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«ABITURIYENT-2013» loyihasi turkumidan

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ELEMENTAR MATEMATIKA

TENGLAMALAR

IV QISM

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«SHARQ»
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Mazkur qo‘llanmadan akademik litsey, kollej yoki oliy o‘quv yurtlarining kirish imtihonlariga tayyorlanayotgan, maktabda olgan bilimlarini qisqa vaqt ichida tiklab olishga harakat qilayotgan o‘quvchilar foydalanishlari mumkin. Qo‘llanmada keltirilgan test topshiriqlari abituriyentlarni mantiqiy fikrlash, javobni tez va to‘g‘ri ishlashga o‘rgatadi. Unda misol-masalalarni yechishning mantiqiy tomonlari, tez va aniq hisoblash yo‘llari, shuningdek, noan’anaviy usullardan oqilona foydalanish sirlari mujassam.

Besh qismdan iborat qo‘llanmaning ushbu 4-qismi tenglamalarga bag‘ishlangan. Mazkur to‘plam 17 mavzuni o‘z ichiga oladi.

Har qaysi mavzuda yetarlicha misol va masalalar, ularni tez va aniq bajarishni ta’minlaydigan usullar keltirilgan. Bundan tashqari, o‘quvchilarga qulaylik tug‘dirish maqsadida to‘plamda mustaqil yechish uchun ko‘plab misol va masalalar ham berilgan.

Ushbu to‘plam akademik litsey, kollej va oliy o‘quv yurtlarining kirish imtihonlariga tayyorlanayotgan o‘quvchi hamda abituriyentlarga mo‘ljallangan.

SHARTLI BELGILAR

— e’tibor bering, yangi mavzu

— mavzuga taalluqli mashqlar

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So'zboshi

Aziz o'quvchi! Shuni yaxshi bilingki, Siz O'zbekistonning ko'rar ko'zi, kelajagisiz! Fan cho'qqilarini zabt etishingizda mazkur qo'llanma Sizga beminnat ko'makchi bo'lishiga ishonamiz.

Bilamizki, oliy o'quv yurtlariga kirish imtihonlari test sinovlari asosida o'tkaziladi. Bu sinovlar iqtidorli va yetuk bilimga ega bo'lgan yoshlarni yurtimizdagi nufuzli o'quv maskanlariga o'qishga kirish imkoniyatlarini kafolatlaydi. Sir emaski, ko'pgina oliygohlarning kirish imtihonida matematika yetakchi fan sanaladi. O'z kelajagining poydevorini qurayotgan yoshlar uchun bu fanni chuqur o'rganish muhim.

Qo'lingizdagi qo'llanma **5 ta nashrlardan iborat qismlarga ajratilgan bo'lib,** u oliy o'quv yurtlarining kirish imtihonlariga tayyorlanayotgan abituriyentlar va mustaqil o'rganuvchilar uchun mo'ljallab tuzilgan.

1-qismda – Arifmetik hisoblash va soddalashtirishlar;

2-qismda – Algebraik ratsional va irratsional ifodalar hamda soddalashtirishlar;

3-qismda – Logarifmik va trigonometrik ifodalarni hisoblash hamda soddalashtirishlar;

4-qismda – Tenglamalar;

5-qismda – Tengsizliklarni yechish usullari va funksiyalar haqida.

Yodgor va Erkin Sharifboyevlarning mazkur «Elementar matematika» to'plamlarini boshqa qo'llanmalardan asosiy farqi shundaki, to'plam nazariy qoidalar bilan cheklanibgina qolmay, olgan bilim va malakalaringizni amalda qanday qo'llashingizni ham to'la yoritib bera oladi. Qo'llanmada bir qator qulayliklar mavjud. Har bir mavzu matematik atama, ta'rif-u tavsiflar bilan yoritilib, shu mavzuga doir bir qancha misol va masalalar, bundan tashqari, ularni yechish usullari ham keltirilgan. O'quvchilarning bilimlarini mustahkamlash zaruriyati inobatga olinib, mavzu so'ngida mustaqil bajarish uchun mashqlar ham berilgan. Har bir mashqdan so'ng ishlash uchun joy qoldirilgani foydalanuvchiga yana bir qulaylik. Kitob so'ngida keltirilgan test namunalari orqali abituriyentlar oliy o'quv yurtlarining kirish imtihonlariga tayyorgarlik darajasini tekshirib ko'rishi, lozim bo'lganda ayrim mavzularga qaytishi mumkin.

O'ylaymizki, ushbu qo'llanma oliy o'quv yurtlariga kirishga hozirlik ko'rayotgan abituriyentlarga test sinovlariga tayyorlanishlariga tayanch manba bo'lib xizmat qiladi.

Ilm cho'qqilari sari qadam tashlayotganingizda Sizlarga omad hamroh bo'lsin!

TENGLAMALAR HAQIDA TUSHUNCHA

1-MAVZU. TENGLIK, AYNIYAT VA TENGLAMA

1.1. *Ikki ifodaning barobar (=) belgisi bilan bog'langanligi (yozilishi)* ***tenglik*** *deyiladi.*

Masalan:

$17 - 9 = 32 : 4$; $a(x + b) = ax + ab$; $3x + 2(x - 4) = 5x - 8$ kabi. Tenglik to'g'ri tenglik bo'lishi mumkin; masalan: $19 - 9 = 5 \cdot 2$; noto'g'ri bo'lishi mumkin, masalan: $x^2 + 4 = x^2 - 1$; yoki tenglik tenglamadagi harflarning ba'zi bir qiymatlaridagina o'rinli bo'lishi mumkin. Masalan: $x^2 - 5x = 6$ tenglik x ning faqat $x = 6$ va $x = -1$ qiymatlaridagina o'rinli bo'lib, boshqa qiymatlarda o'rinli emas.

Tenglikda qatnashgan ifodalar ***tenglik hadlari*** *deyiladi. Tenglik hadlarini tenglik belgisining bir tomonidan ikkinchi tomoniga teskari ishora bilan o'tkazish mumkin.*

Masalan:

$A - C + 4 = b - 8$ ni $A = b + C - 12$ deb yoki $A - C - b = -12$ shu kabi $A - C - b + 12 = 0$ yozish mumkin.

Tenglikdagi ba'zi bir munosabatlar:

1) $a = b$ bo'lsa $b = a$;
2) $a = b$ va $b = c$ bo'lsa, $a = c$ bo'ladi;
3) $a = b$ bo'lsa,

($c \neq 0$ uchun) $a \pm c = b \pm c$; $ac = bc$; $\frac{a}{c} = \frac{b}{c}$ bo'ladi.

4) $a = b$ va $c = d$ bo'lsa, $a \pm c = b \pm d$, $ac = bd$ va $\frac{a}{c} = \frac{b}{d}$ $(c \neq 0, d \neq 0)$ bo'ladi.

1.2. *Agar tenglik ifodada qatnashgan harflarning mumkin bo'lgan barcha qiymatlarida o'rinli bo'lsa, bunday tenglik* ***ayniyat*** *deyiladi va (≡) belgisi bilan birlashtiriladi.*

Masalan:

$5(x - 2) + 20 \equiv 5x + 10$; $2[3 - 4x(x + 1)] \equiv 2x(x - 4) - 10x^2 + 6$; $a(a - 3b) \equiv a[2 - (3b - a)] - 2a$; $(a + b)^2 \equiv a^2 + 2ab + b^2$

1.3. *Agar tenglik ifodada qatnashgan harflarning ba'zi bir qiymatlaridagina o'rinli bo'lsa, bunday tenglik* ***tenglama*** *deyiladi.*

Masalan:

$$2x - 8 = 0$$

bu tenglama, chunki yolg'iz $x = 4$ qiymatidagina tenglik o'rinli bo'ladi. ($2 * 4 - 8 = 0$, $0 = 0$) o'zgaruvchi x ning boshqa qiymatlarida $2x - 8 = 0$ tenglik o'rinli bo'lmaydi. Shu kabi $x^2 = 3x + 4$ ham tenglama, chunki $x = 4$ va $x = -1$ dagina to'g'ri tenglik saqlanadi.

Tenglamadagi noma'lumning (o'zgaruvchi harfning) tenglikni qanoatlantiradigan ya'ni uni ayniyatga aylantiradigan (to'g'ri tenglik hosil qiladigan) son qiymatlari ***tenglama yechimlari*** *yoki* ***ildizlari*** *deyiladi. Bu ildizlarni aniqlash tenglamani yechish deyiladi.*

Tenglamaning ikkala tomoniga bir xil sonni qo'shish, ayirish, noldan farqli songa ko'paytirib, bo'lish mumkin, lekin o'zgaruvchili ifodaga ko'paytirib, bo'lish mumkin emas.

Masalan:

$2x^2 = 4x + 30$ tenglikning ikki tomonini 0,5 ga ko'paytirsak, $x^2 = 2x + 15$ hosil bo'ladi, bu tenglama bilan boshlang'ich berilgan tenglama ildizlari $x = 5$ va $x = -3$, $5x - 2 = 13$ tenglamani olsak bu tenglamaning ildizi $x = 3$. Agar tenglamaning ikki tomonini $(x - 2)$ ga ko'paytirib soddalashtirsak, $x^2 - 5x = -6$ tenglamaga kelinadi, bu tenglama ildizlari $x = 3$ va $x = 2$ bo'lib, $x = 2$ esa berilgan boshlang'ich tenglama ildizi emas, bunday ildizni tenglamaning ***chet ildizi*** deyiladi. Agar $(x - 3)^2 = 2(x - 3)$ tenglamaning ikki tomonini $(x - 3)$ ga bo'lsak, $x - 3 = 2$ yoki $x = 5$ yechim olamiz, boshlang'ich tenglamada esa ildizlari $x = 3$ va $x = 5$ bo'lib bitta $x = 3$ ildizi yo'qoldi.

Tenglama ildizlari bitta, bir nechta, cheksiz ko'p bo'lishi yoki umuman ildizga ega bo'lmasligi mumkin.

Masalan:

1) $2x - 3 = x + 5$ tenglama bitta $x = 8$ ildizga ega;

2) $x^3 = 3x^2 + 4x$ tenglama esa $x = 0$, $x = -1$ va $x = 4$ uchta yechimga ega;

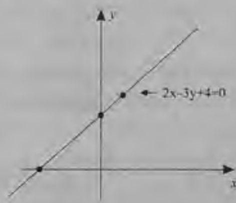
3) $3(x - 2) + 1 = 3x + 2$ tenglama x ning hech bir qiymatida to'g'ri tenglikka aylanmaydi. Demak tenglama yechimga ega emas.

4) $4(x + 2) - 5 = 2(3x - 1) - (2x - 5)$ tenglama cheksiz ko'p yechimga ega. Chunki x o'zgaruvchining o'rniga ixtiyoriy son qo'ysak, to'g'ri tenglik hosil bo'ladi.

Tenglamada ikkita va undan ko'p o'zgaruvchi bo'lsa, bunday tenglamalar cheksiz ko'p yechimga ega bo'ladi. Bunday tenglamalar mexanik va geometrik ma'noga ega.

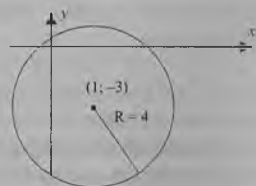
Masalan:

$ax + by + c = 0$ ikki o'zgaruvchili tenglama tekislikdagi ***to'g'ri chiziqning umumiy tenglamasi*** deyiladi. Xususiy holda $2x - 3y + 4 = 0$ tenglamani qarasak, $\left(0; \frac{4}{3}\right)$, $(-2;0)$ va $(1;2)$ kabi juft qiymatlar tenglama yechimi bo'lib, tenglamani qanoatlantiruvchi bunday juft qiymatlar o'rni berilgan tenglamaning geometrik tasviri to'g'ri chiziqni beradi.



1-chizma

Yana $x^2 + y^2 + 6y - 2x - 6 = 0$ tenglamani qanoatlantiruvchi tekislikdagi nuqtalarning o'rni markazi (1; – 3) nuqtada va radiusi $R = 4$ bo'lgan aylanani ifodalaydi.



2-chizma

Tenglama yechimi noma'lumlar olishi mumkin bo'lgan qiymatlari qanday sohada tenglama tekshirilishiga bog'liq.

<u>Masalan:</u>

$x^3 - x^2 = 6x$ tenglama natural sonlar sohasida bitta $x = 3$ yechimga ega. Agar tenglama Z to'plamda (ya'ni butun sonlar to'plamida) qarasak, $x = -2$, $x = 0$ va $x = 3$ yechimlarga ega. $x^2 - 5 = 0$ tenglama ratsional sonlar sohasida yechimga ega emas, irratsional sonlar sohasida $x = \pm\sqrt{5}$ yechimlarga ega. Agar $2x^3 + x^2 + 8x + 4 = 0$ tenglamani ratsional sonlar sohasida qaralsa, faqat $x = -0{,}5$ yechimga ega, agar bu tenglamani kompleks sonlar to'plamida qaralsa, $x = -0{,}5$ va $x = 2i$, $x = -2i$ ildizlarga ega bo'ladi.

1.4. *Agar ikkita tenglamaning barcha ildizlari soni bir xil va qiymati o'zaro teng bo'lsa, u holda bu ikki tenglama* ***<u>teng kuchli (ekvivalent) tenglamalar</u>*** *deyiladi.*

<u>Masalan:</u>

1)$11x - 4 = 6x + 21$ va $2(1 + 3x) = 7x - 2(x - 3{,}5)$ tenglamalar teng kuchli, chunki yolg'iz $x = 5$ tenglamalarni ayniyatga aylantiradi.

2) $x^2 = 4x + 5$ tenglama bilan $x^2 - 7x + 10 = 0$ tenglamalar teng kuchli emas, chunki tenglama ildizlari soni bir xil ikkita bo'lgani bilan, qiymatlari o'zaro teng emas Birinchi tenglama ildizlari

$$x = 5 \text{ va } x = -1$$

bo'lsa, ikkinchi tenglamada

$$x = 5 \text{ va } x = 2.$$

Berilgan tenglamaga biror o'zgarmas sonni ikki tomoniga qo'shish va ayirish bilan yoki tenglamaning ikki tomonini nolga teng bo'lmagan *songa ko'paytirish bilan, shu kabi tenglamaning hadlarini bir tomondan ikkinchi tomonga o'tkazish bilan hosil qilingan tenglamalar* ***o'zaro teng kuchli tenglamalar*** *deyiladi.*

Tenglamada o'xshash hadlarga nisbatan soddalashtirish amallarini bajarganda ham teng kuchli tenglama hosil bo'ladi. Shu kabi tenglamaning ikki tomoniga shu tenglamaning aniqlanish sohasida bo'lgan bir xil ifodalarni qo'shishda ham teng kuchli tenglamalar hosil bo'ladi. Ya'ni $f(x)=\varphi(x)$ tenglama bilan $f(x)+g(x)=\varphi(x)+g(x)$ tenglamalar teng kuchli.

Tenglama yechimi tenglamaning aniqlanish sohasida, ya'ni tenglamada qatnashgan o'zgaruvchilarning tenglamani ma'noga ega qiladigan barcha qiymatlari sohasida bo'lishi kerak. Bu tenglamada qatnashgan ifodalarning ko'rinishiga bog'liq.

Masalan:

$\frac{x}{x-1}=3+\frac{1}{2+x}$ tenglama mos ravishda $x=1$ va $x=-2$ da mavjud emas, bu tenglamaning aniqlanish sohasi $x\in\left]-\infty;-2\right[\cup\left]-2;1\right[\cup\left]1;+\infty\right[$.

Shunday qilib tenglama yechimga ega, yoki yechimi mavjud emasligini ko'rsatish, mavjud bo'lsa yechimlarini aniqlab berish ***tenglamani yechish*** *deyiladi.*

Tenglama yechimlar to'plamini aniqlashda ko'p hollarda bu tenglamaga, ya'ni teng kuchli tenglamaga keltiriladi, bunda quyidagi tasdiqlar o'rinli:

1) $f(x)=\varphi(x)$ tenglama bilan $f(x)-\varphi(x)=0$ yoki $\varphi(x)-f(x)=0$ tenglamalar teng kuchli.

2) Ixtiyoriy $a\in R$ $(a\neq 0)$ son uchun $f(x)=\varphi(x)$ tenglama va $f(x)\pm a=\varphi(x)\pm a$, $af(x)=a\varphi(x)$ tenglamalar teng kuchli.

3) Ixtiyoriy $a>0$ son uchun $f(x)=\varphi(x)$ va $a^{f(x)}=a\varphi^{(x)}$ teng kuchli

4) Biror D sohada $f(x)>0$, $\varphi(x)>0$ bo'lsa, u holda $f(x)=\varphi(x)$ tenglama bilan $[f(x)]^n=[\varphi(x)]^n$ tenglamalar $(n\in N)$ teng kuchli.

5) Agar $g(x)$ funksiya, $f(x)=\varphi(x)$ tenglama aniqlanish sohasida ma'noga ega va hech bir nuqtasida nolga teng bo'lmasa, $f(x)=\varphi(x)$ va $f(x)g(x)=\varphi(x)g(x)$, tenglamalar teng kuchli.

Shu kabi tasdiqlarga amal qilgan holda yechimlari topilsa, u holda tenglamaning yechimlarini tekshirish shart emas.

Boshqa hollarda tenglamaning yechimlari ichida chet ildiz hosil bo'lishi yoki biror yechim yo'qolishidan qutulish uchun topilgan yechimlarni tekshirish kerak. Tekshirishning ikkita asosiy usuli bor:

1) Topilgan yechimlarning har birini berilgan tenglamaga qo'yish;

2) Soddalashtirishning hamma qadamlarda hosil bo'lgan tenglama berilgan tenglamaga teng kuchli tenglama ekanligiga ishonch hosil qilish bilan; Masalan: 1) $x^2+3x+\sqrt{x+1}=\sqrt{x+1}-2$ tenglamani yechish uchun o'ng tomonidagi hadlarini chap tomoniga o'tkazsak, $x^2+3x+2=0$ tenglama hosil bo'lib yechimlari $x=-2$ va $x=-1$, bunda $x=-2$ boshlang'ich tenglamaga yechim emas.

3) $(x-4)^2 = 2(x-4)$ tenglamani yechishda tenglamani $\frac{1}{x-4}$ ga ko'paytirsak, $x-4=2$ yoki $x=6$ yechim hosil bo'ladi. Boshlang'ich tenglamani soddalashtirsak $x^2-10x+24=0$ bo'lib yechimlari $x=6$ va $x=4$.

Demak, $x=4$ yechim yo'qolgan.

Agar tenglamada o'zgaruvchining transendent funksiyalari qatnashsa, bunday tenglamalar ***transendent tenglamalar*** *deyiladi.*

Masalan: $3^{(x+1)}=0,(3)$; $\log_2^2 x+3\log_2 x=4$; $2\cos x-\sin^2 x=2$ kabi.

Bir o'zgaruvchili tenglamalarni yechishni eng sodda tenglamalardan boshlaymiz.

Tenglama yechimlarini aniqlash metodlarini o'rganish bilan chegaralanganimiz uchun, quyidagi temalarni qaytarib eslab, bilib olishimiz kerak bo'ladi, chunki har bir tenglamani yechishda bu tenglamalar juda kerak bo'ladi.

1. Ratsional sonlar, ratsional sonlardagi bog'lanishlar, amallarni bajarish tartibi, ifodalarni hisoblash kabi.

2. Algebraik ifodalar, algebraik ifodalarda amallar bajarish, ko'phadni ko'paytuvchilarga ajratish, ko'phadlarda amallar. Ratsional va irratsional algebraik ifoda qiymatini aniqlash kabi.

3. Trigonometrik ko'rsatkichli va logarifmik funksiyalar va undagi asosiy bog'lanishlar, munosabatlar, trigonometrik va logarifmik ifodalarni soddalashtirish ifoda qiymatlarini aniqlash kabi.

4. Tenglamada qatnashgan elementar funksiyalarning aniqlanish sohasini eslash kerak, bunda:

1. Ko'pxad ko'rinishidagi $y=a_1x^n+a_2x^{(n-1)}+\ldots+a_{(n-1)}x+a_n$ funksiyaning aniqlanish sohasi $X\in R$, yani $D\{x : x\in(-\infty;+\infty)\}$.

Masalan:

$y=x^4-5x^3+x^2-6x+9$ funkiya aniqlash sohasi $D\{x : x\in(-\infty;+\infty)\}$.

2. Kasr algebraik $y=\frac{A}{\varphi(x)}$. funksiyada maxraj nol bo'lmasligi va funksiya mavjud bo'lishi kerak.

Masalan:

$y=\frac{x^2+5x+6}{x^2+4}$. da $D\{x : x\in(-\infty;+\infty)\}$. $y=\frac{2x^2-x+5}{x^2-4x-5}$ funksiyada aniqlanish sohada $x^2-4x-5\neq 0$.

3. $y=\sqrt[k]{\varphi(x)}$ + irratsional funksiyada:

3.1. Ildiz ko'rsatkichi $k=2n$ juft bo'lgan $y=\sqrt[2n]{\varphi1(x)}$ irratsional funksiyada ildiz ostidagi funksiya $\varphi1(x)$ mavjud va $\varphi1(x)\geq 0$ bo'lishi kerak.

Masalan:

$$y=\sqrt[4]{\frac{3x+1}{2x^2-4x+9}}$$

da $\varphi 1(x)=\frac{3x+1}{2x^2-4+9}$ funksiya haqiqiy sonlar to'plamida ($X\in R$da) mavjud bo'lib, irratsional $y=\sqrt[4]{\varphi 1(x)}$ funksiyaning aniqlash sohasida $\frac{3x+1}{2x^2-4x+9}\geq 0$ shart bajarilishi kerak.

3.2. Ildiz ko'rsatkich $k=2n+1$ bo'lgan $y=\sqrt[2n+1]{\varphi 1(x)}$ funksiya aniqlanish sohasida, ildiz ostidagi $\varphi 1(x)$ funksiya mavjudligi talab qilinadi.

Masalan:

$$y=\sqrt[3]{\frac{x^2+5x+6}{x^2+1}}$$ da

$\varphi 2(x)=\frac{x^2+5x+6}{x^2+1}$ funksiya $X\in R$ da mavjud bo'lganligi uchun berilgan irratsional funksiyaning aniqlanish sohasi $D\{x: x\in(-\infty;+\infty)\}$.

$$y=\sqrt[5]{\frac{x^2+5x-1}{2x+4}}$$

irratsional funksiyaning aniqlanish sohasida $2x+4\neq 0$ talab qilinadi.

4. Ko'rsatkichli $y=a^{\varphi(x)}$ funksiya aniqlanish sohasida $\varphi(x)$ funksiyaning mavjudligi talab qilinadi. Masalan: $y=\left(\frac{1}{3}\right)^{\sqrt{2x-1}}$ funksiya aniqlanish sohasida $2x-1\geq 0$ bo'lishi talab qilinadi.

5. Logarifmik $y=\log_a\varphi(x)$ $(a>0,\ a\neq 1)$

Funksiya aniqlash sohasida $\varphi(x)$ mavjud va $\varphi(x)>0$ bo'lishi talab qilinadi.

Masalan:

$$y=\lg\frac{3x+1}{4x^2+1}$$

funksiyada $3x+1>0$ bo'lish talab qilinadi.

$y=\log_2\frac{x^2-2x+5}{x^2+2x-3}$ funksiya aniqlash sohasida $x^2+2x-3\neq 0$ va $\frac{x^2 2x+5}{x^2+2x-3}>0$ bo'lishi talab qilinadi.

6. $y = \sin\varphi(x)$ yoki $y = \cos\varphi(x)$ funksiyalarda $\varphi(x)$ ning mavjudligi talab qilinadi.

Masalan:

$y = \sin(x^2 - 4x + 5)$ funksiya aniqlanish sohasi $D\{x : x \in (-\infty; +\infty)\}$.

$y = \cos\dfrac{4x+1}{x+2}$ funksiya aniqlanish sohasida $x + 2 \neq 0$ talab qilinadi.

7. $y = \mathrm{tg}\varphi(x)$ funksiyada $\varphi_{(x)} \neq \dfrac{\pi}{2} + k\pi,\ k \in Z$ talab qilinsa, $y = \mathrm{ctg}\varphi(x)$ funksiyada $\varphi(x) \neq k\pi,\ k \in Z$ talab qilinadi. Masalan: $y = tg\left(2x + \dfrac{\pi}{5}\right)$ funksiya aniqlash sohasida $x \neq \dfrac{3\pi}{20} + \dfrac{k\pi}{2},\ k \in Z$ talab qilinadi.

8. $y = \arcsin\varphi(x)$ yoki $y = \arccos\varphi(x)$ funksiyalarda $|\varphi(x)| \leq 1$ talab qilinadi.

Masalan:

$y = \arcsin\dfrac{2x}{3}$ funksiyada $-\dfrac{3}{2} \leq x \leq \dfrac{3}{2}$ talab qilinadi.

9. $y = \mathrm{arctg}\varphi(x)$ yoki $y = \mathrm{arcctg}\varphi(x)$ funksiyalarida $\varphi(x)$ funksiyaning mavjudligi talab qilinadi.

Masalan:

$y = \mathrm{arctg}\dfrac{2x}{x^2+1}$ funksiya aniqlash sohasi $D\{x : x \in (-\infty; +\infty)\}$

$y = arcctg\sqrt{x+1}$ funksiyada $x + 1 \geq 0$ bo‘lish talab qilinadi.

10. Agar tenglama elementar funksiyalarning algebraik yig‘indisi ko‘rinishda bo‘lsa, tenglama mavjudlik sohasi, har bir funksiya aniqlanish sohasining umumiy qismidan iborat bo‘ladi. Masalan: $y = ln(x - 3) + \sqrt{2-x}$ ifoda birinchi funksiya aniqlanish sohasi $x - 3 > 0$, ikkinchi qo‘shiluvchidagi funksiyaning aniqlanish sohasi $2 - x \geq 0$ bo‘lib, bu sohalarning umumiy qisimi $x \in]-\infty; 2] \cap]3; +\infty[$, demak berilgan funksiyalarning aniqlanish sohasi $x \in \varnothing$. $y = \sqrt[4]{x-1} + \dfrac{4}{\sqrt{x-1}}$ da qo‘shiluvchi funksiyalarning aniqlanish sohasi mos ravishda $x - 1 \geq 0$ va $3 - x > 0$. U holda berilgan funksiyaning aniqlanish sohasi

$x \in [1; +\infty[\cap]-\infty; 3[1; 3[$. Shu kabi $y = \dfrac{\sin 2x}{\log_2(4x-1)}$ funksiyaning aniqlanish sohasida $4x - 1 \neq 1$ va $4x - 1 > 0$ bo‘lishi kerak, yani $D\{x : x \neq 0{,}5, x > 0{,}25\}$.

Eslatma. ***Elementar funksiyalar mavjud bo'ladigan argumentning qiymatlar to'plamini (funksiyaning aniqlanish sohasini) aniqlash, tengsizliklar temasidan keyin beriladi.***

Quyidagi funksiyalarning aniqlanish sohasini eslang.

1. $y=\sqrt[4]{x-1}+\sqrt{1-x}$, 2. $y=\dfrac{2x-1}{2x^2-3x+1}$, 3. $y=\sqrt[3]{\dfrac{3x}{x+1}}$,

4. $y=\sqrt{\dfrac{x-1}{2x+1}}$, 5. $y=log_2\dfrac{2}{x-2}$, 6. $y=cos\dfrac{4}{x-1}$, 7. $y=\arcsin(3x+1)$

Misol:

$(2x-1)(x+5)(x^2-5)=0$,(1); $(2x-1)(x+5)=0$ (2);

$(x^2+5)=0$ (3); $2x^2+8=0$ (4); $9x^2=25$ (5); $3x^2=5$ (6)

(1)va (2) ratsional sonlar to'plamida teng kuchli,

(3) va (4) esa haqiqiy sonlar to'plamida teng kuchli,

(5) va (6) tenglamalar esa o'zaro teng kuchli emas, sababi yechimlar soni bir xil bo'lganligi bilan qiymatlari har xil.

a ning qanday qiymatlarida quyidagi juft tenglamalar teng kuchli tenglamalar bo'ladi.

1) $2x+3=12$ bilan $2x+3=12(3a-0{,}5)-6$ tenglama

2) $4-3x=5$ tenglama bilan $-(3x-4)=3a-8$ tenglama

▲ Birinchi misoldagi ikkinchi tenglama chap tomoni birinchi tenglama chap tomoniga teng. Endi o'ng tomonlarini tenglashtirsak, masala hal bo'ladi.

$12=12(3a-0{,}5)-6$ yoki $12=36a-6-6$ dan $a=0{,}(6)$ da tenglamalar teng kuchli bo'ladi.

Ikkinchi misoldagi ikkinchi tenglamani

$-3x+4=3a-8$ deb yozish mumkin, u holda berilgan tenglamalar teng kuchli bo'lishi uchun $5=3a-8$ yoki $a=\dfrac{13}{3}$ bo'lishi kerak. Javob: $a=4{,}(3)$

Misol:

$2\dfrac{1}{4}-1\dfrac{1}{2}(4x-1)=3(0{,}25-0{,}5x)-3\left(1\dfrac{1}{2}x-1\right)$ tenglik ayniyatmi?

Yechish: O'ng va chap tomonlarini soddalashtiramiz.

$\dfrac{9}{4}-6x+\dfrac{3}{2}=\dfrac{3}{4}-\dfrac{3}{2}x-\dfrac{9}{2}x+3$ yoki $\dfrac{15}{4}-6x=\dfrac{15}{4}-6x$, demak ifoda ayniyat ekan.

2-MAVZU. CHIZIQLI (BIRINCHI TARTIBLI) BIR NOMA'LUMLI TENGLAMALAR

2.1. *Noma'lum o'zgaruvchi birinchi darajada qatnashgan:*

$$ax = b, \quad (1) \; a \neq 0, \; a,b \in R$$

tenglamaga ***chiziqli (yoki birinchi tartibli) bir noma'lumli tenglama*** *deyiladi. (1) chiziqli bir noma'lumli tenglamaning eng sodda (kanonik) yoki normal ko'rinishi bo'lib, bunda a va b ixtiyoriy ma'lum sonlar, x-noma'lum o'zgaruvchi, b-ozod had, a esa* ***noma'lum oldidagi koeffisiyent*** *deyiladi.*

(1) ko'rinishidagi chiziqli tenglamada:

1) Agar $a \neq 0$, bo'lsa, tenglama yagona

$$x = b : a \; (2)$$

yechimga ega;

2) $a > 0$, $b > 0$ yoki $a < 0$, $b < 0$ bo'lsa, tenglama yechimi musbat bo'ladi;

3) $a > 0$, $b < 0$ yoki $a < 0$, $b > 0$ bo'lsa tenglama yechimi manfiy bo'ladi;

4) Agar $a \neq 0$, va $b = 0$ bo'lsa, tenglama faqat $x = 0$ yechimga ega;

5) Agar $a = 0$, va $b \neq 0$ bo'lsa, tenglama yechimga ega emas;

6) Agar $a = 0$ va $b = 0$ bo'lsa, tenglama cheksiz ko'p yechimga ega bo'ladi. (Ya'ni ixtiyoriy son tenglama ildizi bo'ladi.)

Tenglama ildizi topilsin:

1) $-4x = -2$

►(2) formuladan foydalanamiz: $x = (-2):(-4) = \frac{-2}{-4} = 0,5$ ◄

2) $0,5x - 1\frac{3}{4} = 0$

►Tenglamada ozod hadni o'ng tomonda yozib, ozod had va koeffisiyentlarni oddiy kasrga keltirib, (2) formuladan foydalanamiz:

$$\frac{1}{2}x = \frac{7}{4} \text{ dan } x = \frac{7}{4} : \frac{1}{2} = \frac{7 \cdot 2}{4 \cdot 1} = \frac{7}{2} = 3,5$$ ◄

3) $-1,(6) = 1,25x$

►Oldingi misoldagi kabi amallar bajaramiz

$1\frac{25}{100}x = -1\frac{6}{9}$ yoki $\frac{5}{4}x = -\frac{5}{3}$ bo'lib tenglama yechimi

$$x = -\frac{5}{3} : \frac{5}{4} = -\frac{5*4}{3*5} = -1,(3)$$ ◄

Sodda (kanonik) ko'rinishdagi chiziqli tenglamaga keltirish mumkin bo'lgan tenglamalarni ko'rib chiqamiz.

2.2. Quyidagi chiziqli tenglamani

$$ax + c = d, \quad (3) \quad (a \neq 0)$$

normal ko‘rinishga keltirish uchun (ildizini topish uchun) ozod hadlarni tenglamaning bir tomoniga yozib olamiz.

$ax = d - c$, bunda:

1) $a \neq 0$ va $c \neq d$ da tenglama yechimi $x = (d - c) : a$.
2) Agar $a \neq 0$, $c = d$ bo‘lsa, tenglama $x = 0$ yechimga ega;
3) Agar $a = 0$, $c \neq d$ bo‘lsa, tenglama yechimga ega emas.
4) Agar $a = 0$, $c = d$ bo‘lsa, tenglama cheksiz ko‘p yechimga ega.

<u>Misollar:</u>

Tenglama yechimi topilsin:

1) $0,5x + 2\frac{1}{4} = -1,2$

Oldin o‘nli va aralash kasrlarni oddiy kasrda yozib, ozod hadlarda amal bajarsak, tenglama normal ko‘rinishga keladi. $\frac{1}{2}x + \frac{9}{4} = -\frac{6}{5}$ dan $\frac{1}{2}x = -\frac{6}{5} - \frac{9}{4}$ yoki $\frac{1}{2}x = -\frac{69}{20}$ berilgan tenglama ildizi $x = -\frac{69}{10} = -6,9$

2) Agar

$$-1\frac{2}{3}x - 2,(3) = 2,25$$

tenglama yechimi x bo‘lsa, $4x$ nimaga teng.

Oldingi misoldagi kabi amallar bajarsak, $-\frac{5}{3}x - 2\frac{1}{3} = \frac{9}{4}$ dan $-\frac{5}{3}x - \frac{55}{12}$ bo‘lib, tenglamaning berilgan shartdagi yechimi $x = -11$.

3) $3,2 - 1\frac{1}{4} = -1,5x$ tenglamaning chapdagi amallarni bajarsak, $\frac{39}{20} = -\frac{3}{2}x$ bo‘lib, tenglama yechimi $x = -1,3$.

2.3.

$$ax + d = cx, \quad (4) \quad (a \neq c)$$

Tenglamada o‘zgaruvchi qatnashgan hadlarni tenglamaning bir tomonida yozsak, tenglama kanonik $d = cx - ax$ ko‘rinishga kelinadi.

(4) ko‘rishidagi tenglamada:

1) $a \neq c$ da tenglama birdan-bir $x = d : (c - a)$ yechimga ega;

2) $a \neq c$ va $d \neq 0$ da bitta $x = 0$ yechimga ega;
3) $a = c$ va $d \neq 0$ da tenglama yechimga ega emas;
4) $a = c$ va $d = 0$ da tenglama cheksiz ko'p yechimga ega.

Misollar:

Tenglama ildizi topilsin.

1) $4x + 28 = -2x$

Yechish: $4x$ qo'shiluvchini ishorasini o'zgartirib tenglamaning o'ng qismiga o'tkazib, soddalashtirsak, tenglama kanonik ko'rinishga kelinadi.

$28 = -2x - 4x \Rightarrow -6x = 28$ dan $x = 28 : (-6) = -4,(6)$

2) $2,5 + 1\frac{1}{4}x = 1,5x$

Yechish: Ozod had va koeffisiyentlarini oddiy kasrda yozib, oldingi misoldagi kabi amallar bajaramiz.

$\frac{25}{10} + \frac{5}{4}x = \frac{15}{10}x \Rightarrow x = \frac{5}{2} : \frac{1}{4} \Rightarrow$ tenglama yechimi $x = 10$.

3) $3x - 2\frac{2}{3}x = 0,1(6)$

Tenglamaning chap tomonidagi amalni bajarsak, tenglama kanonik ko'rinishga kelinadi. $3x - \frac{8}{3}x = \frac{1}{6}$. U holda tenglama yechimi $x = 0,5$.

2.4. *Bir noma'lumli chiziqli tenglamaning $ax + b = cx + d$ (5) umumiy ko'rinishidagi tenglama yechimini aniqlashda o'zgaruvchi hadlarni tenglamining bir tomonida, ozod hadlarni ikkinchi tomonida yozsak, tenglama* ***kanonik ko'rinishga*** *keladi.*

$ax - cx = d - b$, bu tenglamada:
1) $a \neq c$ va $d \neq b$, bu tenglama bitta yechimga ega: $x = (d - b) : (a - c)$
2) $a \neq c$ va $d = b$ bo'lsa, tenglama $x = 0$ yechimga ega.
3) $a = c$ va $d \neq b$ bo'lsa tenglama yechimga ega emas;
4) $a = c$ va $d = b$ da tenglama cheksiz ko'p yechimga ega;

Misollar:

Tenglama yechimi topilsin.

1) $4,75x + 1\frac{1}{4} = 2,5 - 2\frac{2}{3}x$

Oldin o'nli va aralash kasrlarni oddiy kasr ko'rinishida yozib, keyin o'zgaruvchi qo'shiluvchilarni tenglamaning bir tomonida, ozod hadlarini tenglamaning ikkinchi tomonida yozib soddalashtirsak, tenglama kanonik ko'rinishga kelinadi.

$$4\frac{75}{100}x + \frac{5}{4} = \frac{25}{10} - \frac{8}{3}x \Rightarrow 4\frac{3}{4} + \frac{8}{3}x = \frac{5}{2} - \frac{5}{4} \Rightarrow \frac{19}{4}x + \frac{8}{3}x = \frac{5}{4} \Rightarrow \frac{89}{12}x = \frac{5}{4}$$

bo'lib berilgan tenglama yechimi $x = \frac{5}{4} : \frac{89}{12}$, $x = \frac{15}{89}$

2) $3{,}185x+2\frac{2}{3}=2{,}(6)-1\frac{5}{7}x$

Tenglama ozod hadlari bir biriga teng bo'lgani uchun tenglama yechimi nolga teng.

3) $2(3(2-x)-5(2x+1))=4x-(2(x+5)-4)$

Oldin qavslarni ochib soddalashtirib olamiz.

$2(6-3x-10x-5)=4x-(2x+10-4)$ yoki $2(1-13x)=4x-(2x+6)$ yoki $2-26x=4x-2x-6$ noma'lum qatnashgan ifodalarni tenglamaning bir tomonida, ozod hadlarni ikkinchi tomonida yozib tenglama yechimi topiladi.

$$-26x-2x=-6-2\Rightarrow -28x=-8 \text{ dan } x=\frac{2}{7}$$

4) $\dfrac{5x-0{,}2}{4}=\dfrac{1\frac{3}{4}+0{,}5x}{5}$

Tenglamani yechishda $a:b=c:d$ proporsiyaning $a\cdot d=b\cdot c$ xossasidan foydalanamiz:

$5(5x-0{,}2)=4\left(\frac{7}{4}+0{,}5x\right)$ qavslarni ochib soddalashtiramiz:

$$25x-1=7+2x\Rightarrow 23x=8 \text{ dan } x=\frac{8}{23}.$$

$$\frac{9x-0{,}7}{4}-\frac{5x-1{,}5}{7}=\frac{7x-1{,}1}{3}-\frac{5(0{,}4-2x)}{6}$$

Tenglamani maxrajidagi sonlarning eng kichik umumiy bo'linuvchisiga EKUK(4,7,6) = 84 ko'paytiramiz. Keyin qavslarni ochib ifoda soddalashtiriladi.

$21\left(9x-\frac{7}{10}\right)-12(5x-1{,}5)=28\left(7x-\frac{11}{10}\right)-14\cdot 5(0{,}4x-2x)\Rightarrow 189x-$

$-14{,}7-60x+18=196x-30{,}8-28+140x$ yoki $129x+3{,}3=336x-$ $-58{,}8$ soddalashtirsak, $-207x=-62{,}1$ bo'lib, berilgan tenglama yechimi $x=0{,}3$

7) $\frac{x}{3}+\frac{x}{15}+\frac{x}{35}+\frac{x}{63}+\frac{x}{99}=10$

Noma'lumlar oldidagi koeffisiyentlarni hisoblaymiz.

$\frac{1}{3}+\frac{1}{15}+\frac{1}{35}+\frac{1}{63}+\frac{1}{99}=\frac{1}{3}+\frac{1}{2}\left(\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{9}\right)+\right.$

$+\left(\frac{1}{9}-\frac{1}{11}\right)\Big)=\frac{1}{3}+\frac{1}{2}\left(\frac{1}{3}-\frac{1}{11}\right)=\frac{1}{3}+\frac{4}{33}=\frac{5}{11}$, u holda berilgan tenglama $\frac{5}{11}x=10$ ko'rinishida bo'lib, yechim $x = 22$.

8) $$(2x-0,5)\left(1\frac{1}{3}x+2\frac{2}{3}\right)\left(1,5x-2\frac{1}{4}\right)=0$$

Bu ko'rinishdagi tenglama yechimini aniqlashda qavslar ochilmaydi. Har qaysi ko'paytuvchilarni nolga tenglab yechimlar topiladi. (Sababi ko'paytmali ifoda nolga teng bo'lishi uchun kamida bittasi nolga teng bo'lishi yetarli edi). Demak, berilgan tenglama quyidagi uchta tenglamaga ekvivalent bo'ladi:

1) $2x - 0,5 = 0$ dan $x = 0,25$;

2) $1\frac{1}{3}x+2\frac{2}{3}=0$ dan ikkinchi yechim $x = -2$;

3) Uchinchi yechim $1,5x-2\frac{1}{4}=0$ dan $x = 1,5$

Shunday qilib berilgan tenglamaning yechimlari $x = 0,25$ $x = -2$; $x = 1,5$ ekan.

2.5. *Tenglamada noma'lum sonni bildiruvchi harflardan boshqa biron ma'lum sonlardan iborat bo'lgan harflar ham qatnashsa, bunday tenglama* ***harfiy tenglama*** *deyiladi. Tenglama shu ma'lum harflarga nisbatan yechiladi.*

Tenglamada noma'lum x topilsin:

1) $$\frac{3x}{a}-\frac{a}{2}=\frac{2x+a}{2a}-a,\ (a\neq 0)$$

Tenglamaning ikki tomonini $2a$ ga ko'paytiramiz:

$6x - a^2 = 2x + a - 2a^2 \Rightarrow 4x = a - a^2$ bo'lib, tenglama yechimi $x=\frac{a-a^2}{4}$.

2) $a(b - x) + 2x(a + b) = b(2x + 3a)$, $(a \neq 0)$

Qavs ochib soddalashtiriladi:

$ab - ax + 2ax + 2bx = 2bx + 3ab$ dan $ax = 2ab$ bo'lib, bundan $x = 2b$

3-MAVZU. MODULLI TENGLAMALAR

Noma'lum o'zgaruvchi modul belgisi ichida qatnashgan tenglamaga modulli tenglama $|f(x)| = g(x)$ *deyiladi.*

Modulli tenglama

$$\begin{cases} f(x) = y(x) \\ f(x) \geq 0 \end{cases} \text{ va } \begin{cases} -f(x) = y(x) \\ f(x) < 0 \end{cases}$$

tenglamalarga teng kuchli bo'ladi.

3.1. Bir noma'lumli chiziqli modulli tenglamani o'rganishdan boshlaymiz.

Chiziqli bir noma'lumli modulli tenglama yechimini aniqlash usullaridan bittasi oraliqlar usuli. Bunda modul ta'rifidan foydalanamiz:

$$|a| = \begin{cases} a, \ a \geq 0 \\ -a, \ a < 0 \end{cases}.$$

$$|5| = 5, |-12| = -(-12) = 12 \text{ va } |a| \geq 0$$

kabi asosiy xossalaridan foydalanamiz.

<u>*Misol:*</u>

Quyidagi ifodani $x > 2$ da modulsiz yozing:

1) $|x-2| + |x| + 5x - 3$

Ta'rifdan foydalanamiz, $x > 2$ da $(x-2) + x + 5x - 3 = 7x - 5$

2) Ifoda modulsiz yozilsin:

$$|1-x| + |x-3| + |x+2| + 5$$

Har bir modulli qo'shiluvchi nol bo'ladigan nuqtalar yordamida son o'qini oraliqlarga bo'lib tekshiramiz. Bu qiymatlar $x = 1$, $x = 3$ va $x = -2$ bo'lganligi uchun son o'qini 4 qismga bo'lamiz:

$(-\infty; -2)$,$(-2; 1)$,$(1; 3)$ va $(3; \infty)$ oraliqlarga bo'lib ifoda ifoda ishorasi tekshiriladi va modulsiz yoziladi:

a) $x < -2$ da: $-\infty < x < -2$ oraliqda $(1-x)$ ifoda musbat, $(x-3)$ ifoda manfiy, $(x+2)$ ifoda ham manfiy berilgan ifoda modulsiz quyidagicha yoziladi:

$(1-x) - (x-3) - (x+2) + 5 = 1 - x - x + 3 - x - 2 + 5 = -3x + 7;$

b) $-2 < x < 1$ da:

$(1-x) - (x-3) + (x+2) + 5 = 1 - x - x + 3 + x + 2 + 5 = -x + 11;$

v) $1 < x < 3$ da:

$-(1-x) - (x-3) + (x+2) + 5 = -1 + x - x + 3 + x + 2 + 5 = x + 9$

g) $x > 3$ da:

$-(1-x) + (x-3) + (x+2) + 5 = -1 + x + x - 3 + x + 2 + 5 = 3x + 3$

3) Agar $a < b < c$ bo'lsa $|a-b| + |c-b| - |a+c|$ ifodani soddalashtiring. $0 > b > a$ bo'lganligi uchun $a - b < 0$, shu sababli $|a-b| = -(a-b)$. Shu kabi

$0 > c > b$ uchun $c - b > 0$, u holda $|c - b| = c - b$. Demak $|a - b| + |c - b| - |a + c| = -(a - b) + c - b + (a + c) = 2c$.

Tenglama yechimi topilsin:

1) $|2x - 3| = 4$

Tenglamadagi $|2x - 3|$ ifoda $x = 1,5$ da nol bo'ladi. U holda son o'qini $(-\infty; 1,5)$ va $(1,5; \infty)$ oraliqlarga ajratib tenglamani tekshiramiz:

a) $x < 1,5$ da: ya'ni $(-\infty; 1,5)$ oraliqda $(2x - 3)$ ifoda manfiy, tenglamani $-(2x - 3) = 4$, deb yozish mumkin, bundan $-2x = 1$ $x = -0,5$ yechimni olamiz.

b) $x \geq 1,5$ da

Ya'ni $(1,5; \infty)$ da $(2x - 3)$ ifoda musbat, demak tenglamani yozish mumkin.

$2x - 3 = 4$ yoki bundan $x = 3,5$. Demak, berilgan tenglamaning yechimlari $x = -0,5$ va $x = 3,5$ ekan.

2) $|3x - 5| = -2$ tenglamani yeching.

Tenglama yechimga ega emas, chunki ifoda moduli manfiy bo'lishi ($|a| \geq 0$ edi) mumkin emas.

3) $|2x + 3| = 6 + 5x$ tenglamani yeching.

Tenglamani $(-\infty; -1,5)$ va $(-1,5; \infty)$ oraliqlarda tekshiramiz:

a) $x < -1,5$ da $-(2x + 3) = 6 + 5x$ bundan $-7x = 9$ yoki $x = -\frac{9}{7}$ bo'lib,

bu tenglama yechimi emas, chunki $-\frac{9}{7} > -1,5$

b) $x > -1,5$ da $2x + 3 = 6 + 5x$ bo'lib, bundan $-3x = 3$ yoki $x = -1$ $(-1 > -1,5)$ tenglama yechimi.

4) $|x| - |x - 2| = 2$

Bu tenglamada birinchi qoshiluvchi $x = 0$ da, ikkinchi qo'shiluvchi $x = 2$ da nolga teng. Demak, tenglamani $(-\infty; 0)$, $(0; 2)$ va $(2; +\infty)$ oraliqlarda tekshiramiz.

a) $x < 0$ da: $-x - (-(x - 2)) = 2$ ko'rinishda bo'lib, qavs ochilsa, $-x + x - 2 = 2$ yoki $0 * x = 4$ bo'lib, bu oraliqda yechim mavjud emas.

b) $0 < x \leq 2$ da: $x - (-(x - 2)) = 2$ bo'lib, bundan $x + x - 2 = 2$ yoki $2x = 4$, $x = 2$ yechim hosil bo'ladi.

c) $x > 2$ da: $x - (x - 2) = 2$ yoki $0 * x = 0$, cheksiz ko'p yechimga ega. Demak, tenglama uchun $[2; +\infty)$ oraliqdagi barcha qiymatlar yechim bo'ladi.

