8}{9}(2x+3y-z+2)=0,$$

ko‘rinishga ega, yoki $7x+15y-53z+25=0$.

11-misol. $M(4,3,-1)$ *nuqtadan o‘tib,* Ox *o‘qni to‘g‘ri burchak ostida kesuvchi to‘g‘ri chiziq tenglamasini tuzing.*

Yechish. Masala shartiga ko‘ra izlanayotgan to‘g‘ri chiziq $M(4,3,-1)$ nuqtadan o‘tib, Ox o‘qiga perpendikulyar va uni kesganligi sababli, u $N(4,0,0)$ nuqtadan o‘tadi. Demak, M va N nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzsak yetarli. 8-masala yechimidan foygalanib, izlangah tenglamaga ega bo‘lamiz:

$$\frac{x-4}{0}=\frac{y-3}{-3}=\frac{z+1}{1}.$$

12-misol. $\frac{x-1}{1}=\frac{y+1}{2}=\frac{z}{3}$ *to‘g‘ri chiziqning* $x+y+2z-5=0$ *tekislikdagi proyeksiyasi tenglamasini tuzing.*

Yechish. Berilgan to‘g‘ri chiziqning tenglamasini mos ravishda Oxy va Oxz tekisliklariga proeksiyalash natijasi bo‘lgan tekisliklar tenglamalari sistemasi ko‘rinishida yozamiz:

$$\frac{x-1}{1}=\frac{y+1}{2}, \text{ yoki } 2x-y-3=0,$$
$$\frac{x-1}{1}=\frac{z}{3}, \text{ yoki } 3x-z-3=0.$$

Demak, berilgan to‘g‘ri chiziqdan o‘tuvchi tekisliklar dastasi tenglamasi quyidagicha yoziladi:

$$2x-y-3+\lambda(3x-z-3)=0, \text{ yoki } (2+3\lambda)x-y-\lambda z-3(1+\lambda)=0.$$

Tekisliklarning perpendikulyarlik shartidan foydalanib, ushbu tekisliklar dastasidan berilgan to‘g‘ri chiziqni ko‘rsatilgan tekislikga proyeksiyalovchi tekislikni tanlaymiz. Shunga ko‘ra, quyidagiga ega bo‘lamiz:

$1\cdot(2+3\lambda)+1\cdot(-1)+2\cdot(-\lambda)=0$, yoki $\lambda+1=0$, bundan $\lambda=-1$.

Shunday qilib, proyeksiylovchi tekislikning tenglamasi quyidagi ko‘rinishga ega:

$$2x-y-3+(-1)(3x-z-3)=0, \text{ yoki } x+y-z=0.$$

Izlanayotgan proyeksiyani ikki tekislikning, ya’ni berilgan va proyeksiyalovchi tekisliklar kesishishi sifatida aniqlash mumkin:

$$\begin{cases} x + y + 2z - 5 = 0, \\ x + y - z = 0. \end{cases}$$

Bu to‘g‘ri chiziq tenglamasini kanonik shaklga keltiramiz. Buning uchun §1, 8-misoldagiga o‘xshash ish bajarib, quyidagiga ega bo‘lamiz:

$$\frac{x}{1} = \frac{y - \frac{5}{3}}{-1} = \frac{z - \frac{5}{3}}{0}.$$

2.7. Nazorat savollari va masalalar

1. Ikki tekislik orasidagi burchak qanday formula bilan topiladi?
2. Agar normal vektorlari $\boldsymbol{N}_1 = \{A_1, B_1, C_1\}$, $\boldsymbol{N}_2 = \{A_2, B_2, C_2\}$ ko‘rinishda berilsa, u holda ikki tekislikning parallelik va perpendikulyarlik shartlari qanday ifodalanadi?
3. Uch tekislikning kesishish sharti nimadan iborat? Ular qachon bir nuqtada kesishadi?
4. Tekisliklar normal vektorlarning komplanarlik sharti uch tekislik kesishining yetarli sharti bo‘ladimi?
5. $M_0(x_0, y_0, z_0)$ nuqtadan o‘tuvchi tekislikning normal vektori $\boldsymbol{N} = \{A, B, C\}$ bo‘lsin. $M_1(x_1, y_1, z_1)$ nuqtadan tekislikgacha bo‘lgan masofani topish formulasi qanday bo‘ladi?
6. Berilgan $M_1(x_1, y_1, z_1)$ nuqtaning $Ax + By + Cz + D = 0$ tekislikka nisbatan joylshish holatini qanday qilib aniqlash mumkin?
7. Qaysi formula orqali tekislik va to‘g‘ri chiziq orasidagi burchak aniqlanadi?
8. Tekislik va to‘g‘ri chiziqlarning parallellik va perpendikulyarlik shartlari qanday ifodalanadi?
9. Tekislik va to‘g‘ri chiziqlarning kesishish nuqtasini qanday aniqlash mumkin?
10. Fazodagi tekislik va to‘g‘ri chiziqlarlar orasidagi burchak qanday aniqlanadi?
11. Fazodagi tekislik va to‘g‘ri chiziqlarlarning parallellik va perpendikulyarlik shartlari nimadan iborat?
12. Fazodagi ikki to‘g‘ri chiziqning komplanarlik sharti qanday ifodalanadi?
13. Berilgan nuqtadan o‘tib, biror tekislikka parallel bo‘lgan tekislik tenglamasi qanday tuziladi?
14. Berilgan nuqtadan o‘tib, biror tekislikka perpendikulyar bo‘lgan tekislik tenglamasi qanday tuziladi?

15. $M(1,2,3)$ nuqtadan o‘tib, $\frac{x-2}{1}=\frac{y-3}{-1}=\frac{z+1}{2}$ va $\frac{x-1}{2}=\frac{y+3}{3}=\frac{z-1}{1}$ to‘g‘ri chiziqlarga parallel tekislik tenglamasini tuzing.

16. Uchta $M_1(1,3,0)$, $M_2(2,0,-4)$, $M_3(0,3,4)$ nuqtalardan o‘tuvchi tekislik tenglamasini tuzing.

17. Ikki tekislikning kesishish chizig‘idan o‘tuvchi tekisliklar dastasining tenglamasi qanday tuziladi?

18. $\boldsymbol{s}=\{l,m,n\}$ vektorga parallel va $M_0(x_0,y_0,z_0)$ nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

19. $M_1(2,3,-2)$ va $M_2(1,2,3)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

20. $M_0(x_0,y_0,z_0)$ nuqtadan o‘tib $Ax+By+Cz+D=0$ tekislikka perpendikulyar bo‘lgan tekislik tenglamasi qanday tuziladi?

21. $M(1,-1,-1)$ nuqtadan o‘tib, birinchisiga Ox o‘qi, ikkinchisiga esa Oy o‘qi tegishli bo‘lgan tekisliklar orasidagi burchakni hisoblang.

22. Berilgan $x+5y+9z-13=0$, $3x-y-5z+1=0$ tekisliklarning kesishish chizig‘i va $M(0,2,1)$ nuqtadan o‘tuvchi tekislik tenglamasini tuzing.

23. Berilgan $2x+3y-z+5=0$ tekislikning koordinata o‘qlari bilan hosil qilgan burchaklarini toping.

24. Qanday shartda $Ax+By+Cz+D=0$ tekislik Ox va Oy o‘qlarini bir xil burchak ostida kesadi?

25. $3x+y-z+5+\lambda(x-2y+3z-10)=0$ tekisliklar dastasi orasidan $2x+3y+z+8=0$ tekislikka perpendikulyar tekislikni toping .

26. Kanonik shakilda berilgan ikki to‘g‘ri chiziq orasidagi burchakni toping.

27. Berilgan $x+y-z+4=0$, $2x+2y-2z+17=0$ tekisliklar orasidagi masofani toping.

28. Berilgan $A_1x+B_1y+C_1z+D_1=0$, $A_2x+B_2y+C_2z+D_2=0$ tekisliklarga parallel va $M_0(x_0,y_0,z_0)$ nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

29. $\frac{x}{1}=\frac{y-3}{2}=\frac{z-2}{1}$, $\frac{x-3}{1}=\frac{y+1}{2}=\frac{z-2}{1}$ parallel to‘g‘ri chiziqlar orasidagi masofani toping.

30. $\begin{cases}4x-y-z+12=0,\\ y-z-2=0\end{cases}$ va $\begin{cases}3x-2y+16=0,\\ 3x-z=0\end{cases}$ to‘g‘ri chiziqlar orasidagi burchakni toping.

§3. IKKINCHI TARTIBLI SIRTLAR

3.1. Sfera

Ikkinchi tartibli sirtlar sirtning joriy koordinatalariga nisbatan ikkinchi darajali algebraik tenglamalar orqali beriladi.

Quyidagi tenglamani qaraymiz:

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2, \qquad (1)$$

bu yerda x_0, y_0, z_0, R (>0) – berilgan sonlar. Bu tenglama $Oxyz$ fazodagi berilgan $K(x_0,y_0,z_0)$ nuqtadan R masofada yotuvchi barcha $M(x,y,z)$ nuqtalar to‘plamini aniqlaydi. Bunday nuqtalar to‘plamiga markazi $K(x_0,y_0,z_0)$ nuqtada bo‘lgan va radiusi R ga teng *sfera* deyiladi.

(1) tenglama markazi $K(x_0,y_0,z_0)$ nuqtada va radiusi R ga teng sfera tenglamasi ekanligi ikki nuqta orasidagi masofa formulasini ifodalovchi

$$|KM|=\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}=R$$

munosabatdan klib chiqadi. Agar markaz koordinata boshi bilan ustma-ust tushsa, ya’ni $x_0=0$, $y_0=0$, $z_0=0$ bo‘lsa, u holda sfera tenglamasi quyidagi ko‘rinishga ega bo‘ladi

$$x^2+y^2+z^2=R^2. \qquad (2)$$

Sfera bilan ixtiyoriy tekislik kesishish chizig‘i aylanadan iborat. Agar tekislik sfera markazidan o‘tsa, u holda kesishish chizig‘i radiusi sfera radiusiga teng eng katta aylanadan iborat bo‘ladi.

