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Matematik fizika tenglamalaridan masalalar to‘plami

o‘quv-usuliy qo‘llanma

O’ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI
BUXORO DAVLAT UNIVERSITETI

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MATEMATIK FIZIKA TENGLAMALARIDAN MASALALAR TO‘PLAMI

(o‘quv-usuliy qo‘llanma)

Buxoro – 2011

Ushbu qo‘llanma B5460100 – matematika, B5480100 –amaliy matematika va informatika ta`lim yo‘nalishi talabalariga va “Matematik fizika” yo‘nalishi magistrlariga mo‘ljallangan.

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So‘z boshi

Matematik fizika tenglamalari fani nazariy va amaliy ahamiyatga ega. Mexanika, fizika, texnika va boshqa sohalarda uchraydigan turli jarayonlar matematik fizika tenglamalari orqali ifodalanadi. Fanning maqsadi matematik fizikaning klassik tenglamalari deb ataluvchi to‘lqin, Laplas, hamda issiqlik tarqalish tenglamalarini tekshirish va ularga qo‘yiladigan asosiy masalalarni yechishdan iborat.

Bu tenglamalarni o‘rganish talabalarda tegishli jarayonlar haqida tasavvurga ega bo‘lishlariga imkon beradi. Ayni paytda ularni mantiqiy fikrlashga, to‘gri xulosalar chiqarishga o‘rgatadi.

Matematik fizika tenglamalari hozirgi zamon matematikasining muhim sohalaridandir. U matematikaning bir necha sohalari, jumladan matematik analiz, funksiyalar nazariyasi, integral va differentsial tenglamalar nazariyasi, funksional analiz, fizika, texnika fanlari bilan uzviy bog‘liq. Matematik fizika tenglamalari so‘ngi yillarda keng rivoj topib kelyapti. Endigi kunda matematik fizikaning klassik tenglamalaridan tashqari aralash turdagi xususiy hosilali differensial tenglamalar ham o‘rganilib, va u fizikaning ko‘pgina masalalarini hal qilish uchun keng tatbiq qilinmoqda.

Matematik fizika tenglamalari fanining asosiy vazifalariga xususiy hosilali tenglamalar haqida umumiy tushuncha berish, ikkinchi tartibili kvazichiziqli tenglamalarning turlarini aniqlab va ularni kanonik ko‘rinishga keltirish, va matematik fizikaning klassik tenglamalari va integral tenglamalarni o‘rganish, har bir turdagi tenglamalarga asosiy masalaning qo‘yilishi, va bu masalarni yechish usullarini o‘rganishdan iborat. Shu bilan birga bu fanning asosiy mazmuni klassik matematik fizika tenglamalari, integral tenglamalar, aralash turdagi tenglamalarni o‘rganishdir.

Ushbu qo‘llanma matematik fizika tenglamalarini analitik yechish, bu tenglamalarga qo‘yilgan masalalarni, integral tenglamalarni yechish usullariga bag‘ishlangan bo‘lib, bu usullar imkon qadar yoritishga harakat qilingan.

O‘quvchilardan ushbu qo‘llanma bo‘yicha talab va takliflarini kutib qolaman.

1. Xususiy hosilali differensial tenglamalar haqida asosiy tushunchalar. Ikkinchi tartibli xususiy hosilali differensial tenglamalarning klassifikasiyasi. Kanonik ko‘rinishga keltirish

Differensial tenglamalar deb, noma’lumi bir yoki bir necha o‘zgaruvchili funksiya va uning hosilalari qatnashgan tenglamalarga aytiladi.

Agar tenglamada noma’lum funksiya ko‘p o‘zgaruvchining (o‘zgaruvchi 2 tadan kam bo‘lmasligi kerak) funksiyasi bo‘lsa, bunday tenglama xususiy hosilali differensial tenglama deyiladi.

Ta’rif: x,y erkli o‘zgaruvchining $u(x,y)$ noma’lum funksyasi va funksiyaning ikkinchi tartibli xususiy hosilalari orasidagi bog‘lanishga, ikkinchi tartibli xususiy hosilali differensial tenglamalar deyiladi.

Ta’rif: E^2 fazoda ikkinchi tartibli xususiy hosilalari mavjud qandaydir $u(x,y)$ funksiya berilgan bo‘lsin ($u_{xy}=u_{yx}$). U holda

$$F(x,y,u,u_x,u_y,u_{xx},u_{xy},u_{yy})=0 \qquad (1)$$

tenglama umumiy holda berilgan xususiy hosilali differensial tenglama deyiladi.

Bu yerda F - qandaydir funksiya.

Xuddi shunga o‘xshash ko‘p erkli o‘zgaruvchili ikkinchi tartibli xususiy hosilali differensial tenglama quyidagi ko‘rinishda ifodalanadi:

$$F(x_1,x_2,...,x_n,u,u_{x_1},u_{x_2},...,u_{x_n},...,u_{x_i x_j},...)=0. \qquad (2)$$

Ta’rif: Ikkinchi tartibli xususiy hosilali differensial tenglama yuqori tartibli hosilalarga nisbatan chiziqli deyiladi, agarda u yuqori tartibli hosilalarga nisbatan ushbu ko‘rinishga ega bo‘lsa:

$$a_{11}(x,y)\cdot u_{xx}+2a_{12}(x,y)\cdot u_{xy}+a_{22}(x,y)\cdot u_{yy}+F(x,y,u,u_x,u_y)=0. \qquad (3)$$

Ta’rif: Quyidagi ko‘rinishdagi tenglamalarga kvazichiziqli tenglamalar deyiladi:

$$a_{11}(x,y,u,u_x,u_y)\cdot u_{xx}+2a_{12}(x,y,u,u_x,u_y)\cdot u_{xy}+a_{22}(x,y,u,u_x,u_y)\cdot u_{yy}+F(x,y,u,u_x,u_y)=0.$$

(4)

Ta'rif: Tenglama chiziqli deyiladi, agarda u barcha xususiy hosilalarga va noma'lum funksiyaning o'ziga nisbatan ham chiziqli bo'lsa, ya'ni quyidagi ko'rinishga ega bo'lsa,

$$a_{11}(x,y)\cdot u_{xx}+2a_{12}(x,y)\cdot u_{xy}+a_{22}(x,y)\cdot u_{yy}+b_1(x,y)\cdot u_x+b_2(x,y)\cdot u_y+c(x,y)\cdot u+f(x,y)=0. \qquad (5)$$

Ushbu tenglamada $a_{11}(x,y),\ a_{12}(x,y),\ a_{22}(x,y),\ b_1(x,y),\ b_2(x,y),\ c(x,y)$ - (5) tenglamaning koeffitsientlari, $f(x,y)$ - (5) tenglamaning ozod hadi deyiladi va ular oldindan berilgan deb hisoblanadi.

Ta'rif: Agar (5) tenglamada $f(x,y)\equiv 0$ bo'lsa, u holda bu tenglama bir jinsli tenglama deyiladi. Aks holda, agar $f(x,y)\neq 0$ bo'lsa, (5) tenglama bir jinsli bo'lmagan differensial tenglama deyiladi.

Biz x va y erkli o'zgaruvchilarni teskari almashtirish natijasida, ya'ni

$$\xi=\varphi(x,y),\ \eta=\psi(x,y) \qquad (6)$$

berilgan chiziqli tenglamaga ekvivalent bo'lgan va soddaroq ko'rinishga ega bo'lgan tenglamaga ega bo'lishimiz mumkin.

Buning uchun (3) tenglamada x va y erkli o'zgaruvchilardan yangi ξ va η o'zgaruvchilarga o'tamiz:

$$\left.\begin{aligned} u_x&=u_\xi\xi_x+u_\eta\eta_x,\\ u_y&=u_\xi\xi_y+u_\eta\eta_y,\\ u_{xx}&=u_{\xi\xi}\xi_x^2+2u_{\xi\eta}\xi_x\eta_x+u_{\eta\eta}\eta_x^2+u_\xi\xi_{xx}+u_\eta\eta_{xx},\\ u_{xy}&=u_{\xi\xi}\xi_x\xi_y+u_{\xi\eta}\left(\xi_x\eta_y+\xi_y\eta_x\right)+u_{\eta\eta}\eta_x\eta_y+u_\xi\xi_{xy}+u_\eta\eta_{xy},\\ u_{yy}&=u_{\xi\xi}\xi_y^2+2u_{\xi\eta}\xi_y\eta_y+u_{\eta\eta}\eta_y^2+u_\xi\xi_{yy}+u_\eta\eta_{yy}. \end{aligned}\right\} \qquad (7)$$

(7) ifodalarni (3) tenglamaga keltirib qo'yib, ξ va η o'zgaruvchilarga nisbatan (3) tenglamaga ekvivalent bo'lgan quyidagi tenglamani olamiz:

$$\overline{a_{11}}(\xi,\eta)\cdot u_{\xi\xi}+2\overline{a_{12}}(\xi,\eta)\cdot u_{\xi\eta}+\overline{a_{22}}(\xi,\eta)\cdot u_{\eta\eta}+\overline{F}\left(\xi,\eta,u,u_\xi,u_\eta\right)=0, \qquad (8)$$

bu yerda

$$\overline{a}_{11}=a_{11}\xi_x^2+2a_{12}\xi_x\xi_y+a_{22}\xi_y^2,$$

$$\overline{a}_{12}=a_{11}\xi_x\eta_x+a_{12}\left(\xi_x\eta_y+\eta_x\xi_y\right)+a_{22}\xi_y\eta_y,$$

$$\overline{a}_{22}=a_{11}\eta_x^2+2a_{12}\eta_x\eta_y+a_{22}\eta_y^2,$$

Ta’rif: $$a_{11}dy^2 - 2a_{12}dxdy + a_{22}dx^2 = 0 \qquad (9)$$
tenglama (3) tenglamaning xarakteristik tenglamasi deyiladi.

Ta’rif: (9) tenglamaning integrallari esa (3) tenglamaning xarakteristikalari deyiladi.

(9) tenglama quyidagi ikkita tenglamaga ajraladi:

$$\frac{dy}{dx} = \frac{a_{12} + \sqrt{a_{12}^2 - a_{11}\cdot a_{22}}}{a_{11}}, \qquad (10)$$

$$\frac{dy}{dx} = \frac{a_{12} - \sqrt{a_{12}^2 - a_{11}\cdot a_{22}}}{a_{11}}. \qquad (11)$$

(9) yoki (10) va (11) yordamida berilgan (3)-tenglamaning xarakteristikalari topiladi.

Ta’rif: Agar qandaydir D sohada $a_{12}^2 - a_{11}\cdot a_{22} > 0$ bo‘lsa, (3) tenglama giperbolik turga qarashli, agar D sohada $a_{12}^2 - a_{11}\cdot a_{22} < 0$ bo‘lsa, berilgan (3) tenglama elliptik turga qarashli, agar D sohada $a_{12}^2 - a_{11}\cdot a_{22} = 0$ bo‘lsa, parabolik turga qarashli deyiladi.

Shunday qilib, $a_{12}^2 - a_{11}\cdot a_{22}$ ifodaning ishorasiga qarab (3) tenglamani quyidagi kanonik ko‘rinishlarga keltirilishi mumkin ekan.

$a_{12}^2 - a_{11}\cdot a_{22} > 0$ (giperbolik turda), $u_{xx} - u_{yy} = \Phi$ yoki $u_{xy} = \Phi$.

$a_{12}^2 - a_{11}\cdot a_{22} < 0$ (elliptik turda), $u_{xx} + u_{yy} = \Phi$.

$a_{12}^2 - a_{11}\cdot a_{22} = 0$ (parabolik turda) $u_{xx} = \Phi$.

Bu yerda Φ soddalashtirish natijasida hosil bo‘lgan funksiya.

1.1 Ikkinchi tartibli xususiy hosilali differensial tenglamalarni turi saqlanadigan sohada kanonik ko‘rinishga keltirish

Misol. Quyidagi tenglamani kanonik ko‘rinishga keltiraylik:

$$u_{xx}-2u_{xy}-3u_{yy}+u_y=0.$$

$a_{12} = -1,\ a_{11} = 1$, $a_{22} = -3$- tenglama koeffisiyentlari. $\Delta = a_{12}^2 - a_{11}\cdot a_{22}$ ifodaning kiymatini hisoblaymiz. $\Delta = 4 > 0$, demak tenglama giperbolik turga tegishli. (9) xarakteristik tenglamani yechamiz.

$$\frac{dy}{dx}=\frac{-1+2}{1}=1\Rightarrow x-y=C\ ,\ \frac{dy}{dx}=\frac{-1-2}{1}=-3\Rightarrow 3x+y=C$$

Umumiy integrallardan birini ξ va ikkinchisini η bilan belgilab, (7) formulalardan foydalanib hisoblashlarning natijalarini berilgan tenglamaga keltirib qo‘yib, soddalashtirishlardan so‘ng tenglamaning quyidagi kanonik ko‘rinishini hosil qilamiz: $u_{\xi\eta}-\frac{1}{16}(u_{\xi}-u_{\eta})=0$.

Mustaqil bajarish uchun mashqlar

Quyidagi tenglamalarni kanonik ko‘rinishga keltiring:

1. $u_{xx}-6u_{xy}+10u_{yy}+u_x-3u_y=0$
2. $4u_{xx}+4u_{xy}+u_{yy}-2u_y=0$
3. $u_{xx}-xu_{yy}=0$
4. $u_{xx}-yu_{yy}=0$
5. $xu_{xx}-yu_{yy}=0$
6. $yu_{xx}-xu_{yy}=0$
7. $x^2u_{xx}+y^2u_{yy}=0$
8. $y^2u_{xx}+x^2u_{yy}=0$
9. $y^2u_{xx}-x^2u_{yy}=0$
10. $(1+x^2)u_{xx}+(1+y^2)u_{yy}+yu_y=0$
11. $4y^2u_{xx}-e^{2x}u_{yy}=0$
12. $u_{xx}-2\sin x u_{xy}+(2-\cos^2 x)u_{yy}=0$
13. $y^2u_{xx}+2yu_{xy}+u_{yy}=0$
14. $x^2u_{xx}-xu_{xy}+u_{yy}=0$.

1.2 Ko‘p erkli o‘zgaruvchili funksiyalar (n>2) bo‘lgan hol uchun ikkinchi tartibli xususiy hosilali differensial tenglamalarni kanonik ko‘rinishga keltirish

Ko‘p erkli o‘zgaruvchili ikkinchi tartibli xususiy hosilali differensial tenglama qanday kanonik ko‘rinishga keltiriladi? Shu masalani qarab chiqaylik. Ko‘p o‘zgaruvchili chiziqli ikkinchi tartibli xususiy hosilali differensial tenglama quyidagicha berilgan bo‘lsin :

$$\sum_{i,j=1}^{n} A_{ij} \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i,=1}^{n} B_i \frac{\partial u}{\partial x_i} + Cu = f \qquad (12)$$

U holda ushbu tenglamaning xarakteristik tenglamasi ko‘rinishi kvadratik forma bo‘ladi:

$$Q(\lambda_1,...,\lambda_n) = \sum_{i,j=1}^{n} A_{ij}(x)\lambda_i\lambda_j .$$

Har bir fiksirlangan x nuqtada Q kvadratik formani uncha qiyin bo‘lmagan affin almashtirishlari yordamida kanonik ko‘rinishga keltirish mumkin:

$$Q = \sum_{i=1}^{n} \alpha_i \xi_i^2 \qquad (13)$$

Bu yerda α_i lar 1, -1, 0 qiymatlarni qabul qiladi. (13) dagi manfiy va nol koeffisiyentlar Q ni kanonik ko‘rinishga keltirsh usuliga bog‘liq emas. Shunga asosan (12) tenglama klassifikasiyalanadi.

Ta’rif: Agar har bir $x \in D$ nuqtada (13) dagi α_i koeffisiyentlar mos ravishda: hammasi noldan farqli va bir xil ishorali; hammasi noldan farqli va har xil ishorali; va nihoyat hech bo‘lmasi bittasi (hammasi emas) nol bo‘lsa, (12) chiziqli tenglama D sohada elliptik, giperbolik yoki parabolik deyiladi,

Ko‘p erkli o‘zgaruvchili ikkinchi tartibli xususiy hosilali differensial tenglamalardan bittasini kanonik ko‘rinishga keltirish usulini qarab chiqaylik.

Misol. Quyidagi tenglama berilgan bo‘lsin:

$$u_{xx} + 2u_{xy} + 2u_{yy} + 4u_{yz} + 5u_{zz} = 0 .$$

Ushbu tenglamaga mos xarakteristik kvadratik forma $Q = \lambda_1^2 + 2\lambda_1\lambda_2 + 2\lambda_2^2 + 4\lambda_2\lambda_3 + 5\lambda_3^2$ ko‘rinishda bo‘ladi. Bu kvadratik formani, masalan, Lagranj usulidan foydalanib kanonik ko‘rinishga keltiramiz: $Q = (\lambda_1 + \lambda_2)^2 + (\lambda_2 + 2\lambda_3)^2 + \lambda_3^2$. Quyidagi belgilashlar kiritamiz:

$$\mu_1 = \lambda_1 + \lambda_2 ; \ \mu_2 = \lambda_2 + 2\lambda_3 ; \ \mu_1 = \lambda_3 \qquad (*)$$

va natijada Q formani kanonik ko‘rinishga keltiramiz: $Q=\mu_1^2+\mu_2^2+\mu_3^2$.

(*) tengliklardan λ larni topib olamiz. Shunday qilib, $M=\begin{pmatrix}1 & -1 & 2\\ 0 & 1 & -2\\ 0 & 0 & 1\end{pmatrix}$

matrisali quyidagi xosmas affin almashtirishlari: $\lambda_1=\mu_1-\mu_2+2\mu_3$, $\lambda_2=\mu_2-2\mu_3$, $\lambda_3=\mu_3$ Q formani kanonik ko‘rinishga keltiradi: $Q=\mu_1^2+\mu_2^2+\mu_3^2$.

Berilgan differensial tenglamani kanonik ko‘rinishga keltiradigan xosmas affin almashtirishining matrisasi M matrisaga simmetrik bo‘lgan matrisa bo‘ladi: $M^*=\begin{pmatrix}1 & 0 & 0\\ -1 & 1 & 0\\ 2 & -2 & 1\end{pmatrix}$, bu almashtirish quidagi ko‘rinishga ega: $\xi=x$; $\eta=-x+y$; $\zeta=2x-2y+z$.

Shulardan va $u(x,y,z)=v(\xi,\eta)$ belgilashdan foydalanib, quyidagilarni topamiz:

$$u_{xx}=v_{\xi\xi}+v_{\eta\eta}+4v_{\zeta\zeta}-2v_{\xi\eta}+4v_{\xi\zeta}-4v_{\eta\zeta};$$

$$u_{yy}=v_{\eta\eta}+4v_{\zeta\zeta}-4v_{\eta\zeta}; \quad u_{zz}=v_{\zeta\zeta};$$

$$u_{xy}=-v_{\eta\eta}-4v_{\zeta\zeta}+v_{\xi\eta}-2v_{\xi\zeta}+4v_{\eta\zeta}; \; u_{yz}=-2v_{\zeta\zeta}+v_{\eta\zeta}.$$

Topilgan ifodalarni tenglamaga etib qo‘yib, soddalashtirishlar bajargandan so‘ng, berilgan tenglamaning kanonik ko‘rinishini olamiz: $v_{\xi\xi}+v_{\eta\eta}+v_{\zeta\zeta}=0$.