5) $|x - 3| + |x + 2| - |x - 4| = 3$

Berilgan tenglamani $(-\infty; -2)$, $(-2; 3)$, $(3; 4)$ va $(4; \infty)$ oraliqlarda tekshiramiz.

a) $x < -2$, da tenglama:

$$-(x - 3) - (x + 2) + (x - 4) = 3$$

Ko'rinishda bo'lib, qavs ochilib, soddalshtirilsa, $x = -6$ yechimni olamiz.

b) $-2 < x < 3$ da:

$-(x-3)+(x+2)+(x-4)=3$, soddalashtirilsa, $x = 2$ yechim.

c) $3 < x < 4$ da esa:

$$(x-3)+(x+2)+(x-4)=3,$$

qavs ochilsa, $3x = 8$ bo'lib, $x=\frac{8}{3}$ bu son qaralayotgan sohaga tegishli emas, shu sababli bu sohada tenglama yechimi emas.

d) $x > 4$ da: $(x-3)+(x+2)-(x-4)=3$ dan $x-3+x+2-x+4=3$, $x = 0$ bu ham tenglamaga yechim emas. Tenglama yechimlari

$$x=-6 \text{ va } x=2.$$

6) $|x+3| = |(1-|2-x|)|$

Tenglamani birinchi qadamda $(-\infty; -3)$, $(-3; 2)$ va $(2; \infty)$ oraliqda qaraymiz, sababi $|x+3|$ va $|2-x|$ ifodalar mos ravishda (-3) va 2 ga teng.

a) $x < -3$ da tenglama:

$-(x+3) = |1-(2-x)|$ yoki $-(x+3) = |-1+x|$, $x < -3$ ni hisobga olsak, $-x-3 = -(-1+x)$ dan $0 * x = 4$, yechim mavjud emas.

b) $-3 < x < 2$ oraliqda $x+3 = |1-(2-x)|$ yoki $x+3 = |-1+x|$ bo'lib, oraliqni $(-3;1)$ va $(1;2)$ ga ajratib qaraymiz:

b_2) $1 < x < 2$ da: $x+3 = -1+x$, $0 * x = -4$ yechim mavjud emas,

c) $x > 2$ da: $x+3 = |1-(-(2-x))|$ dan $x+3 = |3-x|$, yana oraliqni ikkiga bo'lamiz. $(2; 3)$va $(3; \infty)$ deb :

c_1) $2 < x < 3$ oraliqda $x+3 = (3-x)$ bo'lib, $2x = 0$, $x = 0$ son ko'rsatilgan oraliqda emas, demak $x = 0$ yechim emas.

c_2) $x > 3$ da: $x+3 = -(3-x)$ dan $0 * x = -6$ yechim mavjud emas.

Berilgan tenglama yechimi $x = -1$.

7) $|5-2x| = |3x-5|$

Tenglamani $\left(-\infty; \frac{5}{3}\right)$, $\left(\frac{5}{3}; \frac{5}{2}\right)$, va $\left(\frac{5}{2}; \infty\right)$ oraliqda tekshiramiz.

a) $x < \frac{5}{3}$ da: $5-2x = -(3x-5)$ dan $x = 0$ tenglama yechimi.

b) $\frac{5}{3} < x < \frac{5}{2}$ oraliqda: $5-2x = 3x-5$ bo'lib, $-5x = -10$ yoki $x = 2$ yechimni olamiz.

c) $x > \frac{5}{2}$ da: $-(5-2x) = 3x-5$ $-x = 0$ oraliqda emas. Berilgan tenglama yechimlari $x = 0$ va $x = 2$

4-MAVZU. CHIZIQLI TENGLAMAGA KELADIGAN KASR RATSIONAL TENGLAMALAR

Ko'phadning ko'phadga nisbati ***algebraik ratsional kasr*** *deyiladi. Algebraik ratsional kasr qatnashgan tenglama esa* ***algebraik ratsional kasr tenglama*** *bo'lib, bu ko'rinishdagi tenglamalarni yechishda alohida e'tibor talab qilinadi.*

P(x) va Q(x) chiziqli funksiyalar bo'lsin. U holda, $\frac{P(x)}{Q(x)}$ *ifodaga* ***chiziqli ratsional kasr*** *deyiladi. Chiziqli ratsional kasr qatnashgan tenglamaga* ***kasr chiziqli ratsional tenglama*** *deyiladi. Kasr chiziqli tenglama yechimini topishni xususiy ko'rinishdan boshlaymiz.*

4.1. $\frac{P(x)}{Q(x)}=0.$ (1)

Tenglama yechimini aniqlashda $Q(x) \neq 0$ deb, $P(x) = 0$ tenglama yechiladi.

Misol.

Tenglama yechimi topilsin.

1) $\frac{3(x-1)-2(1+0,5x)}{2x-5}=0$

Kasr chiziqli tenglama yechimini aniqlashda maxrajdagi ifoda nol bo'lmasligini talab qilamiz, ya'ni $2x-5 \neq 0 \Rightarrow x \neq 2,5$ deb, suratini nolga tenglaymiz:

$3(x-1)-2(1+0,5x)=0$, chap tomonini soddalashtirib yechiladi, bunda $x = 2,5$ bo'lib, bu son qo'yilgan shartga qarama-qarshi, shu sababli bu chet ildiz bo'lib, tenglama yechimga ega emas.

2) $\frac{1\frac{1}{4}(2x-0,8)+0,5x}{0,(3)+2x}=0$

Oldin $0,(3)+2x \neq 0$ talab qilamiz, yoki $x \neq -\frac{1}{6}$. Suratini nolga tenglashtirib yechamiz: $1\frac{1}{4}(2x-0,8)+0,5=0$, bundan $2,5x - 1 + 0,5x = 0$ bo'lib, $x=\frac{1}{3}$ tenglama yechimi.

4.2. $\frac{P(x)}{Q(x)}=A,$ (2) $(A \in R, A \neq 0)$ kasr chiziqli tenglamada $Q(x) \neq 0$

shartida $P(x) = A * Q(x)$ tenglamani kanonik ko'rinishdagi tenglamaga keltirish mumkin.

Misol.

Tenglama yechimi topilsin:

1) $\dfrac{3-4x}{x} = 2.$

Tenglama yechimini aniqlashda $x \neq 0$ deb, tenglamaning ikki tomonini x ga ko'paytiramiz: $3 - 4x = 2x$ dan $x = 0{,}5$ tenglama yechimi hosil bo'ladi.

2) $\dfrac{2\left(1{,}5-2\frac{3}{4}x\right)}{3x-2} = 2{,}5$

$3x - 2 \neq 0$ yoki $x \neq \dfrac{2}{3}$ shartida, tenglamani yechamiz: $\dfrac{3-\frac{11}{2}x}{3x-2} = \dfrac{5}{2}.$

Endi proporsiya xossasidan foydalanamiz: $2\left(3-\dfrac{11}{2}x\right) = 5(3x-2)$

soddalashtirsak, tenglama yechimi $x = \dfrac{8}{13}$ ni hosil qilamiz.

3) $\dfrac{0{,}5(x-4)}{3x} = 0{,}1(6)$

$0{,}1(6) = \dfrac{1}{6}$ ni hisobga olib $x \neq 0$ deb, tenglamani $3x$ ga ko'paytirsak:

$0{,}5(x-4) = \dfrac{1}{6} * 3x$ dan $0 * x = 2$, demak, tenglama yechimga ega emas ekan.

4) $\dfrac{\sqrt[3]{81}\cdot(0,(3))^6}{\left(\sqrt[3]{3}\right)^{-1}\cdot 27^{-2/3}} = \dfrac{x}{3\cdot\left(\sqrt[3]{3}\right)^4}.$

Davriy o'nli kasrni $0,(3) = \dfrac{1}{3}$ ko'rinishda yozib va ildiz darajalarini kasr ko'rinishida yozish mumkinligidan foydalanib, proporsiya ko'rinishdagi tenglamani yechimi

$x = 3^{\frac{4}{3}}\cdot 3^{-6}\cdot 3\cdot 3^{\frac{4}{3}}\cdot 3^{\frac{1}{3}}\cdot 3^2 = 3^\circ = 1$ ko'rinishda bo'ladi.

4.3. *Umumiy ko'rinishdagi kasr ratsional tenglama yechimini aniqlashda, maxrajda qatnashgan ifodalarning nol bo'lmasligini talab qilib, ya'ni tenglamaning aniqlanish sohasi topilib, keyin ratsional kasr* *ifodalarda amallar bajarish {3} qoidasidan foydalanib, ifoda soddalashtirib olinadi, keyin yechim topiladi.*

Misol.

Tenglama yechilsin.

1) $\frac{2}{x-1}+3=\frac{x-2}{1-x}$

Tenglama aniqlanish sohasi $]-\infty;1[\cup]1;\infty[$. $x\neq 1$ deb, tenglamaning ikki tomonini $(x-1)$ ga ko'paytiramiz:

$2+3(x-1)=-(x-2)$ qavs ochib soddalashtirilsa, tenglama $4x=3$ kanonik ko'rinishga kelinadi, bundan tenglama yechimi $x=0{,}75$

2) $\frac{7}{3x-2}+\frac{3(x-3)}{3x-2}=2$ yechimini aniqlashda $3x-2\neq 0$ yoki $x\neq\frac{2}{3}$ deb, $3x-2$ umumiy maxrajda birinchi bosqich amali bajariladi.

$\frac{7+3(x-3)}{3x-2}=2$, proporsiyadan $-3x=-2$ yoki $x=\frac{2}{3}$ bu son shartimizga qarama-qarshi bo'lganligi uchun tenglamaga chet ildiz, demak, tenglama yechimga ega emas.

Javob: $x\in\varnothing$.

3) $\frac{5}{3x}-\frac{4x-3}{2(x^2-x)}=\frac{7}{x(4x-4)}$.

Maxrajdagi ifodalar $3x$, $2x(x-1)$ va $4x(x-1)$ ko'rinishida bo'lib, $x\neq 0$, $x\neq 1$ shartlar qo'yiladi.

Berilgan ifoda uchun umumiy maxraj $12x(x-1)$. Birinchi bosqich amalini bajaramiz:

$\frac{5*4(x-1)-6(4x-3)}{12x(x-1)}=\frac{7*3}{12x(x-1)}$ tenglikning ikki tomonini $12x(x-1)$ ga ko'paytiramiz:

$20(x-1)-6(4x-3)=21$ soddalashtirsak, $-4x=23$ bo'lib, tenglama yechimi $x=-5{,}75$.

4) $\frac{12x}{9-x^2}=\frac{x-3}{3+x}-\frac{3+x}{x-3}$

Qisqa ko'paytirish formulasiga asosan

$x^2-9=(x-3)(x+3)$, demak, tenglamani yechishda $x\neq 3$ va $x\neq -3$ ni talab qilamiz. Bu ifodada birinchi bosqich amallarni bajarib

$$12x+(x-3)(x-3)-(3+x)(x+3)=0$$

hosil bo'lgan ifodani soddalashtirsak,

$$12x+x^2-6x+9-9-6x-x^2=0$$

yoki $0*x=0$ tenglama o'zgaruvchining $x=3$ va $x=-3$ dan boshqa ixtiyoriy qiymatlarda yechimga ega, ya'ni $x \in (-\infty; -3) \cup (-3;3) \cup (3;\infty)$.

5) $\left|\dfrac{x-2}{x+1}\right| = \dfrac{x-2}{1+x}$

Tenglamani $(-\infty; -1), (-1;2)$ va $(2;\infty)$ oraliqlarda qaraymiz. Tenglama $x = 2$ da nolga aylanadi, $x = -1$ esa tenglamadagi funksiya uzilish nuqtasi.

Absolyut qiymat *x*ossasidan foydalanib, tenglamaning ko'rsatilgan oraliqlardagi yechimini qidiramiz:

1) $x < -1$ da: $-\left(\dfrac{x-2}{x+1}\right) = \dfrac{x-2}{1+x}$, dan $x - 2 = x - 2$ yoki $0 * x = 0$, demak $(-\infty; -1)$ oraliqdagi ixtiyoriy sonlar tenglama yechimi ekan.

2) $-1 < x < 2$ da: $\left(\dfrac{-x+2}{x+1}\right) = \dfrac{x-2}{1+x}$ bo'lib, $-x + 2 = x - 2$, $x = 2$ yechim.

3) $x > 2$ da: $\dfrac{x-2}{x+1} = \dfrac{x-2}{1+x}$ dan $0 * x = 0$, ya'ni $(2;\infty)$ dagi ixtiyoriy sonlar tenglama yechimi.

Javob: $x < -1$ va $x \geq 2$.

1-, 2-, 3-, 4-MAVZULAR MASHQLARI

Tengliklar ayniyatmi:

1. 1) $x - (4 - 2x) + (3x - 1) = 2(3x - 4)$;

2) $2x - [6x - (5x - 3)] = 3(2x - 1) + 5x$;

3) $2(5{,}3x - 0{,}8) - 0{,}5(1{,}6 - 4{,}2x) = 2x - 1{,}2(5x - 0{,}3)$;

4) $1\frac{1}{3} - 2\frac{2}{3}(3x+1) = 3\left(1\frac{1}{3}x + 2\right) - 2\frac{2}{3}$.

2. 1) $0{,}5[6x - 3(2 - 6x)] = 3[(6x - 5) + 2(3 - x)]$;

2) $3[x - 4(1 - 3x)] = 6(6x - 3) - 3(-2 - x)$;

3) $2(0,7x - 2,1) - (1 - 4x) = 1,8(3x - 1) - 3,4$;

4) $\frac{3}{4}x - 1\frac{1}{2}(2 - 3x) = 1\frac{1}{4} + 2\left(1\frac{3}{4}x - 1\right)$.

Berilgan tenglamalarning qaysi juftliklari $x \in R$ o'zaro teng kuchli tenglamalar bo'ladi:

3. 1) $4x + 6 = 0$ va $1 - x = 0$, 2) $\frac{x-2}{6} + \frac{3x+2}{2} + 6 = 0$ va $x - 4 = 0,5$

3) $x + 1,25x = 1\frac{1}{4}x + 1 - x$ va $x - 0,5 = 0,5 - x$.

4. 1) $47 - [3x - (9 - 5x)] = 0$ va $41 + 3(2 - x) = 4x$;

2) $12x - [15 - (6x - 9)] = 12$ va $9(x - 4) = 9(-x)$;

3) $2[-2 - (x - 4)] = x + 3$ va $0,5[x - 2(4 - 2,5x)] = 2(0,5 - x)$.

a ning qanday qiymatlarida berilgan tenglamalar xaqiqiy sonlar to'plamida teng kuchli tenglamalar bo'ladi.

5. 1) $2 - 3x = 6$ va $-3x + 4 = 2a + 1$,

2) $2x + 5 = 7$ va $7(0,5a + 4) = 5 + 2x$.

6. 1) $4x - 3 = 10$ va $(0,75 + 2,5a)(3 - 4x) = 5$;

2) $2x + 5 = 6$ va $2(7a - 2 + x) = 0,5$.

a soni tenglamaning ildizi bo‘ladimi:

7. 1) $12-2(2x-9)=4(9+x)+6(3-x)$, $a=-12$;

2) $0{,}2(1{,}7-3x)=2x-2(0{,}2x+0{,}5)$, $a=-3$.

8. 1) $3(3x-5)-2(8+3x)=-3x$, $a=3$;

2) $0{,}6+0{,}5(x-2)=0{,}25(2+4x)$, $a=-1{,}8$.

Tenglama yechimi topilsin

9. 1) $6-2x=0$ 2) $0{,}5x=0$ 3) $0{,}1x=4$.

10. 1) $2(3)x-1{,}4=0$, 2) $2{,}4x-2\frac{1}{4}=0$.

11. 1) $-2{,}2x+2\frac{3}{4}=0$, 2) $1{,}(3)x=2{,}(6)$, 3) $1\frac{2}{5}-3{,}2x=0$.

12. 1) $0{,}(6)x-0{,}(2)=0$, 2) $3x-2{,}25=0$, 3) $2\frac{3}{4}x+11=0$.

13. 1) $3x+1{,}2=4$, 2) $2\frac{1}{5}=1{,}5x-3{,}4$, 3) $1\frac{1}{4}-2\frac{2}{5}x=-2$.

14. 1) $0{,}8x+3=1{,}2$, 2) $2\frac{2}{3}-0{,}7x-1{,}5=0$, 3) $-2{,}25+1\frac{1}{2}x=-2\frac{1}{4}$.

15. 1) $7x-1.2=9x$. 2) $2,4+1,6x=-1\frac{1}{3}x$, 3) $2\frac{2}{3}-1,(6)x=2,4x$.

16. 1) $2,5x-5=2\frac{1}{2}x$, 2) $1,(3)x=60+x$, 3) $2,25x-1\frac{3}{4}=1,(6)x$.

17. 1) $5-2(-2x)=4x+7$, 2) $3x-10=5x-4\cdot(-2)$.

18. 1) $4+3(-x)=x(-3)-2(-2)$, 2) $2-1,2x=2,(3)-1\frac{1}{4}x$,

3) $0,(3)x-2,4=1,1(6)+1,2x$.

19. 1) $(0,5x+1,2)-3(1,2-1,5x)=0,3(16x-1)+(10,5x+0,6)$

2) $3\frac{3}{5}-\frac{1}{2}(3x+2)=-\frac{4}{5}(5x+1)-2(0,2x-1)$.

20. 1) $0,7(x-3)-0,5(1-4x)=0,9(3x-1)+0,1$;

2) $\frac{3}{2}(x+2)-1,4\left(1\frac{2}{3}x-3,(3)\right)=0,1(x+1)$

21. 1) $(x-2)(x-3)-(x-1)(x-4)=0$;

2) $(x+1)(x+2)-(x+3)(x+4)=0$.

22. 1) $12x^2 - (4x - 3)(3x + 1) = -2;$

2) $(4x - 3)(3 + 4x) - 2x(8x - 1) = 0;$

3) $2x - (x - 2)(2 + x) = 5 - (x - 1)^2.$

23. 1) $\frac{x+2}{5} = \frac{3x-5}{4};$ 2) $\frac{3x-1}{7} - 12 = \frac{2x-5}{3} - \frac{4x-1}{5};$

3) $6 - \frac{2-5x}{3} = 2x + \frac{6x-4}{5};$ 4) $\frac{4x+0,5}{12} + \frac{x-0,8}{8} + \frac{x+0,2}{6} = 0.$

24. 1) $\frac{2x-1}{3} - \frac{4-x}{2} - x = 1 + \frac{x-3}{6};$

2) $\frac{5x-4}{3} + \frac{3x-2}{6} + \frac{2x-1}{2} = 3x - 2;$

3) $\frac{x-0,5}{4} + \frac{x-0,25}{3} + \frac{x-0,125}{2} = 0;$

4) $\frac{3(1,2-x)}{10} - \frac{5+7x}{4} - x = \frac{9x+0,2}{20} - \frac{4(13x-0,6)}{5}.$

Tenglama ildizlar yig'indisi topilsin

25. 1) $x(2x-1)(3{,}5x+2)=0$; 2) $(x-0{,}5)(1{,}2x+3)(2x-1)=0$.

26. 1) $(x+0{,}5)(2x-3)(2-x)=0$;

2) $\left(1\frac{2}{3}-5x\right)(2x+1)\left(-2{,}5+2\frac{1}{2}x\right)=0.$

0, – 3 va 2 sonlarning qaysi biri quyidagi tenglama yechimlari bo'ladi.

27. 1) $2(1{,}7-0{,}3x)=2x-0{,}2(2x+5)$;

2) $x(0{,}(3)x+1)(1-2x)=0$.

28. 1) $(2x-1)(1+0{,}(3)x)(0{,}45x-0{,}9)=0$;

2) $1{,}5(3x+6)-2(3{,}1-1{,}55x)=4(1{,}8x+0{,}7)$.

Tenglik o'rinli bo'ladigan, o'zgaruvchilarning biror qiymati topilsin.

29. 1) $2x+7y+14+xy=0$; 2) $5x+6y-xy=30$.

30. 1) $4x-3y+xy=12$; 2) $2x^2+3x+5y-16=2xy$.

x o'zgaruvchiga nisbatan tenglama yechilsin

31. 1) $2ab(x-3)-2b(a+x)=2a(3b-4bx)$;

2) $3a(2x-2b)-b(ax-4a)=ab(x+2)$.

32. 1) $b(2a-2x)+x(ab-4)=5(2b-abx)-4x$;

2) $2x(a-4b)-a(4x+6)=4(2bx+a)-a$.

Tenglamada qatnashgan a parametrning tenglama yechimiga ta'siri qanday.

33. 1) $4x+16=a(a-x)$; 2) $a^2x=4x$.

34. 1) $3x-b=3a+ax$; 2) $a+4x=a^2x-2$.

Ifodani modulsiz yozing

35. $|3-2x|-|-x|+|x+5|+3$.

36. $|2+x|-|x-4|-|3x|-4$.

37. $|x|+|x+1|+|x-2|$ ni $x<2$ da

38. $|x|+|1-x|+2|x-2|$ ni $1<x<2$ da

Tengliklar ayniyatmi

39. 1) $|a^2+4|=4+a^2$; 2) $|a+b|=|b|+|a|$.

40. 1) $|a+4|=4+a$; 2) $|a-b|=|b-a|$.

Ifoda xar doim yechimga egami:

41. 1) $|9x-5|=a$; 2) $|x^2-6|=-3$.

42. 1) $|2x+a|=0{,}4$; 2) $x=|ax+1|$.

Tenglama yechimlari topilsin

43. 1) $|-0{,}3x|=0$; 2) $|0{,}5x|=-2$; 3) $|2x|=0{,}5$.

44.) $|-1,(6)x| = 5$; 2) $|0,4x| = 1$; 3) $|5x| = 0,(5)$.

Ifodaning eng kichik va eng katta butun qiymatlari topilsin.

45. 1) $|x-2|+3$ ning $[-4;1]$ da; 2) $|3-x|-2$ ning $[-5;5]$.

46. 1) $x-|4-x|$ ning $[-5;3]$ da; 2) $6-|x+3|$ ning $[-7;4]$.

Tenglamalarni yeching.

47. 1) $|3-1,5x| = 2,5$; 2) $|2+2x| = 6$; $|x-0,(6)| = 0,(3)$.

48. 1) $|0,2x-2| = 3,6$; 2) $|4x+1| = 5$; 3) $|1,1(6)-x| = 0,75$.

49. 1) $2-|x+1| = 1,5$; 2) $|2-x| = 2+3x$.

50. 1) $|x+3|+3,(3) = 2$; 2) $|5x-1| = 9-11x$.

Tenglamalarning barcha yechimlari topilsin.

51. 1) $|x-1|+|x+1| = 2$; 2) $|x+3|+|x-5| = 20$; 3) $-0,2|x| = 10$.

52. 1) $|x+3|+|x-5| = 4$; 2) $|3x-8|-|3x-2| = 6$. 3) $-0,5|x| = -2$.

53. 1) $|(x-|4-x|)|-2x = 4$; 2) $|x-4|-|2x-3| = 2-|3x-2|$.

54. 1) $|7-2x| = |5-3x|+|x+2|$; 2) $|(1-|2-x|)| = |x+3|$.

55. $0 < a < b < c$ bo'lsa $|b+a|-|c-a|+|b-c|$

Ifodani soddalashtiring.

56. $a > b > c > 0$, $|a-b|+|c-a|-|b-c|$, Ifodani soddalashtiring.

57. Agar $a < b < c < 0$ bo‘lsa $|a + b| - |a - c| - |c - b|$

Ifodani soddalashtiring.

58. $0 > \alpha > \beta > \varphi$ bo‘lsa. $|\alpha - \varphi| + |\beta - \varphi| - |\varphi - \alpha| + |\alpha + \beta|$

Ifodani soddalashtiring

59. Agar 1) $a = -3$ va $b = 1$ bo‘lsa, 2) $a = -4$ va $b = -1$ bo‘lsa, koordinata to‘g‘ri chizig‘ida $|a - b|$ ga mos kesmani ko‘rsating.

60. Agar $a > 0$ bo‘lsa, $|a - b| - |b|$ ifodaning qiymati nimaga teng.

Tenglamalarning yechimi topilsin.

61. 1) $\frac{2x-1}{6} = \frac{6-x}{8}$; 2) $\frac{2(x+3)-1,5(4x-5)}{1,2x+2} = 0.$

62. 1) $\frac{x+2}{5} = \frac{3x-5}{4}$; 2) $\frac{1\frac{1}{4}(x-1)-2(1-0,5x)}{3\frac{1}{4}x+2} = 0.$

63. 1) $\frac{3(x-4)}{3-x} + 5 = \frac{x}{3-x}$; 2) $\frac{3x-1}{7} - 12 = \frac{2x-5}{3} - \frac{4x-1}{5}.$

64. 1) $\frac{2x+1}{x-3} + \frac{5-4x}{3-x} = 6$; 2) $12 - \frac{4-5x}{7} = \frac{3x+20}{2} + \frac{11-2x}{5}.$

65. 1) $\dfrac{2x}{x-1}+\dfrac{x+1}{1-x}-\dfrac{5}{2-2x}=3,5;$ 2) $\dfrac{2-x}{x+3}-\dfrac{x+1}{3-x}=\dfrac{3(3x-1)}{x^2-9}.$

66. 1) $\dfrac{4(9+x)}{5x^2-45}+\dfrac{x+3}{5x^2-15x}=\dfrac{x3}{x^2+3x};$

2) $\dfrac{x+1}{2-2x^2}+\dfrac{6}{x+1}-\dfrac{1}{2x-2}=\dfrac{2x-1}{x^2-1}.$

5-MAVZU. DETERMINANT

5.1. Ikkinchli tartibli determinant: $\begin{vmatrix} a & b \\ d & c \end{vmatrix}$, qiymati

$\begin{vmatrix} a & b \\ d & c \end{vmatrix} = a\cdot c - b\cdot d,$ (1) formuladan topiladi. Bu yerda *a*, *b*, *c*, *d* – ***determinant elementlari*** *deyiladi.*

Misol.

Hisoblang:

1) $\begin{vmatrix} -4 & 5 \\ 3 & 6 \end{vmatrix} = (-4)\cdot 6 - 3\cdot 5 = -39.$

2) Tenglama yechilsin: $\begin{vmatrix} 1 & -3x \\ 2 & 5 \end{vmatrix} = 3.$

(1) formuladan foydalanamiz: $1 \cdot 5 - 2 \cdot (-3x) = 3$ yoki $6x = -2$ bo‘lib, tenglama yechimi $x = -0,(3)$.

5.2. Uchinchi tartibli determinant

Uchinchi tartibli determinant: $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$,

qiymati: $\Delta = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$, (2)

formuladan aniqlanadi, Bu yerda , masalan: a_{21} – ikkinchi yo‘l birinchi ustun elementi, a_{31} – esa uchinchi yo‘l birinchi ustun elementi deyiladi va hakozo.

Uchinchi tartibli determinantni uchburchaklar sxemasi shaklidan foydalanib, quyidagicha hisoblash ham mumkin:

Bunda ko‘rsatilgan chiziqlardagi elementlar o‘zaro ko‘paytma deb tushunish kerak, chap tomondagi sxemada uchta ko‘paytma o‘z ishoralari bilan qo‘shiladi, o‘ng tomondagi sxemada bu uchta yig‘indi teskari ishorada olinadi.

<u>*Misol.*</u>

Determinant qiymati topilsin.

$$\Delta=\begin{vmatrix}2 & -3 & 0\\ 5 & 4 & 1\\ 1 & -2 & 6\end{vmatrix}.$$ Birinchi usulda hisoblasak,

$$\Delta=2\cdot\begin{vmatrix}4 & 1\\ -2 & 6\end{vmatrix}-(-3)\cdot\begin{vmatrix}5 & 1\\ 1 & 6\end{vmatrix}+0\cdot\begin{vmatrix}5 & 4\\ 1 & -2\end{vmatrix}=2\cdot(24+2)+3\cdot(30-1)+$$

$+0=2\cdot26+3\cdot29=139.$

Ikkinchi usulda hisoblasak, $\Delta=2\cdot4\cdot6+(-3)\cdot1\cdot1+0\cdot5\cdot$ $\cdot(-2)-(0\cdot4\cdot1+1\cdot(-2)\cdot2+5\cdot(-3)\cdot6)=48-3+0-(0-4-90)=$ $=45+94=139.$

Determinantda asosiy xossalaridan bittasi, agar determinantda biror yo'l (ustun) elementlari nol bo'lsa, yoki biror yo'l (ustun) elementlari boshqa yo'l (ustun) mos elementlariga proporsional bo'lsa, bu determinant qiymati nol bo'ladi.

<u>*Misol.*</u>

Determinant qiymati topilsin.

$$A=\begin{vmatrix}2 & 0 & -9\\ 7 & 0 & 13\\ -24 & 0 & 16\end{vmatrix},\quad B=\begin{vmatrix}2 & -1 & 5\\ 3 & 7 & 12\\ -4 & 2 & -10\end{vmatrix}.$$

Ikkinchi ustun elementlari nol bo'lganligi uchun A determinant qiymati nolga teng. B determinantda birinchi yo'l elementini (-2) ga ko'paytirsak, uchinchi yo'l mos elementlari hosil bo'ladi, ya'ni birinchi yo'l elementlari uchinchi yo'l mos elementlariga proporsional ekan, u holda determinant xossasidan B determinant qiymati ham nolga teng bo'ladi.

6-MAVZU. CHIZIQLI TENGLAMALAR SISTEMASI

Chiziqli tenglamalar sistemasi yechimini aniqlashning bir nechta usuli mavjud bo'lib, ulardan eng sodda va ma'quli determinantlar yordamida (ya'ni, Kramer usuli) aniqlash, chunki:

1) Bu metod yordamida tenglama va noma'lumlar soni n ta ($n \in N$) bo'lganda ham qo'llash mumkin;

2) Sistema yechimi mavjudmi yoki yechim mavjud emasmi, bunga to'la javob beradi;

3) Sistema yechimi mavjud bo'lsa, bu yechimlarni topish formulasi berilgan.

Bu usulni $n = 2$ va $n = 3$ da ko'rsatamiz, n ixtiyoriy natural sonda yechimni aniqlash shu kabi bo'lib, (1) to'la yoritilgan.

6.1. Ikki noma'lumli chiziqli ikkita bir jinsli bo'lmagan tenglamalar sistemasi

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases} \quad (1)$$

ko'rinishda bo'lib, bu yerda a_1, b_1, a_2, b_2 – koeffisiyentlar, c_1, c_2 – ozod hadlar R to'plamda berilgan ma'lum sonlar, x va y noma'lum o'zgaruvchilar.

Sistema yechimi deb, bir juft $(x_0; y_0)$ ga aytiladiki, $x = x_0$, $y = y_0$ da sistemadagi har ikki tenglama ayniyatga aylanishi kerak.

<u>*Masalan:*</u>

$$\begin{cases} x - 2y = 5 \\ 7x - 3y = 13. \end{cases}$$

Sistema uchun (1; – 2) yechim. Haqiqatdan ham $x = 1$, $y = -2$ da $\begin{cases} 1 - 2(-2) = 5 \\ 7 \cdot 1 - 3 \cdot (-2) = 13 \end{cases}$ yoki $\begin{cases} 1 + 4 = 5 \\ 7 + 6 = 13 \end{cases}$ Har ikki tenglama to'g'ri tenglikka aylanadi.

Agar bu sistemaga (7;1) juftlik yechim deb qarasak, $x = 7$, $y = 1$ da: $\begin{cases} 7 - 2 \cdot 1 = 5 \\ 7 \cdot 7 - 3 \cdot 1 = 13 \end{cases}$ dan $\begin{cases} 5 = 5 \\ 46 = 13 \end{cases}$ bo'lib, birinchi tenglama to'g'ri tenglikka aylandi, lekin ikkinchi tenglamada tenglik bajarilmadi, demak, (7;1) tenglamalar sistemasi uchun yechim emas.

(1) Tenglamalar sistemasi yechimini Kramer usulida aniqlashda koeffisiyentlardan quyidagi determinantlarni yozib olamiz.

$$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, \quad \Delta x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}, \quad \Delta y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}, \quad (2)$$

Kramer qoidasi

Agarda:

1) $\Delta \neq 0$ bo'lsa, sistema birdan bir (yagona) yechimga ega va bu yechim $x = \dfrac{\Delta x}{\Delta}$, $y = \dfrac{\Delta y}{\Delta}$ (3) formuladan topiladi.

2) $\Delta = 0$ bo'lib, Δx va Δy lardan kamida bittasi nolga teng bo'lmasa, u holda (1) sistema yechimi mavjud emas.

3) $\Delta = 0$ va $\Delta x = \Delta y = 0$ bo'lsa, (1) sistema cheksiz ko'p yechimga ega.

<u>*Misol.*</u>

Tenglamalar sistemasini yechilsin.

1) $\begin{cases} x - 2y = 5 \\ 7x - 3y = 13 \end{cases}$.

(2) Ko'rinishidagi determinantlarni hisoblab olamiz.

$$\Delta = \begin{vmatrix} 1 & -2 \\ 7 & -3 \end{vmatrix} = -3 + 14 = 11, \quad \Delta x = \begin{vmatrix} 5 & -2 \\ 13 & -3 \end{vmatrix} = -15 + 26 = 11$$

$\Delta y = \begin{vmatrix} 1 & 5 \\ 7 & 13 \end{vmatrix} = 13 - 35 = -22.$ Tenglama yechimi (3) Kramer formulasiga asosan $x = \dfrac{\Delta x}{\Delta} = \dfrac{11}{11} = 1$, $x = \dfrac{\Delta y}{\Delta} = \dfrac{-22}{11} = -2.$

Sistema yechimi $x = 1, y = -2$ ekan, ya'ni $(1; -2)$

2) $\begin{cases} 2x - y = 1 \\ 4x - 2y = 3. \end{cases}$

Oldingi misoldagi kabi hisoblashlarni bajaramiz:

$$\Delta = \begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix} = 0, \quad \Delta x = \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = 1 \neq 0; \quad \Delta y = \begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} = 2 \neq 0$$

Kramer qoidasiga asosan, tenglamalar sistemasining yechimi mavjud emas. Javob: $\varnothing$.

3) $\begin{cases} 0{,}5x - 0{,}(3)y = 1\dfrac{2}{5} \\ x - \dfrac{2}{3}y = 2{,}8 \end{cases}$;

(3) ko'rinishdagi determinantlarni hisoblaymiz:

$$\Delta = \begin{vmatrix} \frac{1}{2} & -\frac{1}{3} \\ 1 & -\frac{2}{3} \end{vmatrix} = 0; \quad \Delta x = \begin{vmatrix} \frac{7}{5} & -\frac{1}{3} \\ \frac{14}{5} & -\frac{2}{3} \end{vmatrix} = 0, \quad \Delta y = \begin{vmatrix} \frac{1}{2} & \frac{7}{5} \\ 1 & \frac{14}{5} \end{vmatrix} = 0$$

Sistema cheksiz ko'p yechimga ega. Bu yechimlarni aniqlashda, sistemadan ixtiyoriy bitta tenglamani olamiz: $0,5x - 0,(3)y = 1\frac{2}{5}$, tenglikni x o'zgaruvchiga nisbatan yozib olamiz: $x = \frac{2}{3}y + \frac{14}{5}$, y o'zgaruvchiga bog'liq holda x topiladi.

Masalan:

$y = 0$ da $x = 2,8$; $y = 3$ da $x = 4,8$; $y = -3$ da $x = 0,8$;...

$\left(\frac{14}{5};0\right), \left(\frac{24}{5};3\right)$ va $\left(\frac{4}{5};-3\right)$ kabi yechimlarni hosil qilamiz.

Bunda tenglama koeffimayetlar R to'plamda ma'lum sonlar

6.2. Uch noma'lumli uchta birinchi tartibli (chiziqli) tenglamalar sistemasini qaraylik

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2; \\ a_3x + b_3y + c_3z = d_3 \end{cases} \quad (4)$$

Bunda:

$a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3, d_1, d_2, d_3 \in R$ – berilgan sonlar. x, y, z – noma'lum o'zgaruvchilar.

Sistema yechimini topishda yana Kramer usulidan foydalanamiz.

Quyidagi determinantlarni yozib olamiz: $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$,

$$\Delta x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \quad \Delta y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \quad \Delta z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}. \quad (5)$$

Agarda:

1) $\Delta \neq 0$ bo'lsa, sistema birdan-bir yechimga ega, bu yechimlar $x = \frac{\Delta x}{\Delta}$ $y = \frac{\Delta y}{\Delta}$, $z = \frac{\Delta z}{\Delta}$. (6) formulada topiladi.

2) Agar $\Delta = 0$ bo'lib, Δx, Δy yoki Δz lardan kamida bittasi nolga teng bo'lmasa, u holda (4) sistema yechimga ega emas.

3) Agar $\Delta=0$ bo'lib, $\Delta x=\Delta y=\Delta z=0$ bo'lsa,
(4) sistema cheksiz ko'p yechimga ega.

Misol:

Tenglamalar sistemasi yechimi topilsin:

$$1)\quad \begin{cases} 4x+3y+z=-9 \\ x-4y+z=4 \\ 2x+y+3z=1 \end{cases}.$$

Tenglamalar sistemasi yechimini topishda
(5) determinantlarni hisoblaymiz:

$$\Delta=\begin{vmatrix} 4 & 3 & 1 \\ 1 & -4 & 1 \\ 2 & 1 & 3 \end{vmatrix}=-48+6+1-(-8+4+9)=-41-5=-46,$$

$$\Delta x=\begin{vmatrix} -9 & 3 & 1 \\ 4 & -4 & 1 \\ 1 & 1 & 3 \end{vmatrix}=92;\quad \Delta y=\begin{vmatrix} 4 & -9 & 1 \\ 1 & 4 & 1 \\ 2 & 1 & 3 \end{vmatrix}=46;$$

$$\Delta z=\begin{vmatrix} 4 & 3 & -9 \\ 1 & -4 & 4 \\ 2 & 1 & 1 \end{vmatrix}=-92.$$

(6) formulaga asosan, sistema yechimlari:

$$x=\frac{\Delta x}{\Delta}=\frac{92}{-46}=-2;\quad y=\frac{\Delta y}{\Delta}=\frac{46}{-46}=-1;\quad z=\frac{\Delta z}{\Delta}=\frac{-92}{-46}=2.$$

$$1)\quad \begin{cases} -3x+y-2z=1 \\ 11x+y+4z=0; \\ x+2y-z=3 \end{cases}$$

Bu sistemada:

$$\Delta=\begin{vmatrix} -3 & 1 & -2 \\ 11 & 1 & 4 \\ 1 & 2 & -1 \end{vmatrix}=(3+4-44)-(-2-24-11)=-37+37=0$$

$$\Delta x=\begin{vmatrix} 1 & 1 & -2 \\ 0 & 1 & 4 \\ 3 & 2 & -1 \end{vmatrix}=9\neq 0$$ Kramer qoidasiga asosan sistema yechimga ega emas.

2) $\begin{cases} x-3y+2z=2 \\ -x+y-2z=-2. \\ 3x-2y+6z=6 \end{cases}$

Bu tenglamalar sistemasida:

$$\Delta=\begin{vmatrix} 1 & -3 & 2 \\ -1 & 1 & -2 \\ 3 & -2 & 6 \end{vmatrix}=6+18+4-(6+4+18)=0,$$

$$\Delta x=\begin{vmatrix} 2 & -3 & 2 \\ -2 & 1 & -2 \\ 6 & -2 & 6 \end{vmatrix}=0,\quad \Delta y=\begin{vmatrix} 1 & 2 & 2 \\ -1 & -2 & -2 \\ 3 & 6 & -6 \end{vmatrix}=0$$

$$\Delta z=\begin{vmatrix} 1 & -3 & 2 \\ -1 & 1 & -2 \\ 3 & -2 & -6 \end{vmatrix}=0.$$

Kramer qoidasiga ko‘ra, tenglamalar sistemasi cheksiz ko‘p yechimga ega. Sistema yechimlar to‘plamini aniqlash uchun ixtiyoriy erkli ikkita tenglamani olamiz:

$$\begin{cases} x-3y+2z=2 \\ -x+y-2z=-2 \end{cases}.$$

Bu sistemada bitta o‘zgaruvchini erkli qilib $\begin{cases} x-3y=2-2z \\ -x+y=-2+2z \end{cases}$, sistemani z o‘zgaruvchiga nisbatan yechamiz: $\Delta=\begin{vmatrix} 1 & -3 \\ -1 & 1 \end{vmatrix}=-2,$

$$\Delta x=\begin{vmatrix} 2-2z & -3 \\ -2+2z & 1 \end{vmatrix}=2-2z-6+6z=4z-4,$$

$$\Delta y=\begin{vmatrix} 1 & 2-2z \\ -1 & -2+2z \end{vmatrix}=-2+2z+2-2z=0$$

U holda sistema yechimlar to‘plami:

$x=\dfrac{\Delta x}{\Delta}=\dfrac{4z-4}{-2}=2-2z,\ y=\dfrac{\Delta y}{\Delta}=\dfrac{0}{-2}=0$ bundan $x=2-2z$, $y=0$ hosil bo‘ladi.

7-MAVZU. CHIZIQLI BIR JINSLI BO'LGAN TENGLAMALAR SISTEMASI

7.1. Uch noma'lumli chiziqli bir jinsli bo'lgan uchta tenglamalar

sistemasi: $\begin{cases} a_1x+b_1y+c_1z=0 \\ a_2x+b_2y+c_2z=0 \\ a_3x+b_3y+c_3z=0 \end{cases}$ (1)

berilgan bo'lsin. Bunda: Koffentiyentlar R to'plamda berilgan sonlar. x, y, z – noma'lum o'zgaruvchilar.

Agar sistemada asosiy determinant $\Delta=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta\neq 0$ bo'lsa, u holda (1) sistema faqat $x=y=z$ yechimga ega.

Agar asosiy determinant $\Delta=0$ bo'lsa, u holda (1) sistema cheksiz ko'p yechimga ega bo'ladi. Bu yechimlar to'plamini aniqlashda mustaqil bo'lgan ikkita tenglamani sistemadan olamiz, masalan: $\begin{cases} a_1x+b_1y+c_1z=0 \\ a_2x+b_2y+c_zz=0 \end{cases}$

sistema yechimlar to'plami

$$\frac{x}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}=\frac{y}{-\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}=\frac{z}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}=k, \quad (2)$$

proporsiyadan topiladi.

Misol:

Tenglamalar sistemasi yechimi topilsin:

1) $\begin{cases} x+y-3z=0 \\ 4x-2y+z=0. \\ x+3y+2z=0 \end{cases}$

Sistema asosiy determinantini hisoblaymiz:

$$\Delta=\begin{vmatrix} 1 & 1 & -3 \\ 4 & -2 & 1 \\ 1 & 3 & 2 \end{vmatrix}=-4+1-36-(6+3+8)=-39-17=-\Delta 56\neq 0$$

Kramer qoidasiga asosan sistema $x=y=z=0$ yechimga ega.

2) $\begin{cases} x+2y-4z=0 \\ 2x-y+2z=0 \\ 3x+3y-6z=0 \end{cases}$.

Sistema asosiy determinantini hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & 2 & -4 \\ 2 & -1 & 2 \\ 3 & 3 & -6 \end{vmatrix} = (6+12-24)-(12+6-24)=0$$

Demak, sistema cheksiz ko'p yechimga ega. Bu yechimlar to'plamini aniqlashda, sistemadan mustaqil $\begin{cases} x+2y-4z=0 \\ 2x-y+2z=0 \end{cases}$ tenglamalarni olamiz, (2) formulaga asosan

$$\frac{x}{\begin{vmatrix} 2 & -4 \\ -1 & 2 \end{vmatrix}} = \frac{y}{-\begin{vmatrix} 1 & -4 \\ 2 & 2 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}} = k$$ bu proporsiyadan

$$x=k\begin{vmatrix} 2 & -4 \\ -1 & 2 \end{vmatrix}=0,\quad y=-k\begin{vmatrix} 1 & -4 \\ 2 & 2 \end{vmatrix}=-10k\quad z=k\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}=-5k.$$

Sistema yechimlar to'plamini $x=0$, $y=-10k$, $z=-5k$.

7.2. Chiziqli bir jinsli bo'lgan tenglamalar sistemasida noma'lumlar soni, tenglamalar sonidan ko'p bo'lsin.

$$\begin{cases} a_1x+b_1y+c_1z=0 \\ a_2x+b_2y+c_2z=0 \end{cases} \quad (3)$$

Bu tenglamalar sistemasi cheksiz ko'p yechimga ega bo'lib, bu yechimlar to'plami (2) formuladan topiladi:

$$x=k\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix},\quad y=-k\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix},\quad z=k\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}. \quad (4)$$

Agar determinantlar bir vaqtda nol bo'lsa, (3) sistema cheksiz ko'p yechimlar to'plamini aniqlashda mustaqil bitta tenglamani olamiz,

<u>Masalan:</u>

$a_1x+b_1y+c_1z=0$ tenglikda ikkita o'zgaruvchiga ixtiyoriy qiymatlar berilib, uchinchi o'zgaruvchi topiladi.

<u>Misol.</u>

Tenglamalar sistemasi barcha yechimlari to'plami topilsin:

1) $\begin{cases} 2x+3y+z=0 \\ x-2y-z=0 \end{cases}$.

Bu sistemada

$$\Delta_1=\begin{vmatrix}3 & 1\\ -2 & -1\end{vmatrix}=-1;\ \Delta_2=\begin{vmatrix}2 & 1\\ 1 & -1\end{vmatrix}=-3;\ \Delta_3=\begin{vmatrix}2 & 3\\ 1 & -2\end{vmatrix}=-7\ (4)$$

formulaga asosan sistemaning yechimlar to'plami: $x = k\Delta_1 = -k$, $y=-k\Delta_2=3k$; $z=k\Delta_3=-7k$ ko'rinishda bo'ladi.

2) $\begin{cases} x+3y-z=0 \\ 2x+6y-2z=0 \end{cases}$.

Tenglamalar sistemasida

$$\Delta_1=\begin{vmatrix}3 & -1\\ 6 & -2\end{vmatrix}=0;\ \Delta_2=\begin{vmatrix}1 & -1\\ 2 & -2\end{vmatrix}=0;\ \Delta_3=\begin{vmatrix}1 & 3\\ 2 & 6\end{vmatrix}=0,$$

Ko'rsatilgan metodga asosan cheksiz yechimlar to'plamini, mustaqil bitta tenglama olinadi: $x+3y-z=0$ dan $z=x+3y$ ko'rinishdagi yechimlar to'plamiga ega bo'lamiz, haqiqatdan ham $x=0$, $y=0$ da $z=0$; $x=3$, $y=-1$ da $z=0$; $x=2, y=-1$ da $z=-1$;... (0; 0; 0) (3; –1; 0) va (2; –1; –1) kabi yechimlar to'plami. Tekshirish $x=2, y=-1$, $z=-1$ ni tenglamalar sistemasiga qo'ysak, $\begin{cases} 2+3\cdot(-1)-(-1)=0 \\ 2\cdot 2+6\cdot(-1)-2\cdot(-1)=0 \end{cases} \Rightarrow \begin{cases} 0=0 \\ 0=0 \end{cases}$ bo'lib, bu juftlar tenglama yechimi ekan.

7.3. Chiziqli bir jinsli bo'lgan tenglamalar sistemasida noma'lumlar soni tenglamalar sonidan kam bo'lsin:

$$\begin{cases} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \\ a_3x+b_3y+c_3=0 \end{cases}\ (5)$$

sistemada asosiy determinantni

olamiz: $\Delta=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$, (6)

Agarda:

1) $\Delta\neq 0$ bo'lsa, (5) sistema birgalikda emas;

2) $\Delta=0$ bo'lsa, sistema yechimlarini aniqlashda

$$x=\frac{u}{t},\ y=\frac{v}{t},\ (7)$$

almashtirishni (5) sistemaga qo'yamiz:

$$\begin{cases} a_1u+b_1v+c_1t=0 \\ a_2u+b_2v+c_2t=0 \quad (8) \\ a_3u+b_3v+c_3t=0 \end{cases}$$

(8) sistema yechimlari yordamida (7) dan (5) sistema yechimlari aniqlanadi. Agar bu sistemadagi asosiy determinant qiymati nol bo'lsa, sistema birgalikda bo'ladi, asosiy determinant qiymati nolga teng bo'lmasa, bu tenglamalar sistemasi birgalikda emas.

Misol.

Tenglamalar sistemasini yechilsin.

$$1)\quad \begin{cases} 2x-3y-6=0 \\ 3x+y-9=0 \\ x+4y-3=0 \end{cases}$$

Sistemadagi determinantni hisoblaymiz:

$$\Delta=\begin{vmatrix} 2 & -3 & -6 \\ 3 & 1 & -9 \\ 1 & 4 & -3 \end{vmatrix}=-6+27-72-(-6-72+27)=0.$$

U holda $x=\frac{u}{t}$, $y=\frac{v}{t}$ almashtirish bajarib, asosiy determinantni hisoblaymiz:

$$\begin{cases} 2u-3v-6t=0 \\ 3u+v-9t=0 \\ u+4v-3t=0 \end{cases};\quad \Delta=\begin{vmatrix} 2 & -3 & -6 \\ 3 & 1 & -9 \\ 1 & 4 & -3 \end{vmatrix}=0,$$

Demak sistema birgalikda bo'lib, sistema yechimlar to'plamini aniqlash uchun sistemadan mustaqil ikkita tenglamalarni olamiz:

$$\begin{cases} 2u-3v-6t=0 \\ 3u+v-9t=0 \end{cases} \text{ bu sistemadan } \Delta_1=\begin{vmatrix} -3 & -6 \\ 1 & -9 \end{vmatrix}=33,$$

$$\Delta_2=\begin{vmatrix} 2 & -6 \\ 3 & -9 \end{vmatrix}=0,\quad \Delta_3=\begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix}=11.$$

(2) formulaga asosan sistema yechimlari $u=k\Delta_1=33k$, $v=-k\Delta_2=0$ $t=k\Delta_3=11k$ ko'rinishda bo'ladi. U holda boshlang'ich berilgan sistema yechimlari $x=\frac{u}{t}=\frac{33k}{11k}=3, y=\frac{v}{t}=0.$

8-MAVZU. PARAMETRLI CHIZIQLI TENGLAMA VA TENGLAMALAR SISTEMASI

Tenglamada noma'lum o'zgaruvchidan boshqa noma'lum sonni ifodalovchi harflar ham qatnashsa, bunday tenglamaga ***parametrli (harfiy) tenglama*** *deyiladi.*

Ko'p hollarda tenglama parametrga nisbatan yechiladi. Bu parametrlarning qatnashishi geometrik va mexanik ma'noga ega.

Masalan:

$2x - ay + 5 = 0$, ikki noma'lumli tenglama, tekislikdagi to'g'ri chiziqni ifoda qiladi, ya'ni tenglama cheksiz yechimlar o'rni to'g'ri chiziq nuqtalarini ifoda qiladi.