1-misol. $x^2+y^2+z^2-2x+y+1=0$ *tenglama bilan berilgan sferaning markazi va radiusi koordinatalarini toping.*

Yechish. Berilgan tenglamani quyidagi ko‘rinishda yozib olamiz:

$$(x^2-2x+1)+\left(y^2+y+\frac{1}{4}\right)+z^2-\frac{1}{4}=0,$$

yoki $(x-1)^2+\left(y+\frac{1}{2}\right)^2+(z-0)^2=\left(\frac{1}{2}\right)^2$.

Shunday qilib, berilgan sfera markazi $C\left(1,-\frac{1}{2},0\right)$ nuqtada, radiusi esa $R=\frac{1}{2}$.

2-misol. $A(1,2,-4)$, $B(1,-3,1)$ *va* $C(2,2,3)$ *nuqtalardan o‘tuvchiva markazi* Oxy *tekislikda bo‘lgan sfera tenglamasini tuzing.*

Yechish. Markazi $K(x_0,y_0,z_0)$ Oxy tekislikda yotuvchi $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2$ sferaga (bunda $z_0=0$) A, B va C

nuqtalar tegishli bo‘lgani uchun ularning koordinatalari izlanayotgan tenglamani ayniyatga aylantiradi, ya’ni:

$$(1-x_0)^2+(2-y_0)^2+(-4)^2=R^2,$$
$$(1-x_0)^2+(-3-y_0)^2+1^2=R^2,$$
$$(2-x_0)^2+(2-y_0)^2+3^2=R^2.$$

Bu yerdan

$$(1-x_0)^2+(2-y_0)^2+16=(1-x_0)^2+(-3-y_0)^2+1,$$
$$(1-x_0)^2+(2-y_0)^2+16=(2-x_0)^2+(2-y_0)^2+9,$$

yoki

$(2-y_0)^2-(-3-y_0)^2=-15$, ya’ni $10y_0=10$;

$(1-x_0)^2-(2-x_0)^2=-7$, ya’ni $2x_0=-4$.

Shunday qilib, $x_0=-2$, $y_0=1$. Demak, izlanayotgan sfera markazi $C(-2{,}1{,}0)$ nuqtada ekan. Sfera radiusi esa,

$$R^2=(1-x_0)^2+(2-y_0)^2+16=(1+2)^2+(2-1)^2+16=26.$$

Yuqoridagilarga asosan, izlanayotgan sfera tenglamasi quyidagi ko‘rinishga ega:

$$(x+2)^2+(y-1)^2+z^2=26.$$

3.2. Ellipsoid

Quyidagi tenglamani qaraymiz:

$$\alpha x^2+\beta y^2+\gamma z^2+\delta=0,$$

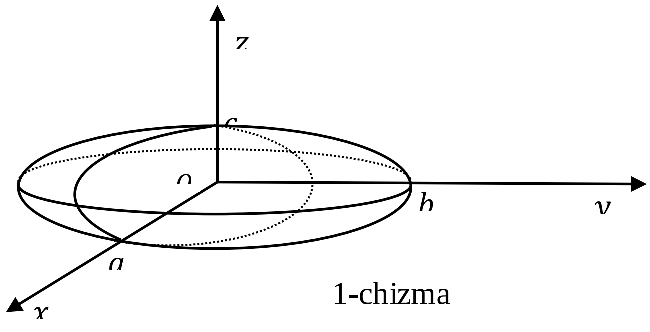
bu yerda $\alpha>0,\beta>0,\gamma>0,\delta<0$. Agar

$$\frac{\delta}{\alpha}=-a^2,\frac{\delta}{\beta}=-b^2,\frac{\delta}{\gamma}=-c^2$$

belgilashlarni kiritsak, u holda berilgan tenglamani quyidagicha yoza olamiz:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1=0. \qquad (3)$$

(3) tenglamaga *ellipsoid* deb ataluvchi sirtning kanonik tenglamasi deyiladi (1-chizma). Bu yerda a,b,c sonlarga ellipsoidning yarim o‘qlari deyiladi va ular ellipsoid sirtining koordinata o‘qlaridan ajratgan kesmalarini bildiradi.



1-chizma

Har qanday ellipsoid sferani Ox, Oy, Oz koordinata o‘qlariga nisbatan koeffitsiyentlari mos ravishda k_1, k_2, k_3 teng bo‘lgan tekis deformatsiya (qisish yok cho‘zish) yordamida hosil qilinadi.

Faraz qilaylik sfera (2) tenglama bilan berilgan bo‘lsin. Tekis deformatsiyadan so‘ng sferaning $M(x, y, z)$ nuqtasi $M'(x', y', z')$ nuqtaga o‘tsin; bunda M nuqtaning x, y, z joriy koordinatalari M' nuqtaning joriy x', y', z' koordinatalariga o‘tadi va quyidagi tengliklar o‘rinli:

$$x' = k_1 x, y' = k_2 y, z' = k_3 z\ .$$

Bu tengliklarni inobatga olib, (2) dan quyidagini hosil qilamiz

$$\frac{(x')^2}{k_1^2} + \frac{(y')^2}{k_2^2} + \frac{(z')^2}{k_3^2} = R^2,$$

yoki

$$\frac{(x')^2}{a^2} + \frac{(y')^2}{b^2} + \frac{(z')^2}{c^2} = 1, \tag{4}$$

Bunda $a = k_1 R, b = k_2 R, c = k_3 R$. (4) tenglama, (2) sferani tekis deformatsiyadan so‘ng hosil qilingan ellipsoid tenglamasidir.

Agar ellipsoidning ikki yarim o‘qi teng bo‘lsa, masalan, $a = b$, u holda bu ellipsoid aylanish ellipsoidi deyiladi:

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{c^2} - 1 = 0.$$

Uni Oxy tekisligiga parallel har qanday $z = h$ tekislik bilan kesilganda, markazi Oz o‘qida bo‘lgan

$$x^2 + y^2 = \left(1 - \frac{h^2}{c^2}\right) a^2, \qquad z = h$$

aylanani hosil qilamiz. Shunday qilib, bu holda Oxz tekisligida yotuvchi

$$\frac{x^2}{a^2} + \frac{z^2}{c^2} - 1 = 0,$$

ellipsni Oz o‘qi atrofida aylantirishidan ellipisoid hosil bo‘ladi.

Agar ellipsoidning uchala yarim o‘qlari o‘zaro teng bo‘lsa, u holda bu sirt sfera bo‘ladi.

Ellipsoidni ixtiyoriy tekislik bilan kesilganda kesimda ellips hosil bo‘ladi.

3-misol. *Quyidagi tenglamani kanonik shakilga keltiring:*

$$4x^2 + 9y^2 + 36z^2 - 8x - 18y - 72z + 13 = 0.$$

Yechish. Bir xil koordinatali hadlarni guruhlaymiz:

$$4(x^2 - 2x) + 9(y^2 - 2y) + 36(z^2 - 2z) = -13.$$

Qavs ichisagi ifodalarni to‘la kvadratga keltiramiz; unda

$$4(x^2-2x+1)+9(y^2-2y+1)+36(z^2-2z+1)=-13+4+9+36,$$

yoki $4(x-1)^2+9(y-1)^2+36(z-1)^2=36.$

Yangi koordinata boshini $O'(1,1,1)$ deb olib, koordinata o‘qlarini parallel ko‘chiramiz. Bu misolda koordinatalarni parallel ko‘chirish formulasi quyidagi ko‘rinishga ega bo‘ladi: $x=x'+1, y=y'+1, z=z'+1$. Yangi koordinatalarda sirt tenglamasi quyidagi ko‘rinishni oladi:

$$4(x')^2+9(y')^2+36(z')^2=36, \text{ yoki } \frac{(x')^2}{9}+\frac{(y')^2}{4}+\frac{(z')^2}{1}=1.$$

Bu tenglama ellipsoidni aniqlaydi; uning markazi yangi $O'(1,1,1)$ koordinatalar boshida, yarim o‘qlari mos ravishda 3, 2 va 1ga teng.

3.3. Giperboloidlar

Kanonik tenglamasi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}-1=0 \qquad (5)$$

ko‘rinishdagi sirtlar *bir pallali giperboloidlar* deb ataladi (2-chizma).

Agar sirtning kanonik tenglamasi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}+1=0, \qquad (6)$$

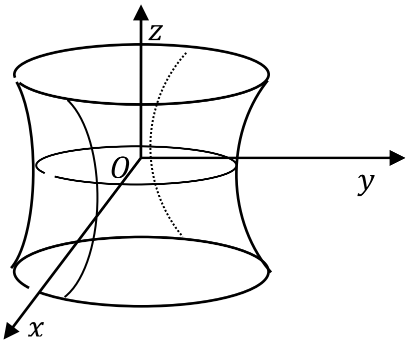
ko‘rinishda bo‘lsa, u holda bunday sirtlar *ikki pallali giperboloidlar* deb ataladi (3-chizma).

Ikkala tipdagi giperboloid uchun ham koordinata tekisliklari *simmetriya tekisliklari* va koordinata boshi esa – *simmetriya markazi* bo‘ladi.

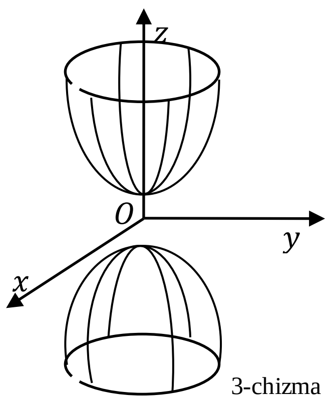
Agar giperboloidning a va b yarim o‘qlari teng bo‘lsa, u holda bu giperboloid *aylanish giperboloidi* deyiladi va u bir pallali giberboloid bo‘lgan holda

$$\frac{x^2}{a^2}-\frac{z^2}{c^2}-1=0, y=0$$

giperbolani,



2-chizma



3-chizma

ikki pallali giperboloid bo‘lgan holda esa

$$\frac{x^2}{a^2}-\frac{z^2}{c^2}+1=0, y=0\ ,$$

giperbolani Oz o‘qi atrofida aylanishidan hosil bo‘ladi.