Mustaqil bajarish uchun mashqlar

Quyidagi tenglamalarni kanonik ko‘rinishga keltiring:

15. $u_{xx}+2u_{xy}-2u_{xz}+2u_{yy}+6u_{zz}=0$
16. $4u_{xx}-4u_{xy}-2u_{zy}+u_y+u_z=0$
17. $u_{xy}-u_{xz}+u_x+u_y-u_z=0$
18. $u_{xx}+2u_{xy}-2u_{xz}+2u_{yy}+2u_{zz}=0$
19. $u_{xx}+2u_{xy}-2u_{xz}-6u_{yz}-u_{zz}=0$
20. $u_{xx}+2u_{xy}+2u_{yy}+2u_{yt}+2u_{zz}+3u_{tt}=0$
21. $u_{xy}-u_{xt}+u_{zz}-2u_{zt}+2u_{tt}=0$
22. $u_{xy}+u_{xz}+u_{xt}+u_{zt}=0$

23. $u_{xx}+2u_{xy}-2u_{zz}-4u_{yz}+2u_{yt}+u_{zz}=0$

24. $u_{xx}+2u_{xz}-2u_{xt}+u_{yy}+2u_{yz}+2u_{yt}+2u_{zz}+2u_{tt}=0$

25. $u_{x_1x_1}+2\sum_{k=2}^{n}u_{x_kx_k}-2\sum_{k=2}^{n}u_{x_kx_{k+1}}=0$

26. $u_{x_1x_1}-2\sum_{k=2}^{n}(-1)^k u_{x_{k-1}x_k}=0$

27. $\sum_{k=2}^{n}ku_{x_kx_k}+2\sum_{l<k}lu_{x_lx_k}=0$

28. $\sum_{k=1}^{n}u_{x_kx_k}+\sum_{l<k}u_{x_lx_k}=0$

29. $\sum_{l<k}u_{x_lx_k}=0$.

2. Xususiy hosilali differensial tenglamalarning umumiy yechimini topish

Ta’rif: Xususiy hosilali differensial tenglamaning umumiy yechimi deb, shu tenglamani qanoatlantiradigan funksiyaga aytiladi.

Oddiy differensial tenglamalar kursidan ma’lumki, $n-$tartibli

$$F(x,y,y',...,y^{(n)})=0$$

tenglamaning yechimi n ta ixtiyoriy o‘zgarmasga bog‘liqdir, ya’ni $y=\varphi(x,c_1,...,c_n)$. Bu o‘zgarmaslarni aniqlash uchun noma’lum funksiya $y(x)$ qo‘shimcha shartlarni qanoatlantirishi kerak.

Xususiy hosilali differensial tenglamalar uchun bu masala murakkabroqdir. Bu tenglamalarning yechimi ixtiyoriy o‘zgarmaslarga emas, balki ixtiyoriy funksiyalarga bog‘liq bo‘lib, bu funksiyalar soni tenglamalar tartibiga teng bo‘ladi. Ixtiyoriy funksiyalar argumentlarining soni yechim argumentlari sonidan bitta kam bo‘ladi.

2.1 O‘zgarmas koeffisiyentli xususiy hosilali differensial tenglamalarning umumiy yechimini topish

Misol. Quyidagi tenglamaning umumiy yechimini toping: $u_{xy}=0$.

Dastlab x bo‘yicha, so‘ngra y bo‘yicha integrallaymiz, natijada $u(x,y)=f_1(x)+f_2(y)$ yechimni olamiz. Ko‘rib turganingizdek, xususiy hosilali differensial tenglamaning yechimida tenglama tartibiga teng miqdorda, ya’ni ikkita funksiya qatnashayapti, bu funksiyalar argumenti esa yechim argumentlari sonidan bitta kam.

Misol. Quyidagi tenglamaning ham umumiy yechimini topaylik:

$$u_{xyy}=0.$$

Yuqoridagidek mulohaza yuritsak umumiy yechim:

$$u(x,y)=f_1(x)y+f_2(x)+f_3(y).$$

Misol. Quyidagi tenglamaning ham umumiy yechimini topaylik:

$$u_{xyz}=0.$$

Yuqoridagidek mulohaza yuritsak umumiy yechim:

$$u(x,y,z)=x\cdot y\cdot f_1(x,y)+x\cdot f_2(x,z)+f_3(y,z).$$

Oxirgi misolda, ko‘rib turganingizdek yechimda tenglama tartibiga mos uchta funksiya qatnashaypti, yechim uch o‘zgaruvchili bo‘lgani uchun bu funksiyalar argumenti ikki o‘zgaruvchili.

Mustaqil bajarish uchun mashqlar

Quyida berilgan tenglamalarning umumiy yechimini toping:

1. $u_{xx}-a^2u_{yy}=0$
2. $u_{xx}-2u_{xy}-3u_{yy}=0$
3. $u_{xy}+au_x=0$
4. $3u_{xx}-5u_{xy}-2u_{yy}+3u_x+u_y=2$
5. $u_{xy}+au_x+bu_y+abu=0$
6. $u_{xy}-2u_x-3u_y+6u=2e^{x+y}$
7. $u_{xx}+2au_{xy}+a^2u_{yy}+u_x+au_y=0.$

2.2 Xususiy hosilali differensial tenglamalarning turi saqlanadigan sohada umumiy yechimini topish

Misol. Quyidagi tenglamaning turi saqlanadigan sohani topib, umumiy ychimini aniqlang: $x^2u_{xx}-y^2u_{yy}=0$.

$a_{11}=x^2,\ \ a_{12}=0,\ \ a_{22}=-y^2$ - tenglama koeffisiyentlari. $\Delta=a_{12}^2-a_{11}a_{22}$ ifodaninig qiymatini hisoblaymiz. $\Delta=(xy)^2$, hamma chorakda tenglamamiz giperbolik ekan. Yangi ξ va η o‘zgaruvchilkarga o‘tamiz :

$\xi=xy$, $\eta=\dfrac{x}{y}$ almashtirish yordamida berilgan tenglamani kanonik ko‘rinishga keltiramiz. Kanonik ko‘rinishi quyidagicha: $u_{\xi\eta}-\dfrac{1}{2\xi}u_\eta=0$.

Unda $u_\eta=v$ almashtirsh bajarib tenglamani yechamiz, natijada

$$\begin{cases} \ln v=\dfrac{1}{2}\ln\xi-\ln f(\eta) \\ \quad v=\sqrt{\xi}\,f(\eta) \qquad \Rightarrow\ u=\sqrt{\xi}\,f(\eta)+g(\xi) \\ \quad u_\eta=\sqrt{\xi}\,f(\eta) \end{cases}$$

yechimni olamiz. Dastlabki o‘zgaruvchilarga qaytsak, biz izlayotgan umumiy yechim :

$$u(x,y)=\sqrt{|\,xy\,|}\cdot f\left(\frac{x}{y}\right)+g(xy)$$

bo‘ladi.

Mustaqil bajarish uchun mashqlar

Quyidagi tenglamalarning umumiy yechimini toping.

8. $yu_{xx}+(x-y)u_{xy}-xu_{yy}=0$.
9. $x^2u_{xx}+2xyu_{xy}-3y^2u_{yy}-2xu_x=0$.
10. $x^2u_{xx}+2xyu_{xy}+y^2u_{yy}=0$.
11. $u_{xy}-xu_x+u=0$.
12. $u_{xy}+2xyu_y-2xu=0$.
13. $u_{xy}+u_x+yu_y+(x-1)u=0$.
14. $u_{xy}+xu_x+2yu_y+2xyu=0$.

3. Ikkinchi tartibli giperbolik turdagi differensial tenglamalarga qo‘yilgan Koshi masalasi

Biror fizik jarayonni to‘la o‘rganish uchun, bu jarayonni tasvirlayotgan tenglamalardan tashqari, uning boshlang‘ich holatini (boshlang‘ich shartlarni) va jarayon sodir bo‘ladigan sohaning chegarasidagi holatini (chegaraviy shartlarni) berish zarurdir.

Shunday qilib, aniq fizik jarayonni ifodolovchi yechimni ajratib olish uchun qo‘shimcha shartlarni berish zarurdir. Bunday qo‘shimcha shartlar boshlang‘ich va chegaraviy shartlardan iboratdir.

Jarayon sodir bo‘layotgan soha $G \subset R^n$ bo‘lib, S uning chegarasi bo‘lsin. S ni bo‘laklari silliq sirt hisoblaymiz.

Differensial tenglamalar uchun, asosan, 3 turdagi masalalar bir biridan farq qiladi.

a) Koshi masalasi. Bu masala, asosan giperbolik va parabolik turdagi tenglamalar uchun qo‘yiladi; G soha butun R^n fazo bilan ustma ust tushadi, bu holda chegaraviy shartlar bo‘lmaydi.

b) Chegaraviy masala elliptik turdagi tenglamalar uchun qo‘yiladi; S da chegaraviy shartlar beriladi, boshlang‘ich shartlar tabiiy bo‘lmaydi.

c) Aralash masala giperbolik va parabolik turdagi tenglamalar uchun qo‘yiladi; $G \subset R^n$ bo‘lib, boshlang‘ich va chegaraviy shartlar beriladi.

Har qanday masalaning mohiyati berilgan $\varphi \in E_\varphi$ funksyailarga asosan uning $u \in E_u$ yechimini topishdan iboratdir, bu yerda E_u va E_φ - metrikalari ρ_u va ρ_φ bo‘lgan qandaydir metrik fazolardir. Bu fazolar masalaning qo‘yilishi bilan aniqlanadi. Masalaning yechimi tushunchasi aniqlangan bo‘lib, har bir $\varphi \in E_\varphi$ elementlar yagona $u = R(\varphi) \in E_u$ yechim mos kelsin.

Agar $\forall\varepsilon > 0$ uchun shunday $\delta(\varepsilon) > 0$ sonni ko‘rsatish mumkin bo‘lib, $\rho_\varphi(\varphi_1,\varphi_2) \le \delta(\varepsilon)$ tengsizlikdan $\rho_u(u_1,u_2) \le \varepsilon$ tengsizlik kelib chiqsa, masala (E_u, E_φ) fazolar juftida turg‘un masala deyiladi.

Bunda $u_i = R(\varphi_i)$, $u_i \in E_u$, $\varphi_i \in E_\varphi$, $i = 1,2,...$ masalaning yechimi berilgan shartlar (boshlang‘ich va chegaraviy shartlar, tenglamaning koeffisientlari, ozod hadi va h.k.) ga uzluksiz bog‘liq bo‘ladi.

Agar tekshirilayotgan masala uchun ushbu

1) ixtiyoriy $\varphi \in E_\varphi$ uchun $u \in E_u$ yechim mavjud;
2) u yechim yagona;
3) masala (E_u, E_φ) fazolar juftligida turg‘un shartlar bajarilsa, masala (E_u, E_φ) fazolar juftligida korrekt (to‘g‘ri) qo‘yilgan yoki to‘g‘ridan to‘g‘ri korrekt masala deyiladi.

Aks holda masala korekt qo‘yilmagan masala deyiladi, ya’ni yuqoridagi talablardan kamida bittasi bajarilmay qoladi.

Yechim boshlang‘ich shartlarga uzluksiz bog‘liq bo‘lmasligi ham mumkin.

3.1 Koshi masalalarini yechish

Masala. Quyidagi Koshi masalasini yeching:

$$xu_{xx} - u_{yy} + \frac{1}{2}u_x = 0 \;;$$

$$u\big|_{y=0} = x, \qquad u_y\big|_{y=0} = 0, \qquad x > 0.$$

Dastlab, tenglamani kanonik ko‘rinishga keltiramiz. $\Delta = a_{12}^2 - a_{11}a_{22}$ ifodaninig qiymatini hisoblaylik. $\Delta = x$, $x > 0$ bo‘lgani uchun tenglama giperbolik. Yangi ξ va η o‘zgaruvchilkarga o‘tamiz :

$\xi = 2\sqrt{x} + y$, $\eta = 2\sqrt{x} - y$ almashtirish yordamida berilgan tenglamani kanonik ko‘rinishga keltiramiz. Kanonik ko‘rinishi quyidagicha: $u_{\xi\eta} = 0$. Berilgan tenglamaninig umumiy yechimi : $u(x,y) = f\left(2\sqrt{x} + y\right) + g\left(2\sqrt{x} - y\right)$.

Bu yechimlar orasidan Koshi shartlarini qanoatlantiruvchi yechimni topamiz. Bu uchun quyidagi tenglamalar sistemasini yechamiz :

$$\begin{cases} f\left(2\sqrt{x}\right)+g\left(2\sqrt{x}\right)=x \\ f_y'\left(2\sqrt{x}\right)-g_y'\left(2\sqrt{x}\right)=0 \end{cases}.$$

Natijada $f\left(2\sqrt{x}\right)=g\left(2\sqrt{x}\right)=\frac{x}{2}$ yechimlarni olamiz, shu natijalarni keltirb umumiy yechimga qo‘ysak, Koshi masalasining yechimini olamiz : $u(x,y)=x+\frac{y^2}{4},\ \ \mathrm{x}>0,\ \ |\,\mathrm{y}\,|<2\sqrt{\mathrm{x}}$.

Mustaqil bajarish uchun mashqlar

Quyidagi Koshi masalalarini yeching :

1. $u_{xy}=0$;

$$u\big|_{y=x^2}=0,\qquad u_y\big|_{y=x^2}=\sqrt{|x|},\qquad |x|<1.$$

2. $u_{xy}+u_x=0$;

$$u\big|_{y=x}=\sin x,\qquad u_x\big|_{y=x}=1\qquad |x|<\infty.$$

3. $u_{xx}-u_{yy}+2u_x+2u_y=0$;

$$u\big|_{y=0}=x,\qquad u_y\big|_{y=0}=0,\qquad |x|<\infty.$$

4. $u_{xx}-u_{yy}-2u_x-2u_y=4$;

$$u\big|_{x=0}=-y,\qquad u_x\big|_{x=0}=y-1,\qquad |y|<\infty.$$

5. $u_{xx}+2u_{xy}-3u_{yy}=2$;

$$u\big|_{y=0}=0,\qquad u_y\big|_{y=0}=x+\cos x,\qquad |x|<\infty.$$

6. $u_{xy}+yu_x+xu_y+xyu=0$;

$$u\big|_{y=3x}=0,\quad u_y\big|_{y=3x}=e^{-5x^2},\qquad x<1.$$

7. $xu_{xx}+(x+y)u_{xy}+yu_{yy}=0$;

$$u\big|_{y=\frac{1}{x}}=x^3,\qquad u_x\big|_{y=\frac{1}{x}}=2x^2,\qquad x>0.$$

8. $u_{xx}+2(1+2x)u_{xy}+4x(1+x)u_{yy}+2u_y=0$;

$$u\big|_{x=0}=y,\qquad u_x\big|_{x=0}=2,\qquad |y|<\infty$$

9. $x^2u_{xx}-y^2u_{yy}-2yu_y=0$;

$$u\big|_{x=1}=y,\quad u_x\big|_{x=1}=y,\qquad y<0.$$

10. $x^2u_{xx}-2xyu_{xy}-3y^2u_{yy}=0$;

$$u\big|_{y=1}=0,\quad u_y\big|_{y=1}=\sqrt[4]{x^7},\quad x>0.$$

11. $yu_{xx}+x(2y-1)u_{xy}-2x^2u_{yy}-\frac{y}{x}u_x=0$

$$u\big|_{y=0}=x^2,\quad u_y\big|_{y=0}=1,\qquad x>0$$

12. $yu_{xx}-(x+y)u_{xy}+xu_{yy}=0$

$$u\big|_{y=0}=x^2,\quad u_y\big|_{y=0}=x,\quad x>0$$

13. $u_{xy}+2u_x+u_y+2u=1,\ 0<x,\ y<1$;

$$u\big|_{x+y=1}=x,\quad u_x\big|_{x+y=1}=x$$

14. $xyu_{xy}+xu_x-yu_y-u=2y,\quad 0<x,\ y<\infty$

$$u\big|_{xy=1}=1-y,\quad u_y\big|_{xy=1}=x-1$$

15. $u_{xy}+\frac{1}{x+y}(u_x+u_y)=2,\quad 0<x,\quad y<\infty$

$$u\big|_{y=x}=x^2,\quad u_x\big|_{y=x}=1+x$$

16. $u_{xx}-u_{yy}+\frac{2}{x}u_x-\frac{2}{y}u_y=0,\quad |x-y|<1,\qquad |x+y-2|<1$

$$u\big|_{y=1}=u_0(x)\qquad u_y\big|_{y=1}=u_1(x),\quad u_0\in C^2(0,2),\quad u_1\in C^1(0,2)$$

17. $2u_{xy}-e^{-x}u_{yy}=4x,\quad -\infty<x,\quad y<\infty$

$$u\big|_{y=x}=x^5\cos x,\quad u_y\big|_{y=x}=x^2+1$$

3.2 Koshining klassik masalasi

$C^2(t>0)\cap C^1(t\ge 0)$ sinfdan shunday $u(x,t)$ funksiya topilsinki, bu funksiya $t>0$ da

$$u_{tt}=a^2\Delta u+f(x,t)$$

tenglamani va quyidagi boshlang‘ich shartlani qanoatlantirsin:

$$u\,|_{t=+0}=u_0(x),\ u_t\,|_{t=+0}=u_1(x),$$

Bu yerda f,u_0,u_1 - berilgan funksiyalar.

Bu masalaga **Koshining klassik masalasi** deyiladi.

Agar quyidagi shartlar bajarilsa,

$f \in C^1(t \geq 0),\ u_0 \in C^2(R^1)$, $u_1 \in C^1(R^1)$, n=1;

$f \in C^2(t \geq 0),\ u_0 \in C^3(R^n)$, $u_1 \in C^2(R^n)$, n=2,3;

u vaqtda Koshining klassik masalasining yechimi mavjud, yagona va quyidagi formulalar orqali topiladi:

Dalamber formulasi bilan, agar n=1 bo‘lsa:

$$u(x,t) = \frac{1}{2}\left[u_0(x+at) + u_0(x-at)\right] + \frac{1}{2a}\int_{x-at}^{x+at} u_1(\xi)d\xi + \frac{1}{2a}\int_0^t \int_{x-a(t-\tau)}^{x+a(t-\tau)} f(\xi,\tau)d\xi d\tau . \qquad (1)$$

Puasson formulasi bilan, agar n=2 bo‘lsa:

$$u(x,t) = \frac{1}{2\pi a}\int_0^t \int_{|\xi-x|<a(t-\tau)} \frac{f(\xi,\tau)d\xi d\tau}{\sqrt{a^2(t-\tau)^2 - |\xi-x|^2}} + \frac{1}{2\pi a}\int_{|\xi-x|<at} \frac{u_1(\xi,\tau)d\xi}{\sqrt{a^2t^2 - |\xi-x|^2}} +$$

$$+\frac{1}{2\pi a}\frac{\partial}{\partial t}\int_{|\xi-x|<at} \frac{u_0(\xi,\tau)d\xi}{\sqrt{a^2t^2 - |\xi-x|^2}} . \qquad (2)$$

Kirxgof formulasi bilan, agar n=3 bo‘lsa:

$$u(x,t) = \frac{1}{4\pi a^2}\int_{|\xi-x|<at)} \frac{1}{|\xi-x|} f\left(\xi, t - \frac{|\xi-x|}{a}\right)d\xi + \frac{1}{4\pi a^2 t}\int_{|\xi-x|=at} u_1(\xi)dS + \frac{1}{4\pi a^2}\frac{\partial}{\partial t}\left[\frac{1}{t}\int_{|\xi-x|=at} u_0(\xi)dS\right]. \qquad (3)$$

$n \geq 2$ bo‘lganda ushbu formulalarning o‘rniga quyidagi formuladan ham foydalansa bo‘ladi:

$$u(x,t) = \sum_{k=0}^{\infty}\left[\frac{t^{2k}}{(2k)!}a^{2k}\Delta^k u_0(x_1,...,x_n) + \frac{t^{2k+1}}{(2k+1)!}a^{2k}\Delta^k u_1(x_1,...,x_n) + \frac{a^{2k}}{(2k+1)!}\int_0^t (t-\tau)^{2k+1}\Delta^k f(x_1,...,x_n,\tau)d\tau\right],$$

(4)

bu yerda Δ - Laplas operatori bo‘lib, $k = 0,1,2,...$ marta mos ravishda u_0, u_1, f - funksiyalarga qo‘llanilgan.