Misol.

1) a parametr qanday bo'lganda berilgan nuqtalarini to'g'ri chiziq A(2; – 1) nuqtadan o'tadi.

Masala yechimi:

► $x=2, y=-1$ qiymatni to'g'ri chiziq tenglamasiga qo'yib, a – topiladi: $2 \cdot 2 - a \cdot (-1) + 5 = 0$ dan $a = -9$. Demak $a = -9$ da to'g'ri chiziq A(2; 1) nuqtadan o'tadi ◄

2) $y + bx - 3 = 0$ va $4x - 2y + 1 = 0$ to'g'ri chiziqlar b – parametr qanday bo'lganda o'zaro kesishadi.

►Masala yechimini topish: $\begin{cases} bx + y = 3 \\ 4x - 2y = -1 \end{cases}$ sistema yechimini yagonaligini aniqlash bilan ekvivalent. Kramer qoidasiga asosan, sistema asosiy determinanti nol bo'lmasligi kerak.

$\Delta = \begin{vmatrix} b & 1 \\ 4 & -2 \end{vmatrix} \neq 0$ yoki $-2b - 4 \neq 0$ dan $b \neq -2$ da berilgan to'g'ri chiziqlar kesishadi. ◄

Shu kabi masalalarni ko'p keltirish mumkin, Misollar bilan oydinlashtiramiz.

3) a ning qanday qiymatlarida $\begin{vmatrix} 2 & -3 \\ 4 & a \end{vmatrix}$ ifoda qiymati nolga teng bo'ladi.

►Yechish: Ikkinchi tartibli determinantni hisoblash formulasidan foydalanamiz: $\begin{vmatrix} 2 & -3 \\ 4 & a \end{vmatrix} = 0 \Rightarrow 2a + 12 = 0 \Rightarrow a = -6.$ ◄

4) a va b qanday bo'lganda tenglama $2ax - 5 = 4x + b$ bitta yechimga ega bo'ladi.

▶ *Yechish:* Tenglamani parametrlarga nisbatan yechib olamiz. $2ax - 4x = b + 5$ dan $x = \dfrac{b+5}{2a-4}$, berilgan tenglama $a \neq 2$ va ixtiyoriy b da yagona yechimga ega. ◀

5) b parametr qanday bo'lganda, $\dfrac{3x-b}{2} = \dfrac{4bx+5}{4}$ tenglama yechimi nol bo'ladi.

Yechish: oldingi misoldagi kabi amallar bajaramiz: $2(3x - b) = 4bx + 5$ dan $x = \dfrac{5+2b}{2(3-2b)}$; tenglama $b \neq 1{,}5$ va $b = -2{,}5$ da nol yechimga ega bo'ladi.

6) a va b larning qanday qiymatlarida $\dfrac{4}{(x+3)(x-1)} = \dfrac{a}{x-1} + \dfrac{b}{x+3}$ tenglik ayniyat bo'ladi.

Yechish: $x \neq -3$, $x \neq 1$ deb ifodada birinchi bosqich amalini bajaramiz: $4 = a(x+3) + b(x-1)$ yoki $4 = ax + 3a + bx - b$ noma'lum koeffisiyentlar metodidan foydalanamiz:

$$(a+b)x + 3a - b = 4 \Leftrightarrow \begin{cases} a+b=0 \\ 3a-b=4 \end{cases} \Rightarrow \begin{cases} a=-b \\ -3b-b=4 \end{cases} \Rightarrow \begin{cases} b=-1 \\ a=1 \end{cases}.$$

7) a va b parametrlarning $4(x - a) = ax + b$ tenglama yechimiga ta'siri qanday?

Tenglamani x ga nisbatan yechim yozsak, $x = \dfrac{4a+b}{4-a}$ bo'ladi. Bunda:

a) $a \neq 4$ da yechim $x = \dfrac{4a+b}{4-a}$;

b) $a = 4$ va $b \neq -16$ da tenglama yechimi mavjud emas.

c) $a = 4$ va $b = -16$ da tenglama cheksiz ko'p yechimga ega bo'ladi.

8) $a = \dfrac{1-bx}{bx+1}$

Yechish: Tenglamaning x ga nisbatan yechimini aniqlashda tenglikni shakl almashtirib yozamiz:

$$b(a+1)x = 1 - a$$

Agar:

1) $b = 0$ va $a = 1$ bo'lsa, har qanday x da tenglik o'rinli bo'ladi, ya'ni $x \in R$;

2) $b = 0$ va $a \neq 1$ (yoki $a = -1$)da yechim mavjud emas, ya'ni $x \in \varnothing$;

3) $b \neq 0$ va $a \neq -1$ da $x = \dfrac{1-a}{b(a+1)}$ yechimga ega;

4) $b \neq 0$ va $a = 1$ da $(a \neq -1)\ x = 0$ yechim;

5) $b = 0$ (yoki $a = -1$) va $a = 1$ da cheksiz ko'p yechimga ega.

9) $\dfrac{a+1}{2} - 3b$ ifoda $b = -0,5$ va a ning biror qiymatida 3,5 ga teng, a ning shu qiymatida va $b = 0,41(6)$ qiymatida berilgan algebraik ifoda qiymati nimaga teng?

Yechish: Masalaning birinchi sharti a ni topishda $\dfrac{a+1}{2} - 3\cdot\left(-\dfrac{1}{2}\right) = \dfrac{7}{2}$ tenglikka kelinadi, bu tenglama yechimi $a = 3$. Endi ikkinchi shartdan foydalanishda $0,41(6) = \dfrac{5}{12}$ ni hisobga olsak, ifoda qiymati $\dfrac{3+1}{2} - 3\cdot\dfrac{5}{12} = 2 - \dfrac{5}{4} = \dfrac{3}{4}$ ga teng ekan.

10) a ning qanday qiymatlarida $a - 0,2ax = x - 5$ tenglama cheksiz ko'p yechimga ega bo'ladi.

Yechish: Tenglamani parametrga nisbatan yozib olamiz:

$(-0,2a - 1)x = -5 - a$ dan $x = \dfrac{a+5}{0,2a+1}$, chiziqli tenglama yechimini aniqlash qoidasiga asosan $a = -5$ da tenglama cheksiz ko'p yechimga ega bo'ladi.

11) Agar $\begin{cases} ax - 3y = 3 \\ bx + 2y = 5 \end{cases}$

Tenglama yechimi $x = 3$, $y = 1$ bo'lsa, a va b ning qiymati topilsin.

Yechish: $x = 3$, $y = 1$ yechimlar sistemasidagi tenglamalarga qo'yib, a va b topiladi: $\begin{cases} 3a - 3 = 3 \\ 3b + 2 = 5 \end{cases}$ dan $\begin{cases} a = 2 \\ b = 1 \end{cases}$.

12) a ning qanday qiymatlarida determinantning satrlari proporsional bo'ladi: $\begin{vmatrix} 2 & -1 & 4 \\ 3 & -2 & 0 \\ a & 2 & -8 \end{vmatrix}$

Yechish: Ikkita satr yoki ustun elementlari proporsional bo'lsa, determinant qiymati nol bo'lishi kerak edi:

$$0 = \begin{vmatrix} 2 & -1 & 4 \\ 3 & -2 & 0 \\ a & 2 & -8 \end{vmatrix} = 32 + 0 + 24 + 8a - 0 - 24 \text{ dan } 8a + 32 = 0$$

Demak, $a = -4$ da birinchi va uchinchi satr elementlari proporsional bo'ladi.

13) b ning qanday qiymatida $\begin{cases} 6y-(6+b)x=-2 \\ 2bx-3y=3-b \end{cases}$ tenglamalar sistemasi cheksiz ko'p yechimga ega bo'ladi. Bu yechimlar to'plamini ko'rsating.

<u>Yechish:</u> Berilgan $\begin{cases} -(6+b)x+6y=-2 \\ 2bx-3y=3-b \end{cases}$ sistemaga Kramer qoidasini tatbiq qilamiz: $\Delta = \begin{vmatrix} -6-b & 6 \\ 2b & -3 \end{vmatrix} = 0$ bo'lish kerak shartidan $18 + 3b - 12b = 0$ bo'lib, $b = 2$. $b = 2$ da yordamchi determinantlar ham nol bo'lishi kerak, tekshiramiz: $\Delta_x = \begin{vmatrix} -2 & 6 \\ 1 & -3 \end{vmatrix} = 0, \Delta_y = \begin{vmatrix} -8 & -2 \\ 4 & 1 \end{vmatrix} = 0$

Demak, $b = 2$ da sistema cheksiz ko'p yechimga ega. Bu yechimlar to'plamini aniqlash uchun sistemadan bittasini olamiz:

$4x - 3y = 1$ dan $y = \frac{4}{3}x - \frac{1}{3}$, demak yechimlar to'plami $x = k$, $y = \frac{4}{3}k - 1$ $k \in R$ ko'rinishda bo'ladi.

14) a ning qanday qiymatida $\begin{cases} 2x-y=a \\ x+3y=5 \end{cases}$ Sistema yechimlari uchun $x + y = 1$ tenglik o'rinli bo'ladi.

15) a parametrga nisbatan sistema yechimini aniqlab bering.

$\Delta = \begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} = 7, \ \Delta_x = \begin{vmatrix} a & -1 \\ 5 & 3 \end{vmatrix} = 3a+5, \ \Delta_y = \begin{vmatrix} 2 & a \\ 1 & 5 \end{vmatrix} = 10-a$

Kraemer formulasidan

$$x = \frac{\Delta x}{\Delta} = \frac{3a+5}{7}, \quad y = \frac{10-a}{7}.$$

Endi shartdan foydalanamiz $1 = x + y = \frac{3a+5}{7} + \frac{10-a}{7}$ ifodasini soddalashtirsak $7 = 3a + 5 + 10 - a$ dan $a = -4$. ◄

16) a ning qanday qiymatida $\begin{cases} 5x+2y+z=11, \\ 2x-y+3z=14, \\ ax-3y+5z=32. \end{cases}$ tenglamalar sistemasi yagona yechimga ega bo'ladi.

► Tenglamalar sistemasida asosiy determinant qiymati nolga teng bo'lmasa, sistema yagona yechimga ega bo'ladi

$$\Delta=\begin{vmatrix}5 & 2 & 1\\ 2 & -1 & 3\\ a & -3 & 5\end{vmatrix}=-25+6a-6+a+45-20\neq 0 \text{ dan } 7a-6\neq 0$$

demak, $a\neq\frac{6}{7}$ da sistema yagona yechimga ega bo'ladi ◄

17) a ning qanday qiymatida $\begin{cases}x-2y+4z=4,\\ 2x+y+z=-2,\\ 4x-5y+2z=3.\end{cases}$ tenglamalar sistemasi yechimga ega bo'lmaydi.

► Kramer qoidasini tadbiq qilamiz.

Asosiy deferminantni hisoblaymiz

$$\Delta=\begin{vmatrix}1 & -2 & a\\ 2 & 1 & 1\\ 4 & -5 & 2\end{vmatrix}=2-8-10a-4a+5+8=0\text{, dan } a=0{,}5$$

Endi yordamchi deferminantlarni hisoblaymiz $a=0{,}5$ da

$$\Delta_x=\begin{vmatrix}4 & -2 & 0{,}5\\ -2 & 1 & 1\\ 3 & -5 & 2\end{vmatrix}=8-6+5-4{,}5+20-8=33-18{,}5=14{,}5\neq 0$$

Demak, berilgan tenglamalar sistemasi $a=0{,}5$ da yechimga ega bo'lmaydi. ◄

5-, 6-, 7-, 8-MAVZULAR MASHQLARI

Determinantlarning qiymatini toping

67. 1) $\begin{vmatrix}5 & 4\\ -3 & 2\end{vmatrix}$, 2) $\begin{vmatrix}-2 & 4 & -12\\ 2 & -3 & 0\\ 5 & 4 & 1\end{vmatrix}$, 3) $\begin{vmatrix}4 & 5 & -3\\ 12 & -9 & 23\\ 0 & 0 & 0\end{vmatrix}$.

68. 1) $\begin{vmatrix}7 & -3 & 5\\ 5 & 2 & 1\\ 5 & -1 & 3\end{vmatrix}$, 2) $\begin{vmatrix}4 & 8 & 12\\ 2 & 3 & 4\\ 3 & 4 & 5\end{vmatrix}$, 3) $\begin{vmatrix}2a & 1 & -a\\ a^2 & 2 & a\\ a & -3 & a^2\end{vmatrix}$.

69. *a* qanday bo'lganda determinanning mos yo'l elementlari proporsional bo'ladi. $\begin{vmatrix} 2 & -5 & 1 \\ 3 & 4 & 7 \\ 6 & a & 3 \end{vmatrix}$

70. *a* qanday bo'lganda determinanning mos ustun elementlari proporsional bo'ladi. $\begin{vmatrix} 12 & a & 9 \\ -8 & 4 & 0 \\ 7 & -3,5 & 11 \end{vmatrix}$

71. *a* ning qanday qiymatlarida tenglik o'rinli bo'ladi.

$$\Delta_1 = \begin{vmatrix} 2a & 3 & -4 \\ 0 & 5 & 1 \\ -4 & 1 & 7 \end{vmatrix} \text{ va } \Delta_2 = \begin{vmatrix} 1 & 0 & 3 \\ 2 & a & 5 \\ -4 & 7 & 9 \end{vmatrix} \text{ da } \Delta_1 = \Delta_2$$

72. *a* ning qanday qiymatlarida Δ 1 determinant Δ 2 determinantga ekvivalent bo'ladi

$$\Delta_1 = \begin{vmatrix} 2 & a & 5 \\ 1 & 3 & 6 \\ 0 & 4 & -2 \end{vmatrix} \text{ bilan } \Delta_2 = \begin{vmatrix} 1 & 3 & 2 \\ 0 & 4 & -1 \\ a & 7 & 5 \end{vmatrix}.$$

Tenglamalar yechilsin.

73. $\begin{vmatrix} 2 & x & -4 \\ -1 & 5 & 2 \\ 3 & 1 & -6 \end{vmatrix} - 2\begin{vmatrix} 2x & 5 \\ -1 & 1 \end{vmatrix} = 0$.

74. $\begin{vmatrix} 3x & 4 & 2 \\ 1 & 0 & -1 \\ x & -1 & -3 \end{vmatrix} + 3\begin{vmatrix} 2 & 3 \\ x & 0 \end{vmatrix} + 6 = 0$.

Chiziqli tenglamalar sistemasini yeching:

75. 1) $\begin{cases} 2x + y = 2 \\ -2x - y = 3 \end{cases}$; 2) $\begin{cases} 0,3x - \frac{1}{5}y = 1 \\ -6x + 5y = -19 \end{cases}$; 3) $\begin{cases} 2x + 3y = 7 \\ -4x - 6y = -14 \end{cases}$.

76. 1) $\begin{cases} 3x + 4y = 18 \\ 2x + 5y = 19 \end{cases}$; 2) $\begin{cases} 2x - y = 3 \\ x - 0,5y = 1 \end{cases}$; 3) $\begin{cases} 3x - 5y = 0 \\ 25y - 15x = 0 \end{cases}$.

77. 1) $\begin{cases} x - 2y + 3z = 6, \\ 2x + 3y - 4z = 20, \\ 3x - 2y - 5z = 6. \end{cases}$ 2) $\begin{cases} x - 2y - z = 2, \\ 5x - 10y - 5z = 10, \\ 3x - 6y - 3z = 6. \end{cases}$

78. 1) $\begin{cases} 2x - y + 3z = 14, \\ 10x + 4y + 2z = 22, \\ 7x - 3y + 5z = 32. \end{cases}$ 2) $\begin{cases} 2x - y + z = 1, \\ 3x + 2y - z = 3, \\ 4x - 2y + 2z = 2. \end{cases}$

3) $\begin{cases} 2x+y-z=1, \\ 5x-3y+2z=3, \\ 4x+2y-2z=5. \end{cases}$

Chiziqli bir jinsli bo'lgan tenglamalar sistemasi yechimi topilsin.

79. 1) $\begin{cases} 2x-y=0, \\ 4x-3y=0. \end{cases}$ 2) $\begin{cases} 3x+y=0, \\ 2x-4y=0. \end{cases}$

80. 1) $\begin{cases} 3x-0,5y=0, \\ 4x+1\frac{1}{4}y=0. \end{cases}$ 2) $\begin{cases} 0,5x+0,(3)y=0, \\ -6x-4y=0. \end{cases}$

81. 1) $\begin{cases} 3x+2y+5z=0, \\ 2x+3y-2z=0, \\ x+y+z=0. \end{cases}$ 2) $\begin{cases} 3x-y+6z=1, \\ x+3y+2z=0, \\ -x-4y-2z=0. \end{cases}$

82. 1) $\begin{cases} 2x+3y+2z=0, \\ x+y-z=0, \\ 3x-2y+4z=0. \end{cases}$ 2) $\begin{cases} -x+2y-z=0, \\ 3x-y+2z=0, \\ 2x-4y+2z=0. \end{cases}$

Chiziqli bir jinsli bo'lgan tenglamalar sistemasi yechimlar to'plami topilsin.

83. 1) $\begin{cases} 3x+y-z=0, \\ 2x+y-4z=0. \end{cases}$ 2) $\begin{cases} x-2y+3z=0, \\ -2x+4y-6z=0. \end{cases}$

84. 1) $\begin{cases} x-3y+2z=0, \\ 2x+y-4z=0. \end{cases}$ 2) $\begin{cases} 2x+6y-z=0, \\ x+3y-0,5z=0. \end{cases}$

85. 1) $\begin{cases} 7x-3y+5=0, \\ 5x+2y+1=0, \\ 2x-y+3=0. \end{cases}$ 2) $\begin{cases} x-2y+3=0, \\ 3x+y-4=0, \\ 2x-4y+6=0. \end{cases}$

86. 1) $\begin{cases} 3x+2y+2=0, \\ x-5y-8=0, \\ 4x+2y+1=0. \end{cases}$ 2) $\begin{cases} x+2y+3=0, \\ 2x+3y+4=0, \\ 3x+4y+5=0. \end{cases}$

Parametrli chiziqli tenglama yechimlar to‘plamini ko‘rsating.

87. 1) $ax-2b=x$, 2) $2ax+a=3b-bx$.

88. 1) $5x-b=2ax$, 2) $(4-a)x-b=2a-5x$.

a ning qanday qiymatida yechim mavjud emas.

89. $\begin{cases} x+3y-2z=5, \\ x-2y+0,5z=3, \\ 2x+ay+z=6. \end{cases}$

90. *a* ning qanday qiymatlarida tenglamalar sistemasida $x=0$ bo‘ladi.

$\begin{cases} x-3y=2, \\ 5x+4y+z=1, \\ x-2y+6z=a. \end{cases}$

91. a ning qanday qiymatlarida sistema yagona yechimga ega bo‘ladi.

$$\begin{cases} 4x+3y+az=-9, \\ x-4y+z=1, \\ 2x+y+3z=1. \end{cases}$$

92. a va b parametrlarning qanday qiymatlarida sistema yagona yechimga ega:

$$\begin{cases} 2x+3y-4z=b, \\ ax-2y-5z=6, \\ ax-2y=3z=6. \end{cases}$$

a ning qanday qiymatlarida sitema birgalikda bo‘ladi, bu yechimlar aniqlansin

93. $\begin{cases} x+3y-5=0, \\ x+ay+3,5=0, \\ 2x-6y+7=0. \end{cases}$

94. $\begin{cases} 2x+ay+4=0, \\ x+3y-5=0, \\ 3x+ay+2=0. \end{cases}$

95. a qanday bo‘lganda $ax-y+4=0$ to‘g‘ri chiziq $A(3;-2)$ nuqtadan o‘tadi.

96. *a* parametr qanday bo'lganda $2x + ay - 6 = 0$, to'g'ri chiziq $A(-1;4)$ nuqtadan o'tadi.

97. *a* va *b* parametrlar qanday bo'lgada $2x + ay = 3$ va $bx - 4y + 1 = 0$ to'g'ri chiziqlar A(1;1) nuqtada kesishadi.

98. *a* parametr qanday bo'lganda $2x - ay - 1 = 0$ va $2x + 4y = -5$ to'g'ri chiziqlar bitta nuqtada kesishadi.

99. *a* qanday bo'lganda $2x - 3y = 1$ va $4x + ay = 2$ to'g'ri ustma-ust tushadi.

100. *a* parametr qanday bo'lganda $3x + y - 3 = 0$ va $ax - 2y = 6$ to'g'ri chiziqlar kesishmaydi.

101. *a* ning qanday qiymatlarida $\begin{cases} 3x - 2y = -2 \\ x + 4y - a = 0 \end{cases}$

Sistema yechimlari uchun $2x - y = 2$ tenglik o'rinli bo'ladi.

102. *a* ning qanday qiymatlarida

$$\begin{cases} 2x - y - a = 0 \\ 3y + 2x + 5 = 0 \end{cases}$$

Sistema yechimlari uchun $x - 3y - 2 = 0$ tenglik o'rinli bo'ladi.

9-MAVZU. BIR NOMA'LUM IKKINCHI TARTIBLI (KVADRAT) TENGLAMA

$$ax^2 + bx + c = 0 \quad (1) \quad (a \neq 0)$$

ko'rinishdagi tenglamaga ***bir noma'lum ikkichi tartibli (kvadrat) tenglama*** *deyiladi.*

Bu yerda $a, b, c \in R$ ixtiyoriy berilgan sonlar bo'lib, a va b noma'lum oldidagi koeffisentlar, c-ozod had deyiladi. x-noma'lum o'zgaruvchi.

(1) Tenglamada $a \neq 0$ bo'lib, b yoki c lardan kamida bittasi nol bo'lsa, bu tenglama chala kadrat tenglama deyiladi. Oldin chala kvadrat tenglamani o'rganamiz.

9.1. (1) tenglamada $c = 0$ bo'lsa, tenglama

$$ax^2 + bx = 0. \quad (2)$$

ko'rinishida bo'lib, yechimini aniqlashda gruppalaymiz $x(ax + b) = 0$ tenglama $x = 0$ va $ax + b = 0$ ikkita tenglamaga teng kuchli bo'ladi.

18) Tenglama yechimi topilsin:

1. $2x^2 + 3x = 0,$

► x ni qavsdan tashqariga chiqarsak, tenglama ikkita $x = 0$ va $2x + 3 = 0$ tenglamalarga teng kuchli bo'lib, berilgan tenglama yechimlari $x_1 = 0,$ $x_2 = -1{,}5.$ ◄

2. $1{,}2x - 2{,}(6)\,x^2 = 0,$

► Koeffisetlarni oddiy kasr ko'rinishga keltirib, oldingi misol kabi amallar bajaramiz:

$x\left(\dfrac{6}{5} - \dfrac{8}{3}x\right) = 0,$ dan tenglama yechimlari $x_1 = 0$ va $x_2 = 0{,}45.$ ◄

9.2. (1) tenglamada $b = 0$ bo'lsa, tenglama

$$ax^2 + c = 0, \quad (3)$$

ko'rinishda bo'lib, tenglamada $a \cdot c > 0$ bo'lsa, tenglama haqiqiy yechimga ega emas, yani $x \in \varnothing$. Agar $a \cdot c < 0$ (a va c har xil ishorali bo'lsa), tenglama yechimi

$$x = \pm\sqrt{-\frac{c}{a}} \text{ yoki } x_1 = \sqrt{-\frac{c}{a}}, \; x_2 = -\sqrt{-\frac{c}{a}}.$$

ko'rinishda bo'ladi. Sondan kvadrat ildiz chiqarish usuli [3] da berilgan. Eng sodda usul sonni tub ko'paytuvchilarga ajratish bilan.

Masalan:

$$\sqrt{144} = \sqrt{4^2 \cdot 3^2} = 4 \cdot 3 = 12 \quad \text{kabi.}$$

19) Tenglama yechimi topilsin

1. $9 - 4x^2 = 0,$

► Koeffisent ishorasi har xil, demak haqiqiy yechim $x^2 = \dfrac{9}{4}$ dan

$x = \pm\sqrt{\dfrac{9}{4}}$ yoki $x_1 = 1{,}5, x_2 = -1{,}5$ ◄

2. $0{,}5x^2 + 2 = 0$,

► Tenglama koeffisentlari bir xil ishorali bo'lganligi uchun, tenglama haqiqiy yechimi mavjud emas. ◄

3. $x^2 - 1296 = 0$,

► $x^2 = 1296$ tenglama yechimi aniqlashda ozod hadni tub ko'paytuvchilarga ajratamiz $1296 = 9^2 \cdot 4^2$ u holda

$x = \pm\sqrt{1296} = \sqrt{9^2 \cdot 4^2} = \pm 9 \cdot 4$ bo'lib $x_1 = 36, x_2 = -36$.◄

9.3. (1) tenglamada $b = c = 0$ bo'lsa, tenglama

$$a \cdot x^2 = 0, \quad (4)$$

Ko'rinishda bo'lib, $a \neq 0$ bo'lgaligi uchun $x^2 = 0$ dan $x_1 = x_2 = 0$.

20) Tenglama yechimi topilsin.

$9x^2 = 0$.

► Tenglama yechimlari $x^2 = 0$ dan $x_1 = x_2 = 0$. ◄

9.4. (1) *tenglamada koeffisentlar va ozod had nol bo'lmasa bu tenglamaga* ***to'la kvadrat*** *deyiladi.*

(1) *tenglamada a = 1 bo'lsa* ***keltirilgan kvadrat tenglama*** *deyiladi.* (1) *tenglamada a ≠ 1 bo'lsa tenglamani a koeffisentga bo'lib*

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$, da $p = \frac{b}{a}$, $q = \frac{c}{a}$ desak

$$x^2 + px + q = 0, \quad (5)$$

keltilgan kvadrat tenglama hosil bo'ladi.

To'la kvadrat tenglama ildizlari

$$\Delta = b^2 - 4ac, \quad (6)$$

Kvadrat tenglama diskriminant ishorasiga bog'liq bo'ladi.

Agar: 1) $\Delta < 0$ bo'lsa, (1) tenglamaning haqiqiy yechimi mavjud emas,

2) $\Delta = 0$ bo'lsa, (1) tenglama bitta

$$x_1 = x_2 = \frac{b}{2a}. \quad (7)$$

(karrali) yechimga ega,

4) $\Delta > 0$ bo'lsa, (1) tenglama haqiqiy har xil ikkita

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a}, \ x_2 = \frac{-b - \sqrt{\Delta}}{2a}, \quad (8)$$

yechimga ega.

Bunda $\Delta = k^2$ ($k \neq 0$) bo'lsa, (1) tenglama ratsional yechimlarga ega, $\Delta \neq k^2$ bo'lsa u holda (1) tenglama o'zaro qo'shma irratsional yechimlarga ega bo'ladi.

5) Agarda (1) tenglamada a va c koeffisentlar o'rini almashtirishda, (1) tenglama ildizlari qiymatiga teskari qiymatli ildizlarga ega bo'lgan tenglama hosil bo'ladi.

6) Agar (1) tenglamada b ni teskari ishorali ($-b$) bilan almashtirilsa, (1) tenglama ildizlariga qarama-qarshi ishorali ildizli tenglama hosil bo'ladi.

21) Tenglama yechilsin

1. $2x^2 + 5x - 3 = 0.$

► Berilgan tenglama to'la kvadrat tenglama bo'lib, bunda $a = 2$, $b = 5$, $c = -3$. Tenglama diskriminantini hisoblaymiz.

$\Delta = b^2 - 4ac = 5^2 - 4 \cdot 2 \cdot (-3) = 25 + 24 = 49$, 49 soning arifmetik ildizi $\sqrt{49} = 7$ ga teng. U holda berilgan tenglamaning ildizlari (8) formulaga asosan

$$x_1 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-5+7}{2\cdot 2} = 0,5, \quad x_2 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-5-7}{2\cdot 2} = -3. ◄$$

2. $x^2 - 2x + 5 = 0,$

► Bu tenglama diskriminanti

$\Delta = b^2 - 4ac = (-2)^2 - 4 \cdot 1 \cdot 5 = -16 < 0$, bo'lganligi uchun, tenglama haqiqiy yechimi mavjud emas. ◄

3. $8x - x^2 - 16 = 0,$

► Bu tenglamada $\Delta = 8^2 - 4 \cdot (-1) \cdot (-16) = 64 - 64 = 0$, tenglama ildizlari karrali bo'lib, (7) formulaga asosan $x_1 = x_2 = -\dfrac{b}{2\cdot a} = -\dfrac{8}{2\cdot(-1)} = 4.$ ◄

22) k qanday bo'lganda $kx^2 + 2x - 1 = 0$, kvadrat tenglama haqiqiy har xil yechimga ega bo'ladi.

► Berilgan kvadrat tenglama diskriminanti

$$\Delta = b^2 - 4ac = 2^2 - 4 \cdot k \cdot (-1),$$

Musbat bo'lishi kerak, demak $4 + 4k > 0$ yoki $k > -1$ da haqiqiy har xil yechimga ega bo'ladi.

23) φ va β qanday bo'lganda, kvadrat tenglama

$2x^2 + \varphi x + \beta = 0$. Bitta ildizi nol ikkinchi ildizi 2 bo'ladi.

► Kvadrat tenglama ildizini topish qoidasiga ko'ra $\beta = 0$ bo'lishi kerak. Demak, $2x^2 + 2x = 0$, $x \cdot (2x + \varphi) = 0$, $x_1 = 0$ birinchi shart bajarildi. Endi ikkinchi shartdan foydalanamiz $2x + \varphi = 0$ dan $x = -\dfrac{\varphi}{2}$ shartda

$x = x_2 = 2$ endi $2 = -\dfrac{\varphi}{2}$ $\varphi = -4$

Javob: $\varphi = -4, \beta = 0$ ◄

24) Agar $(0,5x - 4)(x + 2) = 0$ bo'lsa, $4 - 0,5x$ qanday qiyatlarni qabul qilishi mumkin.

► Berilgan tenglama ikkita $0,5x - 4 = 0$ va $x + 2 = 0$ tenglamalarga teng kuchli bo'lib, yechimlari $x_1 = 8$ va $x_2 = -2$

U holda $4 - 0,5x$ ifodaning qiymati $x = 8$ da $4 - 0,5 \cdot 8 = 0$, $x = -2$ da $4 - 0,5 \cdot (-2) = 5$, demak, qabul qiladigan qiymati 0 yoki 5. ◄

9.5. Kvadrat tenglama ildizlarinig xossalari:

x_1 va x_2 keltirilgan (5) kvadrat tenglama ildizlari bo'lsin. (1) to'la kvadrat tenglama uchun $\left(p = \dfrac{b}{a},\ q = \dfrac{c}{a}\right)$

1. Viet teoremasi: Keltirilgan kvadrat tenglama ildizlari uchun:

$$\begin{cases} x_1 + x_2 = -p, \\ x_1 \cdot x_2 = q. \end{cases}$$ (9) tenglik o'rinli bo'ladi.

2. $ax^2 + bx + c +$ kvadrat uch hadni, chiziqli ko'paytuvchilar $ax^2 + bx + c = a(x - x_1)(x - x_2)$, (10) ko'rinishda yozish mumkin, bu yerda x_1 va x_2 esa (1) tenglamaning ildizlari.

3. To'la kvadrat ildizlar ayirmasi (kattasidan kichigini ayirmasiga)

$$x_2 - x_1 = \frac{\sqrt{\Delta}}{a}$$

25) Quyidagi tenglamani yechmasdan, ildizlar kublarining yig'indisi topilsin $x^2 - 5x - 14 = 0$,

▶ Berilgan keltirilgan kvadrat tenglamaning ildizlari x_1 va x_2 bo'lsin, Masala shartiga ko'ra $x_1^3 + x_2^3$ ni topish kerak. Viet teoremasidagi (9) formulaga asosan $\begin{cases} x_1 + x_2 = 5 \\ x_1 \cdot x_2 = -14 \end{cases}$

Qisqa ko'paytirish formulasidan foydalanamiz

$x_1^3 + x_2^3 = (x_1 + x_2)(x_1^2 - x_1 x_2 + x_2^2) = (x_1 + x_2)[(x_1 + x_2)^2 - 3x_1 x_2$

Tenglikdan foydalanamiz:

$x_1^3 + x_2^3 = 5 \cdot [5^2 - 3 \cdot (-14)] = 5 \cdot [25 + 42] = 335.$ ◀

26) Ildizlari 0,5 va (– 1,(3)) bo'lgan kvadrat tenglama topilsin.

▶ $x_1 = 0,5 = \frac{1}{2}, x_2 = -1(3) = -\frac{4}{3}$ desak, (9) formulaga asosan

$\begin{cases} x_1 + x_2 = -p \\ x_1 - x_2 = q \end{cases}$ dan $\begin{cases} \frac{1}{2} - \frac{4}{3} = -p \\ \frac{1}{2} \cdot \left(-\frac{4}{3}\right) = q \end{cases}$ yoki $p = \frac{5}{6}$, $q = -\frac{2}{3}$, bu qiymatlarni (5) tenglikka qo'ysak $x^2 + \frac{5}{6}x - \frac{2}{3} = 0$,

Javob: $6x^2 + 5x - 4 = 0.$ ◀

27) Bitta ildizi $\frac{\sqrt{7} + \sqrt{6}}{4}$ bo'lgan kvadrat tenglama topilsin.

▶ Kvadrat tenglamaning ildizlari o'zaro qo'shma bo'ladi, u holda (9) formuladan

$$\begin{cases} x_1 + x_2 = -p \\ x_1 \cdot x_2 = q \end{cases}$$ dan

$$\begin{cases} \dfrac{\sqrt{7}+\sqrt{6}}{4}+\dfrac{\sqrt{7}-\sqrt{6}}{4}=-p, \\ \left(\dfrac{\sqrt{7}+\sqrt{6}}{4}\right)\cdot\left(\dfrac{\sqrt{7}-\sqrt{6}}{4}\right)=q. \end{cases} \text{ bundan } \begin{cases} \dfrac{\sqrt{7}}{2}=-p, \\ \dfrac{1}{16}=q. \end{cases}$$ bo‘lib, bu

qiymatlarni (5) tenglikka qo‘yib soddalashtirsak $16x^2-8\sqrt{7}x+1=0$.

28) Agar 0,4 son $5x^2 + bx + 2 = 0$ tenglamaning bitta ildizi bo‘lsa, tenglamaning ikkinchi ildizi va tenglama koeffisentlar yig‘indisi topilsin.

▶ Tenglama ildizlari $x_1 = 0{,}4$ va x_2 bo‘lsin, u holda

$$\begin{cases} x_1+x_2=-\dfrac{b}{5} \\ x_1\cdot x_2=\dfrac{2}{5} \end{cases} \to \begin{cases} 0{,}4+x_2=-0{,}2b \\ 0{,}4\cdot x_2=0{,}4 \end{cases} \to \begin{cases} 0{,}4+1=-0{,}2b \\ x_2=1 \end{cases} \to \begin{matrix} b=-7 \\ x_2=1 \end{matrix}$$

Demak tenglamaning ikkinchi yechimi 1, noma’lum koeffisent $b = -7$ ekan. U holda tenglama koeffisentlar yig‘indisi

$a + b + c = 5 - 7 + 2 = 0$ ekan.◀

29) Kvadrat uchhadni $x^2+bx-\left(b\sqrt{a}+a\right)$ $(a > 0,\ b > 0)$ chiziqli ko‘paytuvchilarga ajrating.

▶ Oldin tenglama ildizlarini a va b parametrlarga nisbatan topamiz:

$$x^2+bx-\left(b\sqrt{a}+a\right)=0$$

$$\Delta=b^2+4\cdot1\cdot\left(b\sqrt{a}+a\right)=b^2+4b\sqrt{a}+4a=\left(b+2\sqrt{a}\right)^2$$

U holda tenglama ildizlari

$$x_1=\frac{-b+b+2\sqrt{a}}{2}=\sqrt{a},\ x_2=\frac{-b-b-2\sqrt{a}}{2}=-b-\sqrt{a}$$

bo‘lib, berilgan qoidaga asosan

$$x^2+bx-\left(b\sqrt{a}+a\right)=\left(x-x_1\right)\left(x-x_2\right)=\left(x-\sqrt{a}\right)\left(x+b+\sqrt{a}\right).$$◀

30) $3+\sqrt{2}$ son $x^2 - 6x + 7 = 0$ kvadrat tenglama yechim bo‘ladimi (tenglamani yechmasdan isbotlang).

▶ $x_1=3+\sqrt{2}$ tenglama yechimi bo‘lsa, u holda ikkinchi yechim $x_2=3-\sqrt{2}$ bo‘ladi. Chunki koeffisentlar haqiqiy sonlar. Sodda isbotlash usuli Viet teoremasidan foydalanish.

$x_1 \cdot x_2 = p$ yoki $\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)=7$ bo'lish kerak, haqiqatdan $(9-2)=7$

$7 = 7$, demak x_1 tenglamaga yechim ekan ◀

31) $x^2 - bx - \left(ba + a^2\right) = 0$ tenglama ildizlar ayirmasi nimaga teng.

▶ Tenglama diskriminantni hisoblaymiz:

$$\Delta = (-b)^2 - (-4)\left(ba + a^2\right) = b^2 + 4ab + 4a^2 = (b + 2a)^2$$

Kvadrat tenglama ildizlarining 3 – xosasiga asosan

$$x_2 - x_1 = \frac{\sqrt{\Delta}}{a} \text{ edi, demak } x_2 - x_1 = \sqrt{(b+2a)^2} = b + 2a \; ◀$$

32) Ushbu $9x^2 - 6\sqrt{5}x + 2 = 0$.
tenglama ildizlarining o'rta proporsional qiymatini toping

▶ a va b sonlarning o'rta proporsional qiymati $\sqrt{a \cdot b}$ ning arifmetik kvadrat tenglama ko'rinishda

$x^2 - \frac{2\sqrt{5}}{3}x + \frac{2}{9} = 0$ yozsak Viet teoremasiga asosan $x_1 \cdot x_2 = \frac{2}{9}$,

u holda, ildizlarning o'rta proporsional qiymati

$$\sqrt{x_1 \cdot x_2} = \sqrt{\frac{2}{9}} = \frac{1}{3}\sqrt{2} \text{ ga teng. } ◀$$

33) Ushbu $x^2 + x - 6 = 0$, tenglamani yechmasdan, bu tenglama ildizlariga teskari va teskari ishorali qiymatli ildizlarga ega bo'lgan tenglamani yozing.

▶ Kvadrat tenglama ildizlarini aniqlash qoidasidagi (7.4. punktda) 4) va 5) ga asosan masala shartini qanoatlantiruvchi tenglama $6x^2 + x - 1 = 0$ ko'rinishda bo'ladi ◀

9.6. Kvadrat tenglamaning kompleks ildizlari haqida.

To'la kvadrat $ax^2 + bx + c = 0$, (1) ($a \neq 0$) tenglamada diskriminanti $\Delta = b^2 - 4ac$ manfiy bo'lsa haqiqiy yechim mavjud emas edi. Lekin mavhum birlik $i = \sqrt{-1}$ ($i^2 = -1$) tushunchasidan foydalanib, kompleks ildizlarini aniqlash mumkin kompleks son haqida tushuncha ([1],[3]) berilgan. Agar kvadrat tenglamaning koeffisentlarni haqiqiy sonlar bo'lsa, u holda tenglama kompleks yechimlari o'zaro qo'shma ko'rinishda bo'ladi. Yani $x_{1,2} = \alpha \pm \beta i$ ko'rinishda bu yerda $\alpha, \beta \in R$. Agar $\beta = 0$ bo'lsa, tenglama haqiqiy yechimga ega bo'ladi, $\alpha = 0$ da esa mavhum yechimga ega bo'ladi. Bunday yechimlarni misollarda oydinlashtiramiz.

34) Tenglama ildizi topilsin.

1. $x^2 - 2x + 5 = 0$,

► Tenglamada $\Delta = (-2)^2 - 4 \cdot 1 \cdot 5 = 4 - 20 = -16$, Mavhum birlik tushunchasidan $\sqrt{-16} = \sqrt{-1} \cdot \sqrt{16} = 4i$,

u holda $x_{1_2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$ bo'lib $x_1 = 1 + 2i$, $x_2 = 1 - 2i$

Ko'rinishdagi qo'shma kompleks yechimlarga ega.◄

2. $2x^2 + 8 = 0$,

► Berilgan chala kvadrat tenglamani $x^2 = -4$ deb yozish mumkin, bundan tenglama $x_{1_2} = \pm\sqrt{-4} = \pm 2i$ mavhum ildizga ega bo'ladi.◄

35) Ildizi $3 + 5i$ bo'lgan haqiqiy koeffisantli kvadrat tenglama topilsin.

► Berilgan nazariyadan ma'lumki haqiqiy koeffisentli kvadrat tenglama kompleks ildizlari o'zaro qo'shma bo'lar edi. Demak $x_1 = 3 + 5i$ bo'lsa $x_2 = 3 - 5i$ bo'lish kerak, tenglamani $[x - (3 - 5i)] \cdot [x - (3 + 5i)] = 0$ yoki $[(x - 3) + 5i][(x - 3) - 5i] = 0$ ko'rinishda qidiramiz. Qisqa ko'paytirish formulasiga asosan $(x - 3)^2 - (5i)^2 = 0$, qavs ochib soddalashtirsak, qidirgan tenglamani hosil qilamiz

$$x^2 - 6x + 9 - (-25) = 0 \text{ yoki } x^2 - 6x + 34 = 0.$$

3. $x^2 - (2 + i)x + 3 + i = 0$

► Tenglamani koeffisentlari haqiqiy emas, bu tenglamada

$\Delta = [-(2 + i)]^2 - 4 \cdot 1 \cdot (3 + i) = 4 + 4i - 1 - 12 - 4i = -9$, bo'lib

Tenglama ildizlari $x_{1_2} = \frac{(2+i) \pm \sqrt{-9}}{2} = \frac{(2+i) \pm 3i}{2}$ dan

$x_1 = \frac{2 + i + 3i}{2} = 1 + 2i$, $x_2 = \frac{2 + i - 3i}{2} = 1 - i$, bo'lib o'zaro qo'shma emas.

10-MAVZU. IKKINCHI TARTIBLI (KVADRAT) TENGLAMA ILDIZLAR ISHORALARINI TENGLAMA KOEFFISIENTLARI YORDAMIDA ANIQLASH

To‘la kvadrat tenglamada

$$ax^2 + bx + c = 0, \quad (1)\ (a \neq 0)$$

$a > 0$ deb faraz qilishimiz mumkin, agar $a < 0$ bo‘lsa tenglamani (-1) ga ko‘paytirsak a musbat bo‘lib qoladi. (1) tenglama ildizlari x_1 va x_2 bo‘lsin, musbat ildizlarini x^+ va manfiy ildizlarini x^- deb belgilasak, to‘la kvadrat tenglama ildizlar ishoralarini quyidagi jadvaldan aniqlab olish mumkin.

Tenglama diskriminantini hisoblab olamiz:

$$\Delta = b^2 - 4ac$$

To‘la kvadrat tenglama						Chala kvadrat tenglama				
$\Delta > 0, a > 0$				$\Delta = 0, a > 0$		$a > 0$				
$b > 0$		$b < 0$				$c = 0$		$b = 0$		$c=b=0$
$c>0$	$c<0$	$c>0$	$c<0$	$b>0$	$b<0$	$b>0$	$b<0$	$c>0$	$c<0$	
$x_1<0$ $x_2<0$	x^-, x^+ $\|x^-\|>\|x^+\|$	$x_1>0$ $x_2>0$	x^-, x^+ $\|x^+\|>\|x^-\|$	$x_1=x_2<0$	$x_1=x_2>0$	$x_1=0$ $x_2<0$	$x_1=0$ $x_2>0$	Φ	x_1^+, x_2^- $\|x_1^-\|=\|x_2^+\|$	$x_1=x_2=0$

1-jadval

⊕ Ushbu $6x^2 + 2x - 11 = 0$,

Tenglama yechim ildizlar ishorasi qanday.

► Tenglamada

► $\Delta = 2^2 - 4 \cdot 6\cdot(-11) = 268 > 0$, $a = 6 > 0$,

$b = 2 > 0$, $c = -11 < 0$, u holda tenglama yechimlari xar xil ishorali bo‘lib, bunda $|x^-| > |x^+|$. ◄

⊕ Ushbu $x^2 - 2x + c = 0$

Tenglamada C qiymatlari tenglama yechim ishoralariga qanday ta’sir qiladi.

► Tenglama diskriminanti

► $\Delta = (-2)^2 - 4 \cdot 1 \cdot c = 4 \neq 4c$ bo‘lib bunda:

1) $c < 0$ da $\Delta > 0$, tenglamada $a = 1 > 0$, $b = -2$, u holda 1-jadvaldan ko‘rinadiki, tenglama ildizlari har xil ishorali bo‘ladi va $|x^+|>|x^-|$, $a1 > 0$

2) $0 < C < 1$da $\Delta > 0$ uchun tenglamada $a = 1 > 0$, $b = -2$, u holda tenglama ikkita ildizlari musbat ishorali,

3) $c = 1$ da $\Delta = 0$, tenglamada $a > 0$, $b = -2$ u holda tenglama ildizlari musbat va karrali $x_1 = x_2$,

4) $c > 1$da $\Delta < 0$ uchun tenglama haqiqiy yechimlarga ega emas. ◄

⊕ a ning shunday qiymati topilsinki $(a - 7)x - ax^2 - 9 = 0$, tenglama karrali manfiy yechimga ega bo‘lsin.

▶ tenglamani (– 1) ga ko‘paytirib, diskriminantini hisoblaymiz.

$ax^2 - (a - 7)x + 9 = 0$, bunda

$\Delta = [-(a-7)]^2 - 4 \cdot a \cdot 9 = a^2 - 14a + 49 - 36a = a^2 - 50a + 49$.

karrali yechim hosil bo‘ladi agar $\Delta = 0$ bo‘lsa, demak $a^2 - 50a + 49 = 0$ tenglama yechimlari $a_1 = 1$, $a_2 = 49$.

Tenglama ildizlari manfiy bo‘lishi uchun $a = 1$ni olamiz.

$\Delta = 0$, $a = 1 > 0$, $b = -(1 - 7) = 6 > 0$, $c = 9 > 0$, bo‘lib tenglama yechimlari $x_1 = x_2$ va manfiy.◀

9.7. Kvadrat tenglama yechimini topishda qulay usullar.

Ba’zi bir kvadrat tenglamalarni berilgan ko‘rinishiga qarab, (oddiy) qulay usulda yechimlarini aniqlash mumkin, bu usullarni misollarda ko‘rib chiqamiz.

1. To‘g‘ri kelsa qisqa ko‘paytirish formulalaridan foyda-lanish.

▶ $4x^2 - 12x + 9 = 0$,

kvadrat tenglama tenglama chap tomonini

$(a - b)^2 = a^2 - 2ab + b^2$ formula ko‘rinishda yozish mumkin

$4x^2 - 12x + 9 = (2x - 3)^2 = 0$ bo‘lib, bundan

$2x - 3 = 0$ yoki $x = 1{,}5$. Tenglama karrali 1,5 yechimga ega ekan.◀

2. Tenglamada to‘la kvadrat ajratish bilan

$[(a \pm b)^2 = a^2 \pm 2ab + b^2]$ formuladan foydalanib,

<u>Masalan:</u>

$$100x^2 - 160x + 63 = 0$$

$100x^2 - 160x + 63 = (10x - 8)^2 - 1$ Kvadrat tenglamaning chap tomonidan to‘la kvadrat ko‘rinishda bo‘ladi, demak $(10x - 8)^2 = 1$ dan $10x - 8 = \pm 1$ bo‘lib, kvadrat tenglama ildizlari $x_1 = 0{,}9$ va $x_2 = 0{,}7$ ko‘rinishda bo‘ladi.

3. Tenglamadagi ifodalarni qo‘shiluvchilarga ajratib, ko‘paytuvchilarga keltirish bilan:

<u>Masalan:</u>

$$x^2 - 14x + 45 = 0,$$

▶ Tenglamaning chap tomonidagi $-14x$ ni ikkita qo‘shiluvchi ko‘rinishda yozib, ko‘paytma ko‘rinishga keltirish mumkin:

$x^2 - 14x + 45 = x^2 - 5x - 9x + 45 = x(x - 5) - 9(x - 5) = (x - 5)(x - 9) = 0$ bo‘lib, tenglama ikkita $(x - 5) = 0$ va $(x - 9) = 0$ tenglamalarga teng kuchli bo‘lib yechimlari $x_1 = 5$, $x_2 = 9$.◀

4. Viet teoremasidan foydalanish bilan:

<u>Masalan:</u>

$2000x^2 - 2001x + 1 = 0$,

▶ Tenglama ildizlarini topish sodda yo‘li Viet teoremasidan foydalanib, tenglamani keltirgan kvadrat tenglama ko‘rinishda yozib olamiz.

$$x^2 - \frac{2001}{2000}x + \frac{1}{2000} = 0,$$

tenglama ildizlari x_1 va x_2 deb qarasak $\begin{cases} x_1 + x_2 = \dfrac{2001}{2000} \\ x_1 \cdot x_2 = \dfrac{1}{2000} \end{cases}$

Mantiqiy fikr yuritish bilan aniqlash qiyin emas. $x_1 = 1$ va $x_2 = \dfrac{1}{2000}$.