Umumiy holdagi giperboloid $(a \neq b)$ aylanish giperboloididan $(a=b)$, Oxz tekisligiga nisbatan $\frac{b}{a}$ nisbatda tekis siqish (yoki cho‘zish) orqali hosil qilinadi.

Giperboloidni ixtiyoriy tekislik bilan kesganimizda har xil *konik kesimlar* hosil bo‘ladi. Masalan, Oxy tekisligiga parallel $z=h$ tekislik bilan (5) bir pallali giperboloid kesishganda, kesimda

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{h^2}{c^2}-1=0, z=h,$$

ellips hosil bo‘ladi; Oxz tekisligiga parallel $y=h\ (|h| \neq b)$ tekislik bilan (5) bir pallali giperboloid kesishganda, kesimda

$$\frac{x^2}{a^2}-\frac{z^2}{c^2}-1+\frac{h^2}{b^2}=0, y=h$$

giperbola hosil bo‘ladi.

$y=b$ tekislik (5) giperboloidni ikki to‘g‘ri chiziq bo‘ylab kesadi:

$$\frac{x^2}{a^2}-\frac{z^2}{c^2}=0, y=b\ .$$

Bir pallali giperboloidning har bir nuqtasidan uning ikkta to‘g‘ri chiziqli yasovchisilari o‘tadi:

$$\begin{cases}\alpha\left(\frac{x}{a}+\frac{z}{c}\right)=\beta\left(1+\frac{y}{b}\right)\\ \beta\left(\frac{x}{a}-\frac{z}{c}\right)=\alpha\left(1-\frac{y}{b}\right)\end{cases}, \quad \begin{cases}\gamma\left(\frac{x}{a}+\frac{z}{c}\right)=\delta\left(1-\frac{y}{b}\right)\\ \delta\left(\frac{x}{a}-\frac{z}{c}\right)=\gamma\left(1+\frac{y}{b}\right)\end{cases}. \qquad (7)$$

4-misol. Berilgan

$$\frac{x^2}{16}+\frac{y^2}{4}-\frac{z^2}{36}=1$$

giperboloidning $M(4,1,-3)$ *nuqtadan o‘tuvchi to‘g‘ri chiziqli yasovchilarining tenglamasini yozing.*

Yechish. $a=4, b=2, c=6$ yarim o‘qlarga ega bir pallali giperboloid berilgan. (7) formulaga asosan bu giperboloidning ikkita to‘g‘ri chiziqli yasovchilari quyidagi ko‘rinishga ega:

$$\begin{cases}\alpha\left(\frac{x}{4}+\frac{z}{6}\right)=\beta\left(1+\frac{y}{2}\right)\\ \beta\left(\frac{x}{4}-\frac{z}{6}\right)=\alpha\left(1-\frac{y}{2}\right)\end{cases}, \quad \begin{cases}\gamma\left(\frac{x}{4}+\frac{z}{6}\right)=\delta\left(1-\frac{y}{2}\right)\\ \delta\left(\frac{x}{4}-\frac{z}{6}\right)=\gamma\left(1+\frac{y}{2}\right).\end{cases}$$

Shartga ko‘ra, bu chiziqlar $\mathrm{M}(4,1,-3)$ nuqtadan o‘tadi. Shuning uchun quyidagilar o‘rinli:

$$\begin{cases}\alpha\left(1-\frac{1}{2}\right)=\beta\left(1+\frac{1}{2}\right)\\ \gamma\left(1-\frac{1}{2}\right)=\delta\left(1-\frac{1}{2}\right)\end{cases}, \quad \begin{cases}\alpha=3\beta\\ \gamma=\delta.\end{cases}$$

Demak, to‘g‘ri chiziqli yasovchilarining tenglamalari quyidagi ko‘rinishga ega:

$$\begin{cases}\frac{x}{4}+\frac{z}{6}=\frac{1}{3}\left(1+\frac{y}{2}\right)\\ \frac{x}{4}-\frac{z}{6}=3\left(1-\frac{y}{2}\right)\end{cases}, \quad \begin{cases}\frac{x}{4}+\frac{z}{6}=1-\frac{y}{2}\\ \frac{x}{4}-\frac{z}{6}=1+\frac{y}{2}.\end{cases}$$

3.4. Paraboloidlar

Kanonik tenglamasi

$$z=\frac{x^2}{a^2}+\frac{y^2}{b^2} \tag{8}$$

ko‘rinishda berilgan sirtga *elliptik paraboloid* deyiladi (4-chizma).

Kanonik tenglamasi

$$z=\frac{x^2}{a^2}-\frac{y^2}{b^2}. \tag{9}$$

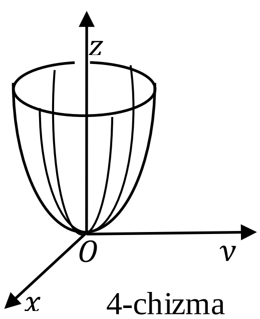
ko‘rinishda berilgan sirtga *giperbolik paraboloid* deyiladi (5-chizma).

Oxz va Oyz tekisliklar (8), (9) paraboloidlarning *simmetriya tekisliklari* bo‘ladi. Ularning kesishish chizig‘i (Oz o′qi) *paraboloidning o‘qi*, o‘q bilan paraboloid sirtining kesishish nuqtasi esa uning *uchi* deyiladi

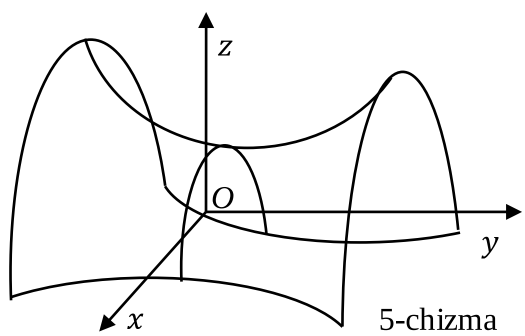
$a=b$ bo‘lganda elliptik paraboloyd *aylanish paraboloidi* deyiladi. Bu sirt

$$z=\frac{x^2}{a^2}, y=0$$

parabolaning Oz o‘qi atrofida aylanishidan hosil qilinadi.



4-chizma



5-chizma

Ikkala paraboloid (elliptik va giperbolik) ham Oxz va Oyz koordinata tekisliklariga parallel tekisliklar bilan kesiganda, kesimda parallel joylashgan va teng parabolalar hosil bo‘ladi.

Masalan, $x = h$ tekislik elliptik paraboloidni

$$z - \frac{h^2}{a^2} = \frac{y^2}{b^2}, \qquad x = h$$

parabolalar bo‘yicha kesadi.

Umumy holda elliptik paraboloidni

$$z = \frac{x^2}{a^2} + \frac{y^2}{a^2}$$

aylanish paraboloidni Oxz tekislikka nusbatan tekis siqish (yoki cho‘-zish) orqali hosil qilish mumkin.

Elliptik paraboloidi Oxy tekislikka parallel tekisliklar bilan ellips-lar bo‘yicha, giperbolik paraboloidda esa – giperbolalar bo‘yicha kesi-shadi. Oxy tekislik giperbolik paraboloidni ikki to‘g‘ri chiziq bo‘ylab kesadi.

(9) giperbolik paraboloidning har bir nuqtasidan uning ikkita to‘g‘-ri chiziqli yasovchilari o‘tadi ($a > 0, b > 0$):

$$\alpha\left(\frac{x}{a} + \frac{y}{b}\right) = \sqrt{2}\beta, \qquad \sqrt{2}\beta\left(\frac{x}{a} - \frac{y}{b}\right) = \alpha z;$$
$$\sqrt{2}\gamma\left(\frac{x}{a} + \frac{y}{b}\right) = \delta z, \qquad \delta\left(\frac{x}{a} - \frac{y}{b}\right) = \sqrt{2}\gamma.$$

5-misol. *Uchi koordinata boshida, o‘qi Oz o‘qidan iborat bo‘lgan va $M_1(-1,-2,2)$,$M_2(1,1,1)$ nuqtalarni o‘zida saqlovchi elliptik para-boloid tenglamasini tuzing.*

Yechish. Elliptik paraboloid tenglamasining (8) umumiy ko‘rini-shidan foydalanamiz. $\mathrm{M}_1(-1,-2{,}2)$ va $\mathrm{M}_2(1{,}1{,}1)$ nuqtalar bu sirtda yotgani uchun

$$\frac{1}{a^2} + \frac{4}{b^2} = 2, \ \frac{1}{a^2} + \frac{1}{b^2} = 1.$$

tengliklar o‘rinli. Bu yerdan esa: $a^2 = \frac{3}{2}$, $b^2 = 3$. Demak, izlanayotgan paraboloid tenglamasi quyidgi ko‘rinishda bo‘ladi:

$$z = \frac{2x^2}{3} + \frac{y^2}{3}.$$

3.5. Konus va silindrik sirtlar

Uchi koordinata boshida va o‘qi Oz koordinata o‘qidan iborat bo‘lgan *ikkinchi tartibli konusning* (6-chizma) kanonik tenglamasi quyidagi ko‘rinshda yoziladi:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0. \tag{10}$$

Xuddi shunga o‘xshash,

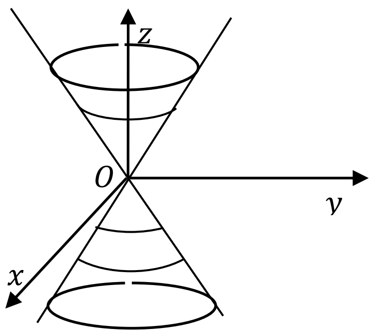
$$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=0,\ -\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=0$$

tenglamalarga uchi koordinata boshida va mos ravishda o‘qlari Oy va Ox koordinatalar o‘qlaridan iborat bo‘lgan *ikkinchi tartibli konusning* kanonik tenglamalari deyiladi.