Masala: $\begin{cases} u_{tt} = u_{xx} + u_{yy} + u_{zz} + ax + bt \\ u(x,y,z,0) = xyz \\ u_t(x,y,z,0) = xy + z \end{cases}$ masalani (4) formula bilan yeching.

$u_0 = xyz$ funksiyaga keraklicha marta Δ operatorni qo‘llaymiz: $\Delta^0 u_0 = u_0 = xyz$; $\Delta^1 u_0 = \Delta u_0(x,y,z) = u_{0xx} + u_{0yy} + u_{0zz} = 0+0+0=0$. Laplas operatorini keyingi qo‘llashlarda ham nol chiqadi, demak hisoblashni shu yerda to‘xtatamiz.

Xuddi shu hisoblashlarni u_1, f funksiyalr uchun ham bajaramiz: $\Delta^0 u_1 = u_1 = xy + z$;

$\Delta^1 u_1 = \Delta^2 u_1 = ... = 0$; $\Delta^0 f = f = ax + bt$; $\Delta^1 f = \Delta^2 f = ... = 0$.

Hisoblashlarni (4) formulaga etib qo‘yamiz, natijada:

$u(x,y,z,t) = xyz + t(xy+z) + \int_0^t (t-\tau)(ax+b\tau)d\tau = xyz + t(xy+z) + \frac{axt^2}{2} + \frac{bt^3}{6}$ yechimni olamiz.

Mustaqil bajarish uchun mashqlar

Quyidagi chegaraviy masalalarni yeching:

a) (n=1)

18. $u_{tt}=u_{xx}+6;\quad u|_{t=0}=x^2,\quad u_t|_{t=0}=4x$
19. $u_{tt}=4u_{xx}+xt;\ u|_{t=0}=x^2,\quad u_t|_{t=0}=x$
20. $u_{tt}=u_{xx}+\sin x;\ u|_{t=0}=\sin x,\quad u_t|_{t=0}=0$
21. $u_{tt}=u_{xx}+e^x;\ u|_{t=0}=\sin x,\quad u_t|_{t=0}=x+\cos x$
22. $u_{tt}=9u_{xx}+\sin x;\ u|_{t=0}=1,\quad u_t|_{t=0}=1$
23. $u_{tt}=a^2u_{xx}+\sin\omega x;\ u|_{t=0}=0,\quad u_t|_{t=0}=0$
24. $u_{tt}=a^2u_{xx}+\sin\omega t;\ u|_{t=0}=0,\quad u_t|_{t=0}=0$

b) (n=2):

25. $u_{tt}=\Delta u+2;\ u|_{t=0}=x;\ u_t|_{t=0}=y$
26. $u_{tt}=\Delta u+6xyt;\ u|_{t=0}=x^2-y^2;\quad u_t|_{t=0}=xy$
27. $u_{tt}=\Delta u+x^3-3xy^2;\ u|_{t=0}=e^x\cos y;\quad u_t|_{t=0}=e^y\sin x$
28. $u_{tt}=\Delta u+t\sin y;\ u|_{t=0}=x^2;\ u_t|_{t=0}=\sin y$
29. $u_{tt}=2\Delta u;\quad u|_{t=0}=2x^2-y^2;\quad u_t|_{t=0}=2x^2+y^2$
30. $u_{tt}=3\Delta u+x^3+y^3;\ u|_{t=0}=x^2;\quad u_t|_{t=0}=y^2$
31. $u_{tt}=\Delta u+e^{3x+4y};\ u|_{t=0}=u_t;\quad u_t|_{t=0}=e^{3x+4y}$
32. $u_{tt}=a^2\Delta u,\ u|_{t=0}=\cos(bx+cy);\ u_t|_{t=0}=\sin(bx+cy)$
33. $u_{tt}=a^2\Delta u,\ u|_{t=0}=r^4;\ u_t|_{t=0}=r^4$, bu yerda $r=\sqrt{x^2+y^2}$
34. $u_{tt}=a^2\Delta u+r^2e^t,\ u|_{t=0}=0;\ u_t|_{t=0}=0$

c) (n=3)

35. $u_{tt}=\Delta u+2xyz,\quad u|_{t=0}=x^2+y^2-2z^2;\quad u_t|_{t=0}=1$
36. $u_{tt}=8\Delta u+t^2x^2,\quad u|_{t=0}=y^2;\quad u_t|_{t=0}=z^2$

37. $u_{tt}=3\Delta u+6r^2,\ \ u|_{t=0}=x^2y^2z^2;\ \ u_t|_{t=0}=xyz$

38. $u_{tt}=\Delta u+6te^{x\sqrt{2}}\sin y\cos z,$

$$u\big|_{t=0}=e^{x+y}\cos z\sqrt{2},\quad u_t\big|_{t=0}=e^{3y+4z}\sin 5x$$

39. $u_{tt}=a^2\Delta u,\qquad u|_{t=0}=u_t|_{t=0}=r^4$ bu yerda $r=\sqrt{x^2+y^2+z^2}$

40. $u_{tt}=a^2\Delta u+r^2e^t,\qquad u|_{t=0}=u_t|_{t=0}=0$ bu yerda $r=\sqrt{x^2+y^2+z^2}$

41. $u_{tt}=a^2\Delta u+\cos x\sin ye^z,\ \ u|_{t=0}=x^2e^{y+z};\ \ u_t|_{t=0}=\sin xe^{y+z}$

42. $u_{tt}=a^2\Delta u+xe^t\cos(3y+4z),\ u|_{t=0}=xy\cos z;u_t|_{t=0}=yze^x$

43. $u_{tt}=a^2\Delta u,\qquad u|_{t=0}=u_t|_{t=0}=\cos r,$ bu yerda $r=\sqrt{x^2+y^2+z^2}$

4. Issiqlik o‘tkazuvchanlik tenglamasi uchun Koshi masalasi

$C^2(t>0)\cap C(t\ge 0)$ sinfdan shunday $u(x,t)$ funksiya topilsinki, bu funksiya $x\in R^n$, $t>0$ da

$$u_t=a^2\Delta u+f(x,t)$$

tenglamani va quyidagi boshlang‘ich shartni qanoatlantirsin:

$$u|_{t=+0}=u_0(x),$$

bu yerda f,u_0 - berilgan funksiyalar.

Bu masalaga issiqlik o‘tkazuvchanlik tenglamasi uchun **Koshining klassik masalasi** deyiladi.

Agar $f\in C^2(t\ge 0)$ funksiya va uning barcha ikkinchi tartibigacha hosilalari har bir $0\le t\le T$ sohada chegaralangan, $u_0\in C(R^n)$ funksiya chegaralangan bo‘lsa, u vaqtda Koshining klassik masalasining yechimi mavjud, yagona va quyidagi Puasson formulasi orqali topiladi:

$$u(x,t)=\frac{1}{\left(2a\sqrt{\pi t}\right)^n}\int\limits_{R^n}u_0(\xi)e^{-\frac{|x-\xi|^2}{4a^2t}}d\xi+\int\limits_0^t\int\limits_{R^n}\frac{f(\xi,\tau)}{\left[2a\sqrt{\pi(t-\tau}\right]^n}e^{-\frac{|x-\xi|^2}{4a^2(t-\tau)}}d\xi d\tau\,. \qquad (1)$$

Quyidagi formuladan ham foydalansa bo‘ladi:

$$u(x,t)=\sum_{k=0}^{\infty}\left[\frac{t^k}{(k)!}\Delta^k u_0(x_1,...,x_n)+\frac{1}{(2k+1)!}\int\limits_0^t(t-\tau)^{2k+1}\Delta^k f(x_1,...,x_n,\tau)d\tau\right]. \qquad (2)$$

<u>**Masala.**</u> $u_t=4u_{xx}+t+e^t,\qquad u|_{t=0}=2.$ Koshi masalasini yeching.

Ushbu misolni yechish uchun (1) formuladan foydalanamiz. Bizning holimizda $a=2$, $u_0(x)=2$, $f(x,t)=t+e^t$ - berilganlar. Shu qiymatlarni (1) formulaga etib qo‘yamiz:

$$u(x,t)=\frac{1}{2\cdot 2\sqrt{\pi t}}\int_{-\infty}^{\infty}2e^{-\frac{(x-\xi)^2}{16t}}d\xi+\int_0^t\int_{-\infty}^{\infty}\frac{\tau+e^\tau}{4\sqrt{\pi(t-\tau)}}e^{-\frac{(x-\xi)^2}{16(t-\tau)}}d\xi d\tau=I_1+I_2, \quad (3)$$

bu yerda $I_1=\frac{1}{2\sqrt{\pi t}}\int_{-\infty}^{\infty}e^{-\frac{(x-\xi)^2}{16t}}d\xi$ va $I_2=\int_0^t\int_{-\infty}^{\infty}\frac{\tau+e^\tau}{4\sqrt{\pi(t-\tau)}}e^{-\frac{(x-\xi)^2}{16(t-\tau)}}d\xi d\tau$. Integralarni alohida-alohida hisoblaymiz.

$$I_1=\frac{1}{2\sqrt{\pi t}}\int_{-\infty}^{\infty}e^{-\frac{(x-\xi)^2}{16t}}d\xi=\left|\begin{array}{l}\frac{x-\xi}{4\sqrt{t}}=\eta \text{ belgilash kiritamiz,}\\ \xi=\text{x - }4\sqrt{\text{t}}\eta\\ d\xi=\text{-}4\sqrt{\text{t}}d\eta\\ \xi=-\infty\to\eta=\infty\\ \xi=\infty\to\eta=-\infty\end{array}\right|=\frac{1}{2\sqrt{\pi t}}\int_{\infty}^{-\infty}\left(\text{-}4\sqrt{\text{t}}e^{-\eta^2}\right)d\eta=$$

$$=\frac{2}{\sqrt{\pi}}\int_{-\infty}^{\infty}e^{-\eta^2}d\eta=\left|\int_{-\infty}^{\infty}e^{-\eta^2}d\eta=\sqrt{\pi}\text{ - Puasson integrali.}\right|=\frac{2}{\sqrt{\pi}}\cdot\sqrt{\pi}=2,$$

demak, $I_1=2$.

$I_2=\int_0^t\int_{-\infty}^{\infty}\frac{\tau+e^\tau}{4\sqrt{\pi(t-\tau)}}e^{-\frac{(x-\xi)^2}{16(t-\tau)}}d\xi d\tau$ - bu integralni hisoblashda ham yuqoridagi kabi fikr yuritib, hisoblashlarni bajaramiz va quyidagi natijani olamiz: $I_2=\frac{t^2}{2}+e^t-1$. Ikkala integralni etib (3) ga qo‘yamiz, natijada quyidagi yechimni olamiz: $u(x,t)=\frac{t^2}{2}+e^t+1$.

Mustaqil bajarish uchun mashqlar

(1) yoki (2) formulalar yordamida quyidagi chegaraviy masalalarni yeching.

a) (n=1)

1. $u_t=4u_{xx}+t+e^t$, $u|_{t=0}=2$
2. $u_t=u_{xx}+3t^2$, $u|_{t=0}=\sin x$
3. $u_t=u_{xx}+e^{-t}\cos x$, $u|_{t=0}=\cos x$

4. $u_t=u_{xx}+e^t\sin x,$ $u|_{t=0}=\sin x$

5. $u_t=u_{xx}+\sin t,$ $u|_{t=0}=e^{-x^2}$

6. $4u_t=u_{xx},$ $u|_{t=0}=e^{2x-x^2}$

7. $u_t=u_{xx},$ $u|_{t=0}=xe^{-x^2}$

8. $4u_t=u_{xx},$ $u|_{t=0}=\sin xe^{-x^2}$

b) (n=2)

9. $u_t=\Delta u+e^t;$ $u|_{t=0}=\cos x\sin y$

10. $u_t=\Delta u+\sin t\sin x\sin y;$ $u|_{t=0}=1$

11. $u_t=\Delta u+\cos t;$ $u|_{t=0}=xye^{-x^2-y^2}$

12. $8u_t=\Delta u+1;$ $u|_{t=0}=e^{-(x-y)}$

13. $2u_t=\Delta u;$ $u|_{t=0}=\cos xy$

c) (n=3)

14. $u_t=2\Delta u+t\cos x;$ $u|_{t=0}=\cos y\sin z$

15. $u_t=3\Delta u+e^t;$ $u|_{t=0}=\sin(x-y-z)$

16. $4u_t=\Delta u+\sin 2z;$ $u|_{t=0}=\frac{1}{4}\sin 2z+e^{-x^2}\cos y$

17. $u_t=\Delta u+\cos(x-y+z);$ $u|_{t=0}=e^{-(x+y-z)}$

18. $u_t=\Delta u;$ $u|_{t=0}=\cos(xy)\sin z$

d) Quyidagi Koshi masalalarini yeching

$$u_t=\Delta u, \qquad u|_{t=0}=u_0(x), \quad x\in R^n$$

bu yerda u_0 quyidagicha aniqlanadi:

19. $u_0=\cos\sum_{k=1}^{n}x_k$

20. $u_0=e^{-|x|^2}$

21. $u_0=\left(\sum_{k=1}^{n}x_k\right)e^{-|x|^2}$

22. $u_0=\left(\sin\sum_{k=1}^{n}x_k\right)e^{-|x|^2}$

23. $u_0=e^{-\left(\sum_{k=1}^{n}x_k\right)^2}$

5. O‘zgaruvchilarni ajratish (Furye) usuli

5.1 Giperbolik turdagi tenglama

Uchlari $x=0$ va $x=l$ nuqtalarda mahkamlangan tor tebtanishi tenglamasi masalasi uchun Furye yoki o‘zgaruvchilarni ajratish usulini bayon qilamiz. Bu masala quyidagi tenglamaga keladi:

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} \qquad (1)$$

Boshlang‘ich shartlar:

$$u|_{t=0} = u_0(x),\ u_t\ |_{t=0} = u_1(x), \qquad (2)$$

Chegaraviy shartlar:

$$u|_{x=0} = 0,\ u|_{x=l} = 0 \qquad (3)$$

Dastlab, (1) tenglamaning xususiy yechimlarini quyidagi korinishda qidiramiz:

$$u(x,t) = X(x)T(t), \qquad (4)$$

bu funksiyalr aynan nolga teng emas va (3) chegaraviy shartlarni qanoatlantirsin.

(4) funksiyani (1) tenglama qo‘yib quyidagi oddiy differensial tenglamalarga kelamiz:

$$T''(t) + a^2\lambda T(t) = 0, \qquad (5)$$

$$X''(x) + \lambda X(x) = 0, \qquad (6)$$

bu yerda $\lambda = const$.

Chegaraviy shartlar quyidagicha bo‘ladi:

$$X(0) = 0, \quad X(l) = 0. \qquad (7)$$

Natijada biz Shturm-Liuvill (6)-(7) masalasiga kelamiz.

Bu masalaning xos sonlari:

$$\lambda_k = \left(\frac{\pi k}{l}\right)^2 \quad k = 1,2,...$$

Va bu xos sonlarga quyidagi xos funksiyalar mos keladi:

$$X_k(x) = \sqrt{\frac{2}{l}} \sin\frac{\pi k x}{l}.$$

$\lambda = \lambda_k$ bo‘lganda (5) tenglama quyidagi umumiy yechimga ega:

$$T_k(t) = a_k \cos\frac{k\pi at}{l} + b_k \sin\frac{k\pi at}{l},$$

shuning uchun

$$u_k(x,t) = X_k(x)T_k(t) = \left(a_k \cos\frac{k\pi at}{l} + b_k \sin\frac{k\pi at}{l}\right)\sin\frac{k\pi x}{l}$$

funksiya har qanday a_k va b_k uchun (1) masalani va (3) chegaraviy shartlarni qanoatlantiradi.

(2)-(3) shartlarni qanoatlantiruvchi (1) masalaning yechimini qator ko‘rinishida qidiramiz:

$$u(x,t) = \sum_{k=1}^{\infty} X_k(x)T_k(t) = \sum_{k=1}^{\infty}\left(a_k \cos\frac{k\pi at}{l} + b_k \sin\frac{k\pi at}{l}\right)\sin\frac{k\pi x}{l} \quad (8)$$

Agar bu qator tekis yaqunlashuvchii bo‘lib, uni hadma-had ikki marta differensiallash mumkin bo‘lsa, u vaqtda qator yig‘indisi (1) tenglamani va (3) chegaraviy shartlarni qanoatlantiradi.

a_k va b_k doimiy koeffisiyentlarni shunday aniqlaymizki (8) qator yig‘indisi (2) boshlang‘ich shartlarni qanoatlantirsin, quyidagi tengliklarga kelamiz:

$$u_0(x) = \sum_{k=1}^{\infty} a_k \sin\frac{k\pi x}{l} \quad (9)$$

$$u_1(x) = \sum_{k=1}^{\infty} \frac{k\pi a}{l} b_k \sin\frac{k\pi x}{l} \quad (10)$$

(9) va (10) formulalar $u_0(x)$ va $u_1(x)$ funksiyalarning (0,*l*) intervalda sinuslar bo‘yicha Furye yoyilmasini beradi. Bu yoyilmalarning koeffisiyentlari quyidagi formulalar bilan topiladi:

$$a_k = \frac{2}{l}\int_0^l u_0(x) sin\frac{k\pi x}{l}dx$$

$$b_k = \frac{2}{k\pi a}\int_0^l u_1(x) sin\frac{k\pi x}{l}dx$$

Masala: Quyidagi masalani Furye usulida yeching.