5. Agar (1) to'la kvadrat tenglama $a + b + c = 0$, bo'lsa, (1) tenglamaning bitta yechimi 1 ga ikkinchi yechimi $x_2 = \dfrac{c}{a}$ ga teng bo'ladi

Masalan:

$$3x^2 + 35x - 38 = 0,$$

► Tenglamada $a + b + c = 3 + 35 - 38 = 0$ demak, tenglamaning bitta yechimi $x = 1$, ikkinchi yechimi $x_2 = \dfrac{c}{a} = -\dfrac{38}{3}$ ◄

6. Agar (1) to'la kvadrat tenglamada $a - b + c = 0$, bo'lsa, (1) tenglamaning bitta yechimi (-1) ga ikkinchi yechim esa $x_2 = -\dfrac{c}{a}$ ko'rinishda bo'ladi

Masalan:

$67x^2 - 105x - 172 = 0,$

► Berilgan tenglama uchun $a - b + c = 67 - (-105) - 172 = 172 - -172 = 0$, demak $x_1 = -1$,

$$x_2 = -\frac{c}{a} = -\frac{-172}{67} = \frac{172}{67}. ◄$$

7. To'la kvadrat tenglama yani o'zgaruvchi kiritib, tenglama yechimini aniqlash mumkin.

Masalan:

$(2x - 1)^2 - 14x + 13 = 0,$

► Tenglamada shakl almashtirib

$$(2x - 1)^2 - 7(2x - 1) + 6 = 0,$$

ko'rinishda yozish mumkin, agar $2x - 1 = y$ desak, $y^2 - 7y + 6 = 0$ kvadrat tenglama ildizlari $y_1 = 6$, $y_2 = 1$, bo'lib almashtirish $2x - 1 = 6$, va $2x - 1 = 1$ teng kuchli tenglamalardan $x_1 = 3{,}5$ va $x_2 = 1$ berilgan tenglama yechimlari hosil bo'ladi. ◄

36) b ning qanday qiymatlarida $x^2 - 2bx = 1 - b^2$, tenglama yechimlari (1;5) oraliqda har xil ildizga ega bo'ladi.

► Berilgan tenglikda ozod hadni tenglikning chap tomoniga o'tkazib, kvadrat tenglama diskriminantini hisoblaymiz:

$$x^2 - 2bx + b^2 - 1 = 0$$

$\Delta = (-2b)^2 - 4(b^2 - 1) = 4b^2 - 4b^2 + 4 = 4$, u holda, $x_{1,2} = \frac{2b \pm 2}{2} = b \pm 1$,

yoki $x_1 = b + 1$, $x_2 = b - 1$ tenglama ildizlari (2;5) oraliqda bo'lishi uchun b parametr (2;4) da o'zgarishi kerak ekan. ◄

37) Agar, ushbu $2x^2 - 14x + c = 0$, tenglamaning bitta ildizi, ikkinchi ildizidan 2,5 martta katta bo'lsa, tenglamaning ikkinchi ildizi va c parametr topilsin.

► Tenglamani keltirilgan tenglama ko'rinishida yozib, masala sharti va Viet teoremasidan foydalanamiz.

$x^2 - 7x + \frac{c}{2} = 0$ bo'lib

$\begin{cases} x_1 + x_2 = 7 \\ x_1 - x_2 = \frac{c}{2} \\ x_1 = \frac{5}{2}x_2 \end{cases}$ sistemada uchinchi tenglikni birinchi tenglikka qo'yamiz:

$\frac{5}{2}x_2 + x_2 = 7$ dan $x_2 = 2$ bu tenglamaning ikkinchi yechimi, endi sistemaning uchinchi tenglikdan $x_1 = \frac{5}{2}x_2 = \frac{5}{2} \cdot 2 = 5$, bu berilgan tenglamaning yana bitta yechimli bo'lib, $x_1 \cdot x_2 = \frac{c}{2}$ dan parameter topiladi. $c = 2x_1 \cdot x_2 = 2 \cdot 2 \cdot 5 = 20$.

38) K ning qanday qiymatida $x^2 + kx + 1 = 0$ va $x^2 + x + k = 0$ tenglamalar bitta umumiy haqiqiy yechimga ega bo'ladi.

► Tenglamalarning umumiy yechimi $x = a$ bo'lsin. U holda

$a^2 + a + k = 0$, va $a^2 + ka + 1 = 0$ bo'lib, birinchi tenglikdan ikkinchi tenglikni ayirsak, $k(1 - a) + a - 1 = 0$, $k \neq 1$ da $a = 1$ bo'ladi.

$a = 1$ ni tenglamaga qo'ysak ,$1 + 1 + k = 0$ yoki $(1 + k + 1 = 0)$ tenglikdan $k = -2$.

Demak $k = -2$ da tenglamalar bir xil bitta haqiqiy $x = 1$ yechimga ega bo'ladi. Agar $k = 1$ desak, tenglamalar $x^2 + x + 1 = 0$ bo'lib, haqiqiy yechim mavjud emas, ◄

39) Ushbu $ax - 20x + c = 0$, tenglama bitta 2 yechimga ega bo'lsa, c parametrlar topilsin.

► Viet teoremasidan foydalanamiz, masala shartiga ko'ra $x_1 = x_2 = 2$, holda $-\frac{-20}{2\cdot a} = 2$ va $\frac{c}{a} = 4$ bu tengliklardan $a = 5$ va $c = 20$ bo'lish kerak ekan. ◄

40) K ning qanday qiymatlarda $2x^2 + (ax + 1)(a - 2) + (x - a - 1) = 0$ Tenglama ildizlari absolyut qiymatlarida

$2x^2 + (ax + 1)(a - 2) + (x - a -) = 0$

tenglama ildizlari absolyut qiymatlari bo'yicha o'zaro teng bo'ladi.

► 1-jadvaldan foydalanamiz $|x_1^+| = |x_2^-|$ uchun $ax^2 + bx + c = 0$ tenglamada $b = 0$, $c < 0$ bo'lishi kerak edi. Tenglamani soddalshtirib olsak $2x^2 + (a^2 - 2a + 1)x - 3 = 0$.

$b = a^2 - 2a + 1$, $c = -3$ bo'lib, $b = 0$ bo'ladi $a = 1$ bo'lsa. Tenglama ildizlari $|x_1^+| = |x_2^-|$ bo'ladi agarda $a = 1$ bo'lsa. ◄

⊕ x_1 va x_2

$$2x^2 - 5x + 1 = 0$$

tenglamaning yechimlari bo'lsa, $\frac{x_2}{x_1} + \frac{x_1}{x_2}$ nimaga teng.

► Viet teoremasidan foydalanib yozamiz:

$$\begin{cases} x_1 + x_2 = \frac{5}{2} \\ x_1 \cdot x_2 = \frac{1}{2} \end{cases}$$

Birinchi tenglikning ikki tomonini kvadratga ko'paytirib ikkinchi tenglikdan foydalanamiz:

$$(x_1 + x_2)^2 = \left(\frac{5}{2}\right)^2 \to x_1^2 + 2x_1x_2 + x_2^2 = \frac{25}{4} \text{ dan}$$

$x_1^2 + x_2^2 = \frac{25}{4} - 2x_1 \cdot x_2 = \frac{25}{4} - 2\cdot\frac{1}{2} = \frac{21}{4}$. Endi $\frac{x_2}{x_1} + \frac{x_1}{x_2}$ ni

hisoblash mumkin $\frac{x_2}{x_1} + \frac{x_1}{x_2} = \frac{x_2^2 + x_1^2}{x_1x_2} = \frac{\frac{21}{4}}{\frac{1}{2}} = \frac{21}{2} = 10{,}5$

Javob: 10,5 ◄

11-MAVZU. KVADRAT TENGLAMAGA KELINADIGAN RATSIONAL TENGLAMALAR HAQIDA

Kvadrat tenglamaga kelinadigan bir o'zgaruvchili ratsional tenglamalarning ba'zi bir ko'rinishlari haqida:

1) Tenglamaning chap tomonini qavsli ko'paytmalar, o'ng tomoni nol bo'lsa qavslar ochilmasdan har biri nolga tenglanadi, berilgan tenglama yechimi, shu har qavsli ifodani nolga aylantiruvchi sonlar to'plamidan iborat bo'ladi.

⊕ Tenglamalar yechimi topilsin.

1) $(0,5x + 3,5) \cdot (x - 7) \cdot (2x^2 + 9) = 0$,

► Tenglamada har bir qavsni nolga tenglashtiramiz:

$$\begin{matrix} 0,5x + 3,5 = 0 & & x = -7 \\ x - 7 = 0 & \text{dan} & x = 7 \\ 2x^2 + 9 = 0 & & 2x^2 + 9 = 0 \end{matrix}$$

haqiqiy yechim yo'q, demak, berilgan tenglamaning yechimlari $x_1 = -7$, $x_2 = 7$◄

2) $(3x + 1) \cdot (2 - 5x)^2 \cdot 4x = 0$,

► Bu tenglamaga teng kuchli tenglamalar $\left[\begin{matrix} 3x+1=0 \\ 2-5x=0 \\ x=0 \end{matrix}\right.$ ko'rinishda

bo'lib, tenglama yechimlari $x_1 = 0$, $x_2 = -\frac{1}{3}$ va $x_3 = 0,4$◄

2) Tenglamada qavsli qo'shiluvchilar bo'lsa, qavslar ochiladi (kerak bo'lsa qisqa ko'paytirish formulalaridan foydalaniladi), hamma ifodalar bir tomonda yozilib, soddalashtirish bilan sodda kvadratik tenglamaga kelinadi.

⊕ Tenglama yechimi aniqlansin.

1) $(3x - 1)(2x - 2) = (4 - x)^2$,

► Tenglamaning chap tomonida qavs ochiladi, o'ng tomoniga qisqa ko'paytirish formulasini tadbiq qilamiz. $6x^2 - 2x - 6x + 2 = 16 - 8x + x^2$ ifodani nolga tenglashtirib yozib, soddalashtirsak berilgan tenglamaga ekvivalent $5x^2 - 14 = 0$ tenglama hosil bo'lib, yechimlari $x_{1,2} = \pm\sqrt{\frac{14}{5}}$

2) $\frac{(4-x)^2}{3} + \frac{x(2-x)}{6} = \frac{(x-3)^2}{2} + \frac{5}{6}$

► Tenglamaning ikki tomonini 6 ga ko'paytirib qavslar ochiladi.

$2(16 - 8x + x^2) + 2x - x^2 = 3(x^2 - 6x + 9) + 5$, qo'shiluvchilarni bir tomonda yozib, soddalashtiriladi

$x^2 - 14x + 32 - 3x^2 + 18x - 32 = 0$ yoki $-2x^2 + 4x = 0$, berilgan tenglama bu chala kvadrat tenglamaga kuchli bo‘lib, yechimlari $x_1 = 0$ va $x_2 = 2$

3) $x(x + 2)^2 - 3(2x^2 + 1) = (x - 1)^3$ ◄

► Qisqa ko‘paytirish formulalaridan foydalanib, qavslar ochilib, soddalashtiriladi.

$x(x^2 + 4x + 4) - 6x^2 - 3 = x^3 - 3x^2 + 3x - 1$, yoki $x^2 + x - 2 = 0$,

Bu teng kuchli tenglama ildizlari $x_1 = -2$, $x_2 = 1$ ◄

3. Tenglamada yangi o‘zgaruvchi kiritib, kanonik ko‘rinishdagi kvadrat tenglamaga keltirish mumkin bo‘lgan hol.

⊕ Tenglama yechimi topilsin.

1) $(x^2 - 8)^2 + 4(x^2 - 8) - 5 = 0$ agar $x^2 - 8 = y$ desak $y^2 + 4y - 5 = 0$

Kanonik ko‘rinishdagi kvadrat tenglamaga kelinadi, bu tenglama yechimlari $y_1 = 1$ va $y_2 = -5$. Bu yordamchi tenglama ildizlarini almashtirishga qo‘yib, boshlang‘ich tenglama yechimlari topiladi.

$x^2 = y + 8$ dan $x^2 = 1 + 8$ bo‘lib $x_1 = 3, x_2 = -3$, endi $y = -5$da $x^2 = -5 + 8$, bo‘lib bundan $x_3 = \sqrt{3}, x_4 = -\sqrt{3}$ ◄

2) $\dfrac{15}{y+3+y^2} = 1 + y + y^2$,

► Berilgan tenglamada $y^2 + y + 1 = z$ deb almashtirish bajarsak tenglama $\dfrac{15}{z+2} = z$, ko‘rinishga kelinadi. $z + 2 \neq 0$ deb tenglamani $(z + 2)$ ga ko‘paytirsak, $z^2 + 2z - 15 = 0$, yordamchi tenglama ildizlari

$z_1 = 3$, $z_2 = -5$ U holda bu sonlarni almashtirishga qo‘ysak,

1) $y^2 + y + 1 = 3$, tenglama ildizlari $y_1 = -2$, $y_2 = 1$

2) $y^2 + y + 1 = -5$ da $y^2 + y + 6 = 0$ $\Delta = -23$ haqiqiy yechim mavjud emas. Demak, berilgan tenglama yechimlari $y_1 = -2$ va $y_2 = 1$ ◄

3) $3\left(\dfrac{x}{x-1}\right) - 3\left(1 - \dfrac{1}{a}\right) = 2,5$

► Tenglamani shakl almashtirib yozib olamiz.

$6\dfrac{x}{x-1} - 6\dfrac{x-1}{x} - 5 = 0$, $x \neq 1$, $x \neq 0$ deb $\dfrac{x}{x-1} = y$ almashtirish bajarsak, tenglama $6y - \dfrac{6}{y} - 5 = 0$, yoki $6y^2 - 5y - 6 = 0$ ko‘rinishga keladi. $(y \neq 0)$ da. Bu tenglama yechimlari $y_1 = \dfrac{3}{2}$ va $y_2 = -\dfrac{2}{3}$ bu sonlarni almashtirishga qo‘ysak

1) $\dfrac{x}{x-1} = \dfrac{3}{2}$ dan $2x = 3x - 3$ yoki $x = 3$

2) $\frac{x}{x-1} = -\frac{2}{3}$ dan $3x = -2x + 2$ yoki $x = \frac{2}{5}$ bo‘lib, tenglama yechimlari $x_1 = 3$, $x_2 = 0{,}4$

11.4. Agar tenglamada modulli qo‘shiluvchi bo‘lsa, u holda tenglama yechimini topishda modul xossalari va modulli tenglama yechimini topish usullarini ham hisobga olish kerak bo‘ladi.

⊕ Tenglama yechimi topilsin.

1) $3|x|^2 + 5x - 2 = 0$

► Modul xossasida $|x^2| = |x|^2 = x^2$ edi, demak berilgan tenglamani $3x^2 + 5x - 2 = 0$ ko‘rinishda yozish mumkin, bu berilgan kvadrat tenglamaga teng kuchli bo‘lib, yechimlari $x_1 = -2$, $x_2 = \frac{1}{3}$ teng.

2) $|3x - x^2 + 5| = -4$,

► Tenglama yechimga ega emas, sababi modulli ifoda manfiy bo‘lishi mumkin emas. ◄

3) $|x^2 - 6x + 5| = 12$,

► Ifoda moduli manfiy bo‘lishi mumkin emasligini hisobga olsak, (chunki $-x^2 + 6x - 5 = 12$, tenglamaning haqiqiy yechimi mavjud emas) tenglama quyidagi tenglamaga teng kuchli bo‘ladi. $x^2 - 6x + 5 = 12$ yoki $x^2 - 6x - 7 = 0$,

Tenglamaga kelinib tenglama yechimi $x_1 = -1$ va $x_2 = 7$.

4) $6x - x^2 - 8 = |x - 4|$,

► $|x - 4|$ ifoda $x = 4$ da nol bo‘ladi, tenglama yechimini topishda oraliq metodidan foydalanamiz.

1. $x < 4$ da $6x - x^2 - 8 = -(x - 4)$ yoki $x^2 - 7x + 12 = 0$ bu yordamchi tenglama yechimlari $x = 3$ va $x = 4$, $x = 4$ ko‘rsatilgan oraliqqa tegishli emas.

2. $x \geq 4$ da $6x - x^2 - 8 = x - 4$ bo‘lib, $x^2 - 5x + 4 = 0$ tenglama yechimi $x = 1$, $x = 4$, bu yerda $x = 1$ ko‘rsatilgan oraliqda emas. Berilgan tenglama yechimlari $x_1 = 3$ va $x = 4$. ◄

5) $4 + |x| - x^2 = |x^2 + 2 - 3x|$

► Modul ichida noma’lum o‘zgaruvchilar qatnashgan, tenglamani. $(-\infty; 0)$,$(0; 1)$,$(1; 2)$ va $(2; +\infty)$ oraliqlarda qaraymiz:

1) $x < 0$ da $x^2 - 3x + 2 = -x - x^2 + 4$ sababi $(x^2 - 3x + 2 = (x - 1)(x - 2)$ yoki $x^2 - x - 1 = 0$ bu tenglama yechimlari $x = \frac{1 \pm \sqrt{5}}{2}$ bo‘lib $x = \frac{1 + \sqrt{5}}{2}$ bu oraliqda emas.

2) $0 < x < 1$ da $4 + x - x^2 = x^2 + 2 - 3x$ yoki $x^2 - 2x - 1 = 0$ bo‘lib bu tenglama yechimlari $x = 1 \pm \sqrt{2} 0 < x < 1$ oraliqda emas.

3) $1 < x < 2$ da $-(x^2 - 3x + 2) = x - x^2 + 4$ yoki $3x - 2 = x + 4$ tenglama yechimi $x = 3$ ko'rsatilgan oraliqda emas.

4) $x > 2$ da $x^2 - 3x + 2 = x - x^2 + 4$ bo'lib, bundan $x^2 - 2x - 1 = 0$ yordamchi tenglama yechimi $x = 1 \pm \sqrt{2}$ bunda $x = 1 - \sqrt{2}$ ko'rsatilgan oraliqda yo'q. Shunday qilib berilgan tenglamaning yechimlari

$$x_1 = \frac{1-\sqrt{5}}{2} \text{ va } x_2 = 1 + \sqrt{2}. ◄$$

11.5. Tenglamada ildizli ifodalar bo'lsa, ildizli qo'shiluvchilarning mavjudligini hisobga olish kerak, bunda topilgan son ildizning mavjudlik to'plamida bo'lish kerak.

⊕ Tenglama yechimi topilsin.

1) $9x + 4\sqrt{x} - 5 = 0$,

► Tenglama mavjudlik sohasi $(0 + \infty)$ tenglamada $\sqrt{x}$ bo'lganligi uchun. Tenglamada $\sqrt{x} = y$, almashtirish bajarsak tenglama

$$9y^2 + 4y - 5 = 0,$$

Ko'rinishdagi kvadrat tenglamaga kelinadi, bu tenglamaning ildizlari

$$y_1 = -1 \text{ va } y_2 = \frac{5}{9}$$

Endi $\sqrt{x} = v$ dan, berilgan tenglama yechimi topiladi.

1) $\sqrt{x} = -1$, yechimi mavjud emas.

2) $\sqrt{x} = \frac{5}{9}$ dan $x = \frac{25}{81}$ yechimni hosil qilamiz. ◄

3) $3x - 10\sqrt{x+1} + 6 = 0$,

► Bu tenglamada $x + 1 \geq 0$ ni talab qilib, shakl almashtirib yozsak

$3(x+1) - 10\sqrt{x+1} + 3 = 0$ $\sqrt{x+1} = y$ almashtirish yordamida $3y^2 - 10y + 3 = 0$, kvadrat tenglamaga kelinadi, bu tenglama ildizlari $y_1 = 3$ va $y_2 = \frac{1}{3}$ bo'lib, almashtirishdan:

1) $\sqrt{x+1} = 3$ dan $x + 1 = 9$ yoki $x_1 = 8$

2) $\sqrt{x+1} = \frac{1}{3}$ dan $x + 1 = \frac{1}{9}$ yoki $x_2 = -\frac{8}{9}$ tenglama yechimlari 8 va $\left(-\frac{8}{9}\right)$

3) $x^2 + 4 - 5\sqrt{x^2 - 2} = 0$

▶ Tenglamada $x^2 - 2 \geq 0$ ni talab qilib, tenglamani $x^2 - 2 - 5\sqrt{x^2 - 2} + 6 = 0$, ko'rinishda yozish mumkin. Agar $\sqrt{x^2 - 2} = y$ desak, tenglama kanonik ko'rinishdagi kvadrat tenglamaga kelinadi.

$y^2 - 5y + 6 = 0$ bu tenglama ildizlari $y_1 = 2$ va $y_2 = 3$ bo'lib, almashtirishdan:

1) $\sqrt{x^2 - 2} = 2$, dan $x^2 - 2 = 4$ yoki $x = \pm\sqrt{6}$

2) $\sqrt{x^2 - 2} = 3$, dan $x^2 - 2 = 9$ yoki $x = \pm\sqrt{11}$ bo'lib, berilgan tenglama yechimlari $x = \pm\sqrt{6}$ va $x = \pm\sqrt{11}$.

11.6. Ifoda algebraik ratsional kasrlar tengligi ko'rinishda berilgan bo'lsa, noma'lum qatnashgan mahraj nol bo'lganligini talab qilib, proporsiyaning xossasidan foydalanishi mumkin.

⊕ Tenglamalar yechilsin.

1) $\dfrac{3x^2 - 14x}{x - 4} = \dfrac{8}{4 - x}$

▶ Oldin $x - 4 \neq 0$ yoki $x \neq 4$ deb, tenglamaning ikki tomonini $(x - 4)$ ga ko'paytiramiz.

$$3x^2 - 14x = -8 \rightarrow 3x^2 - 14x + 8 = 0$$

bu tenglama ildizlari $x_1 = 4$ va $x_2 = \dfrac{2}{3}$ berilgan tenglama uchun $x = 4$ chet ildiz bo'lib, tenglama yechimi $x_2 = \dfrac{2}{3}$ bo'ladi.◀

2) $\dfrac{a - x}{1 - ax} = \dfrac{1 - bx}{b - x}$

▶ Oldin $1 - ax \neq 0, b - x \neq 0$ yoki $x \neq b, \dfrac{n\pi}{3}$ deb, proporsiya xossasidan foydalanamiz

$$(a - x)(b - x) = (1 - ax)(1 - bx),$$

Qavsni ochib soddalashtiramiz.

$ab - bx - ax + x^2 = 1 - ax - bx + abx^2$ dan $(1 - ab)\,x^2 = 1 - ab$, agar $ab \neq 1$ bo'lsa, $x^2 = 1$ yoki $x = \pm 1$, agar $ab = 1$ bo'lsa, $x^2 = 1$ yoki $x = \pm 1$, agar $ab = 1$ bo'lsa $x \in R$. Demak, parametrli tenglamaning yechimlari $ab \neq 1$ da $x_1 = 1, x_2 = -1$; $ab = 1$ da $x \neq b$ dan boshqa barcha haqiqat sonlar.

3) $\dfrac{3 - 7x}{4 + 2x} = \dfrac{1{,}5 - 3{,}5x^2}{x + 2}$

► tenglama yechimini aniqlashda $x+2\neq 0$ deb ◄

Tenglamaning ikki tomonini $2(x+2)$ ga ko'paytiramiz $3-7x=$ $=2(1,5-3,5x^2)$ soddalashtirsak, $7x^2-7x=0$ bo'lib $x_1=0$ bo'lib, $x_1=0$ va $x_2=1$ yechimlarni olamiz:

4) $\frac{1x^2-11}{x-2}=x$,

► $x\neq 2$ deb, tenglamani $(x-2)$ ga ko'paytiramiz: $|x^2-1|-x(x-2)=0$,

Noma'lum modul ichida qatnashganligi uchun, tenglamani $(-\infty;-1)$, $(-1;1)$ va $(1;+\infty)$ oraliqda qaraymiz:

1) $x<-1$ da $x^2-1-x^2+2x=0$ bo'lib $x=\frac{1}{2}$ bu oraliqda emas,

2) $-1<x<1$ da $-(x^2-1)-x^2+2x=0$ yoki $-2x^2+2x+1=0$, bu tenglama yechimlari $x=\frac{1\pm\sqrt{3}}{2}$ bo'lib, bunda $x=\frac{1+\sqrt{3}}{2}$ ko'rsatilgan oraliqda emas.

4) $x>1$ da $x^2-1-x^2+2x=0$ bo'lib, yana $x=\frac{1}{2}$ hosil bo'ladi, bu ko'rsatilgan $(1+\infty)$ oraliqda emas. Natijada berilgan tenglama yechimi $x=0,5\left(1-\sqrt{3}\right)$ bo'ladi. ◄

11.7. Tenglama ratsional algebraik kasrlarning algebrik yig'indisi ko'rinishida berilgan bo'lsa, noma'lum qatnashgan mahrajlarning nol bo'lmasligini hisobga olib, ratsional kasrlar ustida amallar bajarib (bu [3]) da to'la ko'rsatma berilgan), keyin soddalashtirish natijasida tenglamani normal ko'rinishga keltirish mumkin.

⊕ Tenglama yechilsin

1) $\frac{4-x}{x-11}+\frac{3}{x}=\frac{33}{11x-x^2}$,

► Ratsional kasr tenglama yechimi topishda $x-11\neq 0$, $x\neq 0$ deb tenglamani nolga tenglab, tenglamadagi kasrlar uchun eng kichik umumiy bo'linuvchi $x(x-11)$ ga tenglamaning ikki tomonini ko'paytirib soddalashtiramiz $x(4-x)+33+3(x-11)=0$, dan $7x-x^2=0$ yordamchi tenglama yechimi $x=0$ va $x=7$. Bunda $x=0$ berilgan tenglama uchun chet ildiz, tenglama yechimi $x=7$ bo'ladi. ◄

2) $\frac{6}{1-2x}+\frac{27}{2x^2+7x-4}+\frac{2x}{x+4}=-1$,

► Viet teoremasiga asosan $(2x^2+7x-4)$ uch hadni $(x+4)(2x-1)$ ko'rinishda yozish mumkin, endi $(2x^2+7x-4)=(x+4)(2x-1)\neq 0$, yani $x \neq -4$ va $x \neq \frac{1}{2}$ deb, tenglamaning ikki tomonini $(x+4)(2x-1)$ ga ko‘paytiramiz.

$$-6(x+4)+27+2x(2x-1)+(x+4)(2x-1)=0$$

qavslarni ochib soddalashtirsak $6x^2-x-1=0$ hosil bo‘ladi. Bu tenglama yechimlari $x_1=\frac{1}{2}$ va $x_2=-\frac{1}{3}$ Bunda $x=\frac{1}{2}$ berilgan tenglamaga yechim emas, tenglama yechimi $x=-\frac{1}{3}$ ◄

3) $$\frac{2x+2}{4-x^2}+\frac{x^2+2x+4}{x^3+2x^2+4x+8}+\frac{x^2-2x+4}{x^3-2x^2+4x-8}=0,$$

► Tenglamada mahrajdagi ifodalarni ko‘paytma ko‘rinishda yozib olamiz.

$$4-x^2=(2-x)(2+x)x^3+2x^2+4x+8=x^2(x+2)+$$
$$+4(x+2)=(x^2+4)(x+2),$$
$$x^3-2x^2+4x=8=x^2(x-2)+4(x-2)=(x^2+4)(x-2),$$

tenglama $$\frac{2x+2}{(2-x)(2+x)}+\frac{x^2+2x+4}{(x^2+4)(x+2)}+\frac{x^2-4x+4}{(x^2+4)(x-2)}=0,$$

ko‘rinishida bo‘lib, kasr ratsional ifodada eng kichik umumiy bo‘linuvchi $(x^2+4)(x+2)(x-2)$, endi $x \neq 2$, $x \neq -2$ deb tenglamani $(x^2+4)(x+2)(x-2)$ ga ko‘paytiramiz $-(x^2+4)(2x+4)+(x-2)(x^2+2x+4)+(x+2)(x^2-2x+4)=0$ qavslarni ochib soddalashtiramiz $-2x^2-8x-8=0$ yoki $x^2+4x+4=0$ Bu tenglama karrali $x_1=x_2=-2$ yechimga ega. Lekin $x=-2$ berilgan tenglamaga yechim emas, demak berilgan tenglama yechimga ega emas, ya’ni $x \in \phi$ ◄

4) $$\frac{5}{4x^2+4x+4}+\frac{3x}{1-x^3}+\frac{1}{2(1-x)}=0$$

► Bu ifodada eng kichik umumiy bo‘linuvchi $4(1-x^3)$, sababi qisqa ko‘paytirish formulasida $a^3-b^3=(a-b)(a^2+ab+b^2)$ edi. $x \neq 1$ deb tenglamaning $(1-x^3)$ ga ko‘paytiramiz

$$5(1-x)+3\cdot 4\cdot x+2(x^2+x+1)=0,$$

qavslarni ochib soddalashtirsak $2x^2+9x+7=0$ Bu tenglama berilgan tenglamaga teng kuchli bo‘lib, yechimlari

$$x_1=-1 \quad x_2=-3{,}5$$ ◄

5) $$\frac{x-a}{x-b}+\frac{x-b}{x-a}+2=0,$$

▶ Parametrli tenglama yechimni topishda $x \neq b$ va $x \neq a$ deb, tenglamani $(x-b)(x-a)$ ga ko'paytiramiz

$$(x-a)^2+(x-b)^2+2(x-b)(x-a)=0,$$

Qavslarni ochib, soddalashtirsak $4x^2-(4a+4b)x+(a+b)^2=0$

Bu tenlama karrali yechimga ega $x_1 = x_2 = \dfrac{a+b}{2}$

Demak, agar $a \neq b$ bo'lsa berilgan tenglamaning yechimi $x=\dfrac{a+b}{2}$ bo'ladi. Agar $a=-b$ bo'lsa tenglama yechimi mavjud emas.◀

9-, 10-, 11-MAVZULAR MASHQLARI

$2; -0,5; 0,(3)$ va $1\frac{1}{6}$ sonlardan qaysilari quyidagi tenglamaning ildizi:

103. 1) $2x^2-5x+2=0$, 2) $2x^2+3x-2=0$,

3) $6x^2+x-1=0$, 4) $12x^2-20x+7=0$, 5) $2x^2+8=0$.

104. $1; 2i; 0,(3); -0,5; (1-2i)$.

1) $9x^2-6x+1=0$, 2) $0,5x\left(0,5x+\dfrac{1}{4}\right)=0$,

3) $8+2x^2=0$, 4) $x^2-2x+5=0$, 5) $3x^2-2x-1=0$.

105. b xaqiqiy sonning qanday qiymatlarida quyidagi tenglamalar a) kvadrat tenglama b) chiziqli tenglama bo'ladi:

1) $(2b^2-3b-2)x^2+(b^2+5b+4)x+5=0$,

2) $(b^2-b-2)x^2+2x-5=0$.

106. Ildizlari $\left(2-\sqrt{3}\right)$ va $\sqrt{5}$ bo'lgan ratsional koeffisentli chala kvadrat tenglama mavjudmi?

107. Ildizlari (nol bo'lmagan) 1) o'zaro teng bo'lgan;

2) ildizlari 2 va $\sqrt{2}$ bo'lgan chala kvadrat tenglama mavjudmi.

108. Ildizlari 1) 3 va $(2-i)$; 2) $(1-i)$ va $(3+2i)$ bo'lgan ratsional koeffisentli to'la kvadrat tenglama mavjudmi.

Qaysi tenglamalar 1) ratsional ildizga ega; 2) irratsional ildizga ega; 3) Kompleks ildizga ega.

109. 1) $x^2-8x+9=0$, 2) $7x^2-5x-2=0$, 3) $2x^2-9x+13=0$.

110. 1) $9x^2-30x+25=0$, 2) $2x-3x^2+18=0$, 3) $3x^2+12x+11=0$.

Teng kuchli tenglamalarni ko'rsating; (Haqiqiy sonlar to'plamida)

111. 1) $0{,}5x^2-2=0$ va $4=x^2$; 2) $x^2-x=20$ va $(x-4)(x-5)=0$;

3) $(x+5)(x-5)=0$ va $(5-x)(x+5)=0$;

4) $(x-3)(x-2)=0$ va $\dfrac{(x-2)^2(x-3)}{(x-2)}=0$

112. 1) $(x-11)(x+10)+10=0$ va $(x+4)(x-5)=80$,

2) $(x-3)^3=2x(x^2+1)$ va $6+(2x+1)^2=x(x^2+3)$,

3) $(2x+1)^2=4x-3$ va $(2-x)^2+x(x+4)=-2x^2$.

Chala kvadrat tenglama ildizlari topilsin:

113. 1) $0,5(8x^2) = 0$, 2) $(-2x)^2 = -16$,

3) $[-0,(2)x]^2 = 4$, 4) $0,5x^2 - 2x = 0$,

5) $\dfrac{5x^2+9}{6} - \dfrac{4x^2-9}{5} = 3$, 6) $(x+2)^3 + 19 = (x+3)^3$.

114. 1) $6 - 0,1(6)x^2 = 0$, 2) $[1,(3)x]^2 = 0,(1)$,

3) $-2x^2 - 8 = 0$, 4) $2x^2 + 4x = -3(x^2 - 6x)$,

5) $3(x^2 - 11) + 2(74 - 2x^2) = 240$,

6) $(x-3)^3 + 2x(5x+1) = x^3 - (2x-1)^2 - 26$.

To'la kvadrat tenglama yechimlari topilsin:

115. 1) $5x^2 + 7x + 2 = 0$. 2) $8x^2 - 16x + 9 = 0$,

3) $3x^2 - 8x + 4 = 0$, 4) $(3x-1)(2x-2) = (x-4)^2$,

5) $(3x-1)^2 \cdot (2x-2) = (x-4)^2$, 6) $x^2 - 4\sqrt{2}x + 4 = 0$.

116. 1) $9x^2 + 24 + 16 = 0$, 2) $x^2 - 0,1(6)x - 8,5 = 0$,

3) $\sqrt{2}x^2 - 10x + 8\sqrt{2} = 0$, 4) $(x-3)^2 - (x-4)^2 - (x-5)^2 = x + 24$,

5) $(3x-4)^2-(5x+2)(2x+8)=0$, 6) $0{,}6x^2+0{,}8x-7{,}8=0$.

Xarfli koeffisentli tenglama yechilsin:

117. 1) $ax^2-(a+1)x+1=0$, 2) $x+\frac{1}{a}=a+\frac{1}{x}$,

3) $x^2-2(a+b)x+4ab=0$.

118. 1) $x^2-2ax+a^2-b^2=0$,

2) $abx^2-(a^2+b^2)x+ab=0$, 3) $(x-a)^2+(b-x)^2=(a-b)^2$.

Tenglamaning kompleks ildizlari topilsin:

119. 1) $0{,}5x^2+4=0$, 2) $x^2+3x+5=0$, 3) $x^2-(2+i)x+3+i=0$.

120. 1) $x^2-4x+6=0$, 2) $4x^2-4x+5=0$,

3) $x^2-6x+11=0$, 47) $x^2-(2+3i)x+1i-2=0$.

Tenglamani eng qulay usulda yeching:

121. 1) $x^2-8x+16=0$, 2) $4x^2+12x+9=0$,

3) $8x^2+15x-23=0$,4) $10x^2+53x+43=0$,

5) $x^2-32x+260=0$, 6) $x^2-281x+280=0$.

122. 1) $0{,}25x^2-3x+9=0$, 2) $0{,}5x^2-14x+98=0$,

3) $5x^2 + 9x - 14 = 0$, 4) $11x^2 + 14x - 25 = 0$,

5) $9x^2 + 43x + 34 = 0$, 6) $390x^2 - 391x + 1 = 0$.

Quyidagi tenglamalarni yechmay, tenglama ildizlar yig‘indisi va ko‘-paytmasini toping:

123. 1) $5x^2 + 12x + 7 = 0$, 2) $3x^2 + 12x + 11 = 0$, 3) $0,(6)x^2 + 2x - 1 = 0$.

124. 1) $4x^2 - 112x + 55 = 0$,

2) $\sqrt{3}x^2 - 12x - 7\sqrt{3} = 0$, 3) $9x^2 - 30x + 25 = 0$.

Kvadrat uchxadni chiziqli ko‘paytuvchilarga ajrating:

125. 1) $7x - x^2 - 12$, 2) $x^2 - 84x + 41$, 3) $6x^2 + 5x - 6$.

126. 1) $9x - 10 - 2x^2$, 2) $x^2 + (a + 4)x - (2a^2 + a - 3)$,

3) $x^2 - 6x + 11$, 4) $x^2 - 8x + 11$.

Berilgan tenglamani yechmay, tenglama koeffisentlari yordamida uning ildizlari ishorasi qanday bo‘lishini aniqlang:

127. 1) $2 - 20x^2 - 3x = 0$, 2) $2x^2 - 13x + 12 = 0$,

3) $x - 5x^2 - 7 = 0$, 4) $0,5x^2 - x - 4 = 0$.

128. 1) $15x^2 + x - 10 = 0$, 2) $5x^2 - x - 10 = 0$,

3) $4x^2 + 18x + 12 = 0$, 4) $0,3x^2 - 0,5x + 1 = 0$.

Modul qatnashgan kvadrat tenglama yechimi topilsin:

129. 1) $2x^2 - 3|x| = x$, 2) $|3x^2 - 7x + 4| = 3x^2 - 7x + 4$,

3) $|3x^2 - 7x + 6| = 7x - 6 - 3x^2$.

130. 1) $x^2 - |x| = x \cdot |x - 1|$, 2) $|2 - x - x^2| = 2 - x - x^2$,

3) $|3x^2 - 7x + 6| = 3x^2 - 7x + 6$, 4) $x^2 - 6x + |x - 4| + 8 = 0$.

Berilgan ildizlarga asosan ratsional koeffisentli kvadrat tenglamani tuzing:

131. 1) $\frac{5}{7}$ va $(-0,5)$, 2) $(-0,(3))$ va 3, 3) $(-1 + 3i)$, 4) $2 - 2i$.

132. 1) $-\frac{3}{4}$ va $(-0,8(3))$, 2) $(a + b)$ va $(a - b)$,

3) $(3 - i)$, 4) $2 + \sqrt{2}i$.

Berilgan ildizlarga ko‘ra kvadarat tenglamani tuzing:

133. 1) 4 va $(-i)$, 2) -2 va $1 - i$, 3) $2 + 3i$ va $1 - 3i$.

134. 1) 2 va $1 - \sqrt{3}$, 2) $\sqrt{2}$ va $\sqrt{6}$, 3) $2i$ va $2 - 4i$.

a ning qanday qiymatlarida quyidagi ifodalar o‘zaro teng bo‘ladi.

135. 1) $a^2 + 6$ va $3a^2 - a$; 2) $2a^2 - 1$ va $a^2 - 3$.

136. 1) $3a^2 - a$ va $2a^2 - 4a + 4$; 2) $a^2 + a - 3$ va $5(a - 2)$

137. a qanday bo'lganda $3x^2+(a^2+4a-5)x-4a=0$ tenglama ildizlar yig'indisi nol bo'ladi.

138. a qanday bo'lganda $0{,}5x^2+(3a+1)x+a^2-7a+12=0$ tenglama ildizlar ko'paytmasi nol bo'ladi.

139. a qanday bo'lganda $x^2-a^2x+3a=0$ tenglama x_1 va x_1 ildizlarda $x_1+x_2+x_1\cdot x_2$ ifoda eng kichik qiymatni qabul qiladi

140. a ning qanday qiymatlarida $x^2-4ax-a^2+4a=0$, tenglama ildizlar ko'paytmasi eng katta qiymatni qabul qiladi.

141. $ax^2+bx+4=0$, tenglama koeffisentlari qanday bo'lganda tenglama ildizlari (-2) va $(-0{,}25)$ ga teng bo'ladi.

142. $ax^2+6x+c=0$, tenglama koeffisentlari qanday bo'lganda tenglama ildizlari 3 va (-4) ga teng bo'ladi.

143. Kasrlarni qisqartiring:

1) $\dfrac{3x^2+16x-12}{10-13x-3x^2}$, 2) $\dfrac{2x^2-12x+20}{2i(1-3i)}$.

144. 1) $\dfrac{2x^2+9x-5}{4x^2-1}$, 2) $\dfrac{x^2+2x+5}{(2+i)i}$.

Ratsional kasr tenglamalarning yechimi topilsin.

1) $\frac{2x^2}{x-2}=\frac{6-7x}{2-x}$, 2) $\frac{x+2}{x-2}+\frac{x(x-4)}{x^2-4}=\frac{x-2}{x+2}-\frac{4(1+x)}{4-x^2}$,

3) $\frac{10}{x^2-4x-5}+\frac{x}{x+1}=\frac{3}{x-5}$, 4) $\frac{1}{x}+\frac{10}{5x-x^2}-\frac{x-3}{5-x}$.

146. 1) $\frac{x+3}{x-3}=\frac{2x+3}{x}$, 2) $\frac{2x}{x+4}+\frac{27}{2x^2+7x-4}-\frac{6}{1-2x}=-1$,

3) $\frac{1+2x}{6x^2-3x}-\frac{2x-1}{14x^2+7x}=\frac{8}{12x^2-3}$,

4) $\frac{x+1}{x^3-3x^2+x-3}+\frac{1}{x^4-1}=\frac{x-2}{x^3-3x^2x+3}$.

Tenglama ildizlar yig'indisi topilsin:

147. 1) $(x+3{,}5)(x-7)(x^2+9)=0$, 2) $(4-x^2)(3x-1)(2+x)=0$.

148. 1) $(x^2-1)(x-0{,}(6))(1{,}(3)+x)=0$,

2) $(0{,}5x^2-2)(x+1)(3-2x)=0$.

Yangi o'zgaruvchi kiritib, kanonik ko'rinishga keltirib tenglama ildizlar yig'indisini toping:

149. 1) $(3x-4)^2-5(3x-4)+6=0$, 2) $(x^2+2x)^2-2(x^2+2x)-3=0$,

3) $2(x^2+2x+1)^2-(x+1)^2=1$, 4) $2\left(x^2+\frac{1}{x^2}\right)=7\left(x+\frac{1}{x}\right)-9$.

150. 1) $2(x^2+4x)^2+17(x^2+4x)+36=0$,

2) $(x^2-4x+4)^2+2(x-2)^2=3$,

3) $(x^2-5x+7)^2=1+(x-2)(x-3)$, 4) $x^2+3x=\frac{8}{x^2+3x-2}$.

151. $0,(3)\,x^2-x-0,(4)=0$,1 tenglama ildizlariga qarama-qarshi ishorali ildizlarga ega bo'lgan tenglama topilsin.

152. $x^2+2,5x-1,5=0$,1 tenglama ildizlariga teskari va teskari ishorali qiymat ildizlariga ega bo'lgan tenglamani yozing.

153. Bitta (-3) yechimga ega bo'lgan $x^2+ax+c=0$,1 tenglama koeffisentlar yig'indisi topilsin.

154. Agar $[0,1(6)x-0,(3)]\cdot(0,3-1,5x)=0$ bo'lsa $[0,1(6)x-0,(3)]$ ifoda qanday qiymatlarni qabul qiladi.

155. a ning qanday qiymatlarida $ax^2-2ax+a^2=9$, tenglama $(-3;7)$ oraliqda har xil ildizlarga ega bo'ladi.

156. b qanday bo‘lganda $2x^2 + 9x + b = 0$ tenglama ildizlarida $|x\ | > |x^{+}|$ bo‘ladi.

157. b qanday bo‘lganda $4x^2 + bx + 1 = 0$ tenglama karrali musbat ildizga ega bo‘ladi.

158. a va b qanday bo‘lganda $\sqrt{2} \cdot ax^2 - 3bx - 2\sqrt{2} = 0$ tenglama ildizlari turlicha bo‘ladi.

159. $2bx - 0,5x + a(2b + 0,5a) = 0$ tenglama ildizlar ayirmasi nimaga teng.

160. $0,2\sqrt{3}x^2 - \sqrt{12}x + 0,(3) = 0$, tenglama ildizlarning o‘rta arifmetik qiymati nimaga teng.

12-MAVZU. YUQORI TARTIBLI BIR NOMA'LUMLI TENGLAMALAR

Tenglamada qatnashgan noma'lum o'zgaruvchining eng katta darajasi tenglama tartibini beradi. Tenglama ildizlari va ildizlar soni tenglama tartibiga bog'lik. Biz yuqori tartibli tenglamani ba'zi xususiy ko'rnishlaridan boshlab ko'rib chiqamiz.

12.1. Agar $ax^n + b = 0$, (1) $(n \in N)$

tenglamada $(a \neq 0, b \neq 0)$ *bo'lsa, (1) tenglamaga ikki hadli tenglama deyiladi.*

(1) Xaqiqiy koeffitsentli tenglama bo'lsa, bu tenglama

$x^n + \frac{b}{a} = 0$ ga teng kuchli tenglama bo'ladi. $x = \sqrt[n]{\left|\frac{a}{b}\right|}y$ almashtirish bajarsak ($\sqrt[n]{\left|\frac{b}{a}\right|}$ arifmetik ildiz), tenglama $\left|\frac{b}{a}\right|y^n + \frac{b}{a} = 0$ ko'rinishga keladi, bunda:

1) $\frac{b}{a} > 0$ bo'lsa, u holda tenglama $y^n + 1 = 0$ ko'rinishga kelinadi

2) $\frac{b}{a} < 0$ bo'lsa $\left(\left|\frac{b}{a}\right| = -\frac{b}{a}\right)$ tenglama $y^n - 1 = 0$, ko'rnishga kelinadi.

Xususiy hollar:

$n = 2$ da

1) $y^2 + 1 = 0$ yechimi $y = \pm i$,

2) $y^2 - 1 = 0$ yechimi esa $y = \pm 1$

$n = 3$ da

1) $y^3 + 1 = 0$,

Qisqa ko'paytirish formulasidan foydalansak $(y + 1)(y^2 - y + 1) = 0$ bu tenglama $y + 1 = 0$ va $y^2 - y + 1 = 0$

Teng kuchli ikkita tenglamaga ajralib yechimlari:

$y_1 = -1,\ y_{2,3} = 0,5\left(1 \pm \sqrt{3}i\right)$.

2) $y^3 - 1 = 0$, tenglama $y - 1 = 0$ va $y^2 + y + 1 = 0$ tenglamalarga ekvivalent bo'lib, yechimlari $y_1 = 1,\ y_{2,3} = 0,5(-1 \pm \sqrt{3}i)$.

$n = 4$ da

1) $y^4 + 1 = 0$ tenglamani $(y^2 + 1)^2 - 2y^2 = 0$ deb yozish mumkin, bu tenglamani qisqa ko'paytirish formulasiga asosan:

$$\left(y^2 + \sqrt{2}y + 1\right)\left(y^2 - \sqrt{2}y + 1\right) = 0$$

yozish mumkinligidan tenglama yechimlari:

$$y_{1,2} = 0,5\left(-\sqrt{2} \pm \sqrt{2}i\right) \text{ va } y_{3,4} = 0,5\left(\sqrt{2} \pm \sqrt{2}i\right).$$

2) $y^4 - 1 = 0$ tenglamani $(y^2 - 1)(y^2 + 1)^2 = 0$ deb yozish mumkin, bu tenglama yechimlari: $y_{1,2} = \pm 1, y_{3,4} = \pm i$.

Ixtiyoriy n da (1) tenglama ildizlarini aniqlashda kompleks sonlarda amallar bajarlik qoidasidan (bu [1] da to'la ko'rsatilgan) foydalanamiz.

Har qanday A sonni $A = A(\cos\varphi + i\sin\varphi)$ ko'rnishida yozish mumkin.

Xususiy xolda $A \in R$ va $A > 0$ bo'lsa

$A = A(\cos 0 + i\sin 0)$,

$A \in R$ va $A < 0$ bo'lsa

$A = |A|(\cos\pi + i\sin\pi)$, deb yozish mumkin.

$x^n + B = 0, (n \in N, B \in R)$,

Ikki hadli tenglama ildizlari

$$x_{k+1} = \sqrt[n]{|B|}\left(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{2}\right), \quad (2) \quad (k = \overline{0, n-1})$$

Formuladan topiladi.

Tenglama yechimlari topilsin.

1) $3x^6 - 192 = 0$

► Tenglamani $x^6 = 64$ ko'rnishda yozib olish mumkin, tenglama yechimlari (2) formulaga asosan:

$$x_k = \sqrt[6]{|64|}\left(\cos\frac{0 + 2k\pi}{6} + i\sin\frac{0 + 2k\pi}{6}\right) \quad (k = \overline{0,5})$$

$k = 0$ da $x_1 = \sqrt[6]{64}(\cos 0 + i\sin 0) = 2$,

$k = 1$ da $x_2 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$,

$k = 2$ da $x_3 = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$,

$k = 3$ da $x_4 = 2(\cos\pi + i\sin\pi) = -2$,

$k = 4$ da $x_5 = 2\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right) = 2\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$,

$k = 5$ da $x_6 = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) = 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$, ◄

2) $x^5 + 243 = 0$

► Tenglama ildizlarini topish, (-243) dan beshinchi ildiz chiqarish $x = \sqrt[5]{-243}$ bilan teng kuchli, bu tenglama ildizlari (2) formulaga asosan hisoblashda,

$-243 = |243|(\cos 180° + i\sin 180°)$ deb yozish mumkinligidan foydalanamiz.

$$x_{k+1}=\sqrt[5]{|243|}\left(\cos\frac{180°+2k\pi}{5}+i\sin\frac{180+2k\pi}{5}\right)\quad k=0,1,2,3,4$$

$k=0$ da

$$x_1=3\left(\cos\frac{180°}{5}+i\ sin\frac{180°}{5}\right)=3(\cos 36°+i\ sin36°),$$

$k=1$ da

$$x_2=3\left(\cos\frac{540°}{5}+i\ sin\frac{540°}{5}\right)=3(-\cos 72°+i\ sin72°),$$

$k=2$ da

$$x_3=3\left(\cos\frac{900°}{5}+i\ sin\frac{900°}{5}\right)=3(\cos 180°+i\ sin180°)=-3,$$

$k=3$ da

$$x_4=3\left(\cos\frac{1260°}{5}+i\ sin\frac{1260°}{5}\right)=3(\cos 252°+i\ sin252°)=$$

$= -3(\cos 72° + i \sin 72°)$,

$k=4$ da

$$x_5=3\left(\cos\frac{1625°}{5}+i\ sin\frac{1625°}{5}\right)=3(\cos 325°+i\ sin325°)=$$

$= 3(\cos 35° + i \sin 35°)$ ◄

3) $x^3-i=0$

► (2) formuladan foydalanish uchun i sonni

$i = \cos 90° + i\sin 90°$ trigonometrik ko'rnishda yozib olsak, u holda, tenglama ildizlari

$$x_{k+1}=\left(\cos\pi\ \frac{90°+2k\pi}{3}+i\ sin\frac{90°+2k\pi}{3}\right),$$ $k = 0,1,2$ formuladan

topiladi.