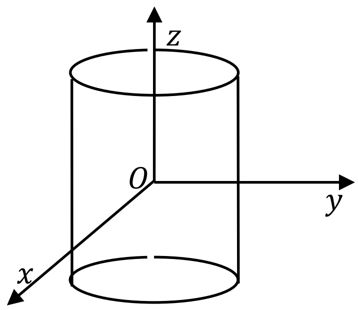
(10) umumiy konus

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{z^2}{c^2}=0$$

doiraviy konusidan Oxz tekisligiga nisbatan tekis siqish(cho‘zish) orqali hosil qiliinadi.



6-chizma



7-chizma

Takidlaymizki, (5) va (6) bir pallali va ikki pallali paraboloidlar (10) konus bilan tabiiy ravishda bog‘langanligi uchun bu konusga *asimptotik konus* deyiladi.

Oz o‘qidan o‘tuvchi har qanday tekislik giperboloidni giperbola bo‘yicha kesadi, konusni esa giperbolaning asimptotalari deb ataluvchi ikki yasovchilar bo‘yicha kesadi. Xususiy holda, masalan, Oxz $(y=0)$ tekislik (5), (6) giperboloidlarni

$$\frac{x^2}{a^2}-\frac{z^2}{c^2}-1=0,\ \frac{x^2}{a^2}-\frac{z^2}{c^2}+1=0,$$

giperbolalar bo‘yicha, (10) konusni esa bu giperbolalarnig asimptotalari bo‘lgan

$$\frac{x}{a}-\frac{z}{c}=0, \frac{x}{a}+\frac{z}{c}=0,$$

ikki to‘g ‘ri chiziq bo‘ylab kesadi.

Silindirik sirtlarning asosan uch turi mavjud. Ularning kanonik tenglamalarini keltiramiz.

Elliptik silindr (7-chizma):

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0.$$

Giperbolik silindr (8-chizma):

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}-1=0.$$

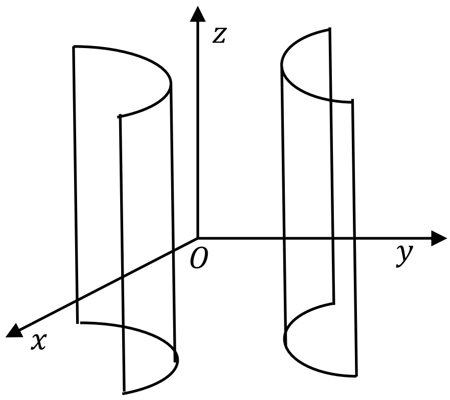
Parabolik silindir (9-chizma):

$$\frac{x^2}{a^2}-pz=0.$$

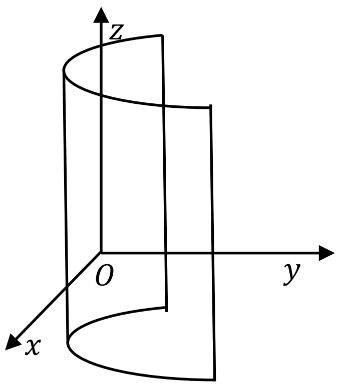
Umumiy elliptik silindr

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}-1=0$$

doiraviy silindrdan Oxz tekisligiga nisbatan tekis siqish (cho‘zish) orqali hosil qiliinadi.



8-chizma



9-chizma

Elliptik, giperbolik, parabolik silindrlar Oxy tekisligini mos ravishda ellips, giperbola, parabolalar bo‘yicha kesadi.

6-misol. *Uchi* $M(0,0,1)$ *nuqtada va yo‘naltiruvchisi esa*

$$\frac{x^2}{16}+\frac{y^2}{9}=1, z=3$$

ellips bo‘lgan sirtning kanonik tenglamasini tuzing.

Yechish. Agar $A(x_0, y_0, z_0)$ ellipsda yotuvchi nuqta bo‘sa, AM yasovchi to‘g‘ri chiziqning tenglamasini tuzamiz. Bu yasovchining tenglamasi quyidagi ko‘rinishga ega:

$$\frac{x}{x_0}=\frac{y}{y_0}=\frac{z-1}{z_0-1}.$$

A nuqta ellipisda yotgani uchun uning koordinatalari ellips tenglamasini qanoatlantiradi, ya’ni

$$\frac{(x_0)^2}{16}+\frac{(y_0)^2}{9}-1=0, z_0=3.$$

Endi

$$\frac{x}{x_0}=\frac{z-1}{z_0-1},\ \frac{y}{y_0}=\frac{z-1}{z_0-1},\ \frac{(x_0)^2}{16}+\frac{(y_0)^2}{9}-1=0, z_0=3$$

tenglamalardan x_0, y_0, z_0 larni yo‘qotib, izlanayotgan konus tenglamasini olamiz:

$$\frac{x^2}{16}+\frac{y^2}{9}-\frac{(z-1)^2}{4}=0.$$

7-misol. $x^2+y^2-2z^2=0$ *konus bilan* $y=2$ *tekislikning kesishish chizig‘ini aniqlang.*

Yechish. $x^2+y^2-2z^2=0$, $y=2$ tenglamalar sistemasidan $x^2+4-2z^2=0$ tenglamani hosil qilamiz, yoki $\frac{z^2}{2}-\frac{x^2}{4}=1$. Shunday qilib, izlanayotgan kesishish chizig‘i giperbola bo‘lib, $y=2$ tekisligida yotadi: uning haqiyqiy o‘qi Oz o‘qiga parallel, mavhum o‘qi esa – Ox o‘qiga parallel.

8-misol. *Yo‘naltiruvchisi* $x^2+y^2=4x, z=0$, *yasovchisi* $\bar{P}(1,1,1)$ *vektorga parallel bo‘lgan silindirik sirtning tenglamasini tuzing.*

Yechish. Izlanayotgan sirtning yasovchisinig tenglamasi quyidagi to‘g‘ri chiziqdan iborat:

$$\frac{x-x_0}{1}=\frac{y-y_0}{1}=\frac{z}{1},$$

bu yerda $(x_0, y_0, 0)$ nuqta $x^2+y^2=4x, z=0$ yo‘naltiruvchi chiziqga tegishli, ya’ni $x_0^2+y_0^2=4x_0$. Bundan x_0, y_0 larni yo‘qotsak, quyidagi silindrik sirt tenglamasini hosil qilamiz: $(x-z)^2+(y-z)^2=4(x-z)$. Bu esa izlanayotgan sirt tenglamasidir.

3.6. Nazorat savollari va masalalar

1. Markazi $M(x_0, y_0, z_0)$ nuqtada va radiusi R ga teng sfera tenglamasi qanday ko‘rinishga ega?
2. Sfera bilan ixtiyoriy tekislikning kesishish cizig‘i nimadan iborat?
3. $x^2+y^2+z^2+ax+by+cz+d=0$ tenglama berilgan. a, b, c, d koefisiyentlarga qanday shartlar qo‘yganda bu tenglama sferani ifodalaydi? Bu sfera markazining koordinatalari qanday va radiusi nimaga teng?
4. Ellipisoidning kanonik tenglamasi qanday ko‘rinishga ega? Ellipsoidning yarim o‘qlari qanday ma‘noga ega?

5. $x^2 + y^2 + z^2 = R^2$ sferani tekis deformasiyadan so‘ng a, b, c yarim o‘qli ellipsoid hosil bo‘lgan. Ox,Oy, Oz koordinata o‘qlari yo‘nalishidagi deformatsiya koeffitsiyentlarini aniqlang.

6. Ellipsoid qachon aylanish ellipsoidi deyiladi? Qaysi chiziqni koordinata o‘qi atrofida aylanishi natijasida aylanish ellipsoidi hosil bo‘ladi?

7. Ixtiyoriy tekislik va ellipsoid qanday chiziq bo‘ylab kesishadi?

8. Bir pallali va ikki pallali giperboloidlar qanday kanonik tenglamalar bilan aniqlanadi?

9. Aylanish giperboloidi qanday hosil qilinadi?

10. Umumiy giperboloidni aylanish giperboloiddan hosil qilish uchun uni Oxz tekislikka nisbatan qanday nisbatda tekis siqish(yoki cho‘zish) kerak?

11. Giperboloidlar koordinata tekisliklariga parallel tekisliklar bilan kesishganda kesimda qanday chiziq hosil bo‘ladi?

12. Bir pallali giperboloidning har bir nuqtasidan qanday to‘g‘ri chiziqli yasovchilar o‘tadi?

13. Elliptik va giperbolik paraboloidlr qanday kanonik tenglamalar bilan beriladi?

14. Paraboloidning o‘qi va uchi nima?

15. Paraboloid qachon aylanish paraboloidi deyiladi?

16. Paraboloidlar koordinata tekisliklariga parallel tekisliklar bilan qanday chiziqlar bo‘yicha kesishadi?

17. Giperbolik paraboloidning har bir nuqtasidan qanday yasovchi to‘g‘ri chiziqlar o‘tadi?

18. Konik sirt (konus) qanday kanonik tenglamalar bilan beriladi?

19. Giperboloidning asimptotik konusi nima?

20. Yasovchilari Oz o‘qiga parallel silindrik sirtlar (elliptik, giperbolik, parobolik) qanday kanonik tenglamalr bilan beriladi?

21. Quyidagi tenglama bilan berilgan sferaning markazi va radiusini toping:

$$x^2 + y^2 + z^2 + 4x + 6y - 4z - 8 = 0.$$

22. Ouyidagi aylananing markazi va radiusini toping:

$$(x - 3)^2 + (y + 2)^2 + (z - 1)^2 = 100,$$
$$2x - 2y - z + 9 = 0.$$

23. $M(1,1,3)$ nuqta quyidagi sferalarga nisbatan qanday joylashgan:

1) $(x - 1)^2 + (y - 2)^2 + z^2 = 19;$

2) $x^2 + y^2 + z^2 - x + y = 0$;

3) $x^2+y^2+z^2-4x+y-2z=0$.

24. Berilgan $\frac{x^2}{a^2}+\frac{z^2}{c^2}=1, y=0$ ellipsni Oz o‘qi atrofida aylanishidan hosil bo‘lgan sirt tenglamasini yozing.

25. Berilgan $\frac{x^2}{9}+\frac{y^2}{4}+\frac{z^2}{25}=1$ ellipsoid $z=3, y=1$ tekisliklar bilan kesishganda kesimda qanday chiziq hosil bo‘ladi?