$u_{tt}=u_{xx}+u,\ (0<x<l);\ \ u|_{x=0}=0,\ u|_{x=l}=t,\ \ u|_{t=0}=0,\ u_t|_{t=0}=\frac{x}{l}.$

Chegaraviy shartlar noldan farqli bo‘lgni uchun, yechimni $u=v+w$ ko‘rinishda qidaramiz, bu yerda $w=\mu_1(t)+\frac{x}{l}(\mu_2(t)-\mu_1(t))$, $\mu_1(t)=0,\ \mu_2(t)=t$. U holda $w(x,t)=\frac{xt}{l}$, yechim esa $u(x,t)=v(x,t)+\frac{xt}{l}$ (*) ko‘rinishda bo‘ladi. Yechimdagi $v(x,t)$ funksiya quyidagi masalani qanoatlantiradi:

$v_{tt}=v_{xx}+v+\frac{xt}{l}$, $(0<x<l)$; $v|_{x=0}=0$, $v|_{x=l}=0$, $u|_{t=0}=0$, $u_t|_{t=0}=0$. (11)

Berilgan tenglamaning $\lambda_n=\left(\frac{\pi n}{l}\right)^2-1$ - xos sonlarini va $\sin\frac{\pi n}{l}x$ xos funksiyalarini aniqlaymiz. Shunga asosan yechimni quyidagi ko‘rinishda qidiramiz:

$$v(x,t)=\sum_{n=1}^{\infty}g_n(t)\sin\frac{\pi n}{l}x. \quad (12)$$

Tenglamaning ozod hadi $f(x,t)=\frac{xt}{l}$ funksiyani Furye qatoriga yoyamiz:

$$f(x,t)=\sum_{n=1}^{\infty}f_n(t)\sin\frac{\pi n}{l}x. \quad (13)$$

$f_n(t)$ - Furye koeffisiyentlarini quyidagi formula yordamida aniqlaymiz: $f_n(t)=\frac{2}{l}\int_0^l f(\xi,t)\sin\frac{\pi n}{l}\xi d\xi=\frac{2}{l}\int_0^l\frac{\xi t}{l}\sin\frac{\pi n}{l}\xi d\xi$. Integralni bo‘laklab integralymiz. Natijada

$$f_n(t)=(-1)^{n+1}\frac{2t}{\pi n}. \quad (14)$$

(12) va (13) funksiyalarni (14) ni hisobga olgan holda (11) masalaga etib qo‘yamiz, natijada noma’lum $g_n(t)$ funksiya uchun quyidagi Koshi masalasini olamiz:

$$\begin{cases} g''_n(t)+\left(\left(\frac{\pi n}{l}\right)^2-1\right)g_n(t)=(-1)^{n+1}\frac{2t}{\pi n}, \\ g'_n(t)=0,\quad g_n(t)=0. \end{cases} \quad (15)$$

(15) masalani yechishda, dastlab, tenglamaning yechimini quyidagi ko‘rinishda qidiring: $g_n(t)=\bar{g}_n(t)+g*_n(t)$, bu yerda $\bar{g}_n(t)$ - berilgan tenglamaga mos bir jinsli tenglamaning umumiy yechimi, $g*_n(t)$ -

berilgan tenglamaning xususiy yechimi bo‘lib, o‘ng tomonga qarab tanlanishi mumkin, bizning holimizda, $g*_n(t)=at+b$ ko‘rinishda qidirish mumkin.

(15) masalani yechib, natijada (11) masalaning yechimini aniqlaymiz:

$$v(x,t)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cdot 2}{\pi n\left(\left(\frac{\pi n}{l}\right)^2-1\right)}\left(t-\frac{\sin\sqrt{\left(\frac{\pi n}{l}\right)^2-1}\,t}{\sqrt{\left(\frac{\pi n}{l}\right)^2-1}}\right)\sin\frac{\pi n}{l}x \,. \tag{16}$$

(16) funksiyani (*) ga etib qo‘yib, berilgan masalaning yechimini olamiz, ya’ni:

$$u(x,t)=\frac{xt}{l}+\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cdot 2}{\pi n\left(\left(\frac{\pi n}{l}\right)^2-1\right)}\left(t-\frac{\sin\sqrt{\left(\frac{\pi n}{l}\right)^2-1}\,t}{\sqrt{\left(\frac{\pi n}{l}\right)^2-1}}\right)\sin\frac{\pi n}{l}x \,.$$

Mustaqil bajarish uchun mashqlar

Quyidagi aralash masalalarni yeching:

1. $u_{tt}=u_{xx}-4u(0<x<1);\ u|_{x=0}=u|_{x=1}=0;\ u|_{t=0}=x^2-x,\ u_t|_{t=0}=0.$
2. $u_{tt}+2u_t=u_{xx}-u(0<x<\pi);\ u|_{x=0}=u|_{x=\pi}=0;\ u|_{t=0}=\pi x-x^2,\ u_t|_{t=0}=0.$
3. $u_{tt}+2u_t=u_{xx}-u(0<x<\pi);\ u_x|_{x=0}=0,\ u|_{x=\pi}=0;\ u|_{t=0}=0,\ u_t|_{t=0}=x.$
4. $u_{tt}+u_t=u_{xx}(0<x<1);\ u|_{x=0}=t, u|_{x=1}=0\ ;\ u|_{t=0}=0\ ,\ u_t|_{t=0}=1-x$
5. $u_{tt}=u_{xx}+u(0<x<2);\ u|_{x=0}=2t,\ u|_{x=2}=0\ ;\ u|_{t=0}=u_t|_{t=0}=0.$
6. $u_{tt}=u_{xx}+u(0<x<l);\ u|_{x=0}=0,\ u|_{x=l}=t,\ u|_{t=0}=0,\ u_t|_{t=0}=\frac{x}{l}$
7. $u_{tt}=u_{xx}+x(0<x<\pi);\quad u|_{x=0}=u|_{x=\pi}=0;\quad u|_{t=0}=\sin 2x,\quad u_t|_{t=0}=0$
8. $u_{tt}+u_t=u_{xx}+1(0<x<1);\ u|_{x=0}=u|_{x=1}=0;\quad u|_{t=0}=u_t|_{t=0}=0.$
9. $u_{tt}-u_{xx}+2u_t=4x+8e^t\cos x\qquad (0<x<\pi/2)\ ;\ u_x|_{x=0}=2t,\ u|_{\frac{\pi}{2}}=\pi\cdot t;\ u|_{t=0}=\cos x,$ $u_t|_{t=0}=2x.$
10. $u_{tt}-u_{xx}-2u_t=4t(\sin x-x)\quad (0<x<\pi/2)\ ;\ u|_{x=0}=3,\ u_x|_{x=\pi/2}=t^2+t,\ ;\ u|_{t=0}=3$ $u_t|_{t=0}=x+\sin x.$

11. $u_{tt}-3u_t=u_{xx}+u-x(4+t)+\cos\frac{3x}{2}$ $(0<x<\pi)$; $u_x|_{x=0}=t+1$,

$u|_{x=\pi}=\pi(t+1)$; $u|_{t=0}=u_t|_{t=0}=x$.

12. $u_{tt}-7u_t=u_{xx}+2u_x-2t-7x-e^{-x}\sin 3x$ $(0<x<\pi)$; $u|_{x=0}=0$,

$u|_{x=\pi}=\pi t$; $u|_{t=0}=0$, $u_t|_{t=0}=x$.

13. $u_{tt}+2u_t=u_{xx}+8u+2x(1-4t)+\cos 3x$ $(0<x<\pi/2)$; $u_x|_{x=0}=t$, $u|_{x=\frac{\pi}{2}}=\frac{\pi\cdot t}{2}$;

$u|_{t=0}=0$, $u_t|_{t=0}=x$.

14. $u_{tt}=u_{xx}+4u+2\sin^2 x$ $(0<x<\pi)$; $u_x|_{x=0}=u_x|_{x=\pi}=0$;

$u|_{t=0}=u_t|_{t=0}=0$.

15. $u_{tt}=u_{xx}+10u+2\sin 2x\cos x$ $(0<x<\pi/2)$; $u|_{x=0}=u_x|_{x=\pi/2}=0$;

$u|_{t=0}=u_t|_{t=0}=0$.

16. $u_{tt}-3u_t=u_{xx}+2u_x-3x-2t$ $(0<x<\pi)$; $u|_{x=0}=0$, $u|_{x=\pi}=\pi t$; $u|_{t=0}=e^{-x}\sin x$,

$u_t|_{t=0}=x$.

5.2 Parabolik turdagi tenglama

Qisqacha bir jinsli ingichka sterjenda issiqlik tarqalish masalasini ko‘rib chiqamiz, uning yon sirti issiqlik o‘tkazmaydi, *x=0* va *x=l* chegaralarida esa nollik temperatura. Shu masala uchun Furye yoki o‘zgaruvchilarni ajratish usulini bayon qilamiz. Bu masala quyidagi tenglamaga keladi:

$$\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}. \qquad (1)$$

Boshlang‘ich shartlar:

$$u|_{t=0}=u_0(x), \qquad (2)$$

Chegaraviy shartlar:

$$u|_{x=0}=0,\ u|_{x=l}=0. \qquad (3)$$

Dastlab, (1) tenglamaning xususiy yechimlarini quyidagi korinishda qidiramiz:

$$u(x,t)=X(x)T(t) \qquad (4)$$

bu funksiyalr aynan nolga teng emas va (3) chegaraviy shartlarni qanoatlantirsin.

(4) funksiyani (1) tenglama qo‘yib quyidagi oddiy differensial tenglamalarga kelamiz:

$$T'(t)+a^2\lambda T(t)=0, \tag{5}$$

$$X''(x)+\lambda X(x)=0, \tag{6}$$

bu yerda $\lambda = const$.

Chegaraviy shartlar quyidagicha bo‘ladi:

$$X(0)=0, \quad X(l)=0 \tag{7}$$

Natijada biz Shturm-Liuvill (6)-(7) masalasiga kelamiz.

Bu masalaning xos sonlari:

$$\lambda_k=\left(\frac{\pi k}{l}\right)^2 \quad k=1,2,...$$

Va bu xos sonlarga quyidagi xos funksiyalar mos keladi:

$$X_k(x)=\sin\frac{\pi kx}{l}.$$

$\lambda=\lambda_k$ bo‘lganda (5) tenglama quyidagi umumiy yechimga ega:

$$T_k(t)=a_k e^{-\left(\frac{k\pi a}{l}\right)^2 t},$$

shuning uchun

$$u_k(x,t)=X_k(x)T_k(t)=a_k e^{-\left(\frac{\pi ka}{l}\right)^2 t}\sin\frac{k\pi x}{l}$$

funksiya har qanday a_k uchun (1) masalani va (3) chegaraviy shartlarni qanoatlantiradi.

(2)-(3) shartlarni qanoatlantiruvchi (1) masalaning yechimini qator ko‘rinishida qidiramiz:

$$u(x,t)=\sum_{k=1}^{\infty}X_k(x)T_k(t)=\sum_{k=1}^{\infty}a_k e^{-\left(\frac{\pi ka}{l}\right)^2 t}\sin\frac{k\pi x}{l} \tag{8}$$

Agar bu qator tekis yaqunlashuvchii bo‘lib, uni *t* had bo‘yicha bir marta *x* bo‘yicha ikki marta differensiallash mumkin bo‘lsa, u vaqtda qator yig‘indisi (1) tenglamani va (3) chegaraviy shartlarni qanoatlantiradi.

a_k doimiy koeffisiyentlarni shunday aniqlaymizki (8) qator yig‘indisi (2) boshlang‘ich shartlarni qanoatlantirsin, quyidagi tengliklarga kelamiz:

$$u_0(x) = \sum_{k=1}^{\infty} a_k \sin\frac{k\pi x}{l} \tag{9}$$

(9) formula $u_0(x)$ funksiyaning (0,*l*) intervalda sinuslar bo‘yicha Furye yoyilmasini beradi. Bu yoyilmaning koeffisiyentlari quyidagi formula bilan topiladi:

$$a_k = \frac{2}{l}\int_0^l u_0(x) sin\frac{k\pi x}{l}dx$$

<u>Masala:</u> Quyidagi masalani Furye usulida yeching.

$$u_t=u_{xx}+u,\ \ 0<x<l,\ \ u|_{x=0}=0,\ \ u|_{x=l}=0,\ u|_{t=0}=13x. \tag{10}$$

Dastlab, (1) tenglamaning xususiy yechimlarini quyidagi korinishda qidiramiz:

$$u(x,t) = X(x)T(t), \tag{4}$$

bu funksiyalr aynan nolga teng emas va chegaraviy shartlarni qanoatlantirsin.

(4) funksiyani (10) masaladagi tenglamaga qo‘yib quyidagi oddiy differensial tenglamalarga kelamiz:

$$T'(t) + \lambda T(t) = 0, \tag{5}$$

$$X''(x) + (\lambda+1)X(x) = 0, \tag{6′}$$

bu yerda $\lambda = const$.

Chegaraviy shartlar quyidagicha bo‘ladi:

$$X(0) = 0, \quad X(l) = 0. \tag{7}$$

Natijada biz Shturm-Liuvill (6′)-(7) masalasiga kelamiz.

Bu masalaning xos sonlari:

$$\lambda_n = \left(\frac{\pi n}{l}\right)^2 - 1$$

Va bu xos sonlarga quyidagi xos funksiyalar mos keladi:

$$X_n(x) = \sin\frac{\pi n x}{l}.$$

$\lambda = \lambda_n$ bo'lganda (5) tenglama quyidagi umumiy yechimga ega:

$$T_n(t) = a_n e^{-\left(\left(\frac{\pi n}{l}\right)^2 - 1\right)t},$$

shuning uchun

$$u_n(x,t) = X_n(x)T_n(t) = a_n e^{-\left(\left(\frac{\pi n}{l}\right)^2 - 1\right)t} \sin\frac{n\pi x}{l}$$

funksiya har qanday a_n uchun berilgan masalani qanoatlantiradi.

Berilgan masalaning yechimini qator ko'rinishida qidiramiz:

$$u(x,t) = \sum_{n=1}^{\infty} X_n(x)T_n(t) = \sum_{n=1}^{\infty} a_n e^{-\left(\left(\frac{\pi n}{l}\right)^2 - 1\right)t} \sin\frac{n\pi x}{l}.$$

a_n doimiy koeffisiyentlarni shunday aniqlaymizki qator yig'indisi boshlang'ich shartlarni qanoatlantirsin, quyidagi tenglikga kelamiz:

$$13 \cdot x = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{l},$$

bu tenglik $u_0(x) = 13x$ funksiyaning $(0,l)$ intervalda sinuslar bo'yicha Furye yoyilmasini beradi. Bu yoyilmaning koeffisiyentlari quyidagi formula bilan topiladi:

$$a_n = \frac{2}{l}\int_0^l 13 \cdot x \cdot sin\frac{n\pi x}{l}dx$$

koeffisiyentlarni aniqlash uchun integralni bo'laklab integrallaymiz, natijada: $a_n = \frac{26 \cdot l}{\pi n} \cdot (-1)^{n+1}$. U vaqtda izlanayotgan yechim quyidagi ko'rinishda bo'ladi:

$$u(x,t) = \frac{26 \cdot l}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-\left(\left(\frac{\pi n}{l}\right)^2 - 1\right)t} \sin\frac{n\pi x}{l}.$$

Mustaqil bajarish uchun mashqlar

Quyidagi aralash masalalarni yeching*:*

17. $u_t = u_{xx}$, $0<x<l$, $u|_{x=0}=0$, $u|_{x=l}=0$, $u|_{t=0}=A=const$.
18. $u_t = u_{xx}$, $0<x<l$, $u|_{x=0}=0$, $u|_{x=l}=0$, $u|_{t=0}=Ax(l-x)$.
19. $u_t = u_{xx}$, $0<x<l$, $u|_{x=0}=0$, $(u_x+hu)|_{x=l}=0$, $u|_{t=0}=u_0(x)$.
20. $u_t = u_{xx}$, $0<x<l$, $(u_x-hu)|_{x=0}=0$, $(u_x+hu)|_{x=l}=0$, $u|_{t=0}=u_0(x)$.

21. $u_t=u_{xx}$, $0<x<l$, $u_x|_{x=0}=0$, $u_x|_{x=l}=0$, $u|_{t=0}=u_0=const.$

22. $u_t=u_{xx}$, $0<x<l$, $u_x|_{x=0}=0$, $u_x|_{x=l}=0$, $u|_{t=0}=\begin{cases} u_0=const, & \text{agar } 0<x<\frac{l}{2} \\ 0, & \text{agar } \frac{l}{2}<x<l \end{cases}.$

$\lim\limits_{t\to\infty} u(x,t)-?$

23. $u_t=u_{xx}$, $0<x<l$, $u_x|_{x=0}=0$, $u_x|_{x=l}=0$, $u|_{t=0}=\begin{cases} \frac{2u_0}{l}x, & \text{agar } 0<x<\frac{l}{2} \\ \frac{2u_0}{l}(l-x), & \text{agar } \frac{l}{2}\le x<l \end{cases},$

bu yerda $u_0=const.$ $\lim\limits_{t\to\infty} u(x,t)-?$

24. $u_t=u_{xx}$, $0<x<1$, $u_x|_{x=0}=0$, $u|_{x=1}=0$, $u|_{t=0}=x^2-1$

25. $u_{xx}=u_t+u$, $0<x<l$, $u|_{x=0}=0$, $u|_{x=l}=0$, $u|_{t=0}=1$

26. $u_t=u_{xx}-4u$, $0<x<\pi$, $u|_{x=0}=0$, $u|_{x=\pi}=0$, $u|_{t=0}=x^2-\pi x$

27. $u_t=u_{xx}$, $0<x<l$, $u_x|_{x=0}=1$, $u|_{x=l}=0$, $u|_{t=0}=0$

28. $u_t=u_{xx}+u+2\sin 2x\sin x$, $0<x<\frac{\pi}{2}$, $u_x|_{x=0}=u|_{x=\frac{\pi}{2}}=u|_{t=0}=0$

29. $u_t=u_{xx}-2u_x+x+2t$, $0<x<1$, $u|_{x=0}=0$; $u|_{x=1}=t$, $u|_{t=0}=e^x\sin\pi x$

30. $u_t=u_{xx}+u-x+25\sin 2x\cos x$, $0<x<\frac{\pi}{2}$, $u|_{x=0}=0$, $u_x|_{x=\frac{\pi}{2}}=1$, $u|_{t=0}=x$

31. $u_t=u_{xx}+4u+x^2-2t-4x^2t+2\cos^2 x$, $0<x<\pi$, $u_x|_{x=0}=0$, $u_x|_{x=\pi}=2\pi t$, $u|_{t=0}=0.$

32. $u_t-u_{xx}+2u_x-u=e^x\sin x-t$ $0<x<\pi$, $u|_{x=0}=1+t$, $u|_{x=\pi}=1+t$, $u|_{t=0}=1+e^x\sin 2x$

33. $u_t-u_{xx}-u=xt(2-t)+2\cos t$, $0<x<\pi$, $u_x|_{x=0}=t^2$, $u_x|_{x=\pi}=t^2$, $u|_{t=0}=\cos 2x.$

34. $u_t-u_{xx}-9u=4\sin^2 t\cos 3x-9x^2-2$, $0<x<\pi$, $u_x|_{x=0}=0$, $u_x|_{x=\pi}=2\pi$, $u|_{t=0}=x^2+2$

35. $u_t=u_{xx}+6u+2t(1-3t)-6x+2\cos x\cos 2x$, $0<x<\frac{\pi}{2}$; $u_x|_{x=0}=1$, $u|_{x=\frac{\pi}{2}}=t^2+\frac{\pi}{2}$; $u|_{t=0}=x.$

36. $u_t=u_{xx}+6u+x^2(1-6t)-2(t+3x)+\sin 2x$, $0<x<\pi$ $u_x|_{x=0}=1$, $u_x|_{x=\pi}=2\pi t+1$, $u|_{t=0}=x.$

37. $u_t=u_{xx}+4u_x+x-4t+1+e^{-2x}\cos^2\pi x$, $0<x<1$, $u|_{x=0}=t$, $u|_{x=1}=2t$, $u|_{t=0}=0.$

6. Integral tenglamalar

Ta'rif: Agar tenglamadagi noma'lum funksiya shu funksiyaning argumenti bo'yicha olinadigan integral ishorasi ostida bo'lsa, bunday tenglama integral tenglama deb ataladi.

Ta'rif: Ushbu integral tenglama Fredgolm[1]ning 1-tur tenglamasi deyiladi:

$$\lambda\int_a^b K(x,y)\varphi(y)dy = f(x) \tag{1}$$

Bunda $\varphi(x)$– noma'lum funksiya, *f(x)* –ozod had va *K(x,y)* tenglamaning yadrosi – ma'lum funksiyalar, integrallash chegaralari a va b berilgan haqiqiy o'zgarmas sonlar.