$k=0$ da $x_1=\left(\cos\frac{90°}{3}+i\ sin\frac{90°}{3}\right)=0,5(\sqrt{3}+i),$

$k=1$ da $x_2=\left(\cos\frac{450°}{3}+i\ sin\frac{450°}{3}\right)=0,5(-\sqrt{3}+i),$

$k=2$ da

$$x_3=\left(\cos\frac{810°}{3}+i\ sin\frac{810°}{3}\right)=\cos 270°n+i\ sin270°=-i$$ ◄

4) $x^4-1+i\sqrt{3}=0$

►Tenglamani $x^4 = 1 - i\sqrt{3}$ ko'rinishda yozib, ozod hadni trigonometrik ko'rnishda

$$1 - i\sqrt{3} = 2\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = 2\left(\cos\frac{5\pi}{3} + i\,sin\frac{5\pi}{3}\right),$$

yozib olsak, tenglama yechimlari

$$x_{k+1} = \sqrt[4]{2}\left(\cos\frac{300° + k360°}{4} + i\,sin\frac{300° + k360°}{4}\right),\quad k = 0,1,2,3$$

$k = 0$ da

$$x_1 = \sqrt[4]{2}\left(\cos\frac{300°}{4} + i\,sin\frac{300°}{4}\right) = \sqrt[4]{2}\left(\cos 75° + i\,sin 75°\right),$$

$k = 1$ da

$$x_2 = \sqrt[4]{2}\left(\cos\frac{660°}{4} + i\,sin\frac{660°}{4}\right) = \sqrt[4]{2}\left(\cos 165° + i\,sin 165°\right) =$$

$$= \sqrt[4]{2}\left(-\cos 15° + i\,sin 15°\right),$$

$k = 2$ da

$$x_3 = \sqrt[4]{2}\left(\cos\frac{1020°}{4} + i\,sin\frac{1020°}{4}\right) = \sqrt[4]{2}\left(\cos 255° + i\,sin 255°\right) =$$

$$= \sqrt[4]{2}\left(-\cos 45° - i\,sin 45°\right) = \sqrt[4]{2}\left(-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)$$

$k = 3$ da

$$x_4 = \sqrt[4]{2}\left(\cos\frac{1380°}{4} + i\,sin\frac{1380°}{4}\right) = \sqrt[4]{2}\left(\cos 345° + i\,sin 345°\right) =$$

$$= \sqrt[4]{2}\left(\cos 15° - i\,sin 15°\right)$$ ◄

12.2. Algebraik

$$ax^2n + bx^n + c = 0,\quad (3)\quad (n \in N)$$

tenglamada ($n \geq 2$) $a \neq 0$, $b \neq 0$ va $c \neq 0$ bo'lsa, bu tenglama uch hadli tenglama deyiladi.

$n = 2$ dagi uch hadli tenglamaga

$$ax^4 + bx^2 + c = 0,\quad (4)$$

bikvadrat tenglama deyiladi

$$x^2 = y,\quad (5)$$

almashtirishda (4) tenglama y ga nisbatan kvadrat tenglamaga kelinadi $ay^2 + by + c = 0$ bu tenglama yechimlari

$$y_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ bo'lib, (5) almashtirishga qo'yib,

(4) tenglama yechimlari aniqlanadi:

$$x_{1,2}=\pm\sqrt{\frac{-b+\sqrt{b^2-4ac}}{2a}},\quad x_{3,4}=\pm\sqrt{\frac{-b-\sqrt{b^2-4ac}}{2a}} \qquad (6)$$

Agar x_1, x_2, x_3, x_4 bikvadrat tenglama yechimlari bo'lsa,

$$x_1+x_2+x_3+x_4=0,\quad x_1\cdot x_2\cdot x_3\cdot x_4=\frac{c}{a}$$

$(ax^4 + bx2 + c)$ uch xadni $ax^4 + bx^2 + c = a(x-x_1)(x-x_2)(x-x_3)\cdot$ $\cdot(x-x_4)$, ko'rinishda yozish mumkin.

⊕ Tenglama yechimlari topilsin.

1) $x^4-5x^2-36=0$

▶ Bikvadrat tenglamada $x^2=y$ desak $y^2-5y-36=0$, tenglamaga kelinadi, bu tenglama yechimlari $y_1=-4$ va $y_2=9$. U holda berilgan tenglama yechimlari (6)ga asosan:

$$x_{1,2}=\pm\sqrt{-4}=\pm 2i,\quad x_{3,4}=\pm\sqrt{9}=\pm 3$$ ◀

2) $4x^4-5x^2-6=0$ tenglamaning a) ratsional yechimlari; b) irratsional yechimlari; c) mavhum yechimlari; d) kompleks yechimlari topilsin.

▶ $x^2=y$ almashtirishda tenglama $4y^2-5y-6=0$ tenglamaga kelinadi, bu tenglama yechimlari $y_1=2$, $y_2=-\frac{3}{4}$ u holda (2) formuladan

$$x_{1,2}=\pm\sqrt{2},\quad x_{3,4}=\pm\sqrt{-\frac{3}{4}}=\pm 0,5\sqrt{3}i$$

Demak tenglamaning:

a) ratsional yechimlari mavjud emas;

b) irratsional yechimlari $x_1=\sqrt{2}$, $x_2=-\sqrt{2}$;

c) mavhum yechimlari $x_3=0,5\sqrt{3}i$, $x_4=-0,5\sqrt{3}i$;

d) kompleks yechimlari mavjud emas. ◀

3) (x^4-3x^2-4) uchhadni chiziqli ko'paytuvchiga ajrating.

▶ Uchhadni tenglama ko'rnishda yozib, ildizlarini topamiz.

$x^4-3x^2-4=0$ da $x^2=y$ desak, $y^2-3y-4=0$ tenglama ildizlari $y_1=4$, $y_2=-1$, u holda (2) formuladan berilgan tenglama yechimlari

$x_{1,2}=\pm\sqrt{4}=\pm 2$, $x_{3,4}=\pm i$, bo'lib berilgan uch had $x^4-3x^2-4=$ $=(x-2)(x+2)(x-i)(x+i)$ ko'rnishdagi ko'paytuvchilarga ajraladi.◀

Uch hadli tenglamada $x^n=y$ desak tenglama $ay^2+by+c=0$ tenglamaga kelinib y_1 va y_2 yechimi topiladi, va (3) ikkita $x^n=y_1$ va $x^n=y_2$ ikki hadli tenglamaga kelinadi, bu tenglamalar yechimi bilan (3) tenglamaning barcha $2n$ yechimi aniqlanadi.

Masalan: $x^8 - 82x^4 + 81 = 0$ tenglamada $x^4 = y$ desak y ga nisbatan kvadrat tenglamaga $y^2 - 82y + 81 = 0$ kelinadi, bu tenglama yechimlari $y_1 = 81$ va $y_2 = 1$, u holda almashtirishdan:

1) $x^4 = 1$ dan $x^2 = \pm 1$ yoki

$x^2 = 1$ da $x_1 = 1, x_2 = -1, x^2 = -1$ da $x_3 = i, x_4 = -i$. Shu kabi

2) $x^4 = 81$ dan $x^2 = \pm 9$ yoki

$x^2 = 9$ da $x_5 = 3, x_6 = -3, x^2 = -9$ da $x_7 = 3i, x_8 = -3i$.

Berilgan tenglama yechimlari $x_1 = 1, x_2 = -1, x_3 = i, x_4 = -i, x_5 = 3, x_6 = -3, x_7 = 3i, x_8 = -3i$.

12.3. Ushbu $(x + a)^4 + (x + b)^4 = c$ (7)

tenglama $x = y - \frac{(a+b)}{2}$, (8)

almashtirish yordamida bikvadrat tenglamaga kelinadi.

Masalan:

$(x + 4)^4 + (x + 6)^4 = 82$ tenglamada $x = y - \frac{(4+6)}{2} = y - 5$ almashtirish bajarsak tenglama $(y - 1)^4 + (y + 1)^4 = 82$ ko'rinishga keladi, qavslarni ochib soddalashtirsak, $y^4 - 4y^3 + 6y^2 - 4y + 1 + y^4 + 4y^3 + 6y^2 + 4y + 1 = 82$ yoki $y^4 + 6y^2 - 40 = 0$,

(2) formulaga asosan yechimlari

$$y_{1,2} = \pm\sqrt{\frac{-6+14}{2}} = \pm 2, \quad y_{3,4} = \pm\sqrt{\frac{-6-14}{2}} = \pm\sqrt{10}i,$$

u holda berilgan tenglama yechimlari ($x = y - 5$ dan)

$x_1 = 2 - 5 = -3, x_2 = -2 - 5 = -7,$

$x_3 = \sqrt{10}i - 5 = -5 + \sqrt{10}i, \quad x_4 = -5 - \sqrt{10}i.$

12.4. $(x + a)(x + b)(x + c)(x + d) = k$ tenglamada $a + b = c + d$ (yoki $a + c = b + d$, shu kabi $a + d = b + c$), tengliklar bajarilsa, bu tenglamani kvadrat tenglamaga keltirish mumkin, buni misolda ko'rib chiqamiz.

Masalan:

$(x - 1)(x + 2)(x - 3)(x - 6) = 16,$

► Bu tenglamada $-1 - 3 = -4$, $2 - 6 = -4$ shart bajariladi. U holda $[(x - 1)(x - 3)][(x + 2)(x - 6)] = 16$ tenglamada qavslarni ochib soddalashtirsak, $(x^2 - 4x + 3)(x^2 - 4x - 12) = 16$ endi $x^2 - 4x + 3 = y$ desak, $y(y - 15) = 16$ yoki $y^2 - 15y - 16 = 0$, ildizlari $y_1 = 16, y_2 = -1$ bo'lib, $(x^2 - 4x + 3) = y$ almashtirishdan

1) $x^2 - 4x + 3 = 16$ yoki $x^2 - 4x - 13 = 0$ ildizlari $x_{1,2} = 2 \pm \sqrt{17}$,

2) $x^2 - 4x + 3 = -1$ yoki $x^2 - 4x + 4 = 0$ ildizlari $x_3 = x_4 = 2$

Natijada berilgan tenglama yechimlari $x_1 = 2 + \sqrt{17}$, $x_2 = 2 - \sqrt{17}$, $x_3 = x_4 = 2$ ◄

12.5. Uchinchi tartibli (to‘rt hadli) tenglama

$$ax^3 + bx^2 + cx + d = 0, \quad (9) \ (a \neq 0)$$

yechimini topishni xususiy holdan boshlaymiz.

12.5.1. $d = 0$ da tenglama kvadrat tenglamaga kelinadi.

$0 = ax^3 + bx^2 + cx = x(ax^2 + bx + c)$ yoki $c = d = 0$ da esa $0 = ax^3 + bx^2 = x^2(ax + b)$ ko‘rnishidagi tenglamaga kelinadi.

12.5.2. Tenglama berilishiga qarab qisqa ko‘paytirish formulalaridan foydalanish mumkin:

(+) Tenglama yechimi topilsin.

1) $8x^3 - 12x^2 + 6x - 9 = 0$

► Tenglamani $0 = (8x^3 - 12x^2 + 6x - 1) - 8 = (2x - 1)^3 - 8$ yoki $(2x - 1)^3 = 8$ deb yozish mumkin , bundan $2x - 1 = \sqrt[3]{8}$ bo‘lib, tenglama yechimi $x = 1{,}5$ ga teng. ◄

2) $x^3 - 5x^2 + 10x - 8 = 0$

► Tenglamani $(x^3 - 8) - 5x(x - 2) = 0$ deb yozsak, qisqa ko‘paytirish formulasi yordamida ifodani ko‘paytma ko‘rinishga keltirib olish mumkin $(x - 2)(x^2 + 2x + 4) - 5x(x - 2) = 0$ yoki $(x - 2)(x^2 - 3x + 4) = 0$ bu berilgan tenglamaga teng kuchli bo‘lib, yechimlari $x_1 = 2, x_2 = 4, x_3 = -1$. ◄

Umumiy holda (9) tenglama yechimlarini aniqlash (12.8) punktdagi usulga asosan topiladi.

12.6. To‘rtinchi tartibli tenglama.

Butun algebraik to‘rtinchi tartibli

$$ax^4 + bx^3 + cx^2 + dx + e = 0, \quad (10)$$

$(a \neq 0)$tenglamada $a = e$, $b = d$ bo‘lsa, ya’ni

$$ax^4 + bx^3 + cx^2 + bx + e = 0, \quad (11)$$

bu tenglamaga (qaytma) simmetrik tenglama deyiladi. Bu tenglama yechimni topishda tenglamani $x^2 \neq 0$ ga bo‘lib, $x + \frac{1}{x} = y$, almashtirish bajarsak, tenglama y ga nisbatan kvadrat tenglamaga kelinadi. (11) tenglama yechimi

$$x_{1,2,3,4} = \frac{t \pm \sqrt{t^2 - 4}}{2} \quad (12)$$

Bu yerda $t = \dfrac{-b \pm \sqrt{b^2 - 4ac + 8a^2}}{2a}$

Agar x_1, x_2, x_3 va x_4 (10) tenglama yechimlari bo‘lsa, bu ifodani $ax^4 + bx^3 + cx^2 + dx + e = a(x - x_1)(x - x_2)(x - x_3)(x - x_4)$ ko‘rinishda yozish mumkin.

⊕ Tenglama yechimi topilsin

$x^4 + 5x^3 + 2x^2 + 5x + 1 = 0,$

► Bu to'rtinchi tartibli qaytma tenglama, tenglamani $x^2 \neq 0$ ga bo'lamiz $\left(x^2+\frac{1}{x^2}\right)+5\left(x+\frac{1}{x}\right)+2=0$, $x+\frac{1}{x}=y$ almashtirish bajarmiz $\left(x+\frac{1}{x}\right)^2=y^2$ dan $x^2+\frac{1}{x^2}=y^2-2$ bo'ladi, almashtirishni tenglamaga qo'ysak $y^2-2+5y+2=0$,bo'lib tenglama yechimi $y_1=1$, $y_2=-5$

$\left(x+\frac{1}{x}=y\right)$ almashtirishdan berilgan tenglama yechimi topiladi

1) $x+\frac{1}{x}=0$, $x^2+1=0$, dan $x_{1,2}=\pm i$

2) $x+\frac{1}{x}=-5$, $x^2+5x+1=0$, dan $x_{3,4}=\frac{-5\pm\sqrt{21}}{2}$

Berilgan tenglamaning yechimni birdan (12) formula yordamida yozish ham mumkin

$$t=\frac{-b\pm\sqrt{b^2-4ac+8a^2}}{2a}=\frac{-5\pm\sqrt{25-4\cdot1\cdot2+8\cdot1}}{2\cdot1}=\frac{5\pm5}{2},$$

$t_1=0$, $t_2=-5$

1) $t_1=0$ da $x_{1,2}=\frac{t\pm\sqrt{t^2-4}}{2}=\frac{0\pm\sqrt{0-4}}{2}=\frac{\pm2i}{2}=\pm i$;

2) $t_2=-5$ da $x_{3,4}=\frac{t\pm\sqrt{t^2-4}}{2}=\frac{-5\pm\sqrt{25-4}}{2}=\frac{-5\pm\sqrt{21}}{2}$

yechimlar hosil bo'ldi ◄

Agar (10) tenglamada $a=e$ va $b=d$ shartlar bajarilmasa

$$\frac{e}{a}=\left(\frac{d}{b}\right)^2 \quad (13)\ (a\neq0,\ b\neq0)$$

shartni tekshiramiz, bu shart bajarilsa tenglamani $x^2\neq0$ ga bo'lib $x+\frac{d}{bx}=y$ almashtirishda tenglama y ga nisbatan kvadrat tenglamaga kelinadi.

⊕ Tenglama yechimi topilsin.

$2x^4+3x^3-13x^2-6x+8=0$,

► Tenglamada $a=1$, $b=3$, $c=-13$, $d=-6$, $e=8$

bunda $\frac{e}{a}=\frac{8}{2}=4$, $\left(\frac{d}{b}\right)^2=\left(\frac{-6}{3}\right)^2=4$ bo'lib, $\frac{e}{a}=\left(\frac{d}{b}\right)^2$ shart bajarildi. Tenglamani $x^2\neq0$ ga bo'lamiz:

$$2x^2+3x-13-\frac{6}{x}+\frac{8}{x^2}=0 \quad \text{yoki} \quad 2\left(x^2+\frac{4}{x^2}\right)+3\left(x-\frac{2}{x}\right)-13=0,$$

$x-\frac{2}{x}=y$ almashtirish bajarsak (bundan $x^2+\frac{4}{x^2}=y^2+4$)

$2(y^2+4)+3y-13=0$ yoki $2y^2+3y-5=0$ bo'lib, tenglama yechimlari $y_1=1$, $y_2=-\frac{5}{2}$, Almashtirishdan

1) $x-\frac{2}{x}=1$ dan $x^2-x-2=0$ tenglama yechimlari $x_1=-1, x_2=2$

2) $x-\frac{2}{x}=-\frac{5}{2}$ dan $2x^2+5x-4=0$ tenglama yechimlari $x_3=\frac{-5+\sqrt{57}}{4}$, $x_4=\frac{-5-\sqrt{57}}{4}$ ◄

Umumiy ko'rnishdagi (10) tenglama yechimini aniqlash, keyingi punktda umumiy holda ko'riladi.

12.7. Tenglamada yangi o'zgaruvchi kiritib, tenglama yechimni aniqlash usuli:

⊕ Tenglama yechimlari topilsin.

1) $(x^2-5x+6)(x^2-5x+4)=120$

► Bu tenglamada $x^2-5x+4=y$ deb almashtirish bajarsak $y(y+2)=$ $=120$ yoki $y^2+2y-120=0$ ga kelinadi, bu tenglama yechimlari $y_1=10$, $y_2=-12$ bo'lib, berilgan tenglama $x^2-5x+4=10$, va $x^2-5x+4=-12$ tenglamalarga teng kuchli bo'lib yechimlari:

$$x_1=6, x_2=-1, \text{ va } x_3=\frac{5+i\sqrt{39}}{2},\ x_4=\frac{5-i\sqrt{39}}{2} \ ◄$$

2) $(x^2+8)^2+4x^2+37=0$;

► Tenglamani shakl almashtirib,

$(x^2-8)^2+4(x^2-8)-5=0$ deb yozish mumkin, endi $x^2-8=y$, desak, y ga nisbatan $y^2+4y-5=0$, tenglamaga kelinib bu tenglama yechimlari $y_1=1, y_2=-5$. U holda tenglama teng kuchli ikkita $x^2-8=1$ va $x^2-8=$ $=-5$ tenglamalarga ajraladi, bu tenglama yechimlari

$x_1=3, x_2=-3$ va $x_3=\sqrt{3}$, $x_4=-\sqrt{3}$ ◄

3) $x^4+6x^3+5x^2-12x+3=0$

► Tenglamaning chap tomoni to'la kvadrat ajratib yozamiz $(x^2+$ $+3x)^2-4(x^2+3x)+3=0$, endi $x^2+3x=y$ desak $y^2-4y+3=0$ tenglamaga kelinadi bu tenglama yechimlari $y_1=1$, $y_2=3$. U holda berilgan tenglama teng kuchli ikkita $x^2+3x=1$ va $x^2+3x=3$ tenglama kelinadi.

Bunda $x^2 + 3x - 1 = 0$ yechimlari $x_{1,2} = \frac{-3 \pm \sqrt{13}}{2}$ va $x^2 + 3x - 3 = 0$ tenglamada yechimlar $x_{3,4} = \frac{-3 \pm \sqrt{21}}{2}$

4) $(x^2 - 2x)^2 - (x - 1)^2 + 1 = 0$

► Bu tenglama yechimini aniqlashda ketma-ket yangi o'zgaruvchi kiritish bilan bajariladi.

$x - 1 = y$ desak $x^2 - 2x = y^2 - 1$ bo'lib, tenglama $(y^2 - 1)^2 - y^2 + 1 = 0$, ko'rinishga kelinadi, yana $y^2 - 1 = t$ desak $t^2 - t = 0$ chala kvadrat tenglamaga kelinadi bu tenglama yechimlari $t = 0$, $t = 1$ bo'lib $y^2 - 1 = t$ almashtirishdan $y^2 - 1 = 0$ va $y^2 - 1 = 1$ bundan $y_{1,2} = \pm 1$, va $y_{1,2} = \pm\sqrt{2}$.
U holda boshlang'ich tenglama yechimlari $x - 1 = y$ almashtirishdan $x_1 = 2, x_2 = 0,\ x_3 = \sqrt{2} + 1,\ x_4 = \sqrt{2} - 1$ ◄

12.8. Yuqori tartibli tenglama butun ko'phad ko'rinishda bo'lsa, tenglama berilishiga qarab ko'paytuvchilarga ajratish usulidan foydalanamiz.

⊕ Tenglamalar yechilsin.

1) $9x^3 - 18x^2 - x + 2 = 0$,

► Tenglama hadlarini gruppalab, ko'paytuvchilarga keltirish mumkin $(9x^3 - 18x^2) - (x - 2) = 0$ dan $9x^2(x - 2) - (x - 2) = 0$ yoki $(x - 2)\times(9x^2 - 1) = 0$ bo'lib, berilgan tenglama teng kuchli ikkita $x - 2 = 0$ va $9x^2 - 1 = 0$ tenglama ajralib yechimlari

$$x_1 = 2,\ x_2 = \frac{1}{3},\ \text{va}\ x_3 = -\frac{1}{3}. ◄$$

2) $2x^4 - 6x^3 + 6x^2 - 2x = 0$,

► Tenglamaning chap tomonida $2x$ ni qavs tashqarisiga chiqarsak, qavs ichidagi ifoda ikki ifoda ayirmasining kubini beradi:

$2x(x^3 - 3x^2 + 3x - 1) = 0$ yoki $2x(x - 1)^3 = 0$, berilgan tenglamaga teng kuchli bo'lgan bu tenglama yechimlari $x_1 = 0$ va karrali $x_2 = x_3 = x_4 = 1$, yechimlarni olamiz. ◄

3) $x^3 - 2x^2 + 16 = 0$,

► Tenglamada shakl almashtirib yozsak, ifoda ko'paytma ko'rinishga keladi $(x^3 + 8) - 2(x^2 - 4) = 0$ endi qisqa ko'paytirish formulalaridan foydalanmiz $(x + 2)(x^2 - 2x + 4) - 2(x - 2)(x + 2) = 0$, yoki $(x + 2)(x^2 - 4x + 8) = 0$, bu ekvivalent tenglama yechimlari $x_1 = -2$, $x_{2,3} = 2 \pm 2i$. ◄

4) $x^5 + 4x^3 - 2x^4 - 7x^2 + 4 = 0$,

► Tenglamada $-7x^2 = -8x^2 + x^2$ deb qo'shiluvchiga ajratib, keyin gruppalash yo'li bilan, tenglamaning chap tomonini ko'paytma ko'rnishga keltirish mumkin

$0 = x^5 + 4x^3 - 2x^4 - 8x^2 + x^2 + 4 = (x^5 + 4x^3) - (2x^4 + 8x^2) + x^2 + 4 =$
$= x^3(x^2 + 4) - 2x^2(x^2 + 4) + (x^2 + 4) = (x^3 - 2x^2 + 1) \times (x^2 + 4)$

Natijada berilgan tenglama ikkita $x^2 + 4 = 0$ va $x^3 - 2x^2 + 1 = 0$ tenglamalarga ekvivalent bo'ladi. Birinchi tenglama yechimi

$x_{1,2} = \pm 2i.$

Ikkinchi tenglamani $x^3 - x^2 - x^2 + 1 = 0$, deb, yoki $(x - 1)(x^2 - x - 1) = 0$, ko'rinishda yozish mumkinligidan, bu tenglama yechimlari $x_3 = 1$, va

$x_{4,5} = \frac{1 \pm \sqrt{5}}{2}$ Natijada berilgan tenglama yechimlari $x_{1,2} = \pm 2i$, $x_3 = 1$,

va $x_{4,5} = \frac{1 \pm \sqrt{5}}{2}$ ◄

5) $2(x^2 + 6x + 1)^2 + 5(x^2 + 6x + 1) + 2(x^2 + 1)^2 = 0$

► Tenglamaning ikki tomonini $(x^2 + 1)^2$ ga bo'lamiz:

$$2\left(\frac{x^2 + 6x + 1}{x^2 + 1}\right)^2 + 5\frac{x^2 + 6x + 1}{x^2 + 1} + 2 = 0,$$

Agar $\frac{x^2 + 6x + 1}{x^2 + 1} = y$ desak, berilgan tenglama $2y^2 + 5y + 2 = 0$ ko'rinishda bo'lib, bu tenglama yechimlari $y_1 = -2$, $y_2 = -\frac{1}{2}$.

U holda almashtirishdan,

a) $\frac{x^2 + 6x + 1}{x^2 + 1} = -2$ yoki $x^2 + 2x + 1 = 0$ bo'lib, yechimlari $x_1 = x_2 = -1$,

b) $\frac{x^2 + 6x + 1}{x^2 + 1} = -\frac{1}{2}$ yoki $x^2 + 4x + 1 = 0$ bo'lib, yechimlari $x_{3,4} = -2 \pm \sqrt{3}$.

Demak, berilgan tenglama yechimlari:

$x_1 = x_2 = -1, x_3 = -2 + \sqrt{3}, x_4 = -2 - \sqrt{3}$. ◄

6) $x^4 + 4x^3 + 3x^2 + 2x - 1 = 0$

► Tenglamani ko'paytma ko'rinishga keltirish uchun, shakl almashtiring yozamiz:

$(x^4 + 4x^3 + 4x^2) - x^2 + 2x - 1 = 0$,qisqa ko'paytirish formulalaridan ketma-ket ikki marta foydalansak, ifoda ko'paytma ko'rinishga keladi.

$(x^2 + 2x)^2 - (x - 1)^2 = 0$ dan $(x^2 + 2x + x - 1)(x^2 + 2x - x + 1) = 0$ yoki $(x^2 + 3x - 1)(x^2 + x + 1) = 0$ bo'lib, berilgan tenglama yechimlari

$x^2+3x-1=0$ tenglamada $x_{1,2}=\frac{-3\pm\sqrt{13}}{2}$ va $x^2+x+1=0$ yechimlari $x_{3,4}=\frac{-1\pm i\sqrt{3}}{2}$. ◄

12.9. Yuqori tartibli tenglama yechimlarini aniqlashda umumiy metod quyidagi teorema va ularning natijalaridan foydalanish.

__Teorema 1.__ Agar $(\alpha+\beta i)$, $(\beta\neq 0)$ kompleks son n-darajali $f_n(x)=a_0x^n+a_1x^{n-1}+.....+a_n$ ko'phadning ildizi bo'lsa, $\alpha-\beta i$ son ham shu ko'phadning ildizi bo'ladi.

__Teorema 2.__ n-darajali $f_n(x)$ qo'phad $(x-a)$ ko'rinishdagi ikkihadlar va x^2+px+q $(p^2-xq<0)$ ko'rinishdagi kvadrat uchhadlar darajalarining ko'paytmasidan iborat:

$f_n(x)=b_0(x-a)^k....(x^2+px+q)^m...$ $(k,m\in N)$

__Teorema 3.__ (Bezu)

$f_n(x)=x^n+a_1x^{(n-1)}+b_2x^{n-2}+....+b_n$ n-darajali ko'phadni $(x-a)$ ga bo'lganda bo'linma $Q(x)$ va qoldiq $R(x)$ bo'lsin, ya'ni

$f_n(x)=Q(x)(x-a)+R(x)$.

U holda n-darajali $f_n(x)$ ko'phadni $(x-a)$ ga bo'lgandagi qoldiq, ko'phadning $x=a$ dagi xususiy qiymatiga teng, ya'ni $R(a)=f_n(a)$. Agar $f_n(a)=0$ bo'lsa ko'phad $(x-a)$ ga qoldiqsiz bo'linadi.

__Natija 1.__ Agar $x=a$ $(a\in Z)$ butun ratsional algebraik n-darajali ko'phadning yechimi bo'lsa, u holda ko'phaddagi a_n ozod had $x=a$ ga qoldiqsiz bo'linadi.

__Teorema 4.__ $\frac{p}{q}$ qisqarmas kasr $(p\in Z,\ q\in N)$ bo'lsin. $\frac{p}{q}$ ratsional son $f_n(x)=a_0x^n+a_1x^{n-1}+....+a_{n-1}x+a_n$ ko'phadning ildizli bo'lishi uchun p son ozod hadning butun bo'luvchilari, q esa bosh koeffitsent a_0 ning natural bo'luvchisi bo'lishi zarur.

__Natija 2.__ $P\in Z$ soni $f_n(x)$ ko'phadning ildizi bo'lishi uchun P son ozod had a_n ning bo'luvchi bo'lishi zarur.

Ko'phadni ko'paytuvchilarga ajratishdagi teoremalar tatbiqi [3] ishda misollar bilan to'la oydinlashtirilgan. Eslash uchun bitta misol ko'ramiz.

⊕ $f_4(x)=x^4-10x^3+35x^2-50x+24$ ko'phadni ko'paytuvchilarga ajratishda, ozod hadning butun bo'luvchilari $\pm1,\pm2,\pm3,...$ sonlari ketma-ket funksiyaga qo'yib teorema 3 dan foydalanamiz.

$$f_4(1)=1-10+35-50+24=60-60,$$
$$f_4(-1)=1+10+35+50+24=120\neq 0$$

shu kabi hisoblashda $f_4(2)=0, f_4(-2)\neq 0, f_4(3)=0, f_4(-3)\neq 0, f_4(4)=0$ ekanligini ko'rish qiyin emas, demak, $f_4(x)=(x-1)(x-2)(x-3)(x-4)$ ◄

⊕ $\varphi_4(x)=x^4+2x^3+4x^2+3x-10$, ko'phadni ko'paytuvchilarga ajratilsin.

► Ozod had (–10)ning butun bo‘luvchilari ± 1, ± 2, ± 5, ± 10 larni birin-ketin $\varphi_4(x)$, funksiyadagi x ning o‘rniga qo‘yilganda

$$\varphi_4(1)=0,\ \varphi_4(-1)\neq 0,\ \varphi_4(2)\neq 0,\ \varphi_4(-2)=0,\ \varphi_4(5)\neq 0,$$
$$\varphi_4(-5)\neq 0,\ \varphi_4(10)\neq 0,\ \varphi_4(-10)\neq 0,$$

kelib chiqadi, demak $x = 1$ va $x = -2$ ko‘phad ildizlari bo‘lib, ko‘phad $(x-1)(x+2) = x^2 + x - 2$ ga bo‘linadi, bo‘lish amalini bajaramiz

$$\begin{array}{l|l} \boxed{x^4}+2x^3+4x^2+3x-10 & \boxed{x^2}+x-2 \\ \underline{x^4+x^3-2x^2} & x^2+x+5 \\ x^3+6x^2+3x-10 & \\ \underline{x^3+x^2-2x} & \\ 5x^2+5x-10 & \\ -\underline{5x^2+5x-10} & \\ 0 & \end{array}$$

Natijada berilgan ko‘phad

$x^4 + 2x^3 + 4x^2 + 3x - 10 = (x-1)(x+2)(x^2 + x + 5)$ ko‘paytuvchilarga ajraldi ◄

⊕ $2x^5 + 6x^4 - 7x^3 - 21x^2 - 4x - 12 = 0$

1) tenglamaning butun ildizlari, 2) kompleks ildizlari topilsin.

► Tenglamadagi ozod had (–12) ning butun bo‘luvchilari ± 1, ± 2, ± 3, ± 6, ± 12 ni ketma-ket tenglamadagi x ning o‘rniga qo‘yilganda (–2,) 2 va (–3) tenglamani qanoatlantiradi. Demak, tenglamaning butun yechimlari $x_1 = -2$, $x_2 = 2$, $x_3 = -3$. Endi tenglamaning kompleks ildizini toppish uchun, tenglama chap tomondagi ko‘phadni $(x+2)(x-2)(x+3) = x^3 + 3x^2 - 4x - 12$ ko‘phadga bo‘lamiz

$$\begin{array}{l|l} -\boxed{2x^5}+6x^4-7x^3-21x^2-4x-12 & \boxed{x^3}+3x^2-4x-12 \\ \underline{2x^5+6x^4-8x^3-24x^2} & 2x^2+x \\ x^3+3x^2-4x-12 & \\ -\underline{x^3+3x^2-4x-12} & \\ 0 & \end{array}$$

Natijada tenglamani ko‘paytma ko‘rnishda yozish mumkin $(x-2)(x+3)(x+2)(2x^2+1) = 0$ bunda oxirgi ko‘paytma $2x^2 + 1 = 0$ esa $x_{1,2} = \pm i\sqrt{0,5}$ kompleks ildizga ega bo‘ladi. Demak tenglama $x_1 = -2$, $x_2 = 2$, $x_3 = -3$ va $x_{4,5} = i\sqrt{0,5}$ ildizlarga ega. ◄

⊕ $2x^4 - x^3 + 2x^2 + 3x - 2 = 0$, tenglamaning ratsional ildizlari topilsin.

► Ozod (– 2) hadning butun bo‘luvchilari – 1, 1, – 2 va 2. Bosh koeffitsent 2 ning natural bo‘luvchilari 1 va 2. Tenglamaning ratsional ildizlarini $-2, -1, -\frac{1}{2}, \frac{1}{2}$, 1, 2 sonlar orasidan qidiramiz. Bu sonlarni berilgan tenglamaga bevosita qo'yib tekshirish ko'rsatadiki (-1) va $\frac{1}{2}$ soni tenglamaning ildizlari bo'ladi, qolgan sonlar esa tenglamaga ildiz emas. Demak tenglamaning ratsional ildizlari (-1) va 0,5 ekan. ◄

⊕ $4x^4 + 3x^2 - 1 = 0$ tenglamaning 1) ratsional ildizlari, 2) kompleks ildizlari topilsin.

► Ozod had (-1) ning butun bo'luvchilari (-1), 1. Bosh koeffitsent 4 ning natural bo'luvchilari 1,2 va 4. Tenglamaning ratsional ildizlarini -1, $\left(-\frac{1}{2}\right)$, $-\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$ va 1 sonlar orasida qidiramiz. Faqat $\frac{1}{2}$ va $\left(-\frac{1}{2}\right)$ tenglamaning yechimi ekanini aniqlaymiz.

Qolgan yechimlarini aniqlash uchun tenglamani $\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)=x^2-\frac{1}{4}$ ga bo'lamiz, bo'lsak bo'linma $x^2 + 1$ bo'ladi. U holda tenglamani

$$(2x-1)(2x+1)(x^2+1)=0$$

ko'rinishda yozish mumkin, oxirgi ko'paytma ildizlari $\pm i$. Demak berilgan tenglama $\pm 0,5$ ratsional ildizga, $\pm i$ kompleks ildizga ega ◄

⊕ $x^4 - 3x^3 - x^2 + 9x - 6 = 0$ tenglamaning haqiqiy ildizlari topilsin.

► Ozod had (-6) ning butun bo'luvchilari ± 1, ± 2, ± 3 va ± 6 dan faqat 1 va 2 tenglama yechimi ekanligini bilish qiyin emas. Qolgan yechimini topish uchun tenglamani $(x-1)(x-2) = x^2 - 3x + 2$ tenglikdan, tenglama $(x-1)(x-2)(x^2-3) = 0$ ko'rinishga keladi, bundan $x=\pm\sqrt{3}$ tenglama yechimi ekanligi kelib chiqadi.

Javob $x_1 = 1, x_2 = 2, x_{3,4} = \pm\sqrt{3}$. ◄

⊕ $x^3 + 6x^2 - x - 6 = 0$,

► Tenglamadagi ozod had (-6) ning butun bo'luvchilaridan (-6), (-1) va 1 tenglama yechimlari bo'ladi, tenglama uchinchi tartibli bo'lganligi uchun barcha yechimlari $x_1 = -6, x_2 = -1, x_3 = 1$ ◄

⊕ $x^6 - 2x^5 + 3x^4 - 6x^3 - 4x^2 + 8x = 0$ tenglamaning barcha ildizlari topilsin.

► x ni qavsdan chiqarib yozamiz $x(x^5 - 2x^4 + 3x^3 - 6x^2 - 4x + 8) = 0$, $x = 0$ tenglamaga butun yechim. Endi qavs ko'paytmani nolga tenglaymiz $x^5 - 2x^4 + 3x^3 - 6x^2 - 4x + 8 = 0$ teglamadagi ozod had 8 ning ± 1, ± 2, ± 4 va ± 8 butun bo'luvchilari orasidagi (-1), 1 va 2 tenglamaning 2 ratsional ildizlari ekanligiga ishonch hosil qilish mumkin. Shuning uchun tenglamaning chap tomonidagi ko'phad $(x-1)(x+1)(x-2)=x^3-2x^2-x+$ $+2$ ga qoldiqsiz bo'linadi.

Bo'lishni bajarsak $x^5-2x^4+3x^3-6x^2-4x+8=(x-1)(x+1)(x-2)\cdot$ $\cdot(x^2+4)=0$ ni hosil qilamiz. (x^2+4) tenglama ildizlari $\pm 2i$. Natijada berilgan tenglamaning barcha ildizlari $x_1=0$, $x_2=-1$, $x_3=1$, $x_4=2$, va $x_{5,6}=\pm 2i$ ekanligi kelib chiqadi.

Eslatma $x_1, x_2, \ldots, x_n$ sonlar har qanday bo'lganda ham shunday n-tartibli butun algebraik tenglamani yozish mumkinki bu sonlar shu tenglamaning yechimlari bo'ladi. Bu tenglama $(x-x_1)(x-x_2)\ldots(x-x_n)=0$ ko'rinishda bo'lishi mumkin.

⊕ Shunday beshinchi tartibli butun algebraik tenglama topilsinki, uning ildizlari $x_1=2$, $x_2=3$, $x_3=3$, $x_4=-i$ va $x_5=i$ bo'lsin.

► Yuqoridagi tenglikdan foydalanamiz

$(x-2)(x-3)^2(x+i)(x-i)=0$ yoki $(x-2)(x^2-6x+9)(x^2+1)=0$ qavsni ochib soddalashtirsa $x^5-8x^4+22x^3+10x^2+21x+18=0$ tenglamani hosil qilamiz. ◄

12.10. Ratsional kasr algebraik tenglamalar yechimini aniqlash yuqoridagi (11.7) punktdagi kabi aniqlanadi, ratsional algebraik kasrlarda amallar bajariladi (bu [3] to'la misollar bilan oydinlashtirilgan) keyin soddalashtirilsa, kanonik butun ratsional tenglamaga kelinadi, bunda kasr ifoda maxraji nol bo'lmasligini talab qilinadi, chet ildiz foydali bo'lmasligi va ildiz yo'qolmasligiga ahamiyat berish kerak, ya'ni hosil bo'lgan tenglamaning teng kuchli bo'lishligini hisobga olish kerak. Misollarda ko'rib chiqamiz. Oldin ba'zi bir xususiy metodlardan boshlaymiz.

12.10.1.

$$\frac{ax}{\alpha x^2+\beta x+\gamma}+\frac{bx}{\alpha x^2+\beta x+\gamma}=c,\ (c\neq 0)$$

Ko'rnishdagi kasr ratsional algebraik tenglamada yangi o'zgartirish kiritish yordamida kvadrat tenglamaga keltirish mumkin.

Bu tenglamani $\dfrac{a}{\alpha x+\beta+\dfrac{\gamma}{x}}+\dfrac{b}{\alpha x+\beta+\dfrac{\gamma}{x}}=c\quad (x\neq 0)$ ko'rnishda yozib, tenglamada $y=\alpha x+\dfrac{\gamma}{x}$ almashtirish bajariladi.

Misol $\dfrac{2x}{2x^2-5x+3}+\dfrac{13x}{2x^2+x+3}=6$,

► Tenglama chap tomondagi kasrni x ga qisqartiramiz

$$\frac{2}{2x-5+\dfrac{3}{x}}+\frac{13}{2x+1+\dfrac{3}{x}}=6,\ 2x+\frac{3}{x}=y \text{ desak,}$$

$$\frac{2}{y-5}+\frac{13}{y+1}=6,\ (y\neq 5 \text{ va } y\neq -1)$$

Tenglamani $(y-5)(y+1)$ ga ko'paytirsak

$$2(y+1)+13(y-5)=6(y-5)(y+1)$$

holda qavs ochib soddalashtirilsa $2y^2-13y+11=0$ kvadrat tenglamaga kelinadi, bu tenglama ildizlari

$y_1=1, y_2=5{,}5$ bo'lib $\left(2x+\frac{3}{x}=y\right)$, almashtirishdan:

1) $\quad 2x+\frac{3}{x}=1$ yoki $2x^2-x+3=0,\ x_{1,2}=\frac{1\pm i\sqrt{23}}{4}$;

2) $\quad 2x+\frac{3}{x}=5{,}5$ yoki $4x^2-11x+6=0, x_3=2, x_4=0{,}75$ ◄

Shu metodda

$$\frac{ax^2+bx+c}{ax^2+\gamma x+c}\pm\frac{ax^2+\alpha x+c}{ax^2+\beta x+c}=A \text{ va } \frac{ax^2+bx+c}{ax^2+\gamma x+c}=\frac{Ax}{ax^2+\beta x+c}$$

$(A\neq 0,\ a\cdot c\neq 0)$ kabi tenglamalar yechimini aniqlash mumkin.

Misol $\frac{x^2-13x+15}{x^2-14x+15}-\frac{x^2-15x+15}{x^2-16x+15}=-\frac{1}{12}$

► Tenglama chap tomon kasr surat va maxrajini $x\neq 0$ ga bo'lamiz

$\frac{x-13+\frac{15}{x}}{x-14+\frac{15}{x}}-\frac{x-15+\frac{15}{x}}{x-16+\frac{15}{x}}=-\frac{1}{12}$ $x+\frac{15}{x}=y$ deb almashtirish

bajarsak $\frac{y-13}{y-14}-\frac{y-15}{y-16}=-\frac{1}{12}$ $y\neq 14, y\neq 16$ deb tenglamaning ikki

tomonini $12(y-14)(y-16)$ ga ko'paytiramiz, u holda

$$12[(y-13)(y-16)-(y-15)(y-14)]=-(y-14)(y-16)$$

Qavs ochib soddalashtiramiz $y^2-30y+200=0$, bu kvadrat tenglama

yechimlari $y_1=20$ va $y_2=10$. Endi almashtirishdan $\left(x+\frac{15}{x}=y\right)$

1) $\quad x+\frac{15}{x}=20$ yoki $x^2-20x+15=0$ bo'lib yechimlari

$x_{1,2}=10\pm\sqrt{85}$;

2) $x+\frac{15}{x}=10$ yoki $x^2-10x+15=0$ tenglama yechimlari $x_{3,4}=5\pm\sqrt{10}$ ◀

12.10.2. Yangi o‘zgaruvchi kiritish usuli bilan tenglama yechimini topish mumkin.

⊕ Tenglamalar yechilsin

1) $\frac{1}{x^2+2x-3}-\frac{18}{x^2+2x+1}+\frac{18}{x^2+2x+2}=0$

▶ Agar $x^2+2x-3=y$ desak, tenglama quyidagi ko‘rinishda bo‘ladi $\frac{1}{y}-\frac{18}{y+4}+\frac{18}{y+5}=0$ endi $y\neq 0, y\neq -4, y\neq -5$ deb tenglamani $y(y+4)$ $(y+5)$ ga ko‘paytiramiz $(y+4)(y+5)-18y(y+5)+18y(y+4)=0$ qavsni ochib soddalashtirsak, kvadrat tenglamaga kelinadi

$y^2-9y+20=0$, bu tenglama ildizlari $y_1=5$, $y_2=4$. U holda almashtirishdan $(x^2+2x-3=y)$

a) $x^2+2x-3=5$ yoki $x^2+2x-8=0$ bo‘lib yechimlari $x_1=2$, $x_2=-4$,

b) $x^2+2x-3=4$ yoki $x^2+2x-7=0$ bo‘lib, natijada berilgan tenglama ildizlari $x_1=2$, $x_2=-4$, $x_{3,4}=-1\pm2\sqrt{2}$ ◀

2) $\frac{3x}{x^2+x-5}+\frac{x^2+x-5}{x}+4=0$

Agar $\frac{x}{x^2+x-5}=y$ deb almashtirish bajarsak, tenglama $3y+\frac{1}{y}+4=0$ $(y\neq 0)$ yoki $3y^2+4y+1=0$ kvadrat tenglama ildizlari $y_1=-1$ va $y_2=-\frac{1}{3}$ bo‘lib almashtirishdan

a) $\frac{x}{x^2+x-5}=-1$ yoki $x^2+2x-5=0$ yechimlari

$x_{1,2}=-1\pm\sqrt{6}$,

b) $\frac{x}{x^2+x-5}=-\frac{1}{3}$ yoki $x^2+4x-5=0$ yechimlari $x_3=1$, $x_4=-5$ ◀

2) $\frac{x^2-x}{x^2-x+1}-\frac{x^2-x+2}{x^2-x-2}=1$

► Tenglamada $x^2-x-2\neq 0$ yoki $x\neq 2$, $x\neq -1$ deb $x^2-x-2=y$ almashtirish bajarsak tenglama $\frac{y+2}{y+3}-\frac{y+4}{y}=1$ ko'rnishga keladi ($y\neq 0$, $y\neq -3$) tenglamani $y(y+3)$ ga ko'paytirsak, $y(y+2)-(y+4)\cdot(y+3)=y(y+3)$ agar qavs ochib soddalashtirsak, $y^2+8y+12=0$ bo'lib, bu tenglama yechimlari $y_1=-2$, va $y_2=-6$. Endi $x^2-x-2=y$ dan:

a) $x^2-x-2=-2$ yoki $x^2-x=0$, yechimlari $x_1=0$, $x_2=1$,

b) $x^2-x-2=-6$ yoki $x^2-x+4=0$, yechimlari $x_{3,4}=\frac{1\pm i\sqrt{15}}{2}$ ◄

12.10.3. Ratsional algebraik kasrlarda amallar bajarish va ratsional ko'rinishga keltirish qoidasidan foydalanib, tenglama yechimlarni aniqlashni misollarda oydinlashtiramiz

⊕ Tenglamalarni yeching.

1) $\frac{x+3}{9x^2+3x+1}-\frac{1}{3x-1}+\frac{3}{27x^3-1}=0$

► Maxrajdagi ko'phadlar uchun umumiy maxraj $(27x^3-1)$ bo'ladi, chunki $27x^3-1=(3x-1)(9x^2+3x+1)$, endi $3x-1\neq 0$ deb tenglamani $(27x^3-1)$ ga ko'paytiramiz $(3x-1)(x+3)-(9x^2+3x+1)+3=0$, qavslarni ochib soddalashtirsak $6x^2-5x+1=0$

Bu tenglama ildizlari $x_1=\frac{1}{3}$ va $x_2=\frac{1}{2}$; lekin $x=\frac{1}{3}$ berilgan tenglama uchun chet ildiz. Javob $x=0,5$. ◄

2) $\frac{x^2-2x+4}{x^3-2x^2+4x-8}+\frac{x^2+2x+4}{x^3+2x^2+4x+8}=\frac{2x+2}{x^2-4}$,

► Maxrajni ko'paytma ko'rnishda yozib olamiz:

$$\frac{x^2-2x+4}{(x-2)(x^2+4)}+\frac{x^2+2x+4}{(x+2)(x^2+4)}=\frac{2x+2}{(x-2)(x+2)}$$

$x-2\neq 0$, $x+2\neq 0$ deb, tenglamani umumiy maxraj bo'lgan $(x-2)(x+2)(x^2+4)$ ga ko'paytiramiz

$(x+2)(x^2-2x+4)+(x-2)(x^2+2x+4)=(2x+4)(x^2+4)$ qavsni ochib, soddalashtirsak

$$x^2+2x+4=0$$

Kvadrat tenglamaga kelinadi, bu tenglamada haqiqiy yechim mavjud emas, $x_{1,2}=-1\pm i\sqrt{3}$ kompleks ildizga ega.

3) $\left(\frac{x-1}{x-2}\right)^2+\left(\frac{x-1}{x}\right)^2=\frac{40}{9}$

$x-2\neq 0, x\neq 0$ deb, chap tomondagi yig'indini hisoblab olamiz

$$\frac{(x-1)^2}{(x-2)^2}+\frac{(x-1)^2}{x^2}=\frac{x^2(x-1)^2+(x-1)^2(x-2)^2}{x^2(x-2)^2}=$$

$$=\frac{(x-1)^2\left(x^2+x^2-4x+4\right)}{x^2(x-2)^2}=\frac{\left(x^2-2x+1\right)\left(2x^2-4x+4\right)}{\left(x^4-4x^3+4x^2\right)}$$, endi

$x^2-2x=y$ desak, tenglama $\frac{(y+1)(2y+4)}{y^2}=\frac{40}{9}$, $(y\neq 0)$ ko'rinishiga kelinadi. Ifodada qavs ochib soddalashtirilsa, y ga nisbatan kvadrat tenglamaga kelinadi $-11y^2+27y+18=0$, tenglama yechimlari

$$y_{1,2}=\frac{-27\pm\sqrt{1521}}{-22}=\frac{-27\pm 39}{-22}$$ yoki $y_1=3,\ y_2=-\frac{6}{11}$ bo'lib

$x^2-2x=y$ almashtirishdan

a) $x^2-2x=3$ yoki $x^2-2x-3=0$ yechimlari $x_1=-1, x_2=3$,

b) $x^2-2x=-\frac{6}{11}$ yoki $11x^2-22x+6=0$

yechimlari $x_{3,4}=\frac{11\pm\sqrt{55}}{11}$. ◄

4) $\frac{3x+1}{3x^2-3}+\frac{x}{x+2-x^2}=\frac{2}{2x^2-x-1}$

► har bir kasrning maxrajini ko'paytma ko'rnishda yozib olish mumkin

$3x^2-3=3(x-1)(x+1),\ x+2-x^2=-(x^2-x-2)=-(x^2-1-x-1)=$ $=-(x+1)(x-2)$, shu kabi $2x^2-x-1=x^2-x+x^2-1=(x-1)(2x+1)$.