26. Berilgan tenglamalar qanday sirtlarni aniqlaydi?

1) $x^2+y^2-z^2=4$; 2) $x^2-y^2+z^2+4=0$.

27. $x^2-y^2+z^2=4$ giperboloidning (2,4,4) nuqtasidan o‘tuvchi to‘g‘ri chiziqli yasovchisining tenglamasini yozing.

28. $\frac{x^2}{16}-\frac{y^2}{9}=2z$ paraboloidning (4,3,0) nuqtasidan o‘tuvchi to‘g‘ri chiziqli yasovchisi tenglamasini yozing.

29. Uchi kordinatalar boshida va yo‘naltiruvchisi $x^2+y^2=a^2$, $z=c$ bo‘lgan kanonik sirtning tenglamasini yozing.

30. Yo‘naltiruvchisi $y^2=4x, z=0$ chiziqdan iborat va yasovchisi ***P****(1,2,3)* vektorga parallel bo‘lgan silindrik sirt tenglamasini yozing.

III BOB UCHUN TEST SAVOLLARI (3-TEST)

1. Agar $Oxyz$ fazodagi sirtning yasovchisi Oz o‘qiga parallel va yo‘naltiruvchisining tenglamasi $F(x,y)=0$ bo‘lsa, u holda bu sirtning tenglamasi qanday ko‘rinishda bo‘ladi?

$A)$ $F(y,z)=0$; $B)$ $F(x,y)=0$; $C)$ $F(x,z)=0$;
$D)$ $z=F(x,y)$; $E)$ $y=F(x,z)$.

2. Oxy va Oxz koordinata tekisliklarining tenglamalarini ko‘rsating.

$A)$ $z=0; y=0$; $B)$ $x=0; y=0$; $C)$ $z=0; x=0$;
$D)$ $x=0; z=0$; $E)$ $y=0; z=0$.

3. Oy va Oz koordinata o‘qlarining tenglamalarini ko‘rsating.

$A)$ $\begin{cases} x=0 \\ z=0 \end{cases}; \begin{cases} y=0 \\ z=0 \end{cases}$. $B)$ $\begin{cases} y=0 \\ x=0 \end{cases}; \begin{cases} x=0 \\ z=0 \end{cases}$.
$C)$ $\begin{cases} x=0 \\ z=0 \end{cases}; \begin{cases} x=0 \\ y=0 \end{cases}$. $D)$ $\begin{cases} x=0 \\ y=0 \end{cases}; \begin{cases} x=1 \\ z=1 \end{cases}$. $E)$ $\begin{cases} x=1 \\ y=1 \end{cases}; \begin{cases} x=1 \\ z=1 \end{cases}$.

4. Oxz koordinata tekisligiga parallel bo‘lgan tekislik tenglamasi qanday ko‘rinishga ega.

$A)$ $Ax+D=0$; $B)$ $By+D=0$; $C)$ $Cz+D=0$;
$D)$ $Ax+Cz=0$; $E)$ $By+Cz=0$.

5.Tekislik berilgan: $Ax + D = 0$. Bu tekislik qaysi tekislikga parallel?

$A)$ Oxy; $B)$ Oxz; $C)$ $By + D = 0$; $D)$ Oxz; $E)$ $Cz + D = 0$.

6. Oy koordinata o'qiga parallel tekislikni ko'rsating.

$A)$ $Ax + By + D = 0$; $B)$ $Ax + Cz = 0$;
$C)$ $By + Cz = 0$; $D)$ $Ax + By = 0$; $E)$ $Ax + Cz + D = 0$.

7. $z = 4$ tekisligida yotuvchi, markazi Oz o'qida va radiusi $R = 3$ aylana qaysi tenglama bilan berilgan.

$A)$ $x^2 + y^2 = 4, z = 4$; $B)$ $x^2 + y^2 = 9, z = 2$; $C)$ $x^2 + y^2 = 4$, $z = 2$; $D)$ $x^2 + y^2 = 9, z = 4$; $E)$ $x^2 + y^2 = 16, z = 4$.

8. $(x^2 + xy + y^2 + 1)(z - 3) = 0$ tenglamaga qaysi geometrik obraz mos keladi?

$A)$ $z = 3$ tekislik; $B)$ $x = 3$ tekislik; $C)$ $x^2 + y^2 = 1$ aylana;
$D)$ $(x - 1)^2 + y^2 = 4$ aylana; $E)$ $y = 3$ tekislik.

9. $(x - 1)^2 + (y - 2)^2 + (z + 3)^2 = 0$ tenglamaga qaysi geometrik obraz mos keladi?

A) markazi (1,2) nuqtadadagi aylana;
B) markazi (1,2,3) nuqtadadagi aylana;
C) markazi (1,2,–3) nuqtadadagi sfera;
$D)$ yagona M(1,2,–3) nuqta;
E) $M_1(1,2,3)$, $M_2(1,2,-3)$ nuqtalar.

10. $x = a\cos t, y = a\sin t, z = b$ parametrik tenglamalar qanday chiziqni ifodalydi?

$A)$ Oxy tekislikda yotuvchi radiusi a ga teng aylana;
$B)$ Oxz tekislikda yotuvchi radiusi b ga teng aylana;
$C)$ $z = b$ tekislikda yotuvchi radiusi a ga teng aylana;
$D)$ yarim o'qlari a va b bo'lgan ellips;
$E)$ yarim o'qlari $\frac{a}{2}$ va $\frac{b}{2}$ bo'lgan ellips.

11. $5x + 4y - 2z + 10 = 0$ tekislik va unda yotuvchi $M(x_0, y_0, z_0)$ nuqta berilgan $(y_0 = 1, z_0 = 2)$. Berilgan tekislik tenglamasini quyidagi ko'rinishda yozing: $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$.

$A)$ $5(x - 2) + 4(y - 1) + 2(z - 3) = 0$;
$B)$ $5(x + 2) + 4(y - 1) - 2(z - 2) = 0$;
$C)$ $5(x + 2) + 2(y - 1) + 4(z - 2) = 0$;
$D)$ $2(x + 2) + 4(y - 1) - 5(z - 2) = 0$;
$E)$ $4(x + 2) - 2(y - 1) + 5(z - 2) = 0$.

12. Oz o‘qiga parallel, Ox va Oy o‘qlaridan mos ravishda 2 va 3 birlik kesma ajratgan tekislik tenglamasini ko‘rsating.

$A)\ \frac{x}{2}+\frac{y}{3}=1; B)\ \frac{x}{3}+\frac{y}{2}=1; C)\ \frac{x}{2}-\frac{y}{3}=1;\ D)\ 2x+3y=1;$
$E)\ 3x+2y=1.$

13. $M(1,2,3)$ nuqtadan o‘tuvchi va Oz o‘qiga perpendiklyr tekislik tenglamasini ko‘rsating.

$A)\ x=1;\ B)\ \ y=2; C)\ z=3;\ D)\ x=2, x+y=1;$
$E)\ y+z=1.$

14.Tekislik tenglamasi berilgan: $3x+4y-5z-8=0$. Bu tenglamani kesmalarga nisbatan tenglama ko‘rinishiga keltiring.

$A)\ \frac{x}{3}+\frac{y}{4}-\frac{z}{5}=1;\ B)\ \frac{x}{\frac{8}{3}}+\frac{y}{2}+\frac{z}{-\frac{8}{5}}=1; C)\ \frac{x}{4}+\frac{y}{3}-\frac{z}{5}=1;$
$D)\ \frac{x}{\frac{8}{5}}+\frac{y}{2}+\frac{z}{3}=1; E)\ \frac{x}{4}+\frac{y}{5}-\frac{z}{8}=1.$

15. $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ – to‘g‘ri chiziq tenglamasi berilgan. Shu to‘g‘ri chiziqning yo‘naltiruvchi vektorini ko‘rsating.

$A)\ S=\{3,4,2\};\ \ B)\ S=\{3,2,4\};\ \ C)\ S=\{4,3,2\};$
$D)\ S=\{4,2,3\};\ E)\ S=\{2,3,4\}.$

16. l to‘g‘ri chiziqning yo‘naltiruvchi vektori $\boldsymbol{s}=\{1,2,-1\}$ ga teng. Uning yo‘naltiruvchi kosinuslarini toping.

$A)\ \cos\alpha=\frac{1}{\sqrt{6}}, \cos\beta=-\frac{2}{\sqrt{6}}, \cos\gamma=\frac{1}{\sqrt{6}}$;
$B)\ \cos\alpha=\frac{1}{\sqrt{3}}, \cos\beta=-\frac{1}{\sqrt{3}}, \cos\gamma=\frac{1}{\sqrt{3}}$;
$C)\ \cos\alpha=\frac{2}{\sqrt{6}}, \cos\beta=\frac{1}{\sqrt{6}}, \cos\gamma=-\frac{1}{\sqrt{6}}$;
$D)\ \cos\alpha=\frac{1}{\sqrt{6}}, \cos\beta=\frac{2}{\sqrt{6}}, \cos\gamma=-\frac{1}{\sqrt{6}}$;
$E)\ \cos\alpha=\frac{2}{\sqrt{3}}, \cos\beta=\frac{1}{\sqrt{3}}, \cos\gamma=-\frac{1}{\sqrt{3}}$.

17. $2x-y+4z+6=0$ va $x+y-z+6=0$ tekisliklarning kesishishidan hosil bo‘lgan to‘g‘ri chiziqning kanonik tenglamasini toping.

$A)\ \frac{x+\frac{16}{5}}{-7}=\frac{y+\frac{7}{5}}{6}=\frac{z}{5};\ B)\ \frac{x+\frac{7}{5}}{6}=\frac{y+\frac{16}{5}}{-7}=\frac{z}{5}; C)\ \frac{x-\frac{16}{5}}{7}=\frac{y-\frac{7}{5}}{-6}=\frac{z}{5};$
$D)\ \frac{x-16}{7}=\frac{y-7}{6}=\frac{z-2}{5};\ E)\ \ \frac{x-7}{-6}=\frac{y-16}{7}=\frac{z-3}{5}.$

18. $\frac{x-1}{1}=\frac{y+2}{-1}=\frac{z}{-\sqrt{2}}$ to‘g‘ri chiziq koordinata o‘qlari bilan qanday burchaklar tashkil qiladi?