Ta'rif: Ushbu integral tenglama Fredgolmning 2-tur tenglamasi deyiladi:

$$\varphi(x) = f(x) + \lambda\int_a^b K(x,y)\varphi(y)dy \tag{2}$$

Bunda $\varphi(x)$– noma'lum funksiya integral ishorasidan tashqarida ham ishtirok etmoqda. (1) va (2) dagi λ tenglamaning parametri deyiladi.

Bu tenglamalardagi *f(x)* funksiya *I(*$a \le x \le b$*)* kesmada, *K(x,y)* yadro esa *R(*$a \le x \le b, a \le y \le b$*)* yopiq sohada berilgan deb hisoblanadi.

Ta'rif: Agar *I* kesmada $f(x) \equiv 0$ bo'lsa, (2) tenglama quyidagi ko'rinishga keladi:

$$\varphi(x) = \lambda\int_a^b K(x,y)\varphi(y)dy \tag{3}$$

Bunday tenglama bir jinsli integral tenglama deyiladi

Ta'rif: Ushbu integral tenglama Fredgolmning 3-tur tenglamasi deyiladi:

$$\psi(x)\varphi(x) = f(x) + \lambda\int_a^b K(x,y)\varphi(y)dy \tag{4}$$

Agar *I* kesmada

a) $\psi(x) \equiv 0$ bo'lsa, undan (1) tenglama;

[1] Fredgolm Erik Ivar (1866-1927) – mashhur shved matematigi.

b) $\psi(x)\equiv 1$ bo‘lsa, undan (2) tenglama kelib chiqadi

Integral tenglamada ishtirok etadigan noma’lum funksiya ko‘p argumentli, jumladan ikki argumentli bo‘lishi ham mumkin.

Masalan:

$$\varphi(x,y)=f(x,y)+\lambda\int_a^b\int_c^d K(x,y,t_1,t_2)\varphi(t_1,t_2)dt_1dt_2 \qquad (5)$$

bu yerda *f(x,y)* funksiya *R(*$a\le x\le b, c\le y\le d$*)* sohada, *K(x,y,t₁,t₂)* yadro esa *P(*$a\le x\le b, c\le y\le d, a\le t_1\le b, c\le t_2\le d$*)*sohada berilgan deb hisoblanadi; *a,b,c,d* va λ lar berilgan o‘zgarmas haqiqiy sonlardir.

Ta’rif: Ushbu integral tenglama Volterra[2]ning 1-tur tenglamasi deyiladi:

$$\lambda\int_a^x K(x,y)\varphi(y)dy=f(x) \qquad (6)$$

Bunda $\varphi(x)$– noma’lum funksiya, *f(x)* –ozod had *I(*$a\le x\le b$*)* kesmada, va *K(x,y)* tenglamaning yadrosi – *R(*$a\le x\le b, a\le y\le x$*)* yopiq sohada berilgan deb hisoblanadi..

Ta’rif: Ushbu integral tenglama Volterraning 2-tur tenglamasi deyiladi:

$$\varphi(x)=f(x)+\lambda\int_a^x K(x,y)\varphi(y)dy \qquad (7)$$

Bunda $\varphi(x)$– noma’lum funksiya integral ishorasidan tashqarida ham ishtirok etmoqda. (1) va (2) dagi λ tenglamaning parametri deyiladi.

Ta’rif: Agar *I* kesmada $f(x)\equiv 0$ bo‘lsa, (2) tenglama quyidagi ko‘rinishga keladi:

$$\varphi(x)=\lambda\int_a^x K(x,y)\varphi(y)dy \qquad (8)$$

Bunday tenglama bir jinsli integral tenglama deyiladi

Integral tenglamada ishtirok etadigan noma’lum funksiya ko‘p argumentli, jumladan ikki argumentli bo‘lishi ham mumkin.

[2] Volterra Vito (1860-1940) – mashhur italyan matematigi.

Masalan:

$$\varphi(x,y) = f(x,y) + \lambda \int_a^x \int_c^y K(x,y,t_1,t_2)\varphi(t_1,t_2)dt_1 dt_2 \qquad (9)$$

bu yerda *f(x,y)* funksiya *R*($a \le x \le b$, $c \le y \le d$) sohada, *K*(x,y,t_1,t_2) yadro esa *P*($a \le x \le b$, $c \le y \le d$, $a \le t_1 \le x$, $c \le t_2 \le y$)sohada berilgan deb hisoblanadi.

Ta'rif: Fredgolmning 2-tur tenglamasi berilgan bo'lsin:

$$\varphi(x) = f(x) + \lambda \int_a^b K(x,y)\varphi(y)dy \qquad (2)$$

Agar bu tenglamada ishtirok etayotgan yadroni ushbu:

$$K(x,y) = a_1(x)b_1(y) + a_2(x)b_2(y) + ... + a_n(x)b_n(y) \qquad (10)$$

ko'rinishida yozish mumkin bo'lsa, bunday yadro **aynigan yadro**[3] deyiladi.

Integral tenglamalarni yechishning quyidagi usullari mavjud:

1. Differensial tenglamalarga keltirib yechish;
2. Aynigan yadroli integral tenglamalarni chiziqli algebraik tenglamalar sistemasiga keltirib yechish;
3. Aynigan yadroli integral tenglamalarni koeffisiyentlarni tenglash usuli bilan yechish;
4. Ketma-ket yaqinlashish usuli bilan yechish;
5. Rezolventa usuli bilan yechish.

Shu usullardan ba'zilarini misollarda ko'rib chiqamiz.

1-misol. Ushbu tenglama yechilsin:

$$u(x) = x^2 + \lambda \int_0^1 (1 + xt)u(t)dt.$$

Berilgan aynigan yadroli integral tenglamani chiziqli algebraik tenglamalar sistemasiga keltirib yechish usulidan foydalanib yechamiz.

Bu misoldagi λ parametr umumiy holda berilgan bo'lib, $K(x,t) = 1 + xt$ yadro yuqoridagi (10) ko'rinishda ifodalangan. Tenglamaning o'ng tomonidagi integralni ikkiga ajratib,

[3] Aynigan yadro – вырожденное ядро

$$\int_0^1 (1+xt)u(t)dt = \int_0^1 u(t)dt + x\int_0^1 tu(t)dt$$

so‘ngra quyidagicha

$$Q_1 = \int_0^1 u(t)dt, \quad Q_2 = \int_0^1 tu(t)dt$$

belgilaymiz. U holda berilgan integral tenglama

$$u(x) = x^2 + \lambda Q_1 + \lambda Q_2 x \tag{11}$$

ko‘rinishda yoziladi. Noma`lum funksiyaning mana shu ifodasidan foydalanib, Q_1 bilan Q_2 ni hisoblaymiz:

$$Q_1 = \int_0^1 u(t)dt = \int_0^1 \left(t^2 + \lambda Q_1 + \lambda Q_2 t\right)dt = \left[\frac{1}{3}t^3 + \lambda Q_1 + \frac{1}{2}\lambda Q_2 t^2\right]_0^1 = \frac{1}{3} + \lambda Q_1 + \frac{1}{2}\lambda Q_2$$

yoki

$$(1-\lambda)Q_1 - \frac{1}{2}\lambda Q_2 = \frac{1}{3}.$$

Xuddi shuningdek,

$$Q_2 = \int_0^1 u(t)dt = \int_0^1 \left(t^2 + \lambda Q_1 + \lambda Q_2 t\right)dt = \frac{1}{4} + \frac{1}{2}\lambda Q_1 + \frac{1}{3}\lambda Q_3$$

yoki

$$-\frac{1}{2}\lambda Q_1 + \left(1 - \frac{1}{3}\lambda\right)Q_2 = \frac{1}{4}$$

Shunday qilib, quyidagi chiziqli algebraik tenglamalar sistemasi hosil bo‘ldi:

$$\left.\begin{aligned} &(1-\lambda)Q_1 - \frac{1}{2}\lambda Q_2 = \frac{1}{3}, \\ &-\frac{1}{2}\lambda Q_1 + \left(1 - \frac{1}{3}\lambda\right)Q_2 = \frac{1}{4} \end{aligned}\right\}$$

Bu sistemaning yechimini Kramer formulalariga asosan yozamiz:

$$Q_1 = \frac{D_1}{D} \qquad Q_2 = \frac{D_2}{D}$$

Bu erda

$$D = \begin{vmatrix} 1-\lambda & -\frac{1}{2}\lambda \\ -\frac{1}{2}\lambda & 1-\frac{1}{3}\lambda \end{vmatrix} = \frac{1}{12}\left(\lambda^2 - 16\lambda + 12\right) \neq 0,$$

$$D_1 = \begin{vmatrix} \frac{1}{3} & -\frac{1}{2}\lambda \\ \frac{1}{4} & 1-\frac{1}{3}\lambda \end{vmatrix} = \frac{1}{72}(\lambda + 24),$$

$$D_2 = \begin{vmatrix} 1-\lambda & \frac{1}{3} \\ -\frac{1}{2}\lambda & \frac{1}{4} \end{vmatrix} = \frac{1}{12}(3-\lambda),$$

Demak,

$$Q_1 = \frac{D_1}{D} = \frac{1}{6}\cdot\frac{\lambda+24}{\lambda^2-16\lambda+12}, \qquad Q_2 = \frac{D_2}{D} = \frac{3-\lambda}{\lambda^2-16\lambda+12}$$

Bularni izlanayotgan noma`lum funksiyaning (11) ifodasiga qo‘yib, uni quyidagi ko‘rinishda yozamiz:

$$u(x) = x^2 + \frac{\lambda(3-\lambda)}{\lambda^2-16\lambda+12}x + \frac{\lambda(24+\lambda)}{6(\lambda^2-16\lambda+12)},$$

Bu esa berilgan masalaning yechimidir. Yechim ifodasidagi kasrlarning maxraji nolga teng bo‘lmasligi uchun λ parametr

$$\lambda^2-16\lambda+12=0$$

kvadrat tenglamaning ildizi bo‘lmasligi shart, ya`ni $\lambda \neq 8 \pm 2\sqrt{3}$ xususiy holda $\lambda = 2$ deb faraz qilsak, yechim quyidagicha yoziladi:

$$u(x) = x^2 - \frac{x}{8} - \frac{13}{24}$$

<u>**2-misol.**</u> Ushbu tenglamani yeching.

$$u(x,y) = \frac{xy}{2} - \frac{1}{3} + \int_0^1\int_0^1 (xy + t_1t_2)u(t_1,t_2)dt_1dt_2 .$$

Aynigan yadroli ushbu integral tenglamani koeffisiyentlarni tenglash usuli bilan yechamiz.

O‘ng tomondagi qavslarni ochib ikkala intengralni ham qisqacha Q_1 va Q_2 orqali belgilaymiz:

$$u(x,y) = \frac{xy}{2} - \frac{1}{3} + \int_0^1\int_0^1 (xy + t_1t_2)u(t_1,t_2)dt_1dt_2 + \int_0^1\int_0^1 (xy + t_1t_2)u(t_1,t_2)dt_1dt_2 = \frac{xy}{2} - \frac{1}{3} + xyQ_1 + Q_2 ,$$

$$\left(Q_1 + \frac{1}{2}\right)xy + \left(Q_2 - \frac{1}{3}\right) = \alpha\cdot xy + \beta$$

u ning mana shu ifodasini berilgan integral tenglamaga qo‘yamiz:

$$\alpha\cdot xy + \beta = \frac{xy}{2} - \frac{1}{3} + \int_0^1\int_0^1 (xy + t_1t_2)(\alpha\, t_1t_2 + \beta)dt_1dt_2.$$

Bu yerdagi integrallar hisoblab chiqilsa, quyidagi ayniyat

$$\alpha\cdot xy + \beta = \left(\frac{1}{4}\alpha + \beta + \frac{1}{2}\right)xy + \left(\frac{1}{9}\alpha + \frac{1}{4}\beta + \frac{1}{3}\right).$$

Hosil bo‘ladi. Uning ikki tomonidagi xy ning koeffisentlarini o‘zaro hamda ozod hadlarni o‘zaro tenglash natijasida quyidagi tenglamalar

$$\alpha = \frac{1}{4}x + \beta + \frac{1}{2}, \quad \beta = \frac{1}{9}x + \frac{1}{4}\beta - \frac{1}{3}$$

yoki

$$\left.\begin{array}{l} \frac{3}{4}\alpha - \beta = \frac{1}{2}; \\ \frac{1}{9}\alpha - \frac{3}{4}\beta = \frac{1}{3} \end{array}\right\}$$

chiziqli algebraik tenglamalar sistemasi hosil bo‘ladi. Bu sistemaning yechimi

$$\alpha = \frac{6}{65}; \quad \beta = -\frac{28}{65}$$

Demak, integral tenglamaning yechimi

$$u(x, y) = \alpha \cdot xy + \beta = \frac{6}{65}xy - \frac{28}{65}$$

bo‘ladi.

3-misol. Ushbu tenglamani yeching: (Bu yerda ketma-ket yaqinlashish usulidan foydalanamiz).

$$u(x) = x + \int_0^x (t - x)u(t)dt,$$

bunda

$$f(x) = x \text{ va } \lambda = 1$$

Endi quyidagi munosabatlardagi hadlarni hisoblab chiqamiz:

$$u_0(x) = f(x) = x;$$

$$u_1(x) = \int_0^x (t - x)tdt = \left[\frac{t^3}{3} - x\frac{t^2}{2}\right]_{t=0}^{t=x} = \frac{x^3}{3} - \frac{x^3}{2} = -\frac{x^3}{3!};$$

$$u_2(x) = \int_0^x (t - x)\left(-\frac{t^3}{3!}\right)dt = \frac{x^5}{5!};$$

$$u_3(x) = \int_0^x (t - x)\left(-\frac{t^3}{5!}\right)dt = \frac{x^5}{7!};$$

va hokazo. Bu ifodalarning hosil bo‘lishidagi qonuniyat ko‘rinib turibdi. Ularning yig‘indidsini hisoblasak, izlanayotgan yechim hosil bo‘ladi:

$$u(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots = \sin x$$

4-misol. Ushbu tenglama rezol’venta yoradami bilan yechilsin:

$$u(x) = x + \int_0^x (t - x)u(t)dt,$$

Topamiz:

$$K_1 = K(x,t) = t - x = -(x-t)$$

$$K_2(x,t) = \int_t^x (x-s)(s-t)ds = \int_t^x (x-s)(x+s-t-x)ds = \int_t^x (x-s)[(x-t)-(x-s)]ds$$

$$= \int_t^x (x-s)[(x-t)-(x-s)]ds = (x-t)\int_t^x (x-s)ds - \int_t^x (x-s)^2 ds = -(x-t)\left[\frac{1}{2}(x-s)^2\right]_{s=t}^{x} + \frac{1}{3}\left[(x-3)^3\right]_{s-t}^{x}$$

$$= \frac{1}{2}(x-t)^3 - \frac{1}{3}(x-t)^3 = \frac{(x-t)^3}{3!}$$

Xuddi shu usulda $K_3(x,t)$ ni topamiz:

$$K_3(x,t) = -\int_t^x (x-s)\frac{(s-t)^3}{3!}dt = -\frac{1}{3!}\int_t^x (x-t-s+t)(s-t)^3 ds = -\frac{(x-t)^5}{5!}$$

va hokazo bularni $\Gamma(x,t,\lambda) = K_1(x,t) + \lambda K_2(x,t) + \lambda^2 K_3(x,t) + \ldots$ formulaga qo‘yib rezol’ventani topamiz:

$$\Gamma(x,t,\lambda) = -(x-t) + \frac{(x-t)^3}{3!} - \frac{(x-t)^5}{5!} + \ldots = -\sin(x-t)$$

U holda berilgan tenglamaning yechimi

$$u(x) = x - \int_0^x \sin(x-t)tdt$$

bo‘ladi. O‘ng tomondagi integralni hisoblab quyidagi natijani olamiz:

$$u(x) = \sin x$$

Yuqorida ko‘rsatilga usullardan foydalanib quyidagi misollar yechilsin

Mustaqil bajarish uchun mashqlar

a) $\varphi(x) = \lambda\int_0^1 K(x,y)\varphi(y)dy + f(x)$ integral tenglamani quyidagi hollar uchun yeching:

1. K(x,y)=x-1 *f(x)=x*

2. K(x,y)=2e^{x+y} *f(x)=e^x*

3. K(x,y)=x+y-2xy, *f(x)=x+x^2*

b) $\varphi(x) = \lambda\int_{-1}^1 K(x,y)\varphi(y)dy + f(x)$ integral tenglamani quyidagi hollar uchun yeching:

4. K(x,y)=xy+x^2y^2 *f(x)=x^2+x^4*

5. K(x,y)=x^{1/3}+y^{1/3} *f(x)=1-6x^2*

6. $K(x,y)=x^4+5x^3y$, $f(x)=x^2-x^4$

7. $K(x,y)=2xy^3+5x^2y^2$ $f(x)=7x^4+3$

8. $K(x,y)=x^2-xy$ $f(x)=x^2+x$

9. $K(x,y)=5+4xy-3x^2-3y^2+9x^2y^2$ $f(x)=x$

c) $\varphi(x)=\lambda\int_0^{\pi}Kx,y)\varphi(y)dy+f(x)$ integral tenglamani quyidagi hollar uchun yeching:

10. K(x,y)=sin(2x+y), f(x)=π-2x

11. K(x,y)=sin(x-2y) f(x)=cos2x

12. K(x,y)=cos(2x+y), f(x)=sinx

13. K(x,y)=sin(3x+y), f(x)=cosx

14. K(x,y)=siny+ycosx f(x)=1-$\frac{2x}{\pi}$

15. K(x,y)=$\cos^2$(x-y) f(x)=1+cos4x

d) $\varphi(x)=\lambda\int_0^{2\pi}K(x,y)\varphi(y)dy+f(x)$ integral tenglamani quyidagi hollar uchun yeching:

16. $K(x,y)=\cos x\cdot\cos y+\cos 2x\cos 2y,\quad f(x)=\cos 3x$

17. $K(x,y)=\cos x\cdot\cos y+2\sin 2x\cdot\sin 2y,\quad f(x)=\cos x$

18. $K(x,y)=\sin x\cdot\sin y+3\cos 2x\cdot\cos 2y,\quad f(x)=\sin x$

e) Quyidagi integral tenglamalarning barcha xarakteristik qiymatlarini va shu xarakteristikalarga mos xos funksiyalarini toping

19. $$\varphi(x)=\lambda\int_0^{2\pi}\left[\sin(x+y)+\frac{1}{2}\right]\varphi(y)dy$$

20. $$\varphi(x)=\lambda\int_0^{2\pi}\left[\cos^2(x+y)+\frac{1}{2}\right]\varphi(y)dy$$

21. $$\varphi(x)=\lambda\int_0^{1}\left[x^2y^2-\frac{2}{45}\right]\varphi(y)dy$$

22. $$\varphi(x)=\lambda\int_0^{2\pi}\left[\left(\frac{x}{y}\right)^{2/5}+\left(\frac{y}{x}\right)^{2/5}\right]\varphi(y)dy$$

23. $$\varphi(x)=\lambda\int_0^{2\pi}(\sin x\cdot\sin 4y+\sin 2x\cdot\sin 3y+\sin 3x\cdot\sin 2y+\sin 4x\cdot\sin y)\varphi(y)dy$$

f)