Demak tenglamani

$$\frac{2}{(x-1)(2x+1)}+\frac{(x-2)}{(x)(x+1)}-\frac{3x+1}{3(x-1)(x+1)}=0$$

ko'rnishda yozish mumkin.

Maxrajning eng kichik umumiy bo'linuvchisi $3(x-1)(x+1)(2x+1)\cdot$ $\cdot(x-2)$ ga tenglamani ko'paytiramiz, $x\neq 1, x\neq -1, x\neq -0{,}5$ va $x\neq 2$ deb, u holda

$6(x+1)(x-2)+3x(x-1)(2x+1)-(3x+1)(x-2)(2x+1)=0$, qavslarni ochib soddalashtirsak $10x^2-10=0$ bundan $x_1=-1$, $x_2=1$ bu berilgan tenglamaga chet ildiz. Demak tenglamaning yechimi mavjud emas.

12-MAVZU MASHQLARI

Ildizlari $x_1, x_2, x_3 \dots$ bo'lgan tenglamani yozing:

161. 1) 1, – 1 va 2, 2) $2i$, $-2i$ va 3, 3) 0, – 1,2 va 3.

162. 1) 0, – 1, 2 va – 4, 2) 1, – 2 va $2+i$, $2-i$, 3) 0,5, va $1+i$, $1-i$.

Ikki hadli tenglamani kompleks sonlar to'plamida yeching:

163. 1) $0{,}3x^2+9=0$; 2) $0{,}5x^4-8=0$. 3) $x^3+i=0$.

164. 1) $0{,}5x^5-16=0$, 2) $x^4-\sqrt{3}-i=0$, 3) $8x^3+1=0$.

Bikvadrat tenglamani kompleks sonlar to'plamida yechimini toping:

165. 1) $x^4-8x^2-9=0$, 2) $9x^4-40x^2+16=0$.

166. 1) $4x^4-37x^2+9=0$, 2) $x^4+5x^2-36=0$.

Uchhadni chiziqli ko'paytuvchilarga ajrating:

167. 1) x^4-10x^2+25, 2) x^2-13x^2+36.

168. 1) $4x^4-17x^2+4$, 2) $9x^4-32x^2-16$.

Yangi o'zgaruvchi kiritib tenglama barcha yechimlarni aniqlang:

169. 1) $x^6-7x^3-8=0$, 2) $(x^2+2x+1)-6(x^2+1)+5=0$,

3) $(x^2+x+1)(x^2+x+2)=12$, 4) $\dfrac{4x}{x^2+2x+3}+\dfrac{5x}{x^2-5x+3}=-1{,}5$.

170. 1) $x^8 - 17x^4 + 16 = 0$, 2) $2(x^4 + 6x + 9) - 7(x^2 + 3) + 3 = 0$,

3) $(x^2 - 3x + 1)(x^2 - 3x + 3) = 3$, 4) $x^2 + x + 1 = \dfrac{15}{x^2 + x + 3}$.

171. 1) $(x - 2)(x + 1)(x + 4)(x + 7) = 19$,

2) $(x^2 - 1)^2 + 5(x^4 - 1) - 6(x^2 + 1)^2 = 0$.

172. 1) $(x - 1)(x + 2)(x - 3)(x - 6) = 6$, 2) $(x + 5)^4 + (x + 3)^4 = 2$.

173. $(x^2 + 2x - 5)^2 + 2(x^2 + 2x - 5) = x + 5$.

174. $(x^2 - 4x + 6)^2 - 4(x^2 - 4x + 6) = x - 6$.

Ko'paytuvchilarga ajratish usuli bilan tenglamaning barcha yechimlarni toping.

175. 1) $0,(3)\, x^4 - 27 = 0$, 2) $8x^3 + 27 = 0$.

176. 1) $16x^4 - 625 = 0$, 2) $125x^3 + 8 = 0$.

177. 1) $x^4 - x^2 + 6x = 6x^3$, 2) $x^3 - 8x^2 - x + 8 = 0$.

178. 1) $x^4 - 4x^2 + x + 2 = 0$, 2) $x^3 - 5x + 4 = 0$.

179. $x^3 - 8x^2 + 40 = 0$,

180. $2x^4 - x^3 + 2x^2 + 3x - 2 = 0.$

181. $(x^2 - x - 2)^2 + 9(x - x^2) = 0.$

182. $(x^2 + x - 1)^2 - 5x(x^2 + x - 1) + 4x^2 = 0.$

Tenglamaning barcha ildizlarini toping:

183. $x^4 - 3x^3 + x^2 + 3x + 1 = 0$, 2) $2x^4 - 4x^3 + 2x^2 - 4x + 2 = 0.$

184. 1) $2x^2 + 3x^3 - 16x^2 + 3x + 2 = 0,$

2) $2x^4 - 21x^3 + 74x^2 - 105x + 50 = 0.$

Tenglamaning ratsional ildizlarini toping:

185. 1) $x^3 - x^2 - 8x + 6 = 0$, 2) $x^4 - x^3 + x + 2 = 0.$

186. 1) $x^4 - 7x^3 + 14x^2 - 7x + 1 = 0,$

2) $4x^4 + 8x^3 - 3x^2 - 7x + 3 = 0.$

Tenglamaning barcha haqiqiy ildizlarini toping:

187. 1) $2x^3 - 7x^2 + 2x + 3 = 0$, 2) $x^4 - 2x^3 - 8x^2 + 19x - 6 = 0.$

188. 1) $2x^4 + x^3 - 11x^2 + x + 2 = 0$, 2) $x^3 + 8x^2 + 15x + 18 = 0.$

Tenglamaning barcha ildizlarini toping:

189. 1) $4x^4 + 3x^2 - 1 = 0$, 2) $6x^4 + 5x^3 - 38x^2 + 5x + 6 = 0.$

190. 1) $6x^5 + 7x^4 - 3x^3 - 7x^2 + 6x = 0$, 2) $2x^4 - x^3 + 2x^2 + 3x - 2 = 0$.

Berilgan funksiyani chiziqli ko'paytuvchilarga ajrating:

191. 1) $f(x) = x^6 - 2x^5 + 3x^4 - 6x^3\ 4x^2 + 8x$,

2) $f(x) = 8x^4 + 6x^3 - 13x^2 - x + 3$.

191. 1) $f(x) = x^6 - x^5 + 3x^4 - 6x^3 - 4x^2 + 8x$,

2) $f(x) = 8x^4 + 6x^3 - 13x^2 - x + 3$.

192. 1) $f(x) = (x^2 + 4)^2 + 2x^2 + 5$, 2) $x^4 - 2x^2 - 24 = 0$.

Yangi o'zgaruvchi kiritib tenglama barcha ildizlarni toping:

193. $(x - 2)^2 + (x - 2)(x + 1) + (x + 1)^2 = 0$.

194. $(x^2 - 1)^2 + 5(x^4 - 1) - 6(x^2 + 1)^2 = 0$.

195. $2\left(x^2 + \frac{1}{x^2}\right) + 6\left(x + \frac{1}{x}\right) - 4 = 0$.

196. $\frac{8x}{x^2 + x + 3} + \frac{10}{x^2 - 5x + 3} = -3$.

197. $\frac{1}{x(x+2)} - \frac{1}{(x+1)^2} = \frac{1}{12}$.

198. $\left(x-\frac{1}{x}\right)^2-3\left(x-\frac{1}{x}\right)=4$.

Ratsional algebraik tenglamaning barcha ildizlari topilsin.

199. $\frac{4x}{4x^2-8x+7}+\frac{3x}{4x^2-10x+7}=1$.

200. $\frac{x}{3\left(x^2-1\right)}+\frac{2x}{3\left(1-x^4\right)}=\frac{1}{x\left(1+x^2\right)}$.

201. $\frac{1}{2\left(x^2+1\right)}+\frac{7,5}{2\left(x^2+1\right)}+\frac{9}{2x^2\left(x^2-1\right)}$.

202. $\frac{30}{x^2-1}-\frac{13}{x^2+x+1}=\frac{18x+7}{x^3-1}$.

13-MAVZU. IRRATSIONAL TENGLAMALAR

Noma'lum o'zgaruvchi miqdor hech bo'lmaganda bitta qo'shiluvchi haqida irratsional (ildizda) ko'rinishda qatnashgan algebraik tenglamaga irratsional tenglama deyiladi.

Elementar matematikada irratsional tenglamalar haqiqiy sonlar to'plamida qaralib, ildiz ko'rsatkich juft darajali bo'lsa, uning arifmetik ildizi, ildiz ko'rsatkich toq darajali bo'lsa uning algebraik ildizi olinadi.

Irratsional tenglamalar yechimini aniqlashda o'ziga xos alohidaligi, tenglamada qatnashgan irratsional ifodaning mavjud bo'lish sohasini belishimiz zarur.

Irratsional ifoda haqida ma'lumot misollar bilan [3] to'la oydinlashtirib berilgan.

Irratsional $\sqrt[n]{f(x)}$ ifoda $f(x)$-ildiz ostidagi funktsiya, n-ildiz ko'rsatkich) ning mavjudlik sohasi haqida:

1) Agar ildiz ko'rsatkich ($n = 2k$) juft son bo'lsa, u holda $f(x)$ funktsiyaning mavjudligi va $f(x) \geq 0$ bo'lishi talab qilinadi. Masalan: $\sqrt[4]{2x-5}$ da $f(x) = 2x - 5$ funksiya $x \in R$ da mavjud, $x\epsilon\left[\frac{5}{2};+\infty\right)$ da musbat funksiya. Demak berilgan irratsional ifoda aniqlanish sohasi $\left[\frac{5}{2};+\infty\right)$.

$y=\sqrt{\frac{4x}{x+2}}$ da $f(x)=\frac{4x}{x+2}$ funksiyaning aniqlanish sohasida $x \neq -2$ bo'lish kerak; $\frac{4x}{x+2}\geq 0$ bo'lish sharti, oraliq metodidan foydalansak $x \in (-\infty; -2) \cup [0; +\infty)$ da musbat bo'lishi kelib chiqadi. Demak $y=\sqrt{\frac{4x}{x+2}}$ ifodaning aniqlanish sohasi $x \in (-\infty; -2) \cup [0; +\infty)$.

2) Agar ildiz ko'rsatkich ($n = 2k + 1$) toq bo'lsa, $f(x)$ funksiya mavjudligi yetarli. Masalan: $y=\sqrt[5]{6+3x-x^2}$ ifoda $x \in (-\infty; +\infty)$ mavjud. Endi $y=\sqrt[3]{\frac{4x-7}{x^2-4}}$ ifodada $f(x)=\frac{4x-7}{x^2-4}$ funktsiya $x = 2$ va $x = -2$ da mavjud emas, demak berilgan irratsional ifodaning mavjudlik sohasi $x \in (-\infty; -2) \cup (-2;2) \cup (2; +\infty)$.

Agar irratsional tenglama bir nechta irratsional ifodadan iborat bo‘lsa, tenglamaning aniqlash sohasi, tenglamada qatnashgan har qaysi irratsional ifoda aniqlanish sohalarning birlashmasiga (umumiy qismiga) teng. Masalan, tenglamada

$\sqrt[4]{\dfrac{x-2}{x^2+1}}$, $\sqrt{x^2+2x-8}$, va $\sqrt{\dfrac{8}{x-3}}$ irratsional ifodalar qatnashgan bo‘lsa, tenglamaning aniqlanish sohasi $D = D_1 \cap D_2 \cap D_3$, bu yerda D_1 esa $\sqrt[4]{\dfrac{x-2}{x^2+1}}$ ning aniqlash sohasi $x \geq 2$; D_2 esa $\sqrt{x^2+2x-8}$ ifodaning aniqlash sohasi $x \in (-\infty;-4] \cup [2;+\infty)$;

D_3 esa $\sqrt{\dfrac{8}{x-3}}$ ning aniqlanish sohasi $x(3; +\infty)$ dir. U holda tenglamaning aniqlanish sohasi $x \in (3; +\infty)$ bo‘ladi.

Irratsional tenglamaning yechimi aniqlashda, tenglamaning aniqlanish sohasini yozish shart emas, tenglamani biror metod bilan irratsionaldan qutqarib soddalashtirilganda hosil bo‘lgan ratsional tenglama bilan teng kuchliligini ko‘rsatish yetarli, yoki undan ham sodda yo‘l topilgan sonni tenglamaga qo‘yib tekshirish kerak bo‘ladi. Bunda quyidagi teoremalarni hisobga olish kerak.

Teorema 1. Agar $f_1(x) = f_2(x)$ tenglamani kvadratga ko‘tarsak hosil bo‘lgan $[f_1(x)]^2 = [f_2(x)]^2$ tenglama ildizlari $f_1(x) = f_2(x)$ va $f_1(x) = -f_2(x)$ tenglama ildizlariga teng bo‘ladi.

Teorema 2. Agar $f_1(x) = f_2(x)$ tenglamani kubga ko‘tarsak hosil bo‘lgan $[f_1(x)]^3 = [f_2(x)]^3$ tenglama ildizlari $f_1(x) = f_2(x)$ tenglama ildiziga teng bo‘ladi.

Demak berilgan $f_1(x) = f_2(x)$ tenglamani kvadratga (juft darajaga) ko‘targanda hosil bo‘lgan mumkin

($f_1(x) = -f_2(x)$ hisobiga).

Endi $f_1(x) = f_2(x)$ tenglamani kubga (toq darajaga) ko‘targanda hosil bo‘lgan tenglama berilgan tenglamga teng kuchli (ekvivalent) tenglama bo‘lar ekan.

IRRATSIONAL TENGLAMA YECHIMLARINI TOPISH USULLARI

13.1. Tenglamani mantiqiy mulohazar yuritib birdan bir yechimini yozish

⊕ Tenglama yechilsin.

(1) $\sqrt{2x+3}+\sqrt{5-x}=-3$

▶ Tenglama yechimga ega emas, chunki chap tomonida arifmetik ildizlar yig'indisi musbat son, o'ng tomon manfiy son qarama-qarshilik. ◀

(2) $8-\sqrt[4]{x^2-3x}=12$

▶ Tenglama $-\sqrt[4]{x^2-3x}=4$ yoki $\sqrt[4]{x^2-3x}=-4$ bo'lib, tenglama yechimi mavjud emas. ◀

(3) $3\sqrt{4-x}+\sqrt{x-9}=2$

▶ Tenglamada birinchi ifoda $x\le 4$ da, ikkinchi qo'shiluvchi had $x\ge 9$ da mavjud bularning umumiy qismi bo'sh to'plam, demak tenglama yechimi mavjud emas. ◀

(4) $4\sqrt{3-x}+2\sqrt{x-3}=7$

▶ Bu tenglamaning yechimi mavjud emas, chunki tenglamadagi birinchi ifoda $x\le 3$ da ikkinchi qo'shiluvchi had $x\ge 3$ da aniqlangan, bularning umumiy qismi $x=3$, lekin $x=3$ da tenglama $4\cdot 0+2\cdot 0=7$, $0=7$ ko'rinishda bo'lib, tenglamaga yechim emas. ◀

(5) $(x-5)(0,2x+4)\sqrt{0,5x-2}=0$

▶ Ko'paytma nolligidan

1) $x-5=0$, $x=5$; 2) $0,2x+4=0$, $x=-20$;

3) $0,5x-2=0$, $x=4$; bu qiymatlardan tenglama yechimi $x=5$, va $x=4$, $x=-20$ esa chet ildiz, chunki tenglama aniqlanish sohasi $x\ge 4$. ◀

13.2. $\sqrt[n]{f(x)}=\sqrt[n]{\varphi(x)}$ irratsional tenglama ildizlari

$f(x)=\varphi(x)$ tenglama yechimlarida qidiriladi.

⊕ Tenglamani yeching:

(1) $\sqrt[4]{4x^2-3}=\sqrt[4]{3x-2}$.

▶ Irratsional tenglama ildizini

$4x^2-3=3x-2$

tenglama yechimlardan qidiramiz, tenglamani

$4x^2-3x-1=0$ deb yozish mumkin bu tenglama yechimi $x=1$ va $x=-\frac{1}{4}$, bu sonlar tenglamaga qo'yib tekshiramiz $x=1$ da

$\sqrt[4]{4-3}=\sqrt[4]{3-2}$, $x=1$ yechim, $x=-\frac{1}{4}$ da $\sqrt[4]{4\cdot\frac{1}{16}-3}=\sqrt[4]{3\left(-\frac{1}{4}\right)-2}$ bu holda o'ng tomon ma'noga ega emas, demak berilgan tenglamaning yechimi $x=1$. ◄

(2) $\sqrt{3x-4}=\sqrt{x^2-2}$

► Tenglama ildizini $3x-4=x^2-2$ yoki $x^2-3x+2=0$ tenglama ildizlari qidiramiz, bu tenglama ildizlari $x=1$ va $x=2$. Tekshirish qiyin emas $x=2$ tenglama yechimi, $x=1$ tenglamaga yechim emas. ◄

(3) $\sqrt[3]{\frac{2x}{x^2-3}}=1$

► $x\neq\pm\sqrt{3}$ deb tenglamani $\sqrt[3]{2x}=\sqrt[3]{x^2-3}$ ko'rnishda yozish mumkin, bu tenglamani $x^2-2x-3=0$ tenglama yechimlari $x=-1$, va $x=3$ bo'lib, bu sonlar berilgan tenglama uchun ham yechim bo'ladi. ◄

13.3. Darajaga ko'tarib, $\sqrt[n]{f(x)}=\varphi(x)$ tenglama yechimini aniqlash mumkin bo'lgan hol.

(I) Tenglama yechilsin:

(1) $1+\sqrt{2x+7}=x-3$

► Berilgan tenglamani

$\sqrt{2x+7}=x-4$

ko'rinishda yozib, tenglamaning ikki tomonini kvadratga ko'taramiz:

$\left(\sqrt{2x+7}\right)^2=(x-4)^2$ dan

$$2x+7=x^2-8x+16 \text{ yoki } x^2-10x+9=0$$

bu tenglama ildizlari $x=1$, $x=9$. Bundan $x=9$ berilgan tenglamaga yechim bo'lib $x=1$ da $\sqrt{2+7}=1-4$ tenglik bajariladi, $x=1$ chet ildiz bo'lib, $x=9$ berilgan tenglama yechimi. ◄

(2) $\sqrt[3]{x^3-2x-3}=x-1$

►Berilgan tenglamaning ikki tomonini kubga ko'taramiz:

$\left(\sqrt[3]{x^3-2x-3}\right)^3=(x-1)^3$ dan $x^3-2x-3=x^3-3x^2+3x-1$ yoki

$3x^2-5x-2=0$ bo'lib, tenglama yechimlari $x=2$ va $x=-\frac{1}{3}$.

Tekshirish ko'rsatadiki $x=2$ va $x=-\frac{1}{3}$ berilgan tenglama uchun yechim bo'ladi. ◄

(3) $\sqrt[5]{25+\sqrt{x-4}}=2$

► Tenglamani $x \geq 4$ deb darajaga ko'tarib yozsak

$\left(\sqrt[5]{25+\sqrt{x-4}}\right)^5 = 2^5$ yoki

$25+\sqrt{x-4}=32$ yoki $\sqrt{x-4}=7$,tenglikning ikki tomonini kvadratga ko'tarsak $x-4=49$ bundan

$x=53$. Tekshirish $x=53$ da $\sqrt[5]{25+\sqrt{53-4}}=2$ bundan $\sqrt[5]{25+7}=2$ bo'lib $2=2$ kelib chiqadi. Javob: $x=53$. ◄

(4) $\sqrt{x-2}\sqrt{2x+3}=0$

► Tenglikni kvadratga ko'tarish bilan

$(x-2)(2x+3)=0$ tenglamaga kelamiz, bu tenglama ildizlari $x=2$, $x=-1{,}5$. $x=-1{,}5$ da tenglamadagi birinchi ko'paytma ma'noga ega emas, demak berilgan tenglama ildizi $x=2$. ◄

(5) $\sqrt{x^2-9}=x^2-21$

► Tenglikni kvadratga ko'tarsak

$$x^2-9=x^4-42x+441,$$

soddalashtirsak $x^4-43x^2+450=0$, $x^2=y$ almashtirish bajarsak

$$y^2-43y+450=0$$

kvadrat tenglama hosil bo'lib, bu tenglama ildizlari $y_1=25$ va $y_2=18$. $x^2=y$ almashtirishdan $x^2=25$ va $x^2=18$. Bu sonlardan $x^2=18$ chet ildiz chunki tenglamaning o'ng tomoni manfiy bo'lib qoladi. $x^2=25$ dan $x=\pm 5$. Berilgan tenglama ildizlari $x_1=5$, $x_2=-5$. ◄

13.4. Berilgan irratsional tenglamani darajaga ko'tarib, o'z navbatida qisqa ko'paytirish formulalarni tatbiq qilib, tenglamani ratsional tenglama ko'rnishga keltirish mumkin bo'lgan metodni misollarda oydinlashtiramiz.

⊕ Tenglama yeching:

(1) $\sqrt{x-3}-\sqrt{x-6}=1$

► Tenglamani $\sqrt{x-3}=1+\sqrt{x-6}$ ko'rinishda yozib, tenglamaning ikki tomonini kvadratga ko'tarib, qisqa ko'paytirish formulasidan foydalanamiz:

$\left(\sqrt{x-3}\right)^2=\left(1+\sqrt{x-6}\right)^2$ dan $\sqrt{x-6}=1$ yana kvadratga ko'tarsak $x-6=1$ bo'lib $x=7$, tekshirish $x=7$ da $\sqrt{7-3}-\sqrt{7-6}=1$, $2-1=1$, tenglik o'rinli, demak $x=7$ berilgan tenglama yechimi. ◄

(2) $\sqrt{3x+7}-\sqrt{x+1}=2$

► Tenglamani

$$\sqrt{3x+7}=2+\sqrt{x+1}$$

ko'rinishda yozib, kvadratga ko'tarib soddalashtirsak

$$x+1=2\sqrt{x+1}$$

irratsionallikdan ko'tarib soddalashtirsak

$$x^2-2x-3=0$$

tenglamaga kelinadi bu tenglama ildizlari $x=-1$ va $x=3$, tekshirish ko'rsatadiki bu sonlar tenglama yechimi. ◀

(3) $\sqrt{x^2-2x+3}=3-\sqrt{x^2+1}$

▶ Tenglikning ikki tomonini kvadratga ko'tarib soddalashtirsak

$$2x+7=6\sqrt{x^2+1}$$

yana irratsional tenglamaga kelinadi, oldingi metodni qo'llasak

$$32x^2-28x-13=0$$

tenglamaga kelinadi bu tenglama ildizi

$x_{1,2}=\dfrac{7\pm\sqrt{153}}{16}$ berilgan tenglamaga ham yechim bo'ladi. ◀

(4) $\sqrt{x^2+4x+4}+\sqrt{x^2-10x+25}=10$

▶ Tenglamani $\sqrt{(x+2)^2}+\sqrt{(x-5)^2}=10$ ko'rinishda yozish mumkin bundan $|x+2|+|x-5|=10$ modulli tenglamani hosil qilamiz. Agarda:

1) $x<-2$ bo'lsa, u holda tenglama $-(x+2)-(x-5)=10$ ko'rinishda bo'lib bundan $x=-3,5$

2) $-2\le x\le 5$ da $x+2-(x-5)=10$ yoki $7=10$ yechim mavjud emas.

3) $x>5$ da $x+2+x-5=10$ bo'lib yechim $x=6,5$.

$x=-3,5$ va $x=6,5$ ni berilgan tenglamaga qo'yib tekshirsak yechim ekanligi kelib chiqadi. ◀

(5) $\sqrt{3x+1}+\sqrt{2x-1}=\sqrt{4+5x}$

▶ Tenglikning ikki qismini kvadratga ko'tarib soddalashtirsak $\sqrt{(3x+1)(2x-1)}=2$ yoki $\sqrt{6x^2-x-1}=2$ bundan $6x^2-x-5=0$, bu tenglama ildizi $x=1$ va $x=-\dfrac{5}{6}$. Bu sonlarni tenglamaga qo'yib tekshirishda $x=1$ yechim $x=-\dfrac{5}{6}$ esa chet ildiz ekanligi ko'rinadi.

Javob: $x=1$. ◀

13.5. Tenglamani yangi o'zgaruvchi kiritib yechish usuli:

⊕ Tenglamaning yechimi topilsin

(1) $2\sqrt[3]{x^2}+\sqrt[3]{x}-3=0$,

► $\sqrt[3]{x}=y$ deb belgilaymiz, bu holda berilgan tenglama $2y^2+y-$ $-3=0$ kelinadi bunda $y_1=1, y_2=-1{,}5$ bo'lib, almashtirishdan: 1) $\sqrt[3]{x}=1$, $x_1=1$; 2) $\sqrt[3]{x}=-\frac{3}{2}$ dan $x_2=-\frac{27}{8}$. Topilgan sonlarni tenglamaga qo'yib tekshirsak ikkala son ham tenglamani qanoatlantirishini ko'ramiz. Javob: $x_1=1, x_2=-1{,}5$. ◄

(2) $3\sqrt{\frac{x}{x-1}}-3\sqrt{1-\frac{1}{x}}=2{,}5$

► Yangi o'zgaruvchi $\sqrt{\frac{x}{x-1}}=y$ kiritsak $3y-\frac{3}{y}=\frac{5}{2}$ yoki $6y^2-$ $-5y-6=0$ tenglamaga kelamiz, bu tenglama yechimlari $y_1=\frac{3}{2}$, $y_2=-\frac{2}{3}$ bo'lib, almashtirishdan

1) $\sqrt{\frac{x}{x-1}}=\frac{3}{2}$ dan $\frac{x}{x-1}=\frac{9}{4}$ yoki $5x=9, x=1{,}8$

2) $\sqrt{\frac{x}{x-1}}=-\frac{2}{3}$ bo'sh to'plam. $x=1{,}8$ soni berilgan tenglamani qanoatlandiradi. Javob: $x=1{,}8$. ◄

(3) $\sqrt{2-x}-2+\frac{4}{\sqrt{2-x}+3}=0$

► $x\neq 2$ deb, yangi $\sqrt{2-x}=y$ o'zgaruvchi kiritsak, tenglama y^2+ $+y-2=0$, ko'rinishda bo'lib, bu tenglama yechimi $y=1$ va $y=-2$. Almashtirishdan

1) $\sqrt{2-x}=1$ bundan $x=1$, 2) $\sqrt{2-x}=-2$ yechim mavjud emas. Tekshirishdan ko'rinadiki $x=1$ berilgan tenglama yechimi. ◄

13.6. Tenglamani "ifoda qo'shmasiga ko'paytirish" usuli bilan yechish.

⊕ $\sqrt{3x^2+5x+8}-\sqrt{3x^2+5x+1}=1$

► Tenglamani

$$\left(\sqrt{3x^2+5x+8}-\sqrt{3x^2+5x+1}\right)\left(\sqrt{3x^2+5x+8}+\sqrt{3x^2+5x+1}\right)=$$

$=\left(\sqrt{3x^2+5x+8}+\sqrt{3x^2+5x+1}\right)$ ko'rinishda yozib soddalashtirsak

$7=\left(\sqrt{3x^2+5x+8}+\sqrt{3x^2+5x+1}\right)$ ga keladi.

Tenglamadan $\sqrt{3x^2+5x+8}$ ni topib o‘rniga qo‘ysak, tenglama $7=1+2\sqrt{3x^2+5x+1}$ yoki $\sqrt{3x^2+5x+1}=3$ ko‘rinishga keladi. Bundan $3x^2+5x-8=0$ kelib chiqadi. Bu tenglama yechimi $x=1$ va $x=-2\frac{2}{3}$, topilgan sonlar tenglama yechimi bo‘lishini osongina ko‘rish mumkin. Berilgan tenglamani $3x^2+5x=y$ deb yechish ham mumkin. ◄

13.7. Umumlashgan usul :

⊕ Tenglamani yeching:

(1) $\sqrt[3]{2x+19}=x-1$

► Tenglamaning ikki qismini kubga ko‘tarib soddalashtirsak

$$x^3-3x^2+x-20=0$$

tenglamaga kelinadi, bu tenglamaning bitta yechimi $x-4$ bo‘ladi. Tenglamaning chap tomonidagi ko‘phadni $(x-4)$ga bo‘lsak bo‘linma (x^2+x+5) ga teng, berilgan tenglama

$$(x-4)(x^2+x+5)=0$$

ko‘paytma ko‘rinishga kelib, ikkinchi ko‘paytma haqiqiy yechimga ega emas. Tenglama yechimi $x=4$. ◄

(2) $\sqrt[3]{x+34}=1+\sqrt[3]{x-3}$

► tenglamani $\left(\sqrt[3]{x+34}\right)^3=\left(1+\sqrt[3]{x-3}\right)^3$, deb kub formuladan foydalansak

$$x+34=1+3\sqrt[3]{x-3}+3\sqrt[3]{(x-3)^2}+x-3$$

yoki soddalashtirsak

$$\sqrt[3]{(x-3)^2}+\sqrt[3]{x-3}-12=0$$

endi $\sqrt[3]{x-3}=y$ desak

$$y^2+y-12=0$$

tenglama hosil bo‘lib yechimi $y_1=3, y_2=-4$. U holda,

1) $\sqrt[3]{x-3}=3$ bundan $x-3=27, x=30$; 2) $\sqrt[3]{x-3}=-4$ dan $x-3=$ $=-64$ yoki $x=-61$. Bu sonlarni tenglamaga qo‘ysak yechim ekanligi ko‘rinadi.

Javob: $x_1=30, x_2=-61$. ◄

(3) $\dfrac{4}{\sqrt[3]{x}+2}+\dfrac{\sqrt[3]{x}+3}{5}=2$

► $\frac{1}{\sqrt[3]{x}+2}=y$ deb almashtirish bajarsak y ga nisbatan

$$20y^2-9y+1=0$$

tenglamaga kelinadi, bu tenglama yechimi $y_1=\frac{1}{4}$, $y_2=\frac{1}{5}$, endi almashtirishdan:

1) $\frac{1}{\sqrt[3]{x}+2}=\frac{1}{4}$ dan $\sqrt[3]{x}+2=4$ bo'lib bundan $x=8$;

2) $\frac{1}{\sqrt[3]{x}+2}=\frac{1}{5}$ dan $\sqrt[3]{x}+2=5$ yoki $x=27$. Bu 8 va 27 sonlar tenglama yechimi ekanligi isbotlash oson. ◄

(4) $\sqrt[4]{(x-1)^2}-\sqrt[4]{(x+1)^2}=\frac{3}{2}\sqrt[4]{x^2-1}$

► $x^2-1\neq 0$ deb tenglamaning ikki tomonini $\sqrt[4]{x^2-1}$ ifodaga bo'lsak $\sqrt[4]{\frac{(x-1)}{(x+1)}}-\sqrt[4]{\frac{(x+1)}{(x-1)}}=\frac{3}{2}$, endi $\sqrt[4]{\frac{(x-1)}{(x+1)}}=y$ deb yangi o'zgaruvchi kiritsak, y ga nisbatan

$$2y^2-3y-2=0$$

tenglamaga kelinadi, bu tenglama yechimlari

$y_1=2$ va $y_2=-\frac{1}{2}$, u holda almashtirishdan:

1) $\sqrt[4]{\frac{(x-1)}{(x+1)}}=2$ yoki $\frac{x-1}{x+1}=16$ bundan $x=-\frac{17}{15}$;

2) $\sqrt[4]{\frac{(x-1)}{(x+1)}}=-\frac{1}{2}$ yechim mavjud emas. $x=-\frac{17}{15}$ soni tenglamani qanoatlantiradi.

Javob: $x=-1\frac{2}{15}$. ◄

(5) $\sqrt{x+\sqrt{x}}-\sqrt{x-\sqrt{x}}=\frac{3\sqrt{x}}{2\sqrt{x+\sqrt{x}}}$

► Tenglamani shakl almashtirib yozsak,

$$-\sqrt{x-\sqrt{x}}=\frac{3\sqrt{x}}{2\sqrt{x+\sqrt{x}}}-\sqrt{x+\sqrt{x}} \quad \text{dan} \quad -2\sqrt{x^2-x}=\sqrt{x}-2x$$

ifodaning ikki qismini kvadratga ko‘tarib soddalashtirsak $5x-4x\sqrt{x}=0$

bu tenglama yechimlari $x = 0$ va $x=\frac{25}{16}$. Bu sonlarni tenglamaga qo‘yib tekshirsak $x = 0$ chet ildiz $x=\frac{25}{16}$ esa yechim ekanligi ko‘rinadi. ◄

(6) $3x^2+15x+2\sqrt{x^2+5x+1}=2$

► Agar $\sqrt{x^2+5x+1}=y$ desak tenglama

$3y^2 + 2y - 5 = 0$ ko‘rinishga keladi. Bu tenglama yechimi $y_1 = 1$ va $y_2=-\frac{5}{3}$, u holda:

1) $\sqrt{x^2+5x+1}=1$ dan $x^2 + 5x = 0$ yoki $x_1 = 0, x_2 = -5$;

2) $\sqrt{x^2+5x+1}=-\frac{5}{3}$ yechim mavjud emas. Topilgan 0 va –5 sonlar berilgan tenglama yechim bo‘ladi. ◄

13-MAVZU MASHQLARI

Quyidagi irratsional tenglamalarning haqiqiy sonlar to‘plamida aniqlash sohasini ko‘rsating:

203. 1) $\sqrt[3]{x+1}+5=0$, 2) $\sqrt{2x-1}=\sqrt{2-x}$,

3) $\sqrt[4]{8+\sqrt{5-x}}=2$, 4) $\sqrt{x-1}+\sqrt{x+2}-\sqrt{3x-5}=0$.

204. 1) $\sqrt{x-6}+\sqrt{3-x}=4$, 2) $\sqrt[5]{\frac{3}{x^2-3x+2}}=\sqrt[5]{x+4}$,

3) $\sqrt{3x+1}+\sqrt{9-x}=\dfrac{6}{\sqrt{9-x}}$, 4) $\sqrt{x+1}=-4x$.

Tenglamani mantiqiy mulohaza yuritib yeching:

205. 1) $\sqrt{3x-1}+\sqrt{4+x}=-5$, 2) $8-\sqrt{x+\sqrt{5}}=10$,

3) $\sqrt{x-4}+3\sqrt{1-x}=4$.

206. 1) $\sqrt{x-3}+0,5\sqrt{2-x}=5$, 2) $4x\sqrt{2x-5}\cdot\sqrt{1-x}=0$,

3) $\sqrt{2-\sqrt{5x}}+\sqrt{x-5}=0,5$.

Irratsional tenglamaning yechimlarini toping:

207. 1) $\sqrt[3]{x^2-2}=\sqrt[3]{3x+2}$, 2) $\sqrt{6x^2+x+5}=\sqrt{x^2-x-1}$.

3) $\sqrt[3]{25+\sqrt{x^2+3}}=3$, 4) $\sqrt[4]{17+\sqrt[3]{x^2-5}}=2$.

208. 1) $\left(x^2-4\right)\sqrt{x+1}=0$, 2) $21+\sqrt{2x-7}=x$,

3) $\sqrt{1+x\sqrt{x^2+24}}=x+1$, 4) $\sqrt[3]{6+\sqrt{x^2-3x}}=2$.

209. 1) $\sqrt{7+\sqrt[3]{x^2+2x}}=3$, 2) $\sqrt{(x+2)(x+3)}=\sqrt{2}$.

210. 1) $\sqrt{2x-1}\cdot\sqrt{3x+1}=x+1$, 2) $\dfrac{x-2}{\sqrt{2x-7}}=\sqrt{x-4}$.

211. $\sqrt{x-3}+\sqrt{x-6}=1$.

212. $\sqrt{x+1}+\sqrt{x+5}=6$.

213. $\sqrt{x^3-x-1}+\sqrt{2x+1}+2=0$.

214. $\sqrt{x^2-6x+9}-\sqrt{1+4x}+2=0$.

215. $\sqrt{x+1}-\sqrt{2x-12}=\sqrt{9-x}$.

216. $\sqrt{x-5}+\sqrt{x+3}=\sqrt{2x+4}$.

217. $x^2+2\sqrt{x^2+1}=14$.

218. $5\sqrt[4]{x-3}=6\sqrt{x-3}$.

219. $\sqrt{3-\dfrac{1}{x}}=3+\sqrt{\dfrac{x}{3x-1}}$.

220. $x^2+3x-10+3\cdot\sqrt{x(x+3)}=0$.

14-MAVZU. CHIZIQLI BO'LMAGAN IKKI NOMA'LUMLI IKKITA TENGLAMALAR SISTEMASI

Chiziqli bo'lmagan tenglamalar sistemasi yechimini aniqlashda asosan sistema berilishiga qarab, biror metod qo'llab bir noma'lumli tenglamaga keltirib, sitema yechimlari topiladi. Metodlarni misollarda oydinlashtirib, ko'rib chiqamiz.

14.1. Sistemada bitta tenglama chiziqli bo'lsa, o'rniga qo'yish usulidan foydalanamiz.

⊕ Tenglamalar sistemasi yechilsin:

1) $\begin{cases} 2x^2 - xy + 3y^2 - 7x - 12y + 1 = 0, \\ x - y = -1 \end{cases}$

► Sistemadagi ikkinchi tenglamadan $x = y - 1$ ni birinchi tenglamaga qo'yib $2(y-1)^2 - y(y-1) + 3y^2 - 7(y-1) - 12y + 1 = 0$, Soddalashtirilsa $2y^2 - 11y + 5 = 0$ kelinadi, bundan $y_1 = 5$, $y_2 = 0{,}5$ u holda $x = y - 1$ dan $x_1 = 4$, $x_2 = -0{,}5$ Javob (4;5) va (– 0,5;0,5). ◄

2) $\begin{cases} \dfrac{2x-5}{x-2} + \dfrac{2y-3}{y-1} = 2, \\ 3x - 4y = 1 \end{cases}$

► Avval birinchi tenglamada $x \neq 2$, $y \neq 1$ deb umumiy mahraj bersak

$$(y-1)(2x-5) + (x-2)(2y-3) = 2(x-2)(y-1)$$

qavs ochib soddalashtirsak $2xy - 3x - 5y = -7$, endi hosil bo'lgan $\begin{cases} 2xy - 3x - 5y = -7 \\ 3x - 4y = 1 \end{cases}$ sistemada o'rniga qo'yish usulidan foydalanamiz.

$y = \dfrac{3x-1}{4}$ deb, birinchi sistemani soddalashtirsak $6x^2 - 29x + 33 = 0$ bo'lib, bundan $x_1 = 3, x_2 = \dfrac{11}{6}$, u holda $y = \dfrac{3x-1}{4}$ ga asosan

$$y_1 = 2, y_2 = \frac{9}{8}$$

Javob: (3;2) va $\left(\dfrac{11}{6};\dfrac{9}{8}\right)$. ◄

3) $\begin{cases} xy = 2, \\ 2x + 3y = 7 \end{cases}$

► sistema yechimini topishda o'rniga qo'yish usulidan foydalanamiz.

Birinchi tenglamadan $x \neq 0$ deb y ni topib $\left(y=\frac{2}{x}\right)$ ikkinchi tenglamaga qo‘yib soddalashtirsak,

$2x^2-7x+6=0$ ga kelinadi. bundan $x_1=2$, $x_2=\frac{3}{2}$ bo‘lib $y=\frac{2}{x}$ dan $y_1=1$, $y_2=\frac{4}{3}$

Javob: (2;1) va $\left(\frac{3}{2};\frac{4}{3}\right)$. ◄

4) $\begin{cases} \sqrt[3]{x}\cdot\sqrt{y}+\sqrt[3]{y}\cdot\sqrt{x}=12 \\ xy=64 \end{cases}$

► Oldingi misoldagi metodni qo‘llaymiz:

$$\begin{cases} x=\frac{64}{y}\,(y\neq 0 deb) \\ \sqrt{y}\cdot\sqrt[3]{\frac{64}{y}}+\sqrt[3]{y}\cdot\sqrt{\frac{64}{y}}=12 \end{cases} \text{bundan}$$

$4\cdot y^{\frac{1}{6}}+8y^{-\frac{1}{6}}=12$ soddalashtirsak $y^{\frac{1}{3}}-y^{\frac{1}{6}}+2=0$, endi $\sqrt[6]{y}=z$ deb olsak kvadrat tenglamaga kelinadi.

$z^2-3z+2=0$ bundan $z_1=2$, $z_2=1$ bo‘lib $\sqrt[6]{y}=z$ almashtirishdan

$y_1=1$, $y_2=64$ bo‘lib, $x=\frac{64}{y}$ dan $x_1=64$, $x_2=1$

Javob (64;1) va (1;64).

14.2. Tenglamalar sistemasidagi tenglamalarni o‘zaro qo‘shish yoki ayirish bilan soddaroq tenglamaga keltirish mumkin.

⊕ Tenglamalar sistemasi yechilsin

1) $\begin{cases} 2x-y-xy=14 \\ x+2y+xy=-7 \end{cases}$

► Sistemadagi tenglamalarni qo‘shsak (chap tomonini chap tomonga o‘ng tomonini o‘ng tomonga) sodda sistemaga kelinadi

$$\begin{cases} 2x-y-xy=14, \\ 3x+y=7 \end{cases}$$

o‘rniga qo‘yish ($y=7-3x$) .usuldan foydalansak sistema yechimlari

$$(3;-2) \text{ va } \left(-\tfrac{7}{3};14\right). ◄$$

2) $\begin{cases} x^2+y^2+x+y=18 \\ x^2-y^2+x-y=6 \end{cases}$

► Sistemadagi tenglamalarni qo‘shib soddalashtirsak $x^2 + x - 12 = 0$ bundan $x_1 = -4$ va $x_2 = 3$ topiladi. Topilgan x o‘zgaruvchining qiymatlari sistemadagi birinchi tenglamaga qo‘shsak

$$16 + y^2 - 4 + y = 18 \text{ va } 9 + y^2 + 3 + y = 18$$

bunda birinchi tenglamaning yechimlari mos ravishda (–3) va 2; ikkinchi tenglamaniki ham (–3) va ikki bo‘lib, berilgan tenglamalar sistemasi yechimi $(-4;-3),(-4;2),(3;-3)$ va $(3,-3)$.◄

14.3. Tenglamalar sistemasida bittasi yoki ikkitasi ham ko‘paytma ko‘rinishida bo‘lib, o‘ng tomoni nol bo‘lgan hol.

Tenglamalar sistemasi yechimlari topilsin:

1) $\begin{cases} x^2+3xy+2y^2=42, \\ xy-5y+4x-20=0. \end{cases}$

► Sistemadagi ikkinchi tenglamani ko‘paytma ko‘rinishiga keltirish mumkin. $0 = xy + 4x - 5y - 20 = x(y + 4) - 5(y + 4) = (y + 4)(x - 5)$ berilgan tenglamaning sistemasi $\begin{cases} x^2+3xy+2y^2=42, \\ (y+4)(x-5)=0, \end{cases}$ teng kuchli ikkinchi sodda sistemaga kelinadi

$$\begin{cases} x^2+3xy+2y^2=42, \\ y+4=0 \end{cases} \text{ va } \begin{cases} x^2+3xy+2y^2=42, \\ x-5=0 \end{cases}$$

o‘rniga qo‘yish metodi yordamida tenglamalar sistemasi yechimlari topiladi. Bu yechimlar $\left(6+\sqrt{46};-4\right),\left(6-\sqrt{46};-4\right)$ 5;1 va (5; – 8,5) ga teng ◄

2) $\begin{cases} x^2-xy-2x-3y=6, \\ x^2-2xy-3y^2=0 \end{cases}$

► sistemadagi ikkinchi tenglamalarni x ga nisbatan kvadrat tenglama deb, yechimlarini topsak $x_{1.2} = \dfrac{2y \pm \sqrt{16y^2}}{2}$ yoki $x_1 = 3y$ va $x_2 = -y$ bo‘ladi. Bu tenglama Viet teoremasiga asosan

$(x - 3y)(x + y) = 0$ deb yozish mumkin. U holda berilgan tenglamalar sistemasi teng kuchli ikkita sistemaga

$$\begin{cases} x^2 - xy - 2x - 3y = 6 \\ x - 3y = 0 \end{cases} \text{ va } \begin{cases} x^2 - xy - 2x - 3y = 6 \\ x + y = 0 \end{cases}$$

o'rniga qo'yish metodidan foydalasak sistema yechimlari

$$(6;2)\left(-\frac{3}{2};-\frac{1}{2}\right)(-2;2) \text{ va } \left(\frac{3}{2};-\frac{3}{2}\right) \text{ bo'ladi.}$$

3) $\begin{cases} 4x^2 - 4xy + y^2 = 9, \\ x^3 + 2x^2y - 4x - 8y = 0 \end{cases}$

► Sistemadagi ikkita tenglamani ham ko'paytma ko'rinishida yozish mumkin.

$$\begin{cases} (2x-y)(2x-y) = 9 \\ (x^2-4)(x+2y) = 0 \end{cases} \text{ yoki } \begin{cases} 2x - y = \pm 3 \\ (x-2)(x+2)(x+2y) = 0 \end{cases}$$

bu sistema teng kuchli oltita sistemaga kelinadi

1) $\begin{cases} 2x - y = 3 \\ x - 2 = 0 \end{cases}$ yechim, (2;1),

2) $\begin{cases} 2x - y = 3 \\ x + 2 = 0 \end{cases}$ yechim (– 2; – 7),

3) $\begin{cases} 2x - y = 3 \\ x + 2y = 0 \end{cases}$ yechim $\left(\frac{6}{5};-\frac{3}{5}\right)$,

4) $\begin{cases} 2x - y = -3, \\ x + 2 = 0 \end{cases}$ yechim (– 2; – 1),

5) $\begin{cases} 2x - y = -3 \\ x - 2 = 0 \end{cases}$ yechim (2;7)

6) $\begin{cases} 2x - y = -3, \\ x + 2y = 0 \end{cases}$ yechim $\left(-\frac{6}{5};\frac{3}{5}\right)$.

Natijada berilgan tenglamalar sistemasi yechimlari

$$(2;1),(-2;-7),\left(\frac{6}{5};-\frac{3}{5}\right),(-2;-1),(2;7) \text{ va } \left(-\frac{6}{5};\frac{3}{5}\right). \blacktriangleleft$$

14.4. Sistemada tenglamalar chap tomoni x va y o'zgaruvchilarga nisbatan bir jinsli funksiya bo'lib, o'ng tomoni o'zgarmas son bo'lsa $y = zx$ almashtirib bajarib, keyin tenglama chap tomonini ikkinchi tenglamaning chap tomoniga, o'ng tomonini o'ng tomoniga bo'lsak z ga nisbatan tenglamaga kelinadi (funksiya x va y o'zgaruvchilarga nisbatan bir jinsli deyiladi, undagi x va y ning darajalari va xy ko'paytmaning darajali bir xil bo'lsa).

⊕ Tenglamalar sistemasi yechimlari topilsin;

1) $\begin{cases} y^2 - xy = -12 \\ x^2 - xy = 28 \end{cases}$

Sistemadagi tenglamalarning chap tomoni bir jinsli $y = zx$ $(z \neq 0)$ almashtirishni bajaramiz:

$\begin{cases} z^2x^2 - x^2z = -12 \\ x^2 - x^2z = 28 \end{cases}$ sistemadan $\dfrac{x^2z(z-1)}{x^2(1-z)} = \dfrac{-12}{28}$ bundan

$(z \neq 1)\, z = \dfrac{3}{7}$ bo'lib, sistemaning birinchi tenglamasidan

$\dfrac{9}{49}x^2 - \dfrac{3}{7}x^2 = 12$ yechim $x_1 = 7$, $x_2 = -7$, u holda $y = \dfrac{3}{7}x$ almashtirishdan $y_1 = 3$ va $y_2 = -3$. Tenglamalar sistemasi yechimlari (7; 3) va (–7; –3).◄

2) $\begin{cases} 3x^2 - xy + 4y^2 = 14, \\ 2x^2 - xy + 2y^2 = 8 \end{cases}$

► Sistemadagi tenglamalarning chap tomoni x va y ga nisbatan bir jinsli bo'lganligi uchun $y = z \cdot x$ almashtirish bajarsak

$\begin{cases} 3x^2 - x^2z + 4x^2z^2 = 14, \\ 2x^2 - x^2z + 2x^2z^2 = 8. \end{cases}$ bu sistemadan $\dfrac{x^2(3 - z + 4z^2)}{x^2(2 - z + 2z^2)} = \dfrac{14}{8}$

soddalashtirsak $2z^2 + 3z - 2 = 0$ Oldingi misol kabi davom ettiriladi.