$A)\alpha=30°, \beta=60°, \gamma=120°;\ B)\ \alpha=60°, \beta=45°, \gamma=120°;$

$C)\alpha = 60°, \beta = 30°, \gamma = 135°$; $D)\alpha = 45°, \beta = 60°, \gamma = 120°$;
$E)\alpha = 60°, \beta = 120°, \gamma = 135°$.

19. $\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-2}{4}$ to‘g‘ri chiziq bilan Oxy koordinata tekisligining kesishish nuqtasi $M(x_0, y_0, z_0)$ ni toping.

$A)$ $x_0 = 2, y_0 = -\frac{1}{2}$; $B)$ $x_0 = 1, y_0 = \frac{1}{2}$; $C) x_0 = 2, y_0 = 1$;
$D)$ $x_0 = -1, y_0 = 2$; $E)$ $x_0 = -2, y_0 = 1$.

20. $2x - 3y + 6z - 21 = 0$ tekislik tenglamasini normal shaklga keltiring.

$A)$ $\frac{1}{7}x - \frac{4}{7}y + \frac{6}{7}z + 3 = 0$; $B)$ $\frac{2}{7}x - \frac{3}{7}y + \frac{6}{7}z - 3 = 0$;
$C)$ $\frac{2}{9}x - \frac{4}{9}y + \frac{6}{9}z - 3 = 0$; $D)$ $\frac{1}{9}x - \frac{2}{9}y + \frac{2}{3}z - 4 = 0$;
$E)$ $\frac{1}{3}x - \frac{4}{3}y + \frac{2}{3}z - 3 = 0$.

21. Quyidagi tekisliklardan qaysi biri $2x - y + 4z + 5 = 0$ tekislikga parallel.

$A)$ $6x - 12y + 10z - 13 = 0$;
$B)$ $9x - 8y + 15z - 14 = 0$; $C)$ $6x - 8y + 10z + 15 = 0$;
$D)$ $3x - 8y + 10z - 14 = 0$; $E)$ $6x - 4y + 10z - 21 = 0$.

22. Quyidagi tekisliklardan qaysi biri $2x - 5y + z - 6 = 0$ tekislikga perpendikulyar?

$A)$ $4x + 3y - 2z + 1 = 0$;
$B)$ $5x + 3y - 2z + 4 = 0$; $C)$ $3x + y - z + 8 = 0$;
$D)$ $3x + 4y + 5z - 12 = 0$; $E)$ $4x + 3y + 4z - 13 = 0$.

23. $x + y - z + 1 = 0$ va $2x - y + z + 2 = 0$ tekisliklar orasidagi φ burchakni aniqlang.

$A)$ $\varphi = 60°$; $B)$ $\varphi = 45°$; $C)$ $\varphi = 75°$; $D)$ $\varphi = 90°$; $E)$ $\varphi = 55°$.

24. $x - y + 1 = 0$ va $y - z + 2 = 0$ tekisliklar orasidagi o‘tkir φ burchakni aniqlang.

$A)$ $\varphi = 30°$; $B)$ $\varphi = 45°$; $C)$ $\varphi = 75°$; $D)$ $\varphi = 60°$; $E)$ $\varphi = 55°$.

25. $M(3,5,-8)$ nuqtadan $6x - 3y + 2z - 8 = 0$ tekislikgacha bo‘lgan d masofani aniqlang.

$A)$ $d = 4$; $B)$ $d = 5$; $C)$ $d = \frac{5}{2}$; $D)$ $d = \frac{7}{2}$; $E)$ $d = 3$.

26. $3x - 4y + 2z - 10 = 0$ va $6x - 8y + 4z + 10 = 0$ tekisliklar orasidagi d masofani toping.

$A)$ $d = \frac{14}{\sqrt{29}}$; $B)$ $d = \frac{13}{\sqrt{29}}$; $C)$ $d = \frac{12}{\sqrt{29}}$; $D)$ $d = \frac{7}{\sqrt{29}}$; $E)$ $d = \frac{8}{\sqrt{29}}$.

27. $M(2,3,5)$ nuqtadan o‘tib va $\boldsymbol{n} = 4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$ vektorga perpendikulyar bo‘lgan tekislik tenglamasini tuzing.

$A)$ $3x + 4y - 2z - 25 = 0$; $B)$ $4x + 3y + 2z - 27 = 0$;

$C)$ $3x - 4y - 2z - 23 = 0$; $D)$ $4x - 3y - 3z - 25 = 0$;

$E)$ $3x + 4y + 3z - 27 = 0$.

28. $M(2,3,-1)$ nuqtadan o‘tuvchi va $5x - 3y + 2z - 9 = 0$ tekislikka parallel tekislik tenglamasini toping.

$A)$ $3x - 5y + 2z + 2 = 0$; $B)$ $5x + 3y - 2z + 3 = 0$;

$C)$$5x - 3y + 2z + 1 = 0$; $D)$ $4x - 3y + 2z + 2 = 0$;

$E)$ $3x - 4y + 3z + 4 = 0$.

29. Berilgan $x + y + z - 4 = 0$, $y + z - 4 = 0$, $y + z - 2 = 0$ tekisliklarning o‘zaro joylashuvini aniqlang:

$A)$ tekisliklar bir nuqtada kesishadi;

$B)$ tekisliklar to‘g‘ri chiziq bo‘ylab kesishadi;

$C)$ tekisliklar parallel;

$D)$ tekisliklar bir to‘g‘ri chiziqqa parallel, lekin kesishmaydi;

$E)$ berilgan tekisliklardan $x + y + z - 4 = 0$ va $y + z - 4 = 0$ tekisliklari parallel.

30. $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z+1}{\sqrt{2}}$ to‘g‘ri chiziq Oxy koordinata tekisligi bilan qanday φ burchak tashkil qiladi?

$A)$ $\varphi = \frac{\pi}{3}$; $B)$ $\varphi = \frac{\pi}{6}$; $C)$ $\varphi = \frac{3\pi}{4}$; $D)$ $\varphi = \frac{\pi}{2}$; $E)$ $\varphi = \frac{\pi}{4}$.

31. $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ va $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ tekisliklarning parallelik shartini ko‘rsating:

$A)$ $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$; $B)$ $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$;

$C)$ $l_1 m_1 + l_2 m_2 + n_1 n_2 = 0$; $D)$ $l_1 l_2 + m_1 n_2 + n_1 m_2 = 0$;

$E)$ $\frac{l_1}{m_1} = \frac{l_2}{m_2} = \frac{n_1}{n_2}$.

32. $x - y - 2z + 7 = 0$, $x + 3y + 5z - 4 = 0$ tekisliklarnig kesishish chizig‘idan o‘tib va Oy o‘qiga parallel bo‘lgan tekislik tenglamasini tuzing.

$A)$ $3x - z + 18 = 0$; $B)$ $4x - 2z + 15 = 0$;

$C)$$4x - z + 17 = 0$; $D)$ $3x - 2z + 18 = 0$;

$E)$ $4x - 3z + 18 = 0$.

33. $A_1x + B_1x + C_1x + D_1 = 0$ va $A_2x + B_2x + C_2x + D_2 = 0$ tekisliklarning perpendikulyarlik shartini ko‘rsating:

$A)$ $A_1A_2 + B_1B_2 + C_1C_2 = 0$; $B)$ $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$;

$C)\ \frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{D_1}{D_2}$; $D)\ \frac{A_1}{A_2} = \frac{C_1}{C_2} = \frac{D_1}{D_2}$;

$E)\ \boldsymbol{n}_1 = \{A_1, B_1, C_1\}$ va $\boldsymbol{n}_2 = \{A_2, B_2, C_2\}$ normal vektorlarning paralleligi.

34. $Ax + Bx + Cx + D = 0$ tekislik va $\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}$ to‘g‘ri chiziq orasidagi φ burchak qaysi formula bilan aniqlanadi?

$A)\ \sin\varphi = \frac{|Al+Bm+Cn|}{\sqrt{A^2+B^2+C^2}}$; $B)\ \cos\varphi = \frac{|Al+Bm+Cn|}{\sqrt{l^2+m^2+n^2}}$;

$C)\ \sin\varphi = \frac{|Al+Bm+Cn|}{\sqrt{A^2+B^2+C^2}\cdot\sqrt{l^2+m^2+n^2}}$; $D)\ \cos\varphi = \frac{|Al+Bm+Cn|}{\sqrt{A^2+B^2+C^2}}$;

$E)\ \sin\varphi = \frac{|Al+Bm+Cn|}{\sqrt{A^2+B^2+C^2}}$.

35. Parameter n ning qanday qiymatlarida $\frac{x}{2} = \frac{y}{-3} = \frac{z}{n}$ va $\frac{x+1}{3} = \frac{y+5}{2} = \frac{z}{1}$ to‘g‘ri chiziqlari kesishadi?

$A)\ n = 2$; $B)\ n = 3$; $C)\ n = -1$; $D)\ n = -2$; $E)\ n = 1$.

36. $M(1,2,3)$ nuqtadan o‘tib, $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{4}$ to‘g‘ri chiziqga perpendikulyar tekislik tenglamasini tuzing.

$A)\ 2(x-1) + 3(y-2) + 4(z-3) = 0$;

$B)\ 3(x-1) - (y-2) + 4(z-3) = 0$;

$C)\ 2x - y + 4z - 12 = 0$; $D)\ x - 2y + 4z - 12 = 0$;

$E)\ 3x - 2y + 4z - 12 = 0$.

37. Berilgan $M_1(2,0,3)$ va $M_2(1,2,0)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

$A)\ \frac{x-2}{1} = \frac{y-2}{-2} = \frac{z-3}{-3}$; $B)\ \frac{x-2}{-1} = \frac{y}{2} = \frac{z-3}{-3}$;

C) $\frac{x-2}{-1} = \frac{y-1}{2} = \frac{z-3}{3}$; $D)\ \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$; $E)\ \frac{x-2}{1} = \frac{y}{-2} = \frac{z-3}{3}$.