24. a va b parametrlarning qanday qiymatlarida quyidagi tenglama yechimga ega va shu qiymatlardagi yechimini toping:

$$\varphi(x) = 12\int_0^1 \left(xy - \frac{x+y}{2} + \frac{1}{3} \right)\varphi(y)dy + ax^2 + bx - 2?$$

25. a parametrning qanday qiymatlarida quyidagi tenglama yechimga ega:

$$\varphi(x) = \sqrt{15}\int_0^1 \left[y\left(4x^2 - 3x\right) + x(4y^2 - 3y) \right]\varphi(y)dy + ax + \frac{1}{x}?$$

26. λ parametrning qanday qiymatlerida

$$\varphi(x) = \lambda\int_0^{2\pi} \cos(2x - y)\varphi(y)dy + f(x)$$

integral tenglama har qanday $f(x) \in C([0,2\pi])$ uchun yechimga ega, shu yechimni toping.

g) Barcha λ va ozod hadga kiruvchi barcha *a,b,c* parametrlar uchun quyidagi integral tenglamalarning yechimini toping:

27. $\varphi(x) = \lambda\int_{-\pi/2}^{\pi/2} (y\sin x + \cos y)\varphi(y)dy + ax + b$

28. $\varphi(x) = \lambda\int_0^{\pi} \cos(x + y)\varphi(y)dy + a\sin x + b$

29. $\varphi(x) = \lambda\int_{-1}^{1} (1 + xy)\varphi(y)dy + ax^2 + bx + c$

30. $\varphi(x) = \lambda\int_{-1}^{1} (x^2 y + xy^2)\varphi(y)dy + ax + bx^3$

31. $\varphi(x) = \lambda\int_{-1}^{1} \frac{1}{2}(xy + x^2 y^2)\varphi(y)dy + ax + b$

32. $\varphi(x) = \lambda\int_{-1}^{1} \left[5(xy)^{1/3} + 7(xy)^{2/3} \right]\varphi(y)dy + ax + bx^{1/3}$

33. $\varphi(x) = \lambda\int_{-1}^{1} \frac{1 + xy}{1 + y^2}\varphi(y)dy + ax + bx^2$

34. $\varphi(x) = \lambda\int_{-1}^{1} \left(\sqrt[3]{x} + \sqrt[3]{y} \right)\varphi(y)dy + ax^2 + bx + c$

35. $\varphi(x) = \lambda\int_{-1}^{1} (xy + x^2 + y^2 - 3x^2 y^2)\varphi(y)dy + ax + b$

k)

36. *K(x,y)* yadroning xos sonlarini va ularga mos keluvchi xos funksiyalarini toping va barcha λ*,a,b,c* lar uchun quyidagi tenglamani yeching

$$\varphi(x) = \int_{-1}^{1} K(x,y)\varphi(y)dy + f(x)$$

1. $K(x,y)=3x+xy-5x^2y^2$ $\quad f(x)=ax$
2. $K(x,y)=3xy+5x^2y^2$ $\quad f(x)=ax^2+bx$

37. *K(x,y)* yadroning xos sonlarini va ularga mos keluvchi xos funksiyalarini toping va barcha λ*,a,b,c* lar uchun quyidagi tenglamani yeching

$$\varphi(x) = \lambda\int_{-\pi}^{\pi} K(x,y)\varphi(y)dy + f(x)$$

1. $K(x,y)=x\cos y+\sin x\cdot\sin y$ $\quad f(x)=a+b\cos x$
2. $K(x,y)=x\sin y+\cos x$ $\quad f(x)=ax+b$

l) Quyidagi integral tenglamalarni yeching va $R(x,y;\lambda)$rezolventasini toping

38. $\varphi(x) = \lambda\int_{-1}^{1} \sin(x+y)\varphi(y)dy + f(x)$

39. $\varphi(x) = \lambda\int_{-1}^{1} (1-y+2xy)\varphi(y)dy + f(x)$

40. $\varphi(x) = \lambda\int_{-\pi}^{\pi} (x\sin y+\cos x)\varphi(y)dy + ax + b$

41. $\varphi(x) = \lambda\int_{0}^{2\pi} (\sin x\sin y+\sin 2x\sin 2yy)\varphi(y)dy + f(x)$

p) Har qanday λ parametr uchun quyidagi integral tenglamalar yechimga ega bo‘ladigan *a,b,c* parametrlarning barcha qiymatlarini toping:

42. $\varphi(x) = \lambda\int_{-1}^{1} (xy+x^2y^2)\varphi(y)dy + ax^2 + bx + c$

43. $\varphi(x)=\lambda\int_{-1}^{1}(1+xy)\varphi(y)dy+ax^2+bx+c$, bu yerda $a^2+b^2+c^2=1$.

44. $\varphi(x)=\lambda\int_{-1}^{1}\frac{1+xy}{\sqrt{1-y^2}}\varphi(y)dy+x^2+b$

45. $\varphi(x)=\lambda\int_{-1}^{1}(xy-\frac{1}{3})\varphi(y)dy+ax^2-bx+1$

46. $\varphi(x)=\lambda\int_{-1}^{1}(x+y)\varphi(y)dy+ax+b+1$

47. $\varphi(x)=\lambda\int_{0}^{2\pi}\cos(2x+4y)\varphi(y)dy+e^{ax+b}$

48. $\varphi(x)=\lambda\int_{-1}^{1}(\sin x\sin 2y+\sin 2x\sin 4y)\varphi(y)dy+ax^2+bx+c$

49. $\varphi(x)=\lambda\int_{-1}^{1}(1+x^2+y^3)\varphi(y)dy+ax+bx^3$

q) Quyidagi integral tenglamalrnig xos sonlarini va ularga mos keluvchi xos funksiyalarini toping:

50. $\varphi(x_1,x_2)=\lambda\int_{-1}^{1}\int_{-1}^{1}\left[x_1+x_2+\frac{3}{32}(y_1+y_2)\right]\varphi(y_1,y_2)dy_1dy_2$

51. $\varphi(x)=\lambda\int_{|y|<1}(|x|^2+|y|^2)\varphi(y)dy, x=(x_1,x_2)$

52. $\varphi(x)=\lambda\int_{|y|<1}\frac{1+|y|}{1+|x|}\varphi(y)dy, x=(x_1,x_2,x_3)$

Javoblar

1-bo‘lim

1. $u_{\xi\xi}+u_{\eta\eta}+u_{\xi}=0,\ \xi=x,\ \eta=3x+y$. **2.** $u_{\eta\eta}+u_{\xi}=0,\ \xi=x-2y,\ \eta=x$.

3. $u_{\xi\eta}+\dfrac{1}{6(\xi+\eta)}\left(u_{\xi}+u_{\eta}\right)=0,\ \xi=\dfrac{2}{3}x^{\frac{3}{2}}+y,\ \eta=\dfrac{2}{3}x^{\frac{3}{2}}-y$, *x>0*; $u_{\xi\xi}+u_{\eta\eta}-\dfrac{1}{3\xi}u_{\xi}=0,\quad \xi=\dfrac{2}{3}(-x)^{\frac{3}{2}}$,

$\eta=y,\ x<0$. **4.** $u_{\xi\eta}+\dfrac{1}{2(\xi-\eta)}\left(u_{\xi}-u_{\eta}\right)=0,\ \xi=x+2\sqrt{y},\ \eta=x-2\sqrt{y},\ y>0$;

$u_{\xi\xi}+u_{\eta\eta}-\dfrac{1}{\eta}u_{\eta}=0,\ \xi=x,\ \eta=2\sqrt{-y},\ y<0$. **5.** $u_{\xi\xi}-u_{\eta\eta}-\dfrac{1}{\xi}\left(u_{\xi}-u_{\eta}\right)=0,\ \xi=\sqrt{|x|},\ \eta=\sqrt{|y|}$, (*x>0, y>0* yoki *x<0, y<0*) ; $u_{\xi\xi}+u_{\eta\eta}-\dfrac{1}{\xi}\left(u_{\xi}+u_{\eta}\right)=0,\ \xi=\sqrt{|x|},\ \eta=\sqrt{|y|}$ (*x>0, y<0* yoki *x<0, y>0*). **6.**

$u_{\xi\xi}-u_{\eta\eta}+\dfrac{1}{3\xi}u_{\xi}-\dfrac{1}{3\eta}u_{\eta}=0,\ \xi=|x|^{\frac{3}{2}},\ \eta=|y|^{\frac{3}{2}}$, (*x>0, y>0* yoki *x<0, y<0*) ;

$u_{\xi\xi}+u_{\eta\eta}+\dfrac{1}{3\xi}u_{\xi}+\dfrac{1}{3\eta}u_{\eta}=0,\ \xi=|x|^{\frac{3}{2}},\ \eta=|y|^{\frac{3}{2}}$, (*x>0, y<0* yoki *x<0, y>0*).

7. $u_{\xi\xi}+u_{\eta\eta}-u_{\xi}-u_{\eta}=0,\ \xi=\ln|x|,\ \eta=\ln|y|$ (har bir kvadrantda).

8. $u_{\xi\xi}+u_{\eta\eta}+\dfrac{1}{2\xi}u_{\xi}+\dfrac{1}{2\eta}u_{\eta}=0,\ \xi=y^2,\ \eta=x^2$ (har bir kvadrantda).

9. $u_{\xi\eta}+\dfrac{1}{4(\eta^2-\xi^2)}\left(\eta u_{\xi}+\xi u_{\eta}\right)=0,\ \xi=y^2-x^2,\ \eta=y^2+x^2$ (har bir kvadrantda).

10. $u_{\xi\xi}+u_{\eta\eta}-th\xi u_{\xi}=0,\ \xi=\ln\left(x+\sqrt{1+x^2}\right)\ \eta=\ln\left(y+\sqrt{1+y^2}\right)$

11. $u_{\xi\eta}-\dfrac{1}{2(\xi-\eta)}\left(u_{\xi}-u_{\eta}\right)+\dfrac{1}{4(\xi+\eta)}\left(u_{\xi}+u_{\eta}\right)=0,\ \xi=y^2+e^x,\ \eta=y^2-e^x$ (y>0 yoki y<0).

12. $u_{\xi\xi}+u_{\eta\eta}+\cos\xi u_{\eta}=0,\ \xi=x,\ \eta=y-\cos x$. **13. 14.**

15. $u_{\xi\xi}+u_{\eta\eta}+u_{\zeta\zeta}=0,\ \xi=x,\ \eta=y-x,\ \zeta=x-\dfrac{1}{2}y+\dfrac{1}{2}z$. **16.** $u_{\xi\xi}-u_{\eta\eta}+u_{\zeta\zeta}+u_{\eta}=0,\ \xi=\dfrac{1}{2}x,$

$\eta=\dfrac{1}{2}x+y,\ \zeta=-\dfrac{1}{2}x-y+z.$ **17.** $u_{\xi\xi}-u_{\eta\eta}+2u_{\xi}=0,\ \xi=x+y,\ \eta=y-x,\ \zeta=y+z$.

18. $u_{\xi\xi}+u_{\eta\eta}=0,\ \xi=x,\ \eta=y-x,\ \zeta=2x-y+z$. **19.** $u_{\xi\xi}-u_{\eta\eta}-u_{\zeta\zeta}=0,\ \xi=x,\ \eta=y-x,$

$\zeta=\dfrac{3}{2}x-\dfrac{1}{2}y+\dfrac{1}{2}z$. **20.** $u_{\xi\xi}+u_{\eta\eta}+u_{\zeta\zeta}+u_{\tau\tau}=0,\ \xi=x,\ \eta=y-x,\ \zeta=x-y+z,\ \tau=2x-2y+z+t$.

21. $u_{\xi\xi}-u_{\eta\eta}+u_{\zeta\zeta}+u_{\tau\tau}=0,\ \xi=x+y,\ \eta=y-x,\ \zeta=z,\ \tau=y+z+t$.

22. $u_{\xi\xi}-u_{\eta\eta}+u_{\zeta\zeta}+u_{\tau\tau}=0,\ \xi=x+y,\ \eta=y-x,\ \zeta=-2y+z+t,\ \tau=z-t$.

23. $u_{\xi\xi}-u_{\eta\eta}+u_{\zeta\zeta}=0,\ \xi=x,\ \eta=y-x,\ \zeta=2x-y+z,\ \tau=x+z+t$.

24. $u_{\xi\xi}+u_{\eta\eta}=0,\ \xi=x,\ \eta=y,\ \zeta=-x-y+z,\ \tau=x-y+t$.

25. $\sum_{k=1}^{n}u_{\xi_k\xi_k}=0$, $\xi_k=\sum_{l=1}^{k}x_l$, $k=1,2,\dots,n$. **26.** $\sum_{k=1}^{n}(-1)^{k+1}u_{\xi_k\xi_k}=0$, $\xi_k=\sum_{l=1}^{k}x_l$, $k=1,2,\dots,n$.

27. $\sum_{k=1}^{n} u_{\xi_k\xi_k}=0$**,** $\xi_1=x_1$**,** $\xi_k=x_k-x_{k-1}$**,** $k=2,3,...,n$ **. 28.** $\sum_{k=1}^{n} u_{\xi_k\xi_k}=0$**,** $\xi_k=\sqrt{\frac{2k}{k+1}}\left(x_k-\frac{1}{k}\sum_{l<k}x_l\right)$**,** $k=1,2,...,n$ **. 29.** $u_{\xi_1\xi_1}-\sum_{k=2}^{n} u_{\xi_k\xi_k}=0$**,** $\xi_1=x_1-x_2$**,** $\xi_k=\sqrt{\frac{2(k-2)}{k+2}}\left(x_k-\frac{1}{k-2}\sum_{l<k}x_l\right)$**,** $k=3,4,...,n$ **.**

2-bo‘lim

1. $f(y+ax)+g(y-ax)$ **. 2.** $f(x-y)+g(3x+y)$ **. 3.** $f(y)+g(x)e^{-ay}$ **.**

4. $x-y+f(x-3y)+g(2x+y)e^{\frac{3y-x}{7}}$ **. 5.** $[f(x)+g(y)]e^{-bx-ay}$ **. 6.** $e^{x+y}+[f(x)+g(y)]e^{3x+2y}$ **.**

7. $f(y-ax)+g(y-ax)e^{-x}$ **. 8.** $f(x+y)+(x-y)g(x^2-y^2)$ $(x>-y$ yoki $x<-y)$.

9. $f(xy)+|xy|^{\frac{3}{4}}g\left(\frac{x^3}{y}\right)$, (har bir kvadrantda). **10.** $f\left(\frac{x}{y}\right)+xg\left(\frac{x}{y}\right)$, $(x^2+y^2\neq 0)$**.**

11. $xf(y)-f'(y)+\int_0^x (x-\xi)g(\xi)e^{\xi y}d\xi$ **. Ko‘rsatma.** $u_x=v$ belgilash kiritib, $u=xv-v_y$, $v_{xy}-xv_x=0$ munosabatlarni oling. **12.** $yg(x)+\frac{1}{x}g'(x)+\int_0^y (y-\xi)f(\xi)e^{-x^2\xi}d\xi.$ **Ko‘rsatma.** $u_y=v$ belgilash kiritib, $u=\frac{1}{2x}v_x+yv$, $v_{xy}+2xyv_y=0$ munosabatlarni oling.

13. $e^{-y}\left[yf(x)+f'(x)+\int_0^y (y-\eta)g(\eta)e^{-x\eta}d\eta.\right]$ **Ko‘rsatma.** $u_y+u=v$ belgilash kiritib, $u=v_x+yv$, $v_{xy}+v_x+yv_y+yv=0$ munosabatlarni oling. **14.** $e^{-xy}\left[yf(x)+f'(x)+\int_0^y (y-\eta)g(\eta)e^{-x\eta}d\eta.\right]$ **Ko‘rsatma.** $u_y+u=v$ belgilash kiritib, $u=v_x+2yv$, $(v_y+xv)_x+2y(v_y+xv)=0$ munosabatlarni oling.

3-bo‘lim

1. $\frac{4}{5}\left(y^{\frac{5}{4}}-|x|^{\frac{5}{2}}\right)$; $|x|<1$**,** $0<y<1$**. 2.** $\sin y-1+e^{x-y}$; $-\infty<x,\ y>\infty$ **. 3.** $x-y-\frac{1}{2}+\frac{1}{2}e^{2y}$; $-\infty<x$, $y>\infty$ **. 4.** $\frac{1}{2}\left[1-x-3y+(x+y-1)e^{2x}\right]$;$-\infty<x,y<\infty$. **5.** $xy+\frac{3}{2}\sin\frac{2y}{3}\cos\left(x+\frac{y}{3}\right)$;$-\infty<x,y<\infty$.

6. $(y-3x)e^{\frac{x^2+y^2}{2}}$; $x<1, y<3$. 7. $\frac{x^2}{y}$; $x>0, y>0$. 8. $2x+y-x^2$;$-\infty<x,y<\infty$.

9. $\frac{y}{3x}+\frac{2x^2y}{3}$; $x>0, y<0$. 10. $\frac{3}{4}\sqrt{x^7y}\left(\sqrt{y}-\frac{1}{y}\right)$; $x>0, y>0$. 11. x^2+y; $x>0, y<x^2$.

12. x^3+y^3; $y>-x$. 13. $\frac{1}{2}+(4-3y)e^{2(1-x-y)}-\left(2x+\frac{3}{2}\right)e^{2(1-x-y)}$; $R=e^{x-\xi+2(y-\eta)}$. 14. $xy-y$; $R=\frac{\xi y}{x\eta}$.

15. $x-y+xy$; $R=\frac{x+y}{\xi+\eta}$.

16. $\frac{1}{2xy}\left[(x+y-1)u_0(x+y-1)+(x-y+1)u_0(x-y+1)\right]+\frac{1}{2xy}\int_{x-y+1}^{x+y-1}\left[u_0(\xi)+u_1(\xi)\right]\xi d\xi.$

17. $(y-x)(x^2+1)+x^5\cos x$. 18. $(x+2t)^2$ **19.** $x^2+xt+4t^2+\frac{1}{6}xt^3$. **20.** $\sin x$.

21. $xt+\sin(x+t)-(1-cht)e^x$. **22.** $1+t+\frac{1}{9}(1-\cos 3t)\sin x$. **23.** $\frac{1}{a^2\omega^2}(1-\cos a\omega t)\sin\omega x$.

24. $\frac{t}{\omega}-\frac{1}{\omega^2}\sin\omega t$. **25.** $x+ty+t^2$. **26.** $xyt(1+t^2)+x^2-y^2$. **27.** $\frac{1}{2}t^2(x^3-3xy^2)+e^x\cos y+te^y\sin x$. **28.** $x^2+t^2+t\sin y$. **29.** $2x^2-y^2+(2x^2+y^2)t+2t^2+2t^3$.

30. $x^2+ty^2+\frac{1}{2}t^2(6+x^3+y^3)+t^3+\frac{3}{4}t^4(x+y)$. **31.** $e^{3x+4y}\left[\frac{25}{26}ch5t-\frac{1}{25}+\frac{1}{5}sh5t\right]$.

32. $\cos(bx+cy)\cos\left(at\sqrt{b^2+c^2}\right)+\frac{1}{a\sqrt{b^2+c^2}}\sin(bx+cy)\sin\left(at\sqrt{b^2+c^2}\right)$.

33. $(x^2+y^2)^2(1+t)+8a^2t^2(x^2+y^2)\left(1+\frac{1}{3}t\right)+\frac{8}{3}a^4t^4\left(1+\frac{1}{5}t\right)$.

34. $(x^2+y^2+4a^2)(e^t-1-t)-2at^2\left(1+\frac{1}{3}t\right)$. **35.** $x^2+y^2-2z^2+t+t^2xyz$.