Sistema yechimini topishda 2-usul. Sistemadagi birinchi tenglamani (–4) ga ikkinchi tenglamani 7ga ko'paytirib mos hadlarini o'zaro qo'shsak $2x^2 - 3xy - 2y^2 = 0$ yoki $(x - 2y)(2x + y = 0)$

U holda sistema teng kuchli ikkita sodda sistemaga kelinadi:

$\begin{cases} x - 2y = 0 \\ 2x^2 - xy + 2y^2 = 8 \end{cases}$ va $\begin{cases} 2x + y = 0, \\ 2x^2 - xy + 2y^2 = 8 \end{cases}$

O'rniga qo'yish usulidan, sistema yechimlari topiladi:

(2;1), (–2; –1), $\left(\dfrac{\sqrt{6}}{3}; \dfrac{-2\sqrt{6}}{3}\right)$ va $\left(\dfrac{-\sqrt{6}}{3}; \dfrac{2\sqrt{6}}{3}\right)$

14.5. Noma'lumlarga nisbatan sistema tenglamalari simmetrik bo'lgan holda (ya'ni tenglamalardagi o'zgaruvchilarning o'rnini almashtirish bilan tenglama o'zgarmasa)

⊕ Tenglamalar sistemasi yechimi topilsin:

$$\begin{cases} x+y+xy=7 \\ x^2+y^2+xy=13 \end{cases}$$

► Sistema o'zgaruvchilarga nisbatan simmetrik

$$\begin{cases} x+y=v \\ x\cdot y=u \end{cases},(*) \text{ desak sistema } \begin{cases} 4+v=7, \\ v^2-u=13 \end{cases} (**) \text{ ko'rinishga kelinadi.}$$

(**) dan $v^2+v-20=0$ yechimlari $v_1=4$, $v_2=-5$ U holda $u=7-v$ dan $u_1=7-4=3$, $u_2=7-(-5)=12$ endi (*) sistemaga qaytsak

$$\begin{cases} x+y=3 \\ xy=4 \end{cases} \text{ va } \begin{cases} x+y=12 \\ xy=-5 \end{cases}$$

O'rniga qo'yish usulida yechim topildi. ◄

2) $$\begin{cases} x^2+y^2+x+y=8 \\ x^2+y^2+xy=7 \end{cases}$$

► Tenglamalar o'zgaruvchilarga nisbatan simmetrik $\begin{cases} x+y=u \\ xy=v \end{cases}$ (*)

desak sistema $\begin{cases} u^2+u-2v=8, \\ u^2-v=7 \end{cases}$ ko'rinishga keladi $v=u^2-7$ ni birinchi

tenglamaga qo'yib soddalshtirsak $u^2-u-6=0$ hosil bo'ladi, bu tenglama yechimlari $u_1=-2$ va $u_2=3$ u holda $v=u^2-7$ dan $v_1=-3$ va $v_2=2$

Natijada sistema yechimlarini topishda sodda ekvivalent tenglamalar sistemasiga kelinadi.

$\begin{cases} x+y=3; \\ x\cdot y=2 \end{cases}$ va $\begin{cases} x+y=-2, \\ x\cdot y=-3 \end{cases}$ bu sistemalar o'rniga qo'yish usulida

yechimlar (– 3;1), (1;3), (1;2) va (2;1) kelib chiqadi. ◄

3) $$\begin{cases} x^2+y^2-xy=61 \\ x+y-\sqrt{xy}=7 \end{cases} \quad (xy\geq 0)$$

► Tenglamalar o'zgaruvchilarga nisbatan simmetrik

$$\begin{cases} x+y=u \\ \sqrt{xy}=v \end{cases} (*) \text{ desak } \begin{cases} u^2-3v^2=61, \\ u-v=7 \end{cases} (**)$$

Chunki $x^2 + y^2 - xy = (x+y)^2 - 3xy$ ga teng.

(**) sistema yechimlari $u_1 = 13$, $v_1 = 6$ va $u_2 = 8$, $v_2 = 1$ U holda berilgan sistema sodda teng kuchli ikkita sistemaga kelinadi (*) asosan

$$\begin{cases} x+y=13, \\ xy=36 \end{cases} \text{va} \begin{cases} x+y=8, \\ xy=1 \end{cases}$$

O'rniga qo'yish metodidan foydalanish yechimlari

$(4;9),(9;4),\left(4+\sqrt{15},4-\sqrt{15}\right)$ va $\left(4-\sqrt{15};4+\sqrt{15}\right)$ kelib chiqiladi. ◄

4) $\begin{cases} x^3+y^3=7 \\ xy(x+y)=-2 \end{cases}$

► Sistemadagi ikkinchi tenglamani 3 ga ko'paytirib, birinchi tenglamaga qo'shamiz $x^3 + y^3 + 3x^2y + 3xy^2 = 1$ yoki $(x+y)^3 = 1$ dan $x + y = 1$, u holda sistemali sodda ko'rinishga $\begin{cases} x+y=1 \\ xy(x+y)=-2 \end{cases}$ kelinadi, bunda

$x + y = u$, $xy = v$ desak $\begin{cases} u=1 \\ v\cdot u=-2 \end{cases}$ bo'lib sistemadan $u = 1$, $v = -2$,

almashtirishga asosan berilgan tengliklar sistemasi yechimini topish teng kuchli $\begin{cases} x+y=1 \\ x\cdot y=-2 \end{cases}$ sistemaga kelinadi, o'rniga qo'yish usuldan sistema yechimlari topiladi (2; –1) va (–1;2) ◄

14.6. Noma'lum modulda qatnashgan tenglamalar sistemasi yechimini aniqlashda modul tenglama yechimini aniqlsh metodlarini ham hisobga olamiz

⊕ Tenglamalar sistemasi yechimlari topilsin.

1) $\begin{cases} 3y+|x|=7 \\ 2x+2|y-1|=3 \end{cases}$

► Sistema yechimlarini quyidagi shartlarda qidiramiz:

a) $\begin{cases} x\ge 0 \\ y\ge 1 \end{cases}$ da sistema $\begin{cases} x+3y=7 \\ 2x+2y-2=3 \end{cases}$ ko'rinishda bo'lib

bundan $x=\dfrac{1}{4}, y=\dfrac{9}{4}$, bu sistema uchun ham yechim.

b) $\begin{cases} x<0 \\ y<1 \end{cases}$ da sistema $\begin{cases} -x+3y=7 \\ 2x-2y=1 \end{cases}$ ko'rinishida bo'lib, bundan

$x=\dfrac{17}{4}, y=\dfrac{15}{4}$.

Bu sonlar shartga qarama-qarshi bo'lib, sistema uchun yechim emas,

c) $\begin{cases} x>0 \\ y<1 \end{cases}$ da sistema $\begin{cases} x+3y=7 \\ 2x-2y=1 \end{cases}$ bo'lib, topilgan $x=\dfrac{17}{8}$ va

$y=\dfrac{13}{8}$ sonlar shartni bajarmagani uchun chet ildiz.

d) $\begin{cases} x<0 \\ y>1 \end{cases}$ da sistema $\begin{cases} -x+3y=7 \\ 2x+2y=5 \end{cases}$ dan topilgan $x=\dfrac{1}{8}$,

$y=\dfrac{19}{8}$ sonlar sistema uchun chet ildiz. Javob $\left(\dfrac{1}{4};\dfrac{9}{4}\right)$ ◄

2) $\begin{cases} \sqrt{\dfrac{y}{x}}+\sqrt{\dfrac{x}{y}}-\dfrac{7}{\sqrt{xy}}=1 \\ y\sqrt{xy}+x\sqrt{xy}=78 \end{cases}$ $(x>0, y>0)$

► Shartdan $xy \neq 0$ dan birinchi tenglamani $\sqrt{xy}$ ga ko'paytirsak,

$\begin{cases} |y|+|x|-7=\sqrt{xy} \\ y\cdot\sqrt{xy}+x\sqrt{xy}=78 \end{cases}$ $(x > 0, y > 0)$ yoki $\begin{cases} x+y=7+\sqrt{xy}, \\ (x+y)\sqrt{xy}=78 \end{cases}$

Simmetrik tenglamalar sistemasida $\begin{cases} x+y=u, \\ \sqrt{xy}=v \end{cases}$ (*) desak $\begin{cases} u=7+v \\ u\cdot v=78 \end{cases}$

o'rniga qo'yish usulidan foydalanib sistema yechimlari topiladi. $v_1 = 6$, $v_2 = -13$ va $u_1 = 13$, $u_2 = -6$ lekin $v=\sqrt{xy}$ uchun $v_2 = -13$ chet ildiz.

$\sqrt{xy}=v=6$ yoki $xy = 36$, u holda (*) dan $\begin{cases} x+y=13 \\ xy=36 \end{cases}$ o'rniga qo'yish usulidan foydalansak $x_1 = 9$, $x_2 = 4$ va $y_1 = 4$, $y_2 = 9$.

Bu juftliklar tenglama yechimlari ekanligini tekshirish qiyin emas. Javob: (9;4) va (4;9). ◄

14.7. Yangi o'zgaruvchi kiritib, ko'rsatilgan metodlarinng bittasiga keltirish mumkin bo'lgan hol.

Tenglamalar sistemasi yechimlari topilsin:

1) $\begin{cases} \sqrt{2x-y+11}-\sqrt{3x+y-9}=3 \\ \sqrt[4]{2x-y+11}+\sqrt[4]{3x+y-9}=3 \end{cases}$

► Berilgan sistemada $2x - y + 11 = z$, $3x + y - 9 = t$ almashtirish bajarsak, sistema soddalashadi

$$\begin{cases} z^{\frac{1}{2}} - t^{\frac{1}{2}} = 3, \\ z^{\frac{1}{4}} + t^{\frac{1}{4}} = 3 \end{cases} \quad \text{yoki} \quad \begin{cases} \left(z^{\frac{1}{4}} + t^{\frac{1}{4}}\right)\left(z^{\frac{1}{4}} - t^{\frac{1}{4}}\right) = 3 \\ z^{\frac{1}{4}} + t^{\frac{1}{4}} = 3 \end{cases}$$

Bundan $z^{\frac{1}{4}} - t^{\frac{1}{4}} = 3$ u holda $\begin{cases} z^{\frac{1}{4}} - t^{\frac{1}{4}} = 1 \\ z\frac{1}{4} + t\frac{1}{4} = 3 \end{cases}$ sistemadagi

tenglamalarni o'zaro qo'shsak $z\frac{1}{4} = 2$ bo'lib $z = 16$ va $t = 1$. Endi

almashtirishdan $\begin{cases} 2x - y + 11 = 16, \\ 3x + y - 9 = 1 \end{cases}$ sistema yechimlari $x = 3$ va $y = 1$.

Natijada berilgan sistema yechimlari ham (3; 1) ekanligini ishonch hosil qilamiz. ◄

2) $\begin{cases} \dfrac{x}{y} + \dfrac{y}{x} = \dfrac{25}{12}, \\ x^2 + y^2 = 25; \end{cases}$

► $\dfrac{y}{x} = z$ almashtirishda sistemadagi birinchi tenglama

$12z^2 - 25z + 120 = 0$ ko'rinishga keladi. Bundan $z_1 = \dfrac{4}{3}, z_2 = \dfrac{3}{4}$ u holda berilgan sistema teng kuchli sodda ikkita tenglamalar sistemasiga kelinadi.

$$\begin{cases} y = \frac{4}{3}x; \\ x^2 + y^2 = 25 \end{cases} \quad \text{va} \quad \begin{cases} y = \frac{3}{4}x, \\ x^2 + y^2 = 25 \end{cases}$$

O'rniga qo'yish metodidan sistema yechimlari topiladi (± 3; ± 4) va (± 4; ± 3). ◄

3) $\begin{cases} x^2 + y\sqrt{xy} = 336, \\ y^2 + x\sqrt{xy} = 112. \end{cases}$

▶ $y = zx$ almashtirish bajarsak $\begin{cases} x^2 + xz\sqrt{x^2 z} = 336, \\ x^2z^2 + x\sqrt{x^2 z} = 112 \end{cases}$ birinchi tenglamani ikkinchi tenglamaga bo'lamiz $\dfrac{x^2\left(1+z\sqrt{z}\right)}{x^2\left(z^2+\sqrt{z}\right)} = \dfrac{336}{112}$, soddalashtirib gruppalasak $\left(z^{3/2}+1\right)\left(3z^{\frac{1}{2}}-1\right) = 0$ bo'lib $z_1 = -1$ (chet ildiz) $z_2 = \dfrac{1}{9}$, u holda $y = \dfrac{1}{9}x$, $y^2 + x\sqrt{xy} = 112$ o'rniga qo'yish usuli yordamida $x_1 = 18$ $x_2 = -18$ va $y_1 = 2$ $y_2 = -2$ bunda $x = -18$ va $y = -2$ chet ildiz. (18;2) berilgan sistema yechimi ekanligini tekshirish qiyin emas. ◀

14-MAVZU MASHQLARI

Chiziqli bo'lmagan tenglamalar sistemasi yechimlari topilsin:

221. 1) $\begin{cases} x+y=2, \\ xy=-8. \end{cases}$ 2) $\begin{cases} x^2-4y^2=200, \\ x+2y=100. \end{cases}$

222. 1) $\begin{cases} x^2+y^2=10, \\ x+y=4. \end{cases}$ 2) $\begin{cases} x^2-3xy+y^2+2x+3y=6, \\ 2x-y=3. \end{cases}$

223. $\begin{cases} x^2+y^2+xy=21, \\ x+y-\sqrt{xy}=3. \end{cases}$

224. $\begin{cases} x+4y+2\sqrt{xy}=12, \\ x+4y-2\sqrt{xy}=4. \end{cases}$

225. $\begin{cases} x\cdot y-x+y=1, \\ x^2\cdot y-xy^2=30. \end{cases}$

226. $\begin{cases} x^3-y^3=19, \\ x^2y-xy^2=6. \end{cases}$

227. $\begin{cases} \dfrac{x}{y}-\dfrac{y}{x}=\dfrac{3}{2}, \\ x^2+y^2=45. \end{cases}$

228. $\begin{cases} \dfrac{1}{x+y}+\dfrac{1}{x-y}=2, \\ \dfrac{3}{x+y}+\dfrac{4}{x-y}=7. \end{cases}$

229. $\begin{cases} \sqrt[3]{x}-\sqrt[3]{y}=2, \\ xy=27. \end{cases}$

230. $\begin{cases} \sqrt{\dfrac{x}{y}}-\sqrt{\dfrac{y}{x}}=\dfrac{5}{6}, \\ x-y=5. \end{cases}$

231. $\begin{cases} x^2+y^2+x+y=8, \\ x^2+y^2+xy=7. \end{cases}$

232. $\begin{cases} xy - x + y = 5, \\ 2xy + x - y = 4. \end{cases}$

233. $\begin{cases} \dfrac{x}{y} + \dfrac{y}{x} = 3,(3), \\ x^2 - y^2 = 72. \end{cases}$

234. $\begin{cases} x^2 - xy + y^2 = 21, \\ y^2 - 2xy = -15. \end{cases}$

235. $\begin{cases} 2x^2 - 3xy + 5y = 5, \\ (x-y)(y-1) = 0. \end{cases}$

236. $\begin{cases} \sqrt{\dfrac{5x}{x-y}} - \sqrt{\dfrac{x-y}{5x}} = 21, \\ xy + x + y = 11. \end{cases}$

237. $\begin{cases} 2x^2 + 2y^2 = 5xy, \\ 2x + 2y = 3xy. \end{cases}$

238. $\begin{cases} x + y^2 = 11, \\ xy^2 = 18. \end{cases}$

239. $\begin{cases} y - 2|x| + 3 = 0, \\ |y| + x - 3 = 0. \end{cases}$

240. $\begin{cases} 3|x| + 5y + 9 = 0, \\ 2x - |y| - 7 = 0. \end{cases}$

241. $\begin{cases} x^2 + 3xy - 18 = 0, \\ 4y^2 xy - 7 = 0. \end{cases}$

242. $\begin{cases} x^2 + y^2 = 13, \\ xy = 6. \end{cases}$

15-MAVZU. KO'RSATKICH TENGLAMALAR

Daraja ko'rsatkichida noma'lum miqdor qatnashgan tenglama ko'rsatkichli tenglama deyiladi. Ko'rsatkichli tenglama haqiqiy sonlar to'plamida qaraladi.

Ko'rsatkichli tenglama yechish umumiy metodi elementar matematikada mavjud emas. Tenglamaning berilish tiplariga qarab metodlar ishlatiladi, ko'rsatkichli tenglama o'ziga teng kuchli bo'lgan ikkita funksiya tengligi ko'rinishdagi tenglamaga keltirish mumkinligi haqida va funksiya xossalari [1], [4]da berilgan. Ko'rsatkichli tenglama yechimlarini topish usullari

15.1.

$$a^{f(x)} = 1 \ (a > 0, a \neq 1)$$

Ko'rsatkichli tenglama $f(x) = 0$ ga teng kuchli.

⊕ Tenglamalar yechimi topilsin

1) $2^{x^2-5x+6} = 1.$

Berilgan tenglamaga teng kuchli tenglama $x^2 - 5x + 6 = 0$, bo'lib, yechimlari $x_1 = 2$ va $x_2 = 3$. ◄

2) $\sqrt{a^{7-3x}}\sqrt[3]{a^{x+1}}\cdot\sqrt[4]{a^{5x-4}} = 1(a > 0)$

► Ildiz xossalaridan foydalanamiz $a^{\frac{7-3x}{2}+\frac{x+1}{3}+\frac{5x-4}{4}} = 1$ darajada amallarni bajarsak $a^{\frac{x+34}{12}} = 1$, bundan $\frac{34+x}{12} = 0$ berilgan tenglama $x = -34$ yechimga ega. ◄

3) $[2,(3)]^{0,27-3x^2} = 1$

► Berilgan tenglamaga, teng kuchli tenglama $0,27 - 3x^2 = 0$ tenglama ko'rinishida bo'lib, yechimlari $x_1 = 0,3$, $x_2 = -0,3$ ◄

15.2. Bir hil asosga keltirib yechiladigan tenglamalar

2.1. $a^{f(x)} = a^{\varphi(x)}$ $(a > 0, a \neq 1)$

Ko'rinishdagi ko'rsatkichli tenglamaga $f(x) = \varphi(x)$ tenglama teng kuchli tenglama bo'ladi.

⊕ Tenglamalar yechimi topilsin.

1) $3^{x^2-0,5x} = \sqrt{3}$

► Tenglamani $3^{x^2-0,5x} = 3^{\frac{1}{2}}$ deb yozish mumkin, u holda teng kuchli tenglama $x^2 - 0,5x = \frac{1}{2}$ bo'lib, yechimlari $x_1 = 1; x_2 = -0,5$ ◄

2) $\dfrac{3^{\sqrt[3]{x^2}}}{2\cdot 3^{\sqrt[3]{x}+1}}=1,5$.

► Tenglamani kasr ko'rinishisiz yozish mumkin.

$$3^{\sqrt[3]{x^2}}=1,5.6\cdot 3^{\sqrt[3]{x}} \text{ yoki } 3^{\sqrt[3]{x^2}}=3^{\sqrt[3]{x}+2}$$

Bu tenglama teng kuchli tenglama $\sqrt[3]{x^2}=\sqrt[3]{x}+2$ ko'rinishdagi kvadrat tenglama bo'lib $\sqrt[3]{x}=2$ va $\sqrt[3]{x}=-1$ yechimlarga ega, bundan $x_1=8$ va $x_2=-1$ ◄

3) $[0,1(6)]^{x-4}\cdot 0,5=18$

► Bunday davriy o'nli kasr $0,1(6)=\dfrac{1}{6}$ ga teng, u holda $\left[\dfrac{1}{6}\right]^{x-4}=36$ yoki $6^{4-x}=6^2$ bundan $4-x=2$, berilgan tenglama yechimi $x=2$. ◄

4) $8\cdot\sqrt{(0,125)^{4-\frac{x}{3}}}=2^{\sqrt{x-6}}$

► Ifodani shakl almashtirib yozamiz $2^3\left[\left(2^{-3}\right)^{\frac{12-x}{3}}\right]^{\frac{1}{2}}=2^{(x-6)^{\frac{1}{2}}}$,

yoki $2^3\cdot 2^{\frac{x-12}{2}}=2^{(x-6)^{\frac{1}{2}}}$ bu tenglama teng kuchli tenglama

$$3+\frac{x-12}{2}=(x-6)^{\frac{1}{2}}$$

Ifodani soddalashtirsak $x^2-16x+60=0$ bo'lib berilgan tenglamaning yechimlari $x_1=6$ va $x_2=10$ ◄

15.3.

$$a\varphi^{(x)}=b\varphi^{(x)}\,(a\neq b,\ 0<a\neq 1,\ 0<b<\neq 1)$$

ko'rinishidagi tenglamaning $b\varphi^{(x)}$ yoki $(a\varphi^{(x)})$ ga bo'lsak tenglama $\left(\dfrac{a}{b}\right)^{\varphi(x)}=1$ ko'rinishga keladi.

⊕ Tenglama yechimi topilsin

1) $3^{x-1}\cdot 2^{3x-7}=12^{9-x}$,

► 12 ni $12=3\cdot 2^2$ deb yozish mumkinligidan $3^{x-1}\cdot 2^{3x-7}=3^{9-x}\cdot 2^{18-2x}$ yoki $3^{x-1-(9-x)}=2^{18-2x-(3x-7)}$ bo'lib, bundan $3^{2x-10}=2^{25-5x}$ daraja xossasidan foydalanamiz $9^{x-5}=32^{5-x}$ chunki $3^2=9$, $2^5=32$. U holda $\left(\dfrac{9}{32}\right)^{x-5}=1$ dan tenglama yechimi $x=5$◄

1) $0{,}2 \cdot 5^{2x} + 2^{2x} = 25^{x} - 4^{x+1}$

► Tenglamada qo'shiluvchi hadlarni daraja xossalaridan foydalanib

yozamiz $\frac{1}{5} \cdot 5^{2x} + 4^{x} = 5^{2x} - 4^{x+1}$ bundan $5^{2x}\left(1-\frac{1}{5}\right) = 4^{x}(4+1)$

soddalashtirsak $25^{x} \cdot \frac{4}{5} = 4^{x} \cdot 5$ bundan $\left(\frac{25}{4}\right)^{x} = \frac{25}{4}$ berilgan tenglama

yechimi $x = 1$. ◄

2) $9^{x} - 2^{x+\frac{1}{2}} = 2^{x+3,5} - \frac{3^{2x}}{3}$,

► Tenglamada 3 asosli ifodani bir tomonda 2 asosli ifodani ikkinchi tomonda yozib soddalashtirsak

$3^{2x} + 3^{2x-1} = 2^{x+3,5} + 2^{x+\frac{1}{2}}$ dan

$$3^{2x-1}(3+1) = 2^{x+0,5}(2^{3}+1)$$

bo'lib, $3^{2x-1} \cdot 4 = 2^{x+0,5} \cdot 9$ yoki $3^{2x-1-2} = 2^{x+0,5-2}$ bundan $3^{2x-3} = 2^{x-3/2}$ yana daraja xossasidan foydalanamiz.

$$9^{x-\frac{3}{2}} = 2^{x-\frac{3}{2}} \text{ dan } \left(\frac{9}{2}\right)^{x-\frac{3}{2}} = 1$$

natijada berilgan tenglama yechimi $x = 1{,}5$. ◄

15.4.

$$a^{f(x)} = b^{\varphi(x)} \ (a \neq b,\ 0 < a \neq 1,\ 0 < b \neq 1)$$

Ko'rinishdagi tenglama logarifmlash usulidan foydalanib yechimni topish mumkin.

⊕ Tenglamalar yechimi topilsin.

1) $4^{x-1} = 5^{3-x}$

► Ifodaning ikki tomonini o'n asosda logarifmlaymiz $(x-1)\lg 4 = (3-x)\lg 5$ dan $x\lg 4 + x\lg 5 = 3\lg 5 + \lg 4$ logarifm xossasidan $x\lg 20 = \lg(125 \cdot 4)$ bo'lib, tenglama yechimi $x = \frac{\lg 500}{\lg 20}$ ◄

2) $(0{,}5)^{-x} = 3^{x+2}$

► O'nli kasrni oddiy kasrda yozib, daraja xossasidan foydalanamiz

$2^{x} = 3^{x+2}$ dan $\left(\frac{2}{3}\right)^{x} = 9$

Ifodani 3 asosga ko'ra logarifmlaymiz $x\log_3 \frac{2}{3} = \log_3 3^{2}$ logarifm xossalaridan $x\left(\log_3^2-1\right)=2$, demak berilgan tenglama yechimi

$x=\dfrac{2}{\log_3^2-1}$ ◄

3) $25^{-x}=0{,}01$

► Daraja xossalaridan foydalansak $5^{-2x}=10^{-2}$

Ifodani 10 asosda logarifmlaymiz $-2x\lg5=-2\lg10$ yoki $x\lg5=1$ tenglama yechimi $x=(\lg5)^{-1}$. ◄

4) Agar $\lg2=a$ bo'lsa $2^{2x-1}=5^{2-x}$ da x ni a orqali ifoda qiling.

► Tenglikning ikki tomonini o'n asosda logarifmlaymiz

$$(2x-1)\lg2=(2-x)\lg5$$

bunda $\lg5=\lg\dfrac{10}{2}=1-\lg2$ o'rniga qo'ysak $(2x-1)\cdot a=(2-x)(1-a)$

Bundan soddalashtirib berilgan tenglamaning yechimi topiladi

$(2a+1-a)x=a+2-2a$ dan $x=\dfrac{2-a}{a+1}$. ◄

15.5. Umumiy ko'paytuvchini qavsdan tashqariga chiqarish usuli bilan yechiladigan tenglamalar.

⊕ Tenglamalar yechimi topilsin.

1) $2^{2x-5}+2^{2x-7}+2^{2x-9}=42$

► Tenglamaning chap tomonida umumiy ko'paytuvchini qavsdan tashqariga chiqarib soddalashtiramiz $2^{2x-9}(2^4+2^2+1)=42$ dan $2^{2x-9}\cdot21=42$ yoki $2^{2x-9}=2$ bo'lib, berilgan tenglamaning yechimi $x=5$ ◄

2) $9^{\sqrt{x}}+\dfrac{3^{2\sqrt{x}}}{3}-\dfrac{9^{\sqrt{x}}}{9}=11$

► Ifodadagi qo'shiluvchilarni uch asosli qilib yozamiz

$$3^{2\sqrt{x}}+3^{2\sqrt{x-1}}-3^{2\sqrt{-2}}=11,$$

Umumiy ko'paytuvchini qavsdan tashqariga chiqarib soddalashtiramiz

$3^{2\sqrt{x}-2}\left(3^2+3-1\right)=11$ yoki $3^{2\sqrt{x}-2}=1$ bundan berilgan tenglama yechimi $x=1$ ◄

3) $2\cdot3^{3x-1}+27^{x-\frac{2}{3}}=9^{x-1}+2\cdot3^{2x-1}$

► Tenglamadagi qo'shiluvchilarni uch asosli qilib yozib olish mumkin.

$2\cdot3^{3x-1}+3^{3x-2}-3^{2x-2}-2\cdot3^{2x-1}=0$ bundan $3^{3x-2}(2\cdot3+1)-3^{2x-2}(1+$ $+2\cdot3)=0$, dan $3^{3x-2}=3^{2x-2}$ bo'lib, $x=0$ yechim hosil bo'ladi.

15.6. Yangi noma'lum kiritish usuli bilan yechish mumkin bo'lgan holni misollarda oydinlashtiramiz.

Tenglamalar yechimi topilsin:

1) $2\cdot 3^{1+\frac{1}{2x}}=27-\sqrt[x]{3}$

► Ildiz qo'shiluvchini $\sqrt[x]{3}=3^{\frac{1}{x}}$ deb yozish mumkin, u holda

$3^{\frac{1}{x}}+6\cdot 3^{\frac{1}{2x}}-27=0$ agar $3^{\frac{1}{2x}}=y$ desak $y^2+6y-27=0$ tenglama

yechimlari $y_1=3, y_2=-9$ bo'lib $y=3^{\frac{1}{2x}}$ dan;

1) $3^{\frac{1}{2x}}=3$, yoki $\frac{1}{2x}=1$ $x=0,5$ 2) $3^{\frac{1}{2x}}=-9$ yechim mavjud emas.

Javob berilgan tenglama yechimi $x=0,5$ ◄

2) $3^{4\sqrt{x}}-4\cdot 3^{\sqrt{4x}}+3=0$,

► Berilgan tenglamada yangi o'zgaruvchi $3^{2\sqrt{x}}=y$ kiritsak, y ga nisbatan kvadrat tenglama hosil bo'ladi.

$y^2-4y+3=0$ dan $y_1=3, y_2=1$, u holda:

a) $3^{2\sqrt{x}}=3$ yoki $2\sqrt{x}=1 x=0,25$;

b) $3^{2\sqrt{x}}=1 2\sqrt{x}=0 x=0$

Javob: $x_1=0,25, x_2=0$ ◄

3) $4^{x+\sqrt{x^2-2}}-5\cdot 2^{x-1+\sqrt{x^2-2}}=6$

► Tenglamaning ikki tomonini 2 ga ko'paytiramiz:

$2\cdot 2^{2\left(x+\sqrt{x^2-2}\right)}-5\cdot 2^{x+\sqrt{x^2-2}}=12$ agar $2^{x+\sqrt{x^2-2}}=y$ desak, $2\cdot y^2-5y-12=0$

Tenglama ildizlari $y_1=4, y_2=-\frac{3}{2}$ bo'lib, almashtirishdan:

a) $2^{x+\sqrt{x^2-2}}=4$ yoki $x+\sqrt{x^2-2}=2$ irratsional tenglama yechimi $x=1,5$.

b) $2^{x+\sqrt{x^2-2}}=-\frac{3}{2}$ da $x\in\varnothing$ U holda berilgan tenglama yechimi $x=1,5$

4) $25^{x+1}+0,4=11,5^x$

► Daraja xossalaridan foydalanib va $0,4=\frac{2}{5}$ ni hisobga olsak tenglama $125\cdot 5^{2x}-55\cdot 5^x+2=0$ ko'rinishga keladi, bu tenglama yechimlarini topishda $5^x=y$ desak $125y^2-55y+2=0$ hosil bo'ladi bunda,

Δ 3025 – 1000 = 2025 bo'lib, $y_2=\frac{55\pm45}{250}$ dan $y_1=\frac{1}{25}, y_2=\frac{2}{5}$

endi almashtirishdan tenglama yechimlari topiladi.

1) $5^x=\frac{1}{25}$ dan $x_1=-2$, 2) $5^x=\frac{2}{5}$ dan $x\log_5 5=\log_5 2-\log_5 5$

bo'lib, bundan $x_2=\log_5 2-1$. ◄

15.7.

$$A_1 a^x + A_2 b^x + A_3 C^x = 0$$

Bunda A_1, A_2, A_3 istalgan haqiqiy sonlar *a*, *b* va *c* larning istalgan ikkitasining ko'paytmasi uchinchisining kvadratiga teng bo'lsa, tenglamani ko'rsatkichli funksiyalarning bittasiga bo'lib yuborsak, kvadrat tenglamaga kelinadi.

Tenglamalar yechimlari topilsin.

1) $2\cdot 9^x+3\cdot 4^x-5\cdot 6^x=0$

► Tenglamani 4^x ga bo'lsak $2\cdot\frac{9^x}{4^x}+3-5\cdot\frac{2^x\cdot 3^x}{4x}=0$ yoki

$2\left(\frac{3}{2}\right)^{2x}-5\left(\frac{3}{2}\right)^{x}+3=0$ bo'lib $\left(\frac{3}{2}\right)^{x}=y$ desak, $2y^2-5y+3=0$

bundan $y_1=1$, $y_2=\frac{3}{2}$. U holda almashtirishdan:

a) $\left(\frac{3}{2}\right)^{x}=1, x=0,$

b) $\left(\frac{3}{2}\right)^{x}=\frac{3}{2}$ dan $x=1$ javob $x_1=0, x_2=1$ ◄

3) $27^x+12^x=2\cdot 8^x$

► Tenglamaning ikki tomonini 8^x ga bo'lsa $\left(\frac{3}{2}\right)^{3x}+\left(\frac{3}{2}\right)^{x}-2=0$

agar $\left(\frac{3}{2}\right)^{x}=y$ desak $y^3+y-2=0$ tenglamani $(y-1)(y^2+y+2)=0$

deb yozish mumkin, bu tenglamaning haqiqiy yechimi $y=1$. U holda almashtirishdan berilgan tenglama yechimi $x=0$ bo'ladi.

$3\cdot 49^{x^2}+7\cdot 4^{x^2}-9\cdot 14^{x^2}=0$

► Tenglamaning ikki tomonini 4^{x^2} ga bo'lsak

$2\left(\frac{7}{2}\right)^{2x^2}-9\left(\frac{7}{2}\right)^{x^2}+7=0$ agar $\left(\frac{7}{2}\right)^{x^2}=y$ desak $2y^2-9y+7=0$ bu

tenglama yechimlari $y=1$, $y=\frac{7}{2}$.

U holda almashtirishdan berilgan tenglamaning yechimlari topiladi:

a) $\left(\frac{7}{2}\right)^{x^2}=1$ dan $x^2=0$ $x_1=0$

b) $\left(\frac{7}{2}\right)^{x^2}=\frac{7}{2}$ dan $x^2=1$ $x_2=1, x_3=-1$

Berilgan tenglamaning yechimlari $x_1=-1, x_2=0, x_3=1$ ◄

15.8. Umumlashgan hol yoki sun'iy usullar qo'llash bilan tenglama yechimini topish mumkin bo'lgan hol.

Tenglamalar yechimi topilsin.

1) $\left(\sqrt{2-\sqrt{3}}\right)^x+\left(\sqrt{2+\sqrt{3}}\right)^x=4$

► Tenglamani yechishda $\sqrt{2-\sqrt{3}}\cdot\sqrt{2+\sqrt{3}}=1$ tenglikdan foydalanamiz. Bundan $\frac{1}{\sqrt{2+\sqrt{3}}}=\sqrt{2-\sqrt{3}}$, Agar $\left(\sqrt{2-\sqrt{3}}\right)^x=y$

desak $y+\frac{1}{y}=4$ yoki $y^2-4y+1=0$ kvadrat tenglama hosil bo'ladi.

Bu tenglamani yechsak $y_1=2-\sqrt{3}, y_2=2+\sqrt{3}$.

Endi almashtirishdan topamiz:

a) $\left(\sqrt{2-\sqrt{3}}\right)^x=2-\sqrt{3}$ yoki $\left(\sqrt{2-\sqrt{3}}\right)^x=\left(\sqrt{2-\sqrt{3}}\right)^2$ dan $x=2$

b) $\left(\sqrt{2-\sqrt{3}}\right)^x=2+\sqrt{3}$ yoki $\left(\sqrt{2-\sqrt{3}}\right)^x=\frac{1}{\left(2-\sqrt{3}\right)^2}$ bo'lib,

bundan $x=-2$. Natijada berilgan tenglama yechimlari $x_1=2, x_2=-2$. ◄

2) $\sqrt[x]{9}+2x\cdot\sqrt[x]{3}=6x+9\,(x>0)$

► Tenglamada $3^{\frac{1}{x}}=y$ almashtirish bajarib, hosil bo'lgan $y^2+2xy-6x-9=0$

$\blacktriangle = 4x^2 + 4(6x+9) = 4(x+3)^2$ u holda tenglamani y ga nisbatan yechamiz,

bunda $y_2 = \dfrac{-2x \pm 2(x+3)}{2}$ dan $y_1 = 3$, $y_2 = -2x - 3$ almashtirishdan:

a) $\sqrt[x]{3} = 3$ dan $x = 1$ ◀

b) $3^{\frac{1}{x}} = -2x - 3$ da yechim mavjud emas, chunki $a^x > 0$, javob: $x = 1$

3) $2^{2x} \cdot 9^x - 6^{3x-1} + 4^{2x-1} \cdot 3^{4x-2} = 0$,

▶ Darajada xossalaridan foydalanib yozamiz

$$2^{2x} \cdot 3^{2x} - \frac{1}{3} \cdot 2^{3x} \cdot 3^{3x} + \frac{1}{36} \cdot 2^{4x} \cdot 3^{4x} = 0$$

Tenglamaning ikki tomonini $2^{2x} \cdot 3^{2x}$ ga bo'lamiz va 36 ga ko'paytiramiz

$36 - 12 \cdot \dfrac{2^{3x} \cdot 3^{3x}}{2^{2x} \cdot 3^{2x}} + \dfrac{2^{4x} \cdot 3^{4x}}{2^{2x} \cdot 3^{2x}} = 0$ agar $(6^x) = y$ desak $y^2 - 12y + 136 = 0$

bo'lib, tenglama yechimlari $y_1 = y_2 = 6$

Almashtirishdan berilgan tenglama yechimi $x = 1$ ◀

4) $\sqrt[5]{[0,(1)]^{20-0,5x}} + 2 \cdot \sqrt[5]{3^{x-35}} = 21$

▶ Har bir qo'shiluvchini kasr darajada yozamiz. Bunda $0,(1) = \dfrac{1}{9}$ ni

hisobga olamiz

$3^{\frac{-2(20-0,5x)}{5}} + 2 \cdot 3^{\frac{x-35}{5}} = 21$ yoki $3^{\frac{x-40}{5}} + 2 \cdot 3^{\frac{x-35}{5}} = 21$ dan

$3^{\frac{x}{5}} \cdot 3^{-8}(1 + 2 \cdot 3) = 21$ bo'lib $3^{\frac{x}{5}} = 3 \cdot 3^8$ natijada berilgan tenglama

yechimi $x = 45$. ◀

5) $\sqrt[x]{\sqrt[3]{2}} \cdot \sqrt[6]{x\sqrt[x]{4}} \cdot \sqrt[12x]{16 \cdot \sqrt{3^{2x}}} = \sqrt[3x]{6\sqrt{2^x}}$

▶ Tenglamani ildiz ko'rsatkichni kasr darajada yozish $\sqrt[n]{a^k} = a^{\frac{k}{n}}$

xossasidan va $\sqrt[k]{\sqrt[n]{a}} = a^{\frac{1}{kn}}$ xossalardan foydalanib yozamiz

$$2^{\frac{1}{3x}} \cdot 2^{\frac{2}{6x}} \cdot 2^{\frac{4}{12x}} \cdot 3^{\frac{2x}{24x}} = 6^{\frac{1}{3x}} \cdot 2^{\frac{x}{6x}}$$

soddalashtirsak

$$2^{\frac{1}{x}} \cdot 3^{\frac{1}{12}} = 2^{\frac{1}{3x}} \cdot 3^{\frac{1}{3x}} \cdot 2^{\frac{1}{6}}$$

yoki $2^{\frac{1}{x}} \cdot 2^{-\frac{1}{3x}} \cdot 3^{\frac{1}{3x}} = 2^{\frac{1}{6}} \cdot 3^{-\frac{1}{12}}$

bu ifodani yozish mumkin $2^{\frac{2}{3x}}\cdot 3^{-\frac{1}{3x}}=4^{\frac{1}{12}}\cdot 3^{-\frac{1}{12}}$

deb, u holda $\left(\frac{4}{3}\right)^{\frac{1}{3x}}=\left(\frac{4}{3}\right)^{\frac{1}{12}}$ bo‘lib, berilgan tenglama yechimi $x=4$ ◄

15-MAVZU MASHQLARI

Ko‘rsatkichli tenglama yechimlari topilsin.

243. 1) $(0,75)^{x+1}=1\frac{7}{9}$, 2) $4+2^{x-1}=0$, 3) $2^{x^2-6x-2,5}=16\cdot\sqrt{2}$

244. $\sqrt{8^x}=\sqrt[x]{32}$, 2) $0,3^{0,27-3x^2}-1=0$, 3) $1-\left(\frac{1}{2}\right)^{\frac{2}{1-x}}=0$.

245. 1) $\left(\frac{1}{0,125}\right)^x=128$, 2) $\frac{1}{\sqrt[5]{16}}=\sqrt[3]{2^x}$.

246. 1) $\left(\sqrt{3}\right)^{\left(\sqrt[4]{2}\right)^x}=3$, 2) $\frac{2^{5\cdot\sqrt[3]{x}}}{2^{\sqrt[3]{x^2}+2,6}}=8\cdot\sqrt[5]{4}$.

247. $\left(\frac{4}{9}\right)^x\cdot\left(\frac{27}{8}\right)^{x-1}=0,6$.

248. $\sqrt{3^{2-3x}}\cdot\sqrt[3]{3^{x+1}}\cdot\sqrt[4]{9^x}=1$.

249. $11^{-0,00(5)x}=\frac{1}{1331}$.

250. $2 \cdot 2^3 \cdot 2^{2x-1} = 512.$

251. $5^{2x+1} - 7 \cdot 7^x = 25^x + 7^x.$

252. $15^{2x+4} = 27^x \cdot 5^{4x-4}.$

253. $3^{2x+0,5} - 2^{4x-1} = 4^{2x} - 3^{2x-0,5}.$

254. $3^{7x} - 4 \cdot 3^{7x-1} + 5 \cdot 3^{7x-2} + 3^{7x-4} = 57.$

255. $2^{x^2-1} - 3^{x^2} = 3^{x^2-1} - 4 \cdot 2^{x^2}.$

256. $8^x \cdot 3^x - 2^{3(x-1)} \cdot 3^{x-1} = 552.$

257. $2^{x-3} + 8^{0,(3)x-1} - 4^{0,5x-2} = 10.$

258. $6 \cdot 2^{\frac{x-3}{2}} + 2^x = 2.$

259. $\left(\frac{1}{9} \cdot 9^x\right)^x = 3^{2x+6}.$

260. $3^{x+2} + 3^{2x-1} = 4 \cdot 27^{x-1}.$

261. $\sqrt{7^{2x+6}} - \sqrt{49^{x+2}} - 2^{x+5} + 2 \cdot 0,25^{-(1+0,5x)} = 0,$

262. $2^x - 2\cdot(0,5)^{2x} - (0,5)^x = 1$.

263. $2\cdot 2^{2x} - 5\cdot 6^x + 3\cdot 3^{2x} = 0$.

264. $4^{-\frac{1}{x}} + 6^{-\frac{1}{x}} = 6\cdot 9^{-\frac{1}{x}}$.

265. $2^{3x} + 8^{-x} + 3\cdot 2^x - 15\frac{5}{8} + 3\cdot 2^{-x} = 0$.

266. $5^{3x} + 9\cdot 5^x + 27(5^{-3x} + 5^{-x}) = 64$.

267. $\sqrt[x-1]{\sqrt[3]{2^{3x-1}}} - \sqrt[3x-7]{8^{x-3}} = 0$.

268. $\left(\sqrt{5+2\sqrt{6}}\right)^x + \left(\sqrt{5-2\sqrt{6}}\right)^x = 10$.

269. $8^x - \frac{8}{2^{3x}} - 6\left(2^x - \frac{1}{2^{x-1}}\right) = 1$.

270. $3^{x-3} + [0,(3)]^{2-x} - [0,(1)]^{\frac{4-x}{2}} = 99$.

16-MAVZU. LOGARIFMIK TENGLAMALAR

Noma'lum o'zgaruvchi logarifm belgisi ostida yoki logarifm asosida qatnashgan tenglamalarga logarifmik tenglama deyiladi.

Logarifmik tenglamalarni yechish umumiy metodi elementar matematikada mavjud emas. Tenglamalarni tiplarga ajratib, yechish usullarini ko'rib chiqamiz. Logarifmik tenglamalar haqiqiy sonlar to'plamida qaraladi. Logarifmik tenglamalarni yechish usullari tenglamaning tuzilishiga qarab tanlanadi. Bu usullarni misollarni yechish yordamida ko'rib chiqamiz. Logarifmik tenglamalarni yechimiga alohida e'tibor berish kerak. So'ngra topilgan sonlar tenglamaning aniqlanish sohasiga tegishlimi yoki tegishli emasligini aniqlash kerak. Bunda $\log_a f(x) = b$ ga $f(x) > 0$ mavjud bo'lishi va $0 < a \neq 1$ shartlari bajarilishi kerak.

Logarifm haqida to'la malumot [4] da misollar bilan berilgan.

Logarifmik tenglamalarni yechish usullari:

16.1. Logarifmik shartiga asosan yechiladigan tenglamalar

⊕ Tenglamalar yechimi topilsin.

1) $\log_{0,(3)}(-2x) = -2$

► Logarifm shartiga asosan yozamiz, bunda $0,(3) = \frac{3}{9} = \frac{1}{3}$ ni hisobga olib $\left(\frac{1}{3}\right)^{-2} = -2x$ yoki $3^2 = -2x$ dan $x = -4,5$

Tekshirish $x = -4,5$ da

$$\log_{\frac{1}{3}}\left(-4,5(-2)\right) = \log_{\frac{1}{3}} 3^2 = 2\log_{\frac{1}{3}} 3 = -2\log_3 3 = -2,$$

Bu son tenglama yechimi ekan.

2) $\log_x \sqrt[3]{4} = 0,1(6)$

► Tenglama yechimini topishda

$\sqrt[3]{4} = 2^{\frac{2}{3}}$ va $0,1(6) = \frac{1}{6}$ ni hisobga olib, yozamiz $2^{\frac{2}{3}} = x^{\frac{1}{6}}$ yoki

$$\left(2^{\frac{2}{3}}\right)^6 = \left(x^{\frac{1}{6}}\right)^6$$

bundan $x = 2^4 = 16$. Tekshirish $\log_{16} 2^{\frac{2}{3}} = \frac{2}{3}\log_{2^4} 2 = \frac{2}{3}\cdot\frac{1}{4}\log_2 2 = \frac{1}{6}$

javob $x = 16$.◄

3) $\log_x(3x + 4) = 2$

► Logarifm ta'rifiga asosan yozish mumkin.

$3x + 4 = x^2$ yoki $x^2 - 3x - 4 = 0$ bu tenglama ildizlari $x = 4$ va $x = -1$ tekshirish

a) $\log_x(3x + 4)$ ifoda $x = -1$ asos manfiy ma'noga ega emas.

b) $\log_4(3 \cdot 4 + 4) = \log_4 16 = \log 4^2 = 2$ yechim javob: $x = 4$ ◄

16.2. Logarifm xossalaridan foydalanib yechish mumkin bo'lgan tenglamalar:

⊕ Tenglamalar yechimi topilsin:

1) $2^{\log_4(x-8)} = \log_3 81$

► Bunda logarifmning $a^{\log_a N}$ va $\log_a N^k = k\log_a N$ xossalaridan foydalanib, kanonik ko'rinishga keltiriladi.

$2^{\log_{2^2}(x-8)} = \log_3 3^4$ dan $2^{\log_2(x-8)^{\frac{1}{2}}} = 4\log_3 3$ yoki

$\frac{1}{2}log_2(x-8) = 2$ bo'lib bundan $x = 24$. Bu son berilgan tenglamaning yechimi ekanligini o'rniga qo'yish orqali ishonch hosil qilish oson.

Javob: $x = 24$ ◄

2) $2\log_{\log_2 x} 2 = 1$

► Tenglamaning ikki tomonini 2 ga bo'lamiz

$\log_{\log_2 x} 2 = \frac{1}{2}$ logarifm ta'rifiga asosan $2 = (\log_2 x)^{\frac{1}{2}}$ yoki $\log_2 x = 4$ bo'lib, berilgan tenglama yechimi $x = 16$◄

3) $\log_x 81 = 2\left(2 - \sqrt{2} + \frac{2-\sqrt{2}}{\sqrt{2}} + \frac{2-\sqrt{2}}{2} + \ldots\right)$

► O'ng tomondagi qavs ichidagi ifoda maxraji $q = \frac{1}{\sqrt{2}}$ bo'lgan cheksiz kamayuvchi geometrik progressiyani beradi.

Bu yig'indi $S = \frac{b_1}{1-q} = \frac{2-\sqrt{2}}{1-\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}\cdot(2-\sqrt{2})}{\sqrt{2}-1} = \frac{\sqrt{2}\cdot\sqrt{2}(\sqrt{2}-1)}{\sqrt{2}-1} = 2$

demak berilgan tenglama $\log_x 81 = 2 \cdot 2$ logarifm tarifiga ko'ra $3^4 = x^4$ bo'lib bundan $x = 4$, topilgan $x = 4$ berilgan tenglama yechimi bo'ladi.◄

4) $\log_x \sqrt[3]{0,(1)} = -0,1(6)$

► Davriy o'nli kasrni $0,(1) = \frac{1}{9}$, $0,1(6) = \frac{1}{6}$ oddiy kasrda yozamiz, ildiz ko'rsatkichni kasr darajada yozib, logarifm ta'rifidan foydalanamiz

$$\log_x\left(\frac{1}{9}\right)^{\frac{1}{3}}=-\frac{1}{6}$$

dan $\left(\frac{1}{3}\right)^{\frac{2}{3}}=x^{-\frac{1}{6}}$ yoki $3^{-\frac{2}{3}}=x^{-\frac{1}{6}}$ bo'lib bundan

$x=3^4=81$ Tekshirish $x=81$ da $\log_{81}\left(\frac{1}{9}\right)^{\frac{1}{3}}=-\frac{1}{6}$, logarifm xossasidan

$\log_{3^4}3^{\left(-\frac{2}{3}\right)}=-\frac{2}{3}\log_{3^4}3=-\frac{2}{3}\cdot\frac{1}{4}\log_3 3=-\frac{1}{6}$ demak $x=81$ berilgan tenglama yechimi ekan. ◄

5) $\log_3\log_2\log_5(25-x)=0$

► Logarifm ta'rifidan foydalanamiz:

$\log_3[\log_2\log_5(25-x)]=0$ dan $\log_2\log_5(25-x)=1$ yana shu kabi $\log_5(25-x)=2$ dan $25-x=5^2$ yoki $x=0$ ekanligi kelib chiqadi. $x=0$ ni tenglamaga qo'yish bilan yechim ekanligini ishonch hosil qilamiz. ◄

16.3. $\log_a\varphi_{(x)}=\log_a f(x)$ $(0<a\neq 1, f(x)>0, \varphi(x)>0)$ ko'rinishidagi tenglamada logarifm xossasi $f(x)=\varphi(x)$ dan foydalanamiz.

⊕ Tenglamalar yechimi topilsin.