38. $M(-1,2,2)$ nuqtadan o‘tib, $3x + 4y - 2z + 5 = 0$ tekislikga perpendikulyar bo‘lgan to‘g‘ri chiziq tenglamasini tuzing.

$A)\ \frac{x+1}{3} = \frac{y-2}{4} = \frac{z-3}{-2}$; $B)\ \frac{x-1}{3} = \frac{y+2}{4} = \frac{z+2}{2}$;

C) $\frac{x+1}{-3} = \frac{y+2}{4} = \frac{z+2}{2}$; $D)\ \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-2}{4}$; $E)\ \frac{x+1}{-3} = \frac{y-2}{4} = \frac{z+2}{2}$.

39. $A(2,-1,4)$ va $B(3,2,-1)$ nuqtalardan o‘tib, $x + y + 2z - 3 = 0$ tekislikga perpendikulyar bo‘lgan tekislik tebglamasini tuzing.

$A)\ 11x - 6y - z - 20 = 0$; $B)\ 9x - 5y + 2z - 18 = 0$;

$C)\ 5x - 6y - 2z - 17 = 0$; $D)\ 11x - 7y - 2z - 21 = 0$;

$E)\ 9x - 6y - z - 18 = 0$.

40. $P(2,3,-5)$ nuqtadan koordinatalar tekisliklariga perpendikulyarlar tushirilgan. Shu perpendikulyarlar asoslaridan o'tuvchi tekislik tenglamasini tuzing.

$A)$ $5x+2y-3z-30=0$; $B)$ $10x+6y-5z-20=0$;
$C)$ $15x+10y-6z-60=0$; $D)$ $15x-10y-2z-50=0$;
$E)$ $10x-6y-5z-35=0$.

41. $x^2+y^2+z^2-4x+6y+2z-2=0$ sfera $M(x_0,y_0,z_0)$ markazining koordinatalarini va R radiusini aniqlang.

$A)$ $M(-2,3,1)$, $R=3$; $B)$ $M(-2,3,2)$, $R=4$; $C)$ $M(2,3,1)$, $R=4$;
$D)$ $M(2,-3,-1)$, $R=4$; $E)$ $M(3,2,-1)$, $R=3$.

42. Agar $M(4,-1,-3)$ va $N(0,3,-1)$ nuqtalar sfera diametrining chetlari bo'lsa, u holda sfera tenglamasini tuzing.

$A)$ $(x+2)^2-(y-1)^2+(z+2)^2=16$;
$B)$ $(x+3)^2+(y-2)^2+(z-2)^2=9$;
$C)$ $(x-3)^2+(y-1)^2+(z-2)^2=16$;
$D)$ $(x-2)^2+(y-3)^2+(z-2)^2=9$;
$E)$ $(x-2)^2+(y-1)^2+(z+2)^2=9$.

43. Berilgan

$$\begin{cases}(x+3)^2+(y+2)^2+(z-1)^2=100,\\ 2x-2y-z+9=0\end{cases}$$

aylananing markazi $M(x_0,y_0,z_0)$ nuqtaning koordinatalarini va radiusi R ni toping.

$A)$ $M(-1,2,3), R=8$; $B)$ $M(2,-1,3), R=9$;
C) $M(2,3,-1), R=10$; $D)$ $M(-1,3,2), R=9$;
$E)$ $M(3,-2,1), R=10$.

44. $x^2=4y$ tenglama fazoda qanday sirtni ifodalaydi?

$A)$ Yasovchisi Ox o'qiga parallel bo'lgan parobolik silindr;
$B)$ Yasovchisi Oz o'qiga parallel bo'lgan parobolik silindr;
$C)$ Yasovchisi Oy o'qiga parallel bo'lgan elliptik silindr;
$D)$ Yasovchisi Ox o'qiga parallel bo'lgan elliptik silindir;
$E)$ Yasovchisi Oz o'qiga parallel bo'lgan giperbolik silindr.

45. Uchi $M(0,0,1)$ nuqtada, yonaltiruvchiasi esa $\frac{x^2}{25}+\frac{y^2}{9}=1,\ z=3$ ellipsdan iborat bo'lgan sirtning kanonik tenglamasini tuzing.

$A)$ $\frac{x^2}{16}+\frac{y^2}{9}-\frac{(z-2)^2}{4}=0$; $B)$ $\frac{x^2}{25}+\frac{y^2}{9}-\frac{(z-1)^2}{4}=0$;
$C)$ $\frac{x^2}{16}-\frac{y^2}{9}+\frac{(z-2)^2}{25}=0$; $D)$ $\frac{x^2}{16}+\frac{y^2}{25}-\frac{(z-1)^2}{9}=0$;

$E)\ \frac{x^2}{9}-\frac{y^2}{16}-\frac{(z-2)^2}{25}=0.$

46. $4x^2+9y^2+36z^2-8x-18y-72z+13=0$ ellipsoid tenglamasini kanonik shaklga keltiring.

$A)\ \frac{x^2}{12}+\frac{y^2}{16}+\frac{z^2}{9}=1;\ B)\ \frac{x^2}{16}+\frac{y^2}{25}+\frac{z^2}{9}=1;$

$C)\ \frac{x^2}{9}+\frac{y^2}{4}+\frac{z^2}{25}=1;\ D)\ \frac{x^2}{9}+\frac{y^2}{4}+z^2=1;\ E)\ \frac{x^2}{4}+y^2+\frac{z^2}{9}=1.$

47. Uchi koordinatalar boshida, o‘qi Oz bo‘lgan elliptik paraboloid sirtida ikkita $M(-1,-2,2)$ va $N(1,1,1)$ nuqtalar berilgan bo‘lsin. Shu sirt tenglamasini toping.

$A)\ \frac{2x^2}{3}+\frac{y^2}{3}=z;\ B)\ x^2+y^2=3z;\ \ C)\ 2x^2-y^2=3z;$

$D)\ x^2-2y^2=3z;\ E)\ \frac{x^2}{3}+\frac{2y^2}{3}=z.$

48. $x^2-y^2-4x+8y-2z=0$ giperbolik paraboloid tenglamasini kanonik shakilga keltiring.

$A)\ \frac{x^2}{4}-\frac{y^2}{9}=z;\ B)\ \frac{x^2}{9}-\frac{y^2}{4}=z;\ \ C)\ x^2-y^2=2z;$

$D)\ x^2-y^2=4z;\ E)\ \frac{x^2}{16}-\frac{y^2}{9}=z.$

49. $4x^2+y^2-z^2-24x-4y+2z+35=0$ tenglama qanday sirtni aniqlaydi?

$A)\ (x-2)^2+\frac{(y-1)^2}{3}-\frac{(z-1)^2}{4}=1$ – giperboloid;

$B)\ (x-3)^2+\frac{(y-2)^2}{4}-\frac{(z-1)^2}{4}=1$ – giperboloid;

$C)\ \frac{(x-2)^2}{3}+\frac{(y-1)^2}{4}+\frac{(z-1)^2}{4}=1$ – ellipsoid;

$D)\ \frac{(x-2)^2}{4}+\frac{(y-2)^2}{3}+\frac{(z-1)^2}{3}=1$ – ellipsoid;

$E)\ \frac{(x-2)^2}{4}+\frac{(y-2)^2}{3}=z$ – paraboloid.

50. $x^2+4y^2+9z^2+12yz+6xz+4xy-4x-8y-12z+3=0$ tenglama qanday sirtni aniqlaydi?

$A)$ bir pallali giperboloid;

$B)$ ikki pallali giperboloid;

$C)$ elliptik paraboloid;

$D)$ giperbolik paraboloid;

$E)$ ikki parallel to‘g‘ri chiziqlar to‘plami.

GLOSSARIY

Atamaning o‘zbek tilidagi nomi	Atamaning ingliz tilidagi nomi	Atamaning rus tilidagi nomi	Atamaning tavsifi
abssissa	**absitsts**	**абсцисса**	Fazodagi nuqtaning dekart koordinatalaridan birinchisi (nuqta radius vektorining *OX* o‘qidagi proyeksiyasi).
analitik geometriya	**analytic geometry**	**аналитическая геометрия**	Matematikaning koordinatalar usulidan foydalanib geometrik obyektlarni o‘rganaradigan bo‘limi.
applikata	**applicate**	**аппликата**	Uch o‘lchamli fazodagi nuqtaning dekart koordinataladan uchinchisi (nuqta radius vektorining *OZ* o‘qidagi proyeksiyasi).
argument	**argument**	**аргумент**	Erkli o‘zgaruvchi.
asimptota	**asymptote**	**асимптота**	Asimptota – shunday to‘g‘ri chiziqki, bu to‘g‘ri chiziq va egri chiziq nuqtasi orasidagi masofa nuqta chiziq grafigi bo‘ylab cheksiz uzoqlashganda nolga yaqinlashadi.
ellipsning katta yarim	**semi-major axis of the**	**Большая полуось эллипса**	Ellipsning fokuslari yotuvchi simmet-

o‘qi	**ellipse**		riya o‘qi.
vektor	**vector**	**вектор**	Yo‘nalishi va soniy qiymati bilan aniqlanuvchi miqdor (geometrik ifodasi: yo‘naltirilgan kesma).
vektorlar fazosi	**space vector**	**векторное пространство**	Ko‘p o‘lchamli vektorlar fazosi (uch o‘lchamli fazo tushunchasining umumlashmasi).
vektorli ko‘paytma	**vector product**	**векторное произведение**	Ikki vektorning vektorli ko‘paytmasi shunday vektordan iboratki, uning moduli shu vektorlardan yasalgan parallelogram yuziga tehg, yo‘nalishi esa berilgan vektorlar yotgan tekislikka perpendikulyar bo‘lib, o‘ng oriyentasiyaga ega.
giperbola	**hypebola**	**гипербола**	Giperbola – shunday ikkinchi tartibli markaziy chiziqki, uning har bir nuqtasi fokal rariuslari ayirmasining absolyut qiymati o‘zgarmasdir.
giperboloid	**hyperboloid**	**гиперболоид**	Ikkinchi tartibli sirt. Ular giperbolani biror simmetriya o‘qlari atrofida aylantirish natijasida hosil qilinishi mumkin.