36. $y^2+tz^2+8t^2+\frac{8}{3}t^3+\frac{1}{12}t^4x^2+\frac{2}{45}t^6$.

37. $x^2y^2z^2+txy+3t^2(x^2+y^2+z^2+x^2y^2+x^2z^2+y^2z^2)+\frac{3}{2}t^4(3+x^2+y^2+z^2)+\frac{9}{10}t^6$.

38. $e^{x+y}\cos(z\sqrt{2})+te^{3y+4z}\sin 5x+t^3e^{x\sqrt{x}}\sin y\cos z$.

39. $(1+t)(x^2+y^2+z^2)^2+10a^2t^2\left(1+\frac{1}{3}t\right)(x^2+y^2+z^2)+a^4t^4(5+t)$.

40. $(x^2+y^2+z^2+6a^2)(e^t-1-t)-a^2t^2(3+t)$.

41. $\frac{1}{a^2}(1-\cos at)e^z\cos x\sin y+e^{y+z}\left[\frac{1}{a}shat\sin x+\frac{at}{\sqrt{2}}sh(at\sqrt{2})+x^2ch(at\sqrt{2})\right]$.

42. $xy\cos z\cos at+\frac{1}{a}yze^x shat+\frac{x}{1+25a^2}\cos(3y+4z)\left(e^t-\cos 5at-\frac{1}{5a}\sin 5at\right)$.

43. $\left(\cos at+\frac{1}{a}\sin at\right)\cos\sqrt{x^2+y^2+z^2}+\frac{1}{\sqrt{x^2+y^2+z^2}}\sin\sqrt{x^2+y^2+z^2}\left(t\cos at-at\sin at-\frac{1}{a}\sin at\right)$.

4-bo'lim

1. $1+e^t+\frac{1}{2}t^2$. **2.** $t^3+e^{-t}\sin x$. **3.** $(1+t)e^{-t}\cos x$. **4.** $cht\sin x$. **5.** $1-\cos t+(1+4t)^{-\frac{1}{2}}e^{-\frac{x^2}{1+t}}$.

6. $(1+t)^{-\frac{1}{2}}e^{\frac{2x-x^2+t}{1+t}}$. **7.** $x(1+4t)^{-\frac{3}{2}}e^{\frac{-x^2}{1+4t}}$. **8.** $(1+t)^{-\frac{1}{2}}\sin\frac{x}{1+t}e^{-\frac{4x^2+t}{4(1+t)}}$. **9.** $e^t-1+e^{-2t}\cos x\sin y$.

10. $1+\frac{1}{5}\sin x\sin y(2\sin t-\cos t+e^{-2t})$. **11.** $\sin t+\frac{xy}{(1+4t)^3}e^{-\frac{x^2+y^2}{1+4t}}$. **12.** $\frac{t}{8}+\frac{1}{\sqrt{1+t}}e^{-\frac{(x-y)^2}{1+t}}$.

13. $\frac{1}{\sqrt{1+t^2}}\cos\frac{xy}{1+t^2}e^{-\frac{t(x^2+y^2)}{2(1+t^2)}}$. **14.** $\frac{1}{4}\cos x(e^{-2t}-1+2t)\cos y\cos ze^{-4t}$. **15.** $e^t-1+\sin(x-y-z)e^{-9t}$.

16. $\frac{1}{4}\left(1-e^{-t}\right)+\frac{\cos 2y}{\sqrt{1+t}}e^{-t-\frac{x^2}{1+t}}$. **17.** $\frac{1}{3}\cos(x-y+z)\left(1-e^{-3t}\right)+\frac{1}{\sqrt{1+12t}}e^{-\frac{(x+y-z)^2}{1+12t}}$.

18. $\frac{\sin z}{\sqrt{1+4t^2}}\cos\frac{xy}{1+4t^2}e^{-t-\frac{t\left(x^2+y^2\right)}{1+4t^2}}$. **19.** $e^{-nt}\cos\sum_{k=1}^{n}x_k$. **20.** $(1+4t)^{-\frac{n}{2}}e^{-\frac{|x|^2}{1+4t}}$. **21.** $(1+4t)^{-\frac{n+2}{2}}e^{-\frac{|x|^2}{1+4t}}$.

22. $(1+4t)^{-\frac{n}{2}}\sin\frac{\sum_{k=1}^{n}x_k}{1+4t}e^{-\frac{nt+|x|^2}{1+4t}}$. **23.** $\frac{1}{\sqrt{1+4nt}}e^{-\frac{1}{1+4nt}\left(\sum_{k=1}^{n}x_k\right)^2}$.

5-bo‘lim

1. $-\frac{8}{\pi^3}\sum_{k=0}^{\infty}\frac{\sin(2k+1)\pi x}{(2k+1)^3}\cos\left(\sqrt{(2k+1)^2\pi^2+4t}\right)$.

2. $-\frac{8e^{-t}}{\pi}\sum_{k=0}^{\infty}\frac{1}{(2k+1)^3}\left[\cos(2k+1)t+\sin(2k+1)t\right]\sin(2k+1)x$.

3. $8e^{-t}\sum_{k=0}^{\infty}\frac{1}{(2k+1)^2}\left[(-1)^k-\frac{2}{\pi(2k+1)}\right]\sin\frac{(2k+1)}{2}t\cos\frac{(2k+1)}{2}x$.

4. $t(1-x)+\sum_{k=1}^{\infty}e^{-\frac{t}{2}}\frac{1}{(k\pi)^3}\left[2\cos\lambda_k t+\frac{1}{\lambda_k}\sin\lambda_k t-2\right]\sin\pi kx,\quad \lambda_k=\sqrt{(k\pi)^2-\frac{1}{4}}$.

5. $t(2-x)+\sum_{k=1}^{\infty}\left[\frac{4t}{k\pi\lambda_k^2}-\frac{k\pi^3}{\lambda_k^3}\sin\lambda_k t\right]\sin\frac{\pi kx}{2},\quad \lambda_k=\sqrt{\left(\frac{k\pi}{2}\right)^2-1}$.

6. $\frac{xt}{l}+\sum_{k=1}^{\infty}\frac{2(-1)^{k+1}}{\pi k\lambda_k^2}\left[t-\frac{1}{\lambda_k}\sin\lambda_k t\right]\sin\frac{\pi kx}{l},\quad \lambda_k=\sqrt{\left(\frac{k\pi}{l}\right)^2-1}$.

7. $\sin 2x\cos 2t+\sum_{k=1}^{\infty}(-1)^k\frac{2}{k^3}\left[1-\cos kt\right]\sin kx$.

8. $-\sum c_k\left[-1+e^{-\frac{t}{2}}\left(\cos\mu_k t+\frac{1}{\mu_k}\sin\mu_k t\right)\right]\sin(2k+1)\pi x,\quad c_k=\frac{4}{(2k+1)^3\pi^3},\quad \mu_k=\sqrt{(2k+1)^2\pi^2-\frac{1}{4}}$.

Ko‘rsatma. Yechimni $u(x,t)=\sum_{k=1}^{\infty}T_k(t)\sin k\pi x$ qator ko‘rinishida qidiring. **Izoh.** Yechimni $u=v+\omega$ yig‘indi ko‘rinishida qidirish mumkin, bu yerda $v=\frac{1}{2}x(1-x)$ funksiya tenglamani va chegaraviy shartlarni qanoatlantiradi. U holda $u(x,t)=\frac{x(1-x)}{2}-\sum_{k=0}^{\infty}\left(\cos\mu_k t+\frac{1}{2\mu_k}\sin\mu_k t\right)e^{-\frac{t}{2}}\sin(2k+1)\pi x$.

9. $2xt+\left(2e^t-e^{-t}-3te^{-t}\right)\cos x$. **10.** $3+x\left(t+t^2\right)+\left(5te^t-8e^t+4t+8\right)\sin x$.

11. $x(t+1)+\left(\frac{1}{5}e^{\frac{5}{2}t}-e^{\frac{t}{2}}+\frac{4}{5}\right)\cos\frac{3}{2}x$. **12.** $xt+\left(\frac{1}{10}-\frac{1}{6}e^{2t}+\frac{1}{15}e^{5t}\right)e^{-x}\sin 3x$.

13. $xt+\left(1-e^{-t}-te^{-t}\right)\cos 3x$. **14.** $\frac{1}{8}\left(e^{2t}+e^{-2t}\right)-\frac{1}{4}-\frac{t^2}{2}\cos 2x$. **15.** $\frac{1}{9}\sin x(ch3t-1)+\sin 3x(cht-1)$.

16. $xt+\left(2e^{t}-e^{2t}\right)e^{-x}\sin x$. **17.** $\sum_{n=1}^{\infty}a_n e^{-\left(\frac{n\pi a}{l}\right)^2 t}\sin\frac{\pi nx}{l}$, bu yerda $a_n=\frac{2}{l}\int_0^l u_0(x)\sin\frac{\pi nx}{l}dx$,

$u_0(x)=A=const$, bo'lgani uchun $u(x,t)=\frac{4A}{\pi}\sum_{k=0}^{\infty}\frac{1}{2k+1}e^{-\frac{(2k+1)^2\pi^2a^2}{l^2}t}\sin\frac{(2k+1)\pi x}{l}$.

18. $u(x,t)=\frac{8Al^2}{\pi^3}\sum_{k=0}^{\infty}\frac{1}{(2k+1)^3}e^{-\frac{(2k+1)^2\pi^2a^2}{l^2}t}\sin\frac{(2k+1)\pi x}{l}$.

19. $\frac{2}{l}\sum_{n=1}^{\infty}a_n\frac{\sigma^2+\mu_n^2}{\sigma(\sigma+1)+\mu_n^2}e^{-\frac{\mu_n^2a^2t}{l^2}}\sin\frac{\mu_n x}{l}$, bu yerda $a_n=\int_0^l u_0(x)\sin\frac{\mu_n x}{l}dx$, μ_n, $(n=1,2,...)$ - $tg\mu=-\frac{\mu}{\sigma}$,

$\sigma=hl>0$ tenglamaning musbat ildizlari. **20.** $\frac{2}{l}\sum_{n=1}^{\infty}b_n e^{-\frac{\mu_n^2a^2t}{l^2}}\frac{\mu_n\cos\frac{\mu_n x}{l}+\sigma\sin\frac{\mu_n x}{l}}{\sigma(\sigma+2)+\mu_n^2}$, bu yerda

$b_n=\int_0^l u_0(x)\left(\mu_n\cos\frac{\mu_n x}{l}+\sigma\sin\frac{\mu_n x}{l}\right)dx$, μ_n, $(n=1,2,...)$ - $ctg\mu=\frac{1}{2}\left(\frac{\mu}{\sigma}-\frac{\sigma}{\mu}\right)$, $\sigma=hl>0$ tenglamaning musbat ildizlari. **21.** u_0. **22.** $\frac{u_0}{2}+\frac{2u_0}{\pi}\sum_{k=0}^{\infty}(-1)^k\frac{1}{2k+1}e^{-\frac{(2k+1)^2\pi^2a^2t}{l^2}}\cos\frac{(2k+1)\pi x}{l}$, $\lim_{t\to\infty}u(x,t)=\frac{u_0}{2}$.

23. $\frac{u_0}{2}-\frac{4u_0}{\pi}\sum_{k=0}^{\infty}(-1)^k\frac{1}{(2k+1)^2}e^{-\frac{4(2k+1)^2\pi^2a^2t}{l^2}}\cos\frac{2(2k+1)\pi x}{l}$, $\lim_{t\to\infty}u(x,t)=\frac{u_0}{2}$.

24. $\frac{32}{\pi^3}\sum_{n=0}^{\infty}(-1)^n\frac{1}{(2n+1)^3}e^{-\left(\frac{(2n+1)\pi}{2}\right)^2 t}\cos\frac{(2n+1)\pi x}{2}$. **25.** $\frac{4}{\pi}\sum_{k=0}^{\infty}(-1)^k\frac{1}{2k+1}e^{-\left(\left(\frac{(2k+1)\pi}{l}\right)^2+1\right)t}\sin\frac{(2k+1)\pi x}{l}$.

26. $-\frac{8}{\pi}\sum_{k=0}^{\infty}\frac{1}{(2k+1)^2}e^{-(2k+1)^2t}\sin(2k+1)x$. **27.** $x-l+\frac{8l}{\pi^2}\sum_{k=0}^{\infty}\frac{e^{-\lambda_k^2t}}{(2k+1)^2}\cos\lambda_k x$; $\lambda_k=\frac{\pi(2k+1)}{2l}$.

28. $t\cos x+\frac{1}{8}\left(e^{-8t}-1\right)\cos 3x$. **29.** $xt+\sin\pi e^{x-t-\pi^2t}$. **30.** $x+t\sin x+\frac{1}{8}\left(1-e^{-8t}\right)\sin 3x$.

31. $tx^2+\frac{1}{4}\left(e^{4t}-1\right)+t\cos 2x$. **32.** $t+1+\left(1-e^{-t}\right)e^{x}\sin x+e^{x-4t}\sin 2x$.

33. $xt^2+e^{t}+\sin t-\cos t+e^{-3t}\cos 2x$. **34.** $x^2+2e^{9t}+(2t-\sin 2t)\cos 3x$.

35. $x+t^2+\frac{1}{5}\left(e^{5t}-1\right)\cos x+\frac{1}{3}\left(-e^{-3t}+1\right)\cos 3x$.

36. $x^2t+x+\sum_{k=1}^{\infty}\frac{C_{2k-1}}{(2k-1)^2-6}\left(1-e^{-6(2k-1)^2t}\right)\cos(2k-1)x$, $C_{2k-1}=\frac{2}{\pi}\left(\frac{1}{2k+1}-\frac{1}{2k-3}\right)$.

37. $(x+1)t+e^{-2x}\sum_{k=1}^{\infty}\frac{C_k}{k^2\pi^2+4}\left(1-e^{-\left(k^2\pi^2+4\right)t}\right)\sin k\pi x$,

$$C_k=\begin{cases}0, & \text{agar } n=2m\\ \frac{1}{\pi}\left(\frac{2}{2m-1}+\frac{1}{2m+1}+\frac{1}{2m-3}\right), & \text{agar } n=2m-1.\end{cases}$$

6-bo‘lim

1. Agar $\lambda=-2$ bo‘lsa, yechim yo‘q. Agar $\lambda\neq-2$ bo‘lsa, u holda $\varphi(x)=\frac{2x(\lambda+1)-\lambda}{\lambda+2}$.

2. Agar $\lambda \neq \lambda_1$, bu yerda $\lambda_1 = \dfrac{1}{e^2-1}$ bo‘lsa, u holda $\dfrac{e^x}{1-\lambda(e^2-1)}$ bo‘ladi. $\lambda = \lambda_1$ da yechim yo‘q. **3.** Agar $\lambda \neq 2$ va $\lambda \neq -6$ bo‘lsa, u holda $\dfrac{12\lambda^2 x - 24\lambda x - \lambda^2 + 42\lambda}{6(\lambda+6)(2-\lambda)}$. $\lambda = 2$ va $\lambda = -6$ da tenglama yechimga ega emas. **4.** Agar $\lambda \neq \dfrac{3}{2}$ va $\lambda \neq \dfrac{5}{2}$ bo‘lsa, u holda $\dfrac{5(7+2\lambda)}{7(5-2\lambda)}x^2 + x^4$. Agar $\lambda = \dfrac{3}{2}$ bo‘lsa, $Cx + \dfrac{25}{7}x^2 + x^4$, bu yerda C - ixtiyoriy doimiy. $\lambda = \dfrac{5}{2}$ da tenglama yechimga ega emas. **5.** Agar $\lambda \neq \pm\sqrt{\dfrac{5}{12}}$ bo‘lsa, u holda $\dfrac{2\lambda}{12\lambda^2-5}\left(5\sqrt[3]{x} + 6\lambda\right) + 1 - 6x^2$. $\lambda = \pm\sqrt{\dfrac{5}{12}}$ da tenglama yechimga ega emas. **6.** Agar $\lambda \neq \dfrac{5}{2}$ va $\lambda \neq \dfrac{1}{2}$ bo‘lsa, u holda $\dfrac{5(2\lambda-3)}{3(5-2\lambda)}x^4 + x^2$. Agar $\lambda = \dfrac{1}{2}$ bo‘lsa, $Cx^3 + x^2 - \dfrac{5}{6}x^4$, bu yerda C - ixtiyoriy doimiy. $\lambda = \dfrac{5}{2}$ da tenglama yechimga ega emas. **7.** Agar $\lambda \neq \dfrac{5}{2}$ va $\lambda \neq \dfrac{1}{2}$ bo‘lsa, u holda $\dfrac{20\lambda}{1-2\lambda}x^2 + 7x^4 + 3$. Agar $\lambda = \dfrac{5}{2}$ bo‘lsa, $7x^4 + 3 - \dfrac{50}{3}x^2 + Cx$, bu yerda C - ixtiyoriy doimiy. $\lambda = \dfrac{1}{2}$ da tenglama yechimga ega emas. **8.** Agar $\lambda \neq \pm\dfrac{3}{2}$ bo‘lsa, u holda $\dfrac{3(5-2\lambda)}{5(3+2\lambda)}x + x^3$. Agar $\lambda = \dfrac{3}{2}$ bo‘lsa, $\dfrac{1}{5}x + x^3 + Cx^2$, bu yerda C - ixtiyoriy doimiy. $\lambda = -\dfrac{3}{2}$ da tenglama yechimga ega emas. **9.** Agar $\lambda = \lambda_1 = \dfrac{1}{8}$ va $\lambda \neq \dfrac{1}{2}$ bo‘lsa, u holda $C_1 + \dfrac{3}{2}x$. Agar $\lambda = \lambda_2 = \dfrac{5}{8}$ bo‘lsa, $C_2(3x^2-1) - \dfrac{3}{2}x$, bu yerda C_1, C_2 - ixtiyoriy doimiylar. $\lambda = \lambda_3 = \dfrac{3}{8}$ da tenglama yechimga ega emas. Agar $\lambda \neq \lambda_i$, $i = 1,2,3$ bo‘lsa, u holda $\varphi(x) = \dfrac{3x}{3-8\lambda}$. **10.** Agar $\lambda \neq \dfrac{3}{4}$ va $\lambda \neq -\dfrac{3}{2}$ bo‘lsa, u holda $\dfrac{12\lambda}{3-4\lambda}\sin 2x + \pi - 2x$. Agar $\lambda = -\dfrac{3}{2}$ bo‘lsa, $\pi - 2x - 2\sin 2x + C\cos 2x$, bu yerda C - ixtiyoriy doimiy. $\lambda = \dfrac{3}{4}$ da tenglama yechimga ega emas. **11.** Agar $\lambda \neq -\dfrac{3}{4}$ va $\lambda \neq -\dfrac{3}{2}$ bo‘lsa, u holda $\dfrac{3\pi\lambda}{2(2\lambda+3)}\sin x + \cos 2x$. Agar $\lambda = -\dfrac{3}{4}$ bo‘lsa, $\cos 2x - \dfrac{3\pi}{4}\sin x + C\cos x$, bu yerda C - ixtiyoriy doimiy. $\lambda = -\dfrac{3}{2}$ da tenglama yechimga ega emas. **12.** Agar $\lambda \neq \pm\dfrac{3}{2\sqrt{2}}$ bo‘lsa, u holda $\sin x + \dfrac{3\pi\lambda}{8\lambda^2-9}\left(2\lambda\cos 2x + \dfrac{3}{2}\sin 2x\right)$. Agar $\lambda = \pm\dfrac{3}{2\sqrt{2}}$ bo‘lsa, tenglama yechimga ega emas. **13.** λ ning barcha qiymatlarida $\dfrac{\lambda\pi}{2-\lambda\pi}\sin 3x + \cos x$. **14.** Agar $\lambda \neq \pm\dfrac{1}{2}$ bo‘lsa, u holda $1 - \dfrac{2x}{\pi} - \dfrac{\pi^2\lambda}{6(2\lambda+1)}\cos x$. Agar $\lambda = \dfrac{1}{2}$ bo‘lsa, $\dfrac{4}{3} - \dfrac{2x}{\pi} + \left(8 + \pi^2\cos x\right)C$, bu yerda C - ixtiyoriy doimiy. $\lambda = -\dfrac{1}{2}$ da tenglama yechimga ega emas. **15.** Agar $\lambda \neq \dfrac{2}{\pi}$ va $\lambda \neq \dfrac{4}{\pi}$ bo‘lsa, u holda $\cos 4x + 1 + \dfrac{\pi\lambda}{2-\lambda\pi}$. Agar $\lambda = \dfrac{4}{\pi}$ bo‘lsa, $\cos 4x - 1 + C_1\cos 2x + C_2\sin 2x$, bu yerda C_1, C_2 - ixtiyoriy doimiylar. $\lambda = \dfrac{2}{\pi}$ da tenglama yechimga ega emas. **16.** Agar $\lambda \neq \dfrac{1}{\pi}$ bo‘lsa, u holda $\cos 3x$. Agar $\lambda = \dfrac{1}{\pi}$ bo‘lsa, $\cos 3x + C_1\cos x + C_2\cos 2x$, bu yerda C_1, C_2 - ixtiyoriy doimiylar. **17.** Agar $\lambda \neq \dfrac{1}{\pi}$ va $\lambda \neq \dfrac{1}{2\pi}$ bo‘lsa, u holda $\dfrac{\cos x}{1-\lambda\pi}$. Agar $\lambda = \dfrac{1}{2\pi}$ bo‘lsa, $2\cos x + C\sin 2x$, bu yerda C -