1) $\log_{\sqrt{2}}x=\log_{\sqrt{4}}(4x+12)$

Tenglama aniqlanish sohasi $x>0$

► Oldin logarifm xossasidan foydalanamiz

$\log_{2^{\frac{1}{2}}}x=\log_2(4x+12)$ yoki $2\log_2 x=\log_2(4x+12)$ dan $\log_2 x^2=$ $=\log_2(4x+12)$ bo'lib $x^2=4x+12$ kelib chiqadi.

Bu tenglama ildizlari $x=6$ va $x=-2$, bu sonlardan $x=-2$ tenglama aniqlanish sohasiga tegishli emas. $x=6$ tenglamaga yechim ekanligini tekshirish qiyin emas.

2) $3\lg x=2{,}5\lg 0{,}5x$

► Tenglamani logarifm xossasidan foydalanib, shakl almashtirib yozamiz $\lg x^3=\lg\left(\frac{x}{2}\right)^{5/2}$ dan $x^3=\left(\frac{x}{2}\right)^{5/2}$ yoki $2^{5/2}x^3-x^{5/2}=0$

tenglamani ko'paytma ko'rinishida yozish mumkin. $x^{5/2}\left(2^{5/2}x^{\frac{1}{2}}-1\right)=0$

bu tenglama yechimlari $x=\frac{1}{2^5}=\frac{1}{32}$ va $x=0$

Bunda $x = 0$ chet ildiz. $x = \frac{1}{32}$ esa berilgan tenglama yechimi. ◀

3) $$\frac{\lg\left(x^3 - 5x^2 + 19\right)}{\lg(x-2)} = 3$$

▶ $x \neq 2$ va $x \neq 3$ deb (chunki $x = 3$ da $\lg(x - 2)$ nol qiymatni qabul qiladi, kasr mahraji esa nol bo'lishi mumkin emas) tenglikni $\lg(x - 2)$ ga ko'paytirib, logarifm xossasidan foydalanib yozamiz.

$\lg(x^3 - 5x^2 + 19) = \lg(x - 2)^3$ yoki $x^3 - 5x^2 + 19 = (x - 2)^3$

qisqa ko'paytirish formulasidan foydalanib soddalashtirsak $x^2 - 12x + 27 = 0$ bu tenglama ildizlari $x = 3$ va $x = 9$, $x = 3$ esa berilgan tenglama uchun chet ildiz $x = 9$ esa berilgan tenglama yechimi. ◀

4) $\log_4 (x + 12) \log_x 2 = 1$

▶ Logarifmdagi $\log_a N = \frac{\log_b N}{\log_b a}$ formula yordamida

$\log_x 2 = \frac{\log_2 2}{\log_2 x} = \frac{1}{\log_2 x}$ tenglik va $\log_{a^k} N = \frac{1}{k}\log_a N$ xossadan foydalanib yozamiz

$\log_{2^2}(x+12)\cdot\frac{1}{\log_2 x} = 1$ yoki $\log_2 (x+12)^{\frac{1}{2}} = \log_2 x(x \neq 1$ deb), bundan $(x+12)^{\frac{1}{2}} = x$ bo'lib irratsionallikdan qutqarib va soddalashtirilsa $x^2 - x - 12 = 0$ tenglamaga kelinadi bu tenglama yechimlari $x = 4$ va $x = -3$. Bunda $x = -3$ chet ildiz bo'lib, $x = 4$ berilgan tenglama yechimi bo'ladi. ◀

5) $3^{(1+2+3+...+8)\log_3 x} = 27 \cdot x^{30}$

▶ Tenglamadagi qavs ichidagi yig'indi arifmetik progressiya bo'lib yig'indi $\frac{(1+8)\cdot 8}{2} = 36$ u holda tenglamani soddalashtirishda logarifmning $a^{k\log_a x} = x^k$ xossasidan foydalanamiz $3^{36\log_3 x} = 27\cdot x^{30}$ dan $x^{36} = 27 - x^{30}$ yoki $x^6 = 27$ dan $x^3 = 3$ bo'lib tenglama yechimi $x = \sqrt[3]{3}$. ◀

16.4. Potensirlash usuli bilan yechiladigan logarifmik tenglamalar. Tenglamalar yechimi topilsin.

1) $0,5\lg(2x-1) + \lg\sqrt{x-9} = 1$

▶ $\log_a N^k = k \log_a N$ xossadan foydalanamiz, keyin potensirlab, 3-metodni qo'llab soddalashtiramiz:

$\frac{1}{2}\lg(2x-1)+\frac{1}{2}\lg(x-9)=1$ yoki $\frac{1}{2}\lg(2x-1)(x-9)=\lg10$ dan $2x^2-$ $-19x-91=0$ bu tenglama yechimlari $x=13$ va $x=-\frac{14}{4}$ bo'lib, $x=-\frac{7}{2}$ chet ildiz, berilgan tenglama yechimi $x = 13$ ◄

2) $\log_5(x+5)-\log_5(3x+25)=\log_5(x-15)-\log_5 17$

► Logarifmning potensirlash xossasidan foydalansak $(x + 5)17 =$ $= (x - 15)\cdot(3x + 25)$ qavs ochib soddalashtirsak $3x^2 - 37x - 460 = 0$ tenglamaga kelinadi, bu tenglama yechimlari $x = 20$ va $x=-\frac{23}{3}$ dan $x=-\frac{23}{3}$ chet ildiz, $x = 20$ esa berilgan tenglama yechimi. ◄

3) $\lg2 + \lg(4^{x-2} + 9) = 1 + \lg(2^{x-2} + 1)$

► Potensirlash usulidan foydalanishda $\lg10 = 1$ ni hisobga olib yozamiz $\lg2\cdot(4^{x-2} + 9) = \lg10.(2^{x-2} + 1)$ yoki $2(4^{x-2} + 9) = 10(2^{x-2}+ 1)$ soddalashtirsak ifoda ko'rsatkichli tenglamaga kelinadi $2^{2x} -20{,}2^x + 64 = 0$ bu tenglama ildizlari $2^x = 16$ va $2^x = 4$ bo'lib, bundan $x_1 = 4$ va $x_2 = 2$. Berilgan tenglamaga $x = 4$ va $x_2 = 2$ yechimligini tekshirish qiyin emas ◄

4) $\lg\sqrt{5^{x(13-x)}}+11\lg2=11$

► Tenglamaning ikki tomonini 11ga bo'lamiz:

$\frac{1}{11}\lg5^{\frac{x(13-x)}{2}}+\lg2=1$ logarifm xossalaridan foydalanib, keyin potensirlash usulidan foydalanamiz $\lg5^{\frac{x(13-x)}{22}}+\lg2=\lg10$ dan $\lg2\cdot5^{\frac{x(13-x)}{22}}=\lg10$ yoki $2\cdot5^{\frac{x(13-x)}{22}}=10$ dan $\frac{x(13-x)}{22}=1$, soddalashtirsak $x^2 - 13x + 22 = 0$ bu tenglama ildizlari $x_1 = 2$ va $x_2 = 11$ o'z navbatida berilgan tenglamaning yechimi ham bo'ladi. ◄

16.5. Yangi o'zgaruvchi kiritish usuli bilan yechiladigan tenglamalar. Tenglamalar yechimi topilsin.

1) $\log_2^2 x^2-8\log_2 x+3=0$

► Logarifmning $\log_a^k N^n=n^k\log_a^k N$ xossasiga ko'ra $4\log_2^2 x-8\log_2 x+3=0$ tenglamada $\log_2 x = y$ almashtirish bajarsak $4y^2 - 8y + 3 = 0$ kvadrat tenglamaga kelinadi, bu tenglama yechimlari $y_1=\frac{3}{2}$, $y_2=\frac{1}{2}$ almashtirishdan

1) $\log_2 x = \frac{3}{2}$, $x_1 = \sqrt{8}$ va 2) $\log_2 x = \frac{1}{2}$ $x_2 = \sqrt{2}$ bo'lib, $\sqrt{2}$ va $\sqrt{8}$ berilgan tenglama yechimi ekanligiga, tekshirib ishonch hosil qilish mumkin. ◀

2) $4\log_x 3\sqrt{3} - 5 = 4\log_x^2 \sqrt{3}$

▶ Ildiz daraja va logarifm xossalaridan foydalanamiz $4\cdot\frac{3}{2}\log_x 3 - 5 = 4\cdot\frac{1}{4}\log_x^2 3$ Soddalashtirsak logarifmga nisbatan kvadrat tenglamaga $\log_x^2 3 - 6log_x^3 + 5 = 0$ kelinadi, bu tenglama yechimlari $\log_x 3 = 1$ va $\log_x 3 = 5$ bo'lib, bundan berilgan tenglama yechimlari $x_1 = 3$ va $x_2 = \sqrt[5]{3}$ kelib chiqadi. ◀

3) $\lg^2 x + \lg x^2 = \lg^2 - 1$

▶ Agarda $\lg x = y$ desak berilgan tenglama $y^2 + 2\lg + 1 - \lg_2^2 = 0$ kvadrat tenglamaga kelinadi, tenglama yechimlari $y_1 = -1 + \lg 2$ va $y_2 = -1 - \lg 2$ bo'lib, almashtirishdan:

a) $lgx = -1 + \lg 2 = -\lg 10 + \lg 2 = \lg\frac{1}{5}$ dan $x_1 = 0{,}2$

b) $\lg x = -1 - \lg 2 = -\lg 20$ dan $x_2 = 0{,}05$

Javob: $x_1 = 0{,}2$ $x_2 = 0{,}05$ ◀

4) $[\lg(10x)]\cdot[\lg(0{,}1x)] = \lg^3 x - 3$

▶ Logarifm xossalaridan foydalanamiz $[\lg x + 1][\lg x - 1] = 3\lg x - 3$ yoki $\lg^2 x - 1 = 3\lg x - 3$ endi $\lg x = y$ deb yangi o'zgaruvchi kiritsak, y ga nisbatan kvadrat tenglamaga kelinadi $y^2 - 3y + 2 = 0$ bu tenglama ildizlari $y_1 = 1, y_2 = 2$ u holda almashtirishdan a) $\lg x = 2$ bo'lib, $x_1 = 100$ b) $\lg x = 1$ dan $x_2 = 10$ Bu sonlar berilgan tenglamaga yechim bo'ladi. ◀

5) $2\log_3^2 + \left(1 + \frac{1}{2x}\right) - \log_3\left(\sqrt[x]{3} + 27\right) = 0$

▶ Logarifmning xossalari va ta'rifidan foydalanib tenglamani quyidagicha yozish mumkin.

$\log_3 2^2 + \log_3 3^{1+\frac{1}{2x}} - \log_3\left(\sqrt[x]{3} + 27\right) = \log_3 1$ potensirlash usulini qo'llasak $\log_3 \frac{4\cdot 3^{1+\frac{1}{2x}}}{\sqrt[x]{3} + 27} = \log_3 1$ yoki $12\cdot 3^{\frac{1}{2x}} = 3^{\frac{1}{x}} + 27$ $3^{\frac{1}{2x}} = y$ desak $y^2 - 12y + 27 = 0$ bo'lib, bu tenglama ildizlari $y_1 = 3$ va $y_2 = 9$.

U holda almashtirishdan berilgan tenglama yechimlari $x_1 = 0{,}5$ va $x_2 = 0{,}25$ olamiz.◄

16.6. Har xil asosli logarifm qatnashgan tenglamalarda

$\log_a N = \dfrac{log_b N}{\log_b a}$ formula yordamida bir xil asosli logarifmlarga keltiriladi, bunda katta asosli logarifm kichik asosli logarifmga, o'zgaruvchili asosli logarifmni esa o'zgarmas asosli logarifmga keltirib olinadi.

⊕ Tenglamalar yechimi topilsin:

1) $\log_{16} x + \log_4 x + \log_2 x = 7$

► Qo'shiluvchilarni ikki asosli logarifmlarga keltirib olamiz $\log_{2^4} x + \log_{2^2} x + \log_2 x = 7$, yoki $\left(\dfrac{1}{4}+\dfrac{1}{2}+1\right)\log_2 x = 7$ bundan $\log_2 x = 4$ bo'lib, berilgan tenglama yechimi $x = 2^4 = 16$ ◄

2) $\sqrt{\log_3 x^9} - 4\log_9 \sqrt{3x} = 1$

Logarifm xossalaridan foydalanib yozamiz

$\sqrt{9\log_3 x} - 4\cdot\dfrac{1}{2}\log_{3^2} 3x = 1$ yoki $3\sqrt{\log_3 x} - \log_3 3x = 1$

bundan $3(\log_3 x)^{\frac{1}{2}} - 1 - \log_3 x = 1$

soddalashtirsak logarifmli ifodaga nisbatan kvadrat tenglama bo'ladi $\log_3 x - 3(\log_3 x)^{\frac{1}{2}} + 2 = 0$, bu tenglama yechimlari $(\log_3 x)^{\frac{1}{2}} = 1$ va $(\log_3 x)^{\frac{1}{2}} = 2$ bo'lib, bundan, berilgan tenglama yechimlari $x_1 = 3$, $x_2 = 81$ ni olamiz.◄

3) $\sqrt{\log_x \sqrt{5x}} \cdot \log_5 x = -1$

► Tenglikning ikki tomonini kvadratga ko'tarsak

$\log_x \sqrt{5x} \cdot \log_5^2 x = 1$ logarifm xossasidan $\dfrac{1}{2}(\log_x 5 + 1)\log_5^2 x = 1$

Logarifmning $\log_a b \cdot \log_b a = 1$ xossasidan foydalansak logarifmli funksiyaga nisbatan kvadrat tenglamaga kelinadi.

$\log_5 x + \log_5^2 x - 2 = 0$ bu tenglama ildizlari $\log_5 x = 1$ va $\log_5 x = -2$ bo'lib, bundan $x_1 = 5$, $x_2 = 0{,}04$. Bunda $x = 5$ ni tenglamaga qo'shsak tenglamani qanoatlantirmaydi.

$x = 0{,}04$ esa qanoatlantiradi, berilgan tenglama yechimi $x = 0{,}04$ ◄

4) $\log_3 x^3 + \log_2 x^2 = \dfrac{2\lg 6}{\lg 2} + 1$

► Tenglamaning chap tomonidagi ifodaning har bir qo'shiluvchini o'n asosda logarifmlab yozamiz

$$\frac{3lgx}{\lg 3} + \frac{2lgx}{\lg 2} = \frac{2\lg 6}{\lg 2} + 1$$

Endi tenglikning ikki tomonini lg2 · lg3 ga ko'paytirib, keyin soddalashtiramiz (3lg2 + 2lg3)lg$x$ = (2lg6 + lg2)lg3 bundan lg6 = lg2 + lg3 deb yozish mumkinligini hisobga olsak (3lg2 + 2lg3)lg$x$ = (3lg2 + 2lg3)·lg3 dan lg$x$ = lg3 bo'lib, berilgan tenglama yechimi $x = 3$ ga teng bo'ladi.

5) $\log_{3x} 3 = \log_{x^2} 3$

► O'tish formulasidan tenglikning ikki tomonini uch asosli logarifm ko'rinishda yozib olamiz $\dfrac{\log_3 3}{\log_3 3x} = \dfrac{\log_3 3}{\log_3 x^2}$ dan $\log_3 3x = \log_3 x^2$ yoki $3x = x^2$ bu tenglama yechimlari $x_1 = 0$, $x_2 = 3$ dan $x = 3$ berilgan tenglama yechimi bo'ladi. ◄

16.7. Noma'lum o'zgaruvchi daraja va asosda qatnashgan ko'rsatkichli logarifmik tenglamalarning ikki tomonini logarifmlab yechimi topiladi.

Agarda asos yoki darajada logarifm qatnashgan bo'lsa, u holda tenglama shu asosda logarifmlanadi, bu usulni misollarda ko'rib chiqamiz.

⊕ Tenglamalar yechimi topilsin.

1) $x^{3+\lg x} = (0{,}01)^{-2}$

► Tenglamaning ikki tomonini o'n asosda logarifmlaymiz.

(3 + lg$x$)lg$x$ = – 2 lg 0.01 yoki $\lg^2 x + 3\lg x - 4 = 0$ logarifmga nisbatan kvadrat tenglama yechimlari lg$x$ = 1 va lg$x$ = – 4 bo'lib, bundan $x_1 = 1$ va $x_2 = 10^{-4}$ bu tenglamalar yechimlari ekanligini o'rniga qo'yish orqali ishonch hosil qilamiz.

Javob: $x_1 = 1$ va $x_2 = 0{,}0001$ ◄

2) $3 \cdot x^{\log_3 x} = x^{3,(3)}$

► Tenglamaning ikki tomonini uch asosda logarifmlaymiz $\log_3^3 + \log_3 x \cdot \log_3 x = \dfrac{10}{3}\log_3 x$ soddalashtirsak, logarifmga nisbatan kvadrat tenglama hosil bo'ladi $3\log_3^2 x - 10\log_3 x + 3 = 0$.

Bu tenglama ildizlari $\log_3 x = 3$ va $\log_3 x = \dfrac{1}{3}$ bo'lib, bundan $x_1 = 27, x_2 = \sqrt[3]{3}$ bu qiymatlarni tenglamaga qo'yib tekshiramiz:

a) $3\cdot 27^{\log_3 27} = 27^{\frac{10}{3}}$ yoki $3\cdot 27^3 = \left(3^3\right)^{\frac{10}{3}}$ dan $3^{10} = 3^{10}$ demak $x = 27$ yechim,

b) $3\cdot\left(\sqrt[3]{3}\right)^{\log_3 \sqrt[3]{3}} = \left(\sqrt[3]{3}\right)^{\frac{10}{3}}$ yoki $3\cdot\left(3^{\frac{1}{3}}\right)^{\frac{1}{3}} = \left(3^{\frac{1}{3}}\right)^{\frac{10}{3}}$ dan $3^{\frac{10}{3}} = 3^{\frac{10}{3}}$ $x = \sqrt[3]{3}$ ham yechim ekan. ◄

3) $4^{\log_4^2 x} + x^{\log_4^x} = 8$

► Birinchi qo'shiluvchini shakl almashtirib, logarifm xossasidan foydalanib soddalashtiramiz

$$\left(4^{\log_{4^x}}\right)^{\log_{4^x}} + x^{\log_{4^x}} = 8 \text{ dan } (x)^{\log_{4^x}} + x^{\log_{4^x}} = 8$$

yoki $x^{\log_{4^x}} = 4$

Tenglikning ikki tomonini to'rt asosda logarifmlaymiz

$\log_4 x \cdot \log_4 x = \log_4 4$ bo'lib $\log_4^2 x = 1$ bu tenglama yechimi $x_1 = 4$ va $x_2 = \frac{1}{4}$. Bu x_1 va x_2 sonlar berilgan tenglamaning yechimi ekanligiga ishonch hosil qilamiz. ◄

4) $x^{\log_{\sqrt{x}}(1-x)} = 4$

► Tenglamani $x^{\log_x (1-x)^2} = 4$ deb yozish mumkin. Sababi $\log_{\sqrt{x}}(1-x) = \log_{x^{\frac{1}{2}}}(1-x) = 2\log_x(1-x) = \log_x(1-x)^2$

Logarifm xossasidan $(1-x)^2 = 4$ bundan $x = -1$ va $x = 3$ Bu sonlar tenglama aniqlanish sohasiga tegishli emas. Tenglama aniqlanish sohasi $x > 0$. $x \neq 1$ va $1 - x > 0$ yoki $x \in (0;1)$. Demak tenglama yechimi mavjud emas. ◄

16.8. Umumlashgan usul. Tenglamalar yechimi topilsin.

1) $4\log_{25}^2 x = \log_5 x\left[2\log_5\left(\sqrt{x+5}-1\right)\right]$

► Oldin logarifmning $\log_{a^n} N = \frac{1}{n}\log_a N$ xossasidan foydalanib

$\log_{25}^2 x = \left(\log_{5^{2^x}}\right)^2 = \left(\frac{1}{2}\log_5 x\right)^2 = \frac{1}{4}\log_{5^x}^2$ deb yozish mumkin u

holda $\log_5^2 x - \log_5 x\left[2\log_5\left(\sqrt{x+5}-1\right)\right] = 0$ dan

$\log_5 x\left[\log_5 x - 2\log_5\left(\sqrt{x+5}-1\right)\right] = 0$ bunda $\log_5 x = 0$ yoki

$x = 1$, va $\log_5 x - 2\log_5\left(\sqrt{x+5}-1\right) = 0$ dan

$\log_5 x = \log_5\left(\sqrt{x+5}-1\right)^2$ yana logarifm xossasidan

$x = \left(\sqrt{x+5}-1\right)^2$ bu ifoda soddalashtirilsa $\sqrt{x+5} = 3$ bo'lib yechim $x = 4$. O'rniga qo'yish bilan $x = 1$ va $x = 4$ berilgan tenglamaning yechimlari ekanligiga ishonch hosil qilamiz. ◄

2) $x^x + 139x^{-x} - 108x^{-2x} = 32$

► Tenglamaning chap tomonidan umumiy ko'paytuvchini qavsdan tashqariga chiqaramiz $x^{-2x}(x^{3x} + 139x^x - 108) = 32$ bundan

$$x^{3x} - 32x^{2x} + 139x^x - 108 = 0$$

Agar $x^x = y$ desak, tenglama $y^3 - 32y + 139y - 108 = 0$ ko'rinishga keladi $y = 1$ tenglama yechimi chap tomondagi ko'p hadni $(y-1)$ga bo'lsak

$$(y-1)(y^2 - 31y + 108) = 0$$

Bu tenglama yechimlari $y_1 = 1, y_2 = 27$ va $y_3 = 4$ bo'lib almashtirishdan berilgan tenglama yechimlari $x_1 = 1$, $x_2 = 2$ va $x_3 = 3$ ekanligiga ishonch hosil qilish qiyin emas. ◄

3) $\lg 10 + \lg 1 = \frac{1}{5 - lgx} + \frac{2}{1 + lgx}$

► Logarifm ta'rifdan $\lg 10 = 1, \lg 1 = 0$, u holda $\frac{1}{5 - lgx} + \frac{2}{1 + lgx} = 1$

$\lg x \neq 5$ va $\lg x \neq -1$ deb, tenglamani $(1 + \lg x)(5 - \lg x)$ ga ko'paytiramiz

$$1 + \lg x + 2(5 - \lg x) = (5 - \lg x)(1 + \lg x)$$

qavs ochib soddalashtirsak, logarifmga nisbatan kvadrat tenglamaga kelinadi $\lg^2 x - 5\lg x + 6 = 0$ bu tenglama ildizlari $\lg x = 3$ va $\lg x = 2$ bo'lib bundan $x_1 = 1000$ va $x_2 = 100$, bu x_1 va x_2 sonlar berilgan tenglama yechimlari ekanligiga ishonch hosil qilamiz. ◄

4) $\log_2^2 x - \log_2 x = \log_x 2 - \log_x^2 2$

► Tenglamani

$$\log_x^2 + \log_2 x = \log_2^2 x + \log_x^2 2 \text{ deb yozamiz.}$$

Agar $\log_x^2 + \log_2 x = y$ desak $\left(\log_x{}^2 + \log_2 x\right)^2 = y^2$ yoki $\log_x^2 2 + \log_2^2 x = y^2 - 2$ bo'lib, u o'zgaruvchiga nisbatan $y^2 - y - 2 = 0$

tenglamaga kelamiz, bu tenglama ildizlari

$$y_1 = 2, y_2 = -1$$

U holda almashtirishdan

a) $\log_x^2 + \log_2 x = 2, \log_a N = \dfrac{\log_b N}{\log_b a}$ formulaga asosan yozamiz

$\dfrac{1}{\log_2 x} + \log_2 x = 2$ yoki $\log_2^2 x - 2\log_2 x + 1 = 0$ bu tenglama yechimi $\log_2 x = 1$ dan $x = 2$

b) $\log_x 2 + \log_2 x = -1$ dan $\dfrac{1}{\log_2 x} + \log_2 x = -1$ bo‘lib, bundan $\log_2^2 x + \log_2 x + 1 = 0$ bu kvadrat tenglama haqiqiy yechimi mavjud emas. Berilgan tenglama yechimi $x = 2$ ◄

5) $\log_{(x+1)}(x - 0,5) = \log_{(x-0,5)}(x + 1)$

► Tenglama aniqlanish sohasi

$$\begin{cases} x - 0,5 > 0 \\ x + 1 > 0 \\ x - 0,5 \neq 1 \\ x + 1 \neq 1 \end{cases} \text{dan } x > 0,5, x \neq 1,5, x \neq 0$$

Agar $\log_{x+1}(x - 0,5) = y$ desak, berilgan tenglama

$y = \dfrac{1}{y}$ yoki $y^2 = 1$ ko‘rinishda bo‘lib, bundan $y = 1$ va $y = -1$ bo‘ladi.

U holda almashtirishdan

a) $\log_{x+1}(x - 0,5) = 1$ dan $x - 0,5 = x + 1, x \in \varnothing$

b) $\log_{(x+1)}(x - 0,5) = -1$ dan $x - 0,5 = \dfrac{1}{x+1}$ yoki $2x^2 + x - 3 = 0$

bu tenglama ildizlari $x_1 = -1,5$ va $x_2 = 1$ bo‘lib bu qiymatlardan $x_1 = 1,5$ chet ildiz, $x_2 = 1$ esa berilgan tenglama yechimi.◄

6) $\log_2 x \cdot \log_3 x \cdot \log_5 x - \log_3 x \cdot \log_5 x = \log_2 x \cdot \log_3 x + \log_2 x \cdot \log_5 x$

► $\log_a N = \dfrac{\log_b N}{\log_a N}$ formulaga asosan tenglamadagi har bir qo‘shiluvchini ikki asosli logarifmda yozamiz

$$\frac{\log_2^3 x}{\log_2 3 \cdot \log_2 5} = \frac{\log_2^2 x}{\log_2 3 \cdot \log_2^5} + \frac{\log_2^2 x}{\log_2 3} + \frac{\log_2^2 x}{\log_2 5} \text{ yoki}$$

$$\log_2^3 x = (\log_2 5 + \log_2 3 + 1)\log_2^2 x \quad 1 = \log_2 2$$

logarifm xossalaridan foydalanib yozamiz

$$\log_2^3 x = \log_2 30 \cdot \log_2^2 x$$

bundan

$$\left(\log_2 x - \log_2 30\right) \cdot \log_2^2 x = 0$$

tenglama yechimlari:

a) $\log_2^2 x = 0,$ dan $\log_2 x = 0$ bo‘lib $x = 1$

b) $\log_2 x - \log_2 30 = 0$ dan $x = 30$. Bu $x = 1$ va $x = 30$ berilgan tenglamaning yechimlari ekanligiga o‘rniga qo‘yish bilan ishonch hosil qilamiz. ◄

7) $\log_x b + \log_{x\sqrt[4]{6}} x^2 \cdot \sqrt{6} = 4$

► Tenglamadagi logarifmik qo‘shiluvchilarni asosiy formulaga asosan asosga keltirib olamiz

$$\log_x b + \frac{2 + \frac{1}{2}\log_x b}{1 + \frac{1}{4}\log_x b} = 4$$ soddalashtirsak $\log_x b + 2 = 4$ bo‘lib $x^2 = b$

yoki $x = \pm\sqrt{6}$, berilgan tenglama yechimi $x = \sqrt{6}$ bo‘ladi. ($b > 0$ da) ◄

8) $6^{\frac{1}{0,5+\log_9 2}} = \log_{\sqrt[3]{2}} (x-2)$

► Tenglamaning aniqlanish sohasi $x > 2$. Logarifm xossasidan foydalanib, tenglikning chap tomonini shakl almashtirib yozamiz.

$$0,5 + \log_9 2 = \log_3 \sqrt{3} + \log_3 \sqrt{2} = \log_3 \sqrt{6} = \frac{\frac{1}{2}\log_6 6}{\log_6 3} = \frac{1}{\log_6 9}$$

u holda tenglamani yozish mumkin.

$6^{\log_6 9} = \log_2 (x-2)^3$ dan $9 = 3\log_2 (x - 2)$ yoki $3 = \log_2 (x - 2)$ bundan $(x - 2) = 2^3$ yoki $x = 10$ tekshirib ko‘rish qiyin emas $x = 10$ berilgan tenglama yechimi. ◄

17-MAVZU. KO‘RSATKICHLI VA LOGARIFMIK TENGLAMALAR SISTEMASI

Ko‘rsatkichli va logarifmik tenglamalar sistemasi yechimini topish umumiy usuli mavjud emas, sistema yechimini topish tenglamalarning berilishiga qarab, tenglamada qatnashgan funksiyalarning xossalari, o‘tilgan tenglamalar yechimini topish usullaridan foydalanish mumkin, buni misollar yechishda oydinlashtiramiz.

⊕ Tenglamalar sistemasi yechimi topilsin.

1) $\begin{cases} \log_4 x + \log_4 y = 1 + \log_4 9 \\ x + y = 20 \end{cases}$

► Sistemadagi birinchi tenglamani potensirlab yozamiz

2) $\begin{cases} \log_4 xy = \log_4^4 9 \\ x = 20 - y \end{cases}$ dan $\begin{cases} xy = 36 \\ x = 20 - y \end{cases}$

O‘rniga qo‘yish usulidan foydalansak

$y(20-y)=36$ yoki $y^2-20y+36=0$ Tenglama yechimlari $y_1=18, y_2=2$. U holda $x=20-y$ dan $x_1=2, x_2=18$

javob: (2; 18) va (18; 2)

2) $\begin{cases} \sqrt[x]{x+y} = 2 \\ x + y \cdot 5^x = 100 \end{cases}$ o‘rniga qo‘yish usulidan foydalansak

$\sqrt[x]{\frac{100}{5^x}} = 2$ dan $\frac{100^{\frac{1}{x}}}{5} = 2$ yoki $10^{\frac{2}{x}} = 10$ dan $x = 2$ bo‘lib sistemaning ikkinchi tenglamasidan $(2 + y) \cdot 25 = 100$ bundan $y = 2$

javob: (2;2)

3) $\begin{cases} 2^x \cdot 3^y = 24, \\ 2^y \cdot 3^x = 54 \end{cases}$

► Sistemadagi birinchi tenglamani mos ravishda ikkinchi tenglamaga ko‘paytirsak $2^{x+y} \cdot 3^{y+x} = 24 \cdot 54$ yoki $(6)^{x+y} = 2^3 \cdot 3 \cdot 2 \cdot 3^3$ dan $x + y = 4$ endi birinchi tenglamani mos ravishda ikkinchi tenglamaga bo‘lamiz

$2^{x-y} \cdot 3^{y-x} = 2^2 \cdot 3^{-2}$ yoki $\left(\frac{2}{3}\right)^{x-y} = \left(\frac{2}{3}\right)^2$ dan $x - y = 2$. Berilgan tenglama yechimini topish teng kuchli $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$ sistemaga kelindi. Bu sistema yechimi (3;1). Tekshirish qiyin emas. (3;1) berilgan sistema yechimi bo‘ladi. ◄

4) $\begin{cases} \lg(x+y)-\lg(x-y)=2\lg 2+\lg 3 \\ \lg\left(x^2+y^2+10\right)=2+\lg 3 \end{cases}$

► Sistema aniqlanish sohasida potensirlasak

$\begin{cases} \lg\dfrac{x+y}{x-y}=\lg 12 \\ \lg\left(x^2+y^2+10\right)=\lg 300 \end{cases}$ dan $\begin{cases} x+y=12(x-y) \\ x^2+y^2+10=300 \end{cases}$ yoki

$\begin{cases} 11x-13y=0, \\ x^2+y^2=290. \end{cases}$ o‘rniga qo‘yish usulidan foydalansak $x = 13$, $y = 11$ yechimini olamiz. $x = 13$ va $y = 11$ qiymatlar berilgan sistema yechimi ekanligi o‘rniga qo‘yish orqali ishonch hosil qilamiz. ◄

5) $\begin{cases} 3^{\log_3 x}-2^{\log_4 y^2}=77, \\ 3^{\log_3\sqrt{x}}-2^{\log_{16} y^2}=7. \end{cases}$

► Logarifm xossalaridan $3^{\log_3 x}=x, 3^{\log_3\sqrt{x}}=\sqrt{x}$, ,

$2^{\log_4 y^2}=2^{2\cdot\frac{1}{2}\log_2 y}=y$ va $2^{\log_{16} y^2}=2^{\log_2\left(y^2\right)^{\frac{1}{4}}}=\sqrt{y}$ larni

hisobga olsak, u holda $\begin{cases} x-y=77 \\ \sqrt{x}-\sqrt{y}=7 \end{cases}$; yoki $\begin{cases} \sqrt{x}+\sqrt{y}=11, \\ \sqrt{x}-\sqrt{y}=7. \end{cases}$ bu sistema berilgan tenglamalar sistemasiga teng kuchli bo‘lib, sistema yechimi (81;4) ◄

6) $\begin{cases} x^{lgy}=100, \\ \log_y x=2. \end{cases}$

► $x > 0, y > 0, x \neq 1, y \neq 1$ deb, sistamadagi birinchi tenglamani o‘n asosda logarifmlaymiz, ikkinchi tenglamani ta’rifdan foydalanib yozamiz

$\begin{cases} lgy\cdot lgx=2 \\ x=y^2 \end{cases}$ dan $\lg y\cdot\lg y^2=2$ yoki $\lg^2 y=1$ bo‘lib, yechimlari

$y_1=0{,}1, y_2=10$, u holda $x=y^2$ dan:
$x_1=0{,}01, x_2=100$, javob: (0,01;0,1) va (100;10). ◄

7) $\begin{cases} 3^x-4^y=17, \\ 3^{0,5x}+2^y=17 \end{cases}$

► Yangi o'zgaruvchi $3^{0,5x} = u$ va $2^y = v$ desak, sistema

$\begin{cases} u^2 - v^2 = 17, \\ u + v = 17. \end{cases}$ dan $\begin{cases} u - v = 1, \\ u + v = 17 \end{cases}$ o'rniga qo'yish usulidan topar

$u = 9$, $v = 8$ U holda almashtirishdan berilgan sistema yechimi topiladi. $3^{0,5x} = 9$, $x = 4$ va $2^y = 8$ dan $y = 3$. javob: (4;3). ◄

16-, 17-MAVZULAR MASHQLARI

Logarifmik tenglamaning aniqlash sohasini topilsin.

271. 1) $\lg(x + 5) - \lg(x - 15) = \lg(3x + 25) - \lg 15$,

2) $\log_1 x (x^2 - x - 2) = 2$.

272. 1) $\log_{0,5}\left(5x - x^2 - 6\right) + \log_{\sqrt{2}}\left(2x - 1\right) = 4$,

2) $\log_{2x} (2x^2 - 5x - 3) = 1,5$.

273. 1) $\log_3 (x^2 + x - 2) + \log_3 (3x - 1) = 6$,

2) $\lg\sqrt{2 - x} + \lg\sqrt{x + 2} = 0,5$.

274. 1) $\log_{0,5}\log_{0,5}\log_{0,5} x = 0$, 2) $\dfrac{1 - \lg^2 x^2}{lgx - 2\lg^2 x} = \lg x^4 + 5$.

x son nimaga teng.

275. $\log_2 x = \frac{1}{5}\log_2 a - 2\log_2 b - \frac{2}{3}\log_2 c + 2\log_2 3 - 2$.

276. $lgx = \frac{2}{5}\lg 32 - \frac{1}{3}\lg 64 + \lg 5 - \frac{1}{2}\lg 9 - \lg 2 + 1$.

277. $x = 9^{2\log_3 2 + 4\log_{81} 2} - 2 \cdot 100^{\frac{1}{2}\lg 8 - 2\lg 2}$.

278. $x = 10^{2 - lgtg\frac{\pi}{4}} - 100^{1 + lgcos\frac{\pi}{3}}$.

279. 1) $x = \log_{\sqrt{2}} 2\sqrt[4]{2}$, 2) $\log_x \frac{\sqrt[5]{9}}{3} = -0,6$.

280. 1) $\log_{3\sqrt[4]{3}} x = -0,4$, 2) $\log_x \left(3 - 2\sqrt{2}\right) = 2$.

a va *b* ning qanday qiymatlarida tenglik o‘rinli bo‘ladi.

281. $\log_{(2b-a)}(2a - b) = 1$.

282. $\log_{\sqrt{2}}\left(2a - b\right) = 0$.

Logarifmik tenglama yechimlari topilsin.

283. $2^{\log_3 x^2} - 5^{\log_3 x} = 400$.

284. $5^{2(\log_5 2 + x)} - 5^{x + \log_5 2} = 2$.

285. $\log_2 (9 - 2^x) = 10^{\lg(3-x)}$.

286. $\log_3 x^3 + \log_2 x^2 = \dfrac{2\lg 6}{\lg 2} + 1$.

287. $\log_{16} x + \log_4 x + \log_2 x = 3{,}5\log_2 4$.

288. $\log_2 (9^{x-1} + 7) - \log_2 (3^{x-1} + 1) = \log_2 4$.

289. $5\log_2 3 + 2\log_2 \sqrt{(x-2)\sqrt{8}} - 1\dfrac{2}{3}\log_2 27 = 1{,}5$.

290. $2\lg 2 + \left(1 + \dfrac{1}{2x}\right)\lg 3 = \lg\left(\sqrt[x]{3} + 27\right)$.

291. $\log_{1-x} (4x^2 + 4x + 1) = 2$.

292. $x^{4\lg^3 x - \frac{1}{3}\lg x} = 1000 \cdot \sqrt[3]{100}$.

293. $5^{\lg x} - 3^{\lg x - 1} = 3^{\lg x + 1} - 5^{\lg x - 1}$.

294. $\log_3 x + \log_2 x = 1$.

295. $\lg\sqrt{5x-4} + \lg\sqrt{1+x} = 2 + \lg 0{,}18$.

296. $3^{\log_3^2 x} + x^{\log_3 x} = 162$.

297. $\log_a x - \log_{a^2} x + \log_{a^4} x = 0,75$.

298. $(3lgx)^{\lg^2 x + 2lgx^2 + 5} = lgx^3$.

299. $\log_7 \log_4 \log_3^2 (4x-3) = 0$.

300. $\log_3\left(3^{x^2-13x+28} + 0,(2)\right) = \log_5 0,2$.

301. $4\log_x 3\sqrt{3} - 5 = 4\log_x^2 \sqrt{3}$.

302. $x^{3\log_3^3 x - 0,(6)\log_3 x} = 9\sqrt[3]{3}$.

303. $\dfrac{1}{3-lgx} - \dfrac{3}{1+lgx} = 2$.

304. $14^{\log_7 2} \cdot x^{\log_7 4x+1} = 1$.

305. $\lg^3 a^2 x^2 = 8\lg ax$.

306. $\log_8 x + \log_8^2 x + \log_8^3 x + \ldots = 0,5$.

307. $\log_{x+3}(x+1) = \log_{x+1}(x+3)$.

308. $\log_5 \sqrt{2x-4} - \log_5 \sqrt{x+1} - \log_5 \sqrt{x+5} = \log_5 2$.

309. $\log_{3x} 3 = \log_3^2 3x$.

310. $\sqrt{\log_x \sqrt{5x}} \cdot \log_{\frac{1}{5}} x = -1$.

Tenglamalar sistemasini yechimi topilsin:

311. $\begin{cases} y^{x^2-2x-15} = 1 \\ x + y = 1 \end{cases}$

312. $\begin{cases} 3^x \cdot 5^y = 75, \\ 3^y \cdot 5^x = 75. \end{cases}$

313. $\begin{cases} \log_x y + \log_y x = 2, \\ x^2 - y = 20. \end{cases}$

314. $\begin{cases} \lg(x-y) - 2\lg 2 = 1 - \lg(x+y), \\ lgx - \lg 3 = \lg 7 - lgy. \end{cases}$

315. $\begin{cases} 3^x + 3^y = 1, \\ 3^{x+y} = 3. \end{cases}$

316. $\begin{cases} 2^{\log_2(3x-4)} = 8, \\ \log_9\left(x^2 - y^2\right) - \log_9(x+y) = 0,5. \end{cases}$

317. $\begin{cases} \sqrt[y]{4^x} = 32 \cdot \sqrt[x]{8^y}, \\ \sqrt[y]{3^x} = 3 \cdot \sqrt[y]{9^{1-y}}. \end{cases}$

318. $\begin{cases} (x-y)(0,5)^{y-x} = 5 \cdot 2^{x-y}, \\ (x-y)\dfrac{x+y}{7} = 125. \end{cases}$

319. $\begin{cases} \sqrt[x]{3} \cdot \sqrt[y]{1,5} = 0,25, \\ \sqrt[x]{5} \div \sqrt[y]{0,2} = 1. \end{cases}$

320. $\begin{cases} \log_x \log_2 \log_x y = 0, \\ \log_y 9 = 1. \end{cases}$

321. $\begin{cases} \sqrt[x-y]{x+y} = 2\sqrt{3}, \\ (x+y) \cdot 2^{y-x} = 3. \end{cases}$

322. $\begin{cases} x^y = 5x - 4, \\ \log_x 16 = y. \end{cases}$

323. $\begin{cases} \log_x y - \log_y x = \dfrac{3}{2}, \\ x + y = 0{,}75. \end{cases}$

324. $\begin{cases} \left(\sqrt[3]{x}\right)^{\lg y} = 2, \\ y \cdot \sqrt[3]{x} = 20. \end{cases}$

325. $\begin{cases} \sqrt[x-1]{49} = \sqrt[y-1]{343}, \\ 3^y = 9^{2x-y}. \end{cases}$

326. $\begin{cases} 2\log_x 2 + 3\log_y 2 = 0, \\ x^3 - 4y^2 = 0. \end{cases}$

327. $\begin{cases} \sqrt[x]{y} = 2, \\ y^x = 16. \end{cases}$

328. $\begin{cases} 3^x \cdot 2^y = 576, \\ \log_{\sqrt{2}}(y - x) = 4. \end{cases}$

TRIGONOMETRIK TENGLAMALAR

Noma'lum o'zgaruvchi (yoki uning chiziqli funksiya ko'rinishdagi ifodasi) trigonometrik funksiya belgisi ostida qatnashgan tenglamalarga trigonometrik tenglama deyiladi.

Trigonometrik tenglamani yechish, berilgan tenglamani qanoatlantiradigan barcha burchaklarni (yoylarni) topish degani, yani shunday burchaklarni topish kerakki, ularni tenglamadagi o'zgaruvchilarga qo'yilganda tenglama ayniyatga aylanishi kerak.

Trigonometrik tenglama yechimini topishning umumiy metodi mavjud emas. Trigonometrik tenglamaning yechimini tenglama aniqlanish sohasida qidirish kerak.

Trigonometrik tenglama yechimini topishning o'ziga xos tomoni, tenglama aniqlanish sohasini aniqlangandan keyin, trigonometrik funksiyalarning davriligi, juft-toqligi, chorakdagi ishoralarini hisobga olish kerak, asosiy ayniyatlar, formulalar, trigonometrik funksiyalardagi bog'lanishlar (bular [1] to'la yoritib berilgan). Trigonometrik ifodalarni soddalashtirish va hisoblashlar (bular [4] da misollar bilan to'la yoritib berilgan)ni yaxshi eslash kerak.

Tenglama yechimlarini topish usullarida bu kabi ma'lumotlardan qanday foydalanish kerakligini ham yaxshi bilish kerak, aks holda noratsional usul bo'lib, yechimini aniqlash murakkablashishi mumkin, yoki yechim noto'g'ri topilishi mumkin. Bu ma'lumotlardan to'g'ri foydalana olmaslik natijasida chet ildizlar paydo bo'lishi yoki ba'zi ildizlarning yo'qolishi mumkin. Agar topilgan yechimni umumlashtirib, karrali ildizlarni chiqarib soddalashtirib yozishni (bu haqda [4] da misollar ko'rsatilgan) bilmasak, topilgan son qiymat ko'rsatilgan hisobotga to'g'ri kelmasligi mumkin.

Bunday noaniqliklarni misollarda ko'rib chiqamiz.

1. Trigonometrik tenglamani yechishda topilgan son qiymatlar tenglama yechimi ekanligini tekshirishning ikkita usuli mavjud bo'lib, biri topilgan son qiymatni umumiy ko'rinishi bo'yicha tekshirish, ikkinchisi son qiymatlarni (yechimlarni) tenglama davriy bo'yicha tekshirish.

<u>Masalan:</u>

$$3\sin x + \cos 2x = 2$$

Tenglama yechimlari tekshirilsin.

► $\cos 2x = 1 - 2\sin^2 x$ tenglikdan foydalansak, $\sin x$ ga nisbatan $2\sin^2 x - 3\sin x + 1 = 0$ kvadrat tenglamaga kelinadi. Bu tenglama yechimlari

$$x_k = \frac{\pi}{2} + 2k\pi \text{ va } x_n = (-1)^n \frac{\pi}{6} + n\pi,\ k,n \in Z$$

ni umumiy ko'rinish bo'yicha tekshiramiz:

1) $x = x_k$ da $\cos 2\left(\frac{\pi}{2} + 2k\pi\right) + 3\sin\left(\frac{\pi}{2} + 2k\pi\right) = \cos(\pi + 4k\pi) +$

$+3\sin\frac{\pi}{2}=\cos\pi+3\cdot1=-1+3=2$ yechim.

2) $x=x_n$ ga $n=2$m bo'lsa $x_n=\frac{\pi}{6}+2m\pi$ bo'lib,

$$\cos2\left(\frac{\pi}{6}+2m\pi\right)+3\sin\left(\pi/6+2m\pi\right)=\cos\frac{\pi}{3}+3\sin\frac{\pi}{6}=\frac{1}{2}+3\cdot\frac{1}{2}=2$$

agar $\pi=2m+1,\ m\in z$ bo'lsa $x_n=-\frac{\pi}{6}+(2m+1)\pi$ bo'lib,

$$\cos2\left[-\frac{\pi}{6}+(2m+1)\pi\right]+3\sin\left[-\pi/6+(2m+1)\pi\right]=\cos\left(-\pi/3\right)+$$

$$+3\sin\left(\pi-\frac{\pi}{6}\right)=\cos\frac{\pi}{3}+3\sin\frac{\pi}{6}=\frac{1}{2}+3\cdot\frac{1}{2}=2$$

Demak, topilgan son qiymatlar to'plami, berilgan tenglamaning yechimi ekan.

Endi yechimlarni trigonometrik tenglamaning davri bo'yicha tekshiramiz. Tenglamaning davri 2π ga teng. (Odatda trigonometrik tenglamaning davri sifatida, tenglama tarkibidagi trigonometrik funksiyalar eng kichik davri olinadi.) x_k va x_n yechimlardan uzunligi 2π ga teng bo'lgan

$(-\pi;\pi)$ oraliqqa tegishli bo'lgan qiymatlar $\frac{\pi}{2}$, $\frac{\pi}{6}$ va $\frac{5\pi}{6}$ ni tenglamaga qo'yib tekshiramiz:

(x_k dan $\pi/2$ olindi; x_n yechimda $n=2m$ desak $\frac{\pi}{6}$ olindi,

$n=2m+1$ desak $x_n=-\frac{\pi}{6}+(2m+1)\pi$ dan $\frac{5\pi}{6}$ olindi,

$x=\frac{\pi}{2}$ da $\cos2\cdot\frac{\pi}{2}+3\sin\frac{\pi}{2}=-1+3\cdot1=2$,

$x=\frac{\pi}{6}$ da $\cos2\cdot\frac{\pi}{6}+3\sin\frac{\pi}{6}=\frac{1}{2}+3\cdot\frac{1}{2}=2$.

$x=\frac{5\pi}{6}$ da $\cos2\cdot\frac{5\pi}{6}+3\sin\frac{5\pi}{6}=\cos\left(2\pi-\frac{\pi}{3}\right)+3\sin\left(\pi-\frac{\pi}{6}\right)=$

$=\cos\frac{\pi}{3}+3\cdot\sin\frac{\pi}{6}=\frac{1}{2}+3\cdot\frac{1}{2}=2$

Demak, tenglamani yechish natijasida topilgan qiymatlar tenglamaning yechimlari ekan.◄

2. Trigonometrik tenglamalarda topilgan yechimlarda karrali yechimlarni chiqarib, umumlashtirib sodda ko‘rinishda yozish ham katta ahamiyatga ega.

$\sin x \cdot \sin 2x \cdot \sin 4x \cdot \sin 8x = 0$,

► Tenglama yechimlari, ko‘paytmaning nolga teng bo‘lish shartidan:

$x_m = m\pi,\ x_n = \frac{n\pi}{2}, x_l = \frac{l\pi}{2}$ va $x_k = \frac{k\pi}{8}$, $k,l,n,m, \in, Z$: yechimlar to‘plami hosil bo‘ladi.

Bu to‘plamlarning hammasini javob deb ko‘rsatish shart emas. x_k yechimda $k = 8m$ desak, x_m yechimlar, $k = 4l$ bo‘lsa, x_n yechimlar va $k = 2l$ bo‘lsa, x_l yechimlar hosil bo‘ladi.Shuning uchun berilgan tenglamaning javobi deb x_k to‘plamining o‘zini ko‘rsatish yetarli.◄

3. Trigonometrik tenglamani yechish maqsadida biror shakl almashtirishda hosil bo‘lgan tenglamaning aniqlanish sohasi, oldingi tenglama aniqlanish sohasining biror qismidan iborat bo‘lsa (yani aniqlanish soha toraya borsa), u holda berilgan tenglamaning ba’zi ildizlari yo‘qolishi mumkin.

<u>*Masalan:*</u>

$\sin x \cdot \cos x = \sin^2 x$ tenglama aniqlanish sohasi $(-\infty; +\infty)$.

Agar tenglamaning ikki qismini $\sin^2 x$ ga bo‘lib, $\mathrm{ctg}x = 1$ tenglamaga kelsak, bu tenglamaning aniqlanish sohasi $x \neq k\pi\ k \in Z$ yechimlar to‘plami

$$x_k = \frac{\pi}{4} + k\pi$