grafik	**graph (diagram)**	**график**	O‘zgaruvchilar orasidagi berilgan funksional bog‘lanishning geometrik ifodasi (chizma ko‘rinishida).
isbot	**proof**	**доказательство**	Tasdiqning to‘g‘riligi aniqlanadigan mushohadalar zanjiri.
yetarli shartlar	**the sufficient conditions**	**достаточные условия**	Berilgan tasdiq kelib chiqadigan shartlar.
birlik vector	**the unit vector**	**единичный вектор (орт)**	Uzunligi birga teng vektor.
kanonik tenglama	**the canonical equation**	**каноническое уравнение**	Chiziq va sirtnig sodda tenglamasi.
kollinear vektorlar	**collinear vectors**	**коллинеарные векторы**	Bir to‘g‘ri chiziqqa parallel bo‘lgan yoki bir to‘g‘ri chiziqda yotuvchi vektorlar.
komplanar vektorlar	**complanar vectors**	**компланарные векторы**	Bir tekislikda yoki parallel tekisliklarda yotuvchi vektorlar.
koordinata o‘qlari	**coordination axiss**	**координатные осы**	Tekislikda(fazoda) o‘zaro perpendikulyar bo‘lgan to‘g‘ri chiziqlar.
konus (konik sirt)	**cone**	**конус (коническая поверхность)**	Ikkinchi tartibli sirt. Unga mos giperboloid uchun asimptotik konus deb ataladi.

yo‘naltiruv-chi kosi-nuslar	**the direction cosines**	**направляющие косинусы**	Berilgan vektorning koordinata o‘qlari bilan tashkil etgan burchak kosinuslari.
zaruriy shartlar	**the necessary conditions**	**необходимые условия**	Berilgan tasdiqdan kelib chiqadigan shartlar.
zaruriy va yetarli shartlar	**necessary and sufficient conditions**	**необходимые и достаточные условия**	Berilgan tasdiqga ekvivalent (teng-kuchli) shartlar. Teo-remalarning ifodala-nish shakllaridan biri.
normal	**normal**	**нормаль**	Chiziqning berilgan nuqtasigan shu nuq-tagagi urinmaga perpendikulyar o‘tuvchi to‘g‘ri chi-ziq.
determinant	**Determi-nant**	**определитель (детерминант)**	Kvadrat matritsaga ma’lum qoidalar bo‘yicha mos qo‘-yilgan son.
aylana	**circle**	**окружность**	Tekislikda berilgan nuqtadan bir xil masofada joylash-gan nuqtalarining geometrik o‘r ni.
ordinata	**ordinate**	**ордината**	Fazodagi nuqtaning dekart koordinata-laridan ikkinchisi (nuqta radius-vek-torining *OY* o‘qi-dagi proyeksiyasi).
parabola	**parabole**	**парабола**	Parabola – shunday ikkinchi tartibli markazsiz chiziqki,

			uning har bir nuqtasi berilgan nuqta (fokus) va berilgan to‘g‘ri chiziq (direktrisa)gaca bir xil masofada bo‘ladi.
paraboloid	**paraboloid**	**параболоид**	Ikkinchi tartibli sirt. Paraboloidlar elliptik va giperbolik tipda bo‘ladi.
tekis burchak	**plane angle**	**плоский угол**	Tekislikning bir nuqtadan chiquvchi ikki nur (yarim to‘g‘ri chiziqlar) orasidagi qismi o‘lchovi.
to‘gri chiziq	**straight line**	**прямая линия (прямая)**	Faqat o‘zining ikki nuqtasi yoki bitta nuqtasi va yo‘nalishi bilan to‘liq aniqlanuvchi chiziq(berilgan nuqtadan chiquvchi nur – yarim to‘g‘ri chiziqdir).
vector proyeksiyasi	**projection of the vector**	**проекция вектора**	Vektor uzunliginig shu vector va o‘q yo‘nalishi orasidagi burchak kosinusiga ko‘paytmasidan iborat ckalyar miqdor.
skalyar ko‘paytma	**scalar product**	**скалярное произведение**	Ikki vektor uzunliklari bilan ular orasidagi burchak kosinusining ko‘paytmasiga teng bo‘gan skalyar miqdor.

sfera	**sphere**	**сфера**	Fazoda berilgan nuqtadan bir xil masofada joylash-gan nuqtalarning geometrik o‘rni.
teorema	**theorem**	**теорема**	Isbot talab qiluvchi tasdiq(mulohaza).
kollinearlik sharti	**condition of collinearity**	**условие коллинеарности**	Vektorlarninr kolli-nearlik sharti ular-ning mos koordina-talarining propor-sionalligini bildi-radi.
burchak koeffitsiyenti	**the angle coefficient**	**угловой коэффициет**	To‘g‘ri chiziq va absissa o‘qi orasi-dagi burchak tan-gensiga teng miq-dor.
formula	**formula**	**формула**	Miqdorlar orasidagi bog‘lanishning matematik ifodasi (tenglik, tenglama, tengsislik va sh.k.).
silindrik sirt	**cylindrical surface**	**цилиндри-ческая поверхность**	Ikkinchi tartibli sirt. Ularning yasovchi-lari koordinata o‘q-laridan birortasiga parallel bo‘ladi. Si-lindrlar elliptic, gi-perbolik va parabo-lic tipda bo‘ladi.
sonlar o‘qi	**numeric axis**	**числовая ось**	Yo‘nalishga ega bo‘lgan va massh-tab birligi tanlangan kesma (haqiqiy sonlar to‘plamining geometrik tasviri).
ellips	**ellipse**	**эллирс**	Ellips – shunday

			ikkinchi tartibli markaziy chiziqki, uning har bir nuqtasi fokal radiuslari yig‘indisi o‘zgarmasdir. Ularni aylananing koordinata o‘qi bo‘yicha tekis deformatsiyasi natijasi deb qarash mumkin.
ellipsoid	**ellipsoid**	**эллипсоид**	Ikkinch tartibli sirt. Ular sferani koordinata tekisliklariga nisbatan tekis qisish (yoki cho‘zish) natijasida hosil qilinishi mumkin.

TEST SAVOLLARINING JAVOBLARI

I-Test

1	2	3	4	5	6	7	8	9	10
A	**C**	**A**	**C**	**A**	**B**	**A**	**B**	**E**	**C**
11	12	13	14	15	16	17	18	19	20
B	**A**	**B**	**C**	**D**	**B**	**A**	**C**	**E**	**B**
21	22	23	24	25	26	27	28	29	30
C	**A**	**B**	**A**	**D**	**E**	**B**	**C**	**A**	**E**
31	32	33	34	35	36	37	38	39	40
D	**C**	**D**	**C**	**E**	**A**	**B**	**D**	**E**	**C**
41	42	43	44	45	46	47	48	49	50
E	**A**	**B**	**D**	**C**	**E**	**A**	**B**	**C**	**E**

II- Test

1	2	3	4	5	6	7	8	9	10
A	**B**	**A**	**C**	**D**	**B**	**E**	**D**	**B**	**A**
11	12	13	14	15	16	17	18	19	20
C	**D**	**C**	**A**	**B**	**D**	**A**	**B**	**C**	**A**
21	22	23	24	25	26	27	28	29	30
B	**E**	**D**	**A**	**C**	**B**	**A**	**C**	**B**	**C**
31	32	33	34	35	36	37	38	39	40
D	**E**	**C**	**B**	**D**	**A**	**C**	**D**	**B**	**A**
41	42	43	44	45	46	47	48	49	50
C	**B**	**C**	**E**	**A**	**C**	**B**	**D**	**E**	**A**

III- Test

1	2	3	4	5	6	7	8	9	10
B	**A**	**C**	**B**	**D**	**E**	**D**	**A**	**D**	**C**
11	12	13	14	15	16	17	18	19	20
B	**A**	**C**	**B**	**E**	**D**	**A**	**E**	**A**	**B**
21	22	23	24	25	26	27	28	29	30
C	**C**	**D**	**D**	**E**	**A**	**B**	**C**	**D**	**E**
31	32	33	34	35	36	37	38	39	40
B	**C**	**C**	**A**	**E**	**C**	**B**	**A**	**D**	**C**
41	42	43	44	45	46	47	48	49	50
D	**E**	**A**	**B**	**B**	**D**	**A**	**C**	**B**	**E**

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MUNDARIJA

III BOB. FAZODAGI ANALITIK GEOMETRIYADAN MA‘LUMOTLAR

§1. FAZODAGI TEKISLIK VA CHIZIQ TEGLAMALARI. TEKISLIK VA TO‘G‘RI CHIZIQ

§2. TO‘G‘RI CHIZIQ VA TEKISLIKLARGA DOIR ASOSIY MASALALAR

§3. IKKINCHI TARTIBLI SIRTLAR

ОГЛАВЛЕНИЕ

§2. УРАВНЕНИЕ ЛИНИИ НА ПЛОСКОСТИ

§3. ПРЯМАЯ ЛИНИЯ

§4. ЛИНИИ ВТОРОГО ПОРЯДКА

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QAYDLAR UCHUN

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S. OTAKULOV, A. O. MUSAYEV

ANALITIK GEOMETRIYA VA VEKTORLAR ALGEBRASI

Toshkent – «Fan va texnologiyalar nashriyot-matbaa uyi» – 2022

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E-mail: tipografiyacnt@mail.ru Tel: 97-450-11-14, 93-381-22-07.
Bosishga ruxsat etildi 14.06.2022.
Bichimi 60x84 $^{1}/_{16}$. «Times New Roman» garniturasi.
Ofset bosma usulida bosildi.
Shartli bosma tabog‘i 10,75. Nashriyot bosma tabog‘i 10,5.
Tiraji 160. Buyurtma № 78.

«Fan va texnologiyalar nashriyot-matbaa uyi»
bosmaxonasida chop etildi.
Toshkent sh., Foziltepa ko‘chasi, 22 b uy.