ixtiyoriy doimiy. $\lambda=\frac{1}{\pi}$ da tenglama yechimga ega emas. **18.** Agar $\lambda\neq\frac{1}{\pi}$ va $\lambda\neq\frac{1}{3\pi}$ bo‘lsa, u holda $\frac{\sin x}{1-\lambda\pi}$. Agar $\lambda=\frac{1}{3\pi}$ bo‘lsa, $\frac{3}{2}\sin x+C\cos 2x$, bu yerda C - ixtiyoriy doimiy. $\lambda=\frac{1}{\pi}$ da tenglama yechimga ega emas. **19.** $\lambda_1=\frac{1}{\pi}$, $\sin x+\cos x$,1; $\lambda_2=-\frac{1}{\pi}$, $\cos x-\sin x$. **20.** $\lambda_1=\frac{1}{2\pi}$, 1; $\lambda_2=\frac{2}{\pi}$, $\cos 2x$; $\lambda_3=-\frac{2}{\pi}$, $\sin 2x$. **21.** $\lambda_1=-45$, $3x^2-2$; $\lambda_2=\frac{45}{8}$, $15x^2-1$. **22.** $\lambda_1=\frac{3}{8}$, $3x^{\frac{2}{5}}+x^{-\frac{2}{5}}$; $\lambda_2=-\frac{3}{2}$, $3x^{\frac{2}{5}}-x^{-\frac{2}{5}}$. **23.** $\lambda_1=-\frac{2}{\pi}$, $\sin x-\sin 3x$; $\lambda_2=\frac{2}{\pi}$, $\sin 2x+\sin 3x$, $\sin x+\sin 4x$. **24.** $a=-12$, $b=12$, $-12x^2+C_1x+C_2$, bu yerda C_1, C_2 - ixtiyoriy doimiylar. **25.** $a=\sqrt{15}-3$, $C\left[4\sqrt{15}x^2+3\left(1-\sqrt{15}\right)x\right]+\frac{1}{x}-3x$, bu yerda C - ixtiyoriy doimiy **26.** Har qanday λ parametr uchun ushbu tenglama yechimga ega: $\varphi(x)=\lambda\int_0^{2\pi}\cos(2x-y)f(y)dy+f(x)$ **27.** Agar $\lambda\neq\frac{1}{2}$ bo‘lsa, u holda $\frac{\lambda a\pi^3}{12(1-2\lambda)}\sin x+\frac{2\lambda b}{1-2\lambda}+ax+b$. Agar $\lambda=\frac{1}{2}$ da, $a=b=0$ bo‘lsa, va faqat shu holda tenglama yechimga ega bo‘lib, yechim: $\varphi(x)=C_1\cos x+C_2$, bu yerda C_1, C_2 - ixtiyoriy doimiylar. **28.** Agar $\lambda\neq\pm\frac{2}{\pi}$ (a, b -ixtiyoriy) bo‘lsa, u holda $\frac{2(a-2\lambda b)}{2+\lambda\pi}\sin x+b$. $\lambda=\frac{2}{\pi}$ da ixtiyoriy a, b larning qiymatida tenglama yechimga ega: $\varphi(x)=\frac{a\pi-4b}{2\pi}\sin x+b+C_1\cos x$, bu yerda C_1 - ixtiyoriy doimiy; $\lambda=-\frac{2}{\pi}$ da $a\pi+4b=0$ bo‘lsa, va faqat shu holda tenglama yechimga ega bo‘lib, yechim: $\varphi(x)=b+C_2\sin x$, bu yerda C_2 - ixtiyoriy doimiy. **29.** Agar $\lambda\neq\frac{1}{2}$ va $\lambda\neq\frac{3}{2}$ (a, b, c -ixtiyoriy) bo‘lsa, u holda $\frac{2\lambda a+3c}{3(1-2\lambda)}+\frac{3b}{3-2\lambda}x+ax^2$. $\lambda=\frac{1}{2}$ da $a+3c=0$ bo‘lsa, va faqat shu holda tenglama yechimga ega bo‘lib, $\varphi(x)=\frac{3}{2}bx+ax^2+C_1$, bu yerda C_1 - ixtiyoriy doimiy; $\lambda=\frac{3}{2}$ da $b=0$ bo‘lsa, va faqat shu holda tenglama yechimga ega bo‘lib, yechim: $\varphi(x)=ax^2-\frac{1}{2}(a+c)+C_2x$, bu yerda C_2 - ixtiyoriy doimiy. **30.** Agar $\lambda\neq\pm\frac{\sqrt{15}}{2}$ (a, b -ixtiyoriy) bo‘lsa, u holda $\frac{2\lambda(5a+3b)}{15-4\lambda^2}x^2+\frac{4\lambda^2(5a+3b)}{5\left(15-4\lambda^2\right)}x+ax+bx^3$. $\lambda=\frac{\sqrt{15}}{2}$ da $5a+3b=0$ bo‘lsa, va faqat shu holda tenglama yechimga ega bo‘lib, $\varphi(x)=a\left(x-\frac{5}{3}x^3\right)+C_1\left(\sqrt{\frac{5}{3}}x^2+x\right)$, bu yerda C_1 - ixtiyoriy doimiy; $\lambda=-\frac{\sqrt{15}}{2}$ da $5a+3b=0$ bo‘lsa, va faqat shu holda tenglama yechimga ega bo‘lib, yechim: $\varphi(x)=a\left(x-\frac{5}{3}x^3\right)+C_2\left(x-\sqrt{\frac{5}{3}}x^2\right)$, bu yerda C_2 - ixtiyoriy doimiy. **31.** Agar $\lambda\neq 3$ va $\lambda\neq 5$ (a, b -ixtiyoriy) bo‘lsa, u holda $\frac{3a}{3-\lambda}x+\frac{5\lambda b}{3(5-\lambda)}x^2+b$. $\lambda=3$ da $a=0$ bo‘lsa, va faqat shu holda tenglama yechimga ega bo‘lib, $\varphi(x)=b\left(\frac{5}{2}x^2+1\right)+C_1$, bu yerda C_1 - ixtiyoriy doimiy; $\lambda=5$ da $b=0$ bo‘lsa, va faqat shu holda tenglama yechimga ega bo‘lib, yechim: $\varphi(x)=C_2x^2-\frac{3}{2}ax$, bu yerda C_2 - ixtiyoriy doimiy. **32.** Agar $\lambda\neq\frac{1}{6}$ (a,b -ixtiyoriy) bo‘lsa, u holda $\frac{30\lambda a+7b}{7(1-6\lambda)}x^{\frac{1}{3}}+ax$. $\lambda=\frac{1}{6}$ da $5a+7b=0$ bo‘lsa, va faqat shu holda tenglama yechimga ega bo‘lib, $\varphi(x)=-\frac{7}{5}bx+C_1x^{\frac{1}{3}}+C_2x^{\frac{2}{3}}$, bu yerda C_1 va C_2 - ixtiyoriy doimiylar. **33.** Agar $\lambda\neq\frac{2}{\pi}$ va $\lambda\neq\frac{2}{4-\pi}$ (a,b -ixtiyoriy) bo‘lsa, u holda $\frac{2a+\lambda b(4-\pi)}{2-\lambda\pi}+\frac{2}{2-\lambda(4-\pi)}x+bx^2$. $\lambda=\frac{2}{\pi}$ da $a\pi+b(4-\pi)=0$ bo‘lsa, va faqat shu holda tenglama yechimga ega bo‘lib, $\varphi(x)=\frac{\pi}{2(\pi-2)}x+bx^2+C$, bu yerda C - ixtiyoriy doimiy; $\lambda=\frac{2}{4-\pi}$ da tenglama yechimga ega emas. **34.** Agar $\lambda\neq\pm\frac{1}{2}\sqrt{\frac{5}{3}}$ (a,b,c -ixtiyoriy) bo‘lsa, u holda $\frac{5\lambda\left(14a+36\lambda b+42c\right)}{21\left(5-12\lambda^2\right)}x^{\frac{1}{3}}+\frac{28\lambda^2a+30\lambda b+35}{7\left(5-12\lambda^2\right)}+ax^2+bx$. $\lambda=\frac{1}{2}\sqrt{\frac{5}{3}}$ da $15\sqrt{3}b+7\sqrt{5}(a+c)=0$ bo‘lsa, va faqat shu holda tenglama yechimga ega bo‘lib, $\varphi(x)=ax^2+bx+c+C_1\left(x^{\frac{1}{3}}+\sqrt{\frac{3}{5}}\right)$, bu yerda C_1 - ixtiyoriy doimiy; $\lambda=-\frac{1}{2}\sqrt{\frac{5}{3}}$ da $15\sqrt{3}b-7\sqrt{5}(a+3c)=0$ bo‘lsa, va faqat shu holda tenglama yechimga ega bo‘lib, yechim: $\varphi(x)=ax^2+bx+c+C_1\left(x^{\frac{1}{3}}-\sqrt{\frac{3}{5}}\right)$, bu yerda C_2 - ixtiyoriy doimiy. **35.** Agar $\lambda\neq-\frac{15}{8}$ va $\lambda\neq\frac{3}{2}$ (a,b -ixtiyoriy) bo‘lsa, u holda $\frac{30(b-1)\lambda}{15+8\lambda}x^2+\frac{3a\lambda^2}{3-2\lambda}x+\frac{36\lambda^2(b-1)}{(15+8\lambda)(3-2\lambda)}$. $\lambda=-\frac{15}{8}$ da $b=1$ bo‘lsa, va faqat shu holda tenglama yechimga ega bo‘lib, $\varphi(x)=\frac{17}{2}ax+1-20a+C(x^2+1)$, bu yerda C - ixtiyoriy doimiy; $\lambda=\frac{3}{2}$ da $a=b=0$ bo‘lsa, va faqat shu holda tenglama yechimga ega bo‘lib, $\varphi(x)=C_1x+C_2$, bu yerda C_1 va C_2 - ixtiyoriy doimiylar. **36.1.** $\lambda_1=\frac{3}{2}$, $\varphi_1=x$; $\lambda_2=-\frac{1}{2}$, $\varphi_2=3x-4x^2$; agar $\lambda_1\neq\frac{3}{2}$ va $\lambda_2\neq-\frac{1}{2}$ bo‘lsa, $\varphi(x)=\frac{3ax}{3-2\lambda}$ (a-ixtiyoriy); $\lambda=\frac{3}{2}$ da tanglama yechimga ega, agar $a=0$ bo‘lsa va $\varphi(x)=\frac{3}{4}ax+C_2\left(3x-4x^2\right)$, bu yerda C_2 - ixtiyoriy doimiy. **36.2.** $\lambda_1=\frac{1}{2}$, $\varphi_1^{(1)}=x$, $\varphi_1^{(2)}=x^2$; agar $\lambda\neq\frac{1}{2}$ bo‘lsa, $\varphi(x)=\frac{ax^2+bx}{1-2\lambda}$; $\lambda=\frac{1}{2}$ da tenglama yechimga ega, agar $a=b=0$ bo‘lsa, va $\varphi(x)=C_1x^2+C_2x$ bu yerda C_1 va C_2 - ixtiyoriy doimiylar. **37.1.** $\lambda_1=\frac{1}{\pi}$, $\varphi_1=\sin x$; agar $\lambda\neq\frac{1}{\pi}$ bo‘lsa, $\varphi(x)=a+b\cos x+\lambda b\pi x+\frac{2\pi^2\lambda^2b}{1-\pi\lambda}\sin x$; $\lambda=\frac{1}{\pi}$ da tenglama yechimga ega, agar $b=0$ bo‘lsa, va $\varphi(x)=a+C\sin x$ bu yerda C - ixtiyoriy doimiy. **37.2.** $\lambda_1=\frac{1}{\pi}$, $\varphi_1=x$; agar $\lambda\neq\frac{1}{2\pi}$ bo‘lsa, $\varphi(x)=\frac{ax}{1-2\pi\lambda}+b+2\lambda b\pi\cos x$ (bu yerda a,b – ixtiyoriy); $\lambda=\frac{1}{2\pi}$ da tenglama yechimga ega, agar $a=0$ bo‘lsa, va

$\varphi(x)=b(1+\cos x)+Cx$ bu yerda C - ixtiyoriy doimiy. **38.** $\varphi(x)=\lambda\int_0^{\pi}\frac{\sin(x+y)+\lambda\frac{\pi}{2}\cos(x-y)}{\Delta(\lambda)}f(y)dy+f(x)$,

agar $\Delta(\lambda)\neq 0$ bo‘lsa, bu yerda $\Delta(\lambda)=1-\lambda^2\frac{\pi^2}{4}$; $\lambda=\frac{2}{\pi}$ da tenglama yechimga ega, agar $f_1+f_2=0$ bo‘lsa, bu yerda

$f_1=\int_0^{\pi}\cos yf(y)dy$, $f_2=\int_0^{\pi}\sin yf(y)dy$, va yechim: $\varphi(x)=C_1(\sin x+\cos x)+\frac{2}{\pi}f_1\sin x+f(x)$ (C_1 - ixtiyoriy

doimiy); $\lambda=-\frac{2}{\pi}$ da tenglama yechimga ega, agar $f_1-f_2=0$ bo‘lsa va yechim:

$\varphi(x)=C_2(\sin x-\cos x)-\frac{2}{\pi}f_1\sin x+f(x)$ (C_2 - ixtiyoriy doimiy); $R(x,y;\lambda)=\frac{\sin(x+y)+\frac{\lambda\pi}{2}\cos(x-y)}{\Delta(\lambda)}$ - rezolventa.

39. $\varphi(x)=\lambda\int_{-1}^{1}\frac{1-\frac{4}{3}\lambda+y(2x-4\lambda x-1)}{\Delta(\lambda)}f(y)dy+f(x)$, agar $\Delta(\lambda)\neq 0$ bo‘lsa, bu yerda $\Delta(\lambda)=(1-2\lambda)(1-\frac{4}{3}\lambda)$;

$\lambda=\frac{1}{2}$ da tenglama yechimga ega, agar $f_1=3f_2$ bo‘lsa, bu yerda $f_1=\int_{-1}^{1}f(x)dx$, $f_2=\int_{-1}^{1}xf(x)dx$, va yechim:

$\varphi(x)=\left(x-\frac{1}{2}\right)f_1+f(x)+C_1$ (C_1 - ixtiyoriy doimiy); $\lambda=\frac{3}{4}$ da tenglama yechimga ega, agar $f_2=0$ bo‘lsa va yechim:

$\varphi(x)=-\frac{3}{2}f_1+f(x)+C_2(x+1)$ (C_2 - ixtiyoriy doimiy); $R(x,y;\lambda)=\frac{1-\frac{4}{3}\lambda+y(2x-4\lambda x-1)}{\Delta(\lambda)}$ - rezolventa. **40.**

$\varphi(x)=\lambda\int_{-\pi}^{\pi}\left(\frac{x\sin y}{1-2\pi\lambda}+\cos x\right)(ay+b)dy+ax+b=\frac{ax}{1-2\pi\lambda}+2\pi\lambda b\cos x+b$, agar $\lambda\neq\frac{1}{2\pi}$ bo‘lsa (a,b –ixtiyoriy);

$\lambda=\frac{1}{2\pi}$ da tenglama yechimga ega, agar $a=0$ bo‘lsa, yechim: $\varphi(x)=b(\cos x+1)+Cx$ (C - ixtiyoriy doimiy);

$R(x,y;\lambda)=\frac{x\sin y}{1-2\pi\lambda}+\cos x$ - rezolventa.

41. $\varphi(x)=\lambda\int_0^{2\pi}\frac{\sin x\sin y+\sin 2x\sin 2y}{1-\pi\lambda}f(y)dy+f(x)$, agar $\lambda\neq\frac{1}{\pi}$ bo‘lsa; $\lambda=\frac{1}{\pi}$ da tenglama yechimga ega, agar

$\int_0^{2\pi}\sin yf(y)dy=\int_0^{2\pi}\sin 2yf(y)dy=0$ bo‘lsa, yechim: $\varphi(x)=f(x)+C_1\sin x+C_2\sin 2x$ (C_1,C_2 - ixtiyoriy

doimiylar); $R(x,y;\lambda)=\frac{\sin x\sin y+\sin 2x\sin 2y}{1-\pi\lambda}$ - rezolventa. **42.** $b=0,\quad 3a+5c=0$.

43. $a=\frac{3}{\sqrt{10}}$, $b=0$, $c=-\frac{1}{\sqrt{10}}$; $a=-\frac{3}{\sqrt{10}}$, $b=0$, $c=\frac{1}{\sqrt{10}}$. **44.** $a=0,\ \mathrm{b}=-\frac{1}{2}$. **45.** $a=6$.

46. $a=0,\ b=-1$. **47.** a,b - ixtiyoriy. **48.** a,b,c - ixtiyoriy. **49.** $7a+5b=0$. **50.** $\lambda_1=1$, $\varphi_1=4(x_1+x_2)+1$;

$\lambda_2=-1$, $\varphi_2=4(x_1+x_2)-1$. **51.** $\lambda_1=\frac{4\sqrt{3}-6}{\pi}$, $\varphi_1=2+\sqrt{3}(x_1^2+x_2^2)$; $\lambda_2=-\frac{4\sqrt{3}+6}{\pi}$, $\varphi_2=\sqrt{3}(x_1^2+x_2^2)-1$.

52. $\lambda_1=\frac{3}{4\pi}$, $\varphi_1=\frac{1}{1+r}$, bu yerda $r=\sqrt{x_1^2+x_2^2+x_3^2}$.

Foydalanilgan adbiyotlar ro‘yxati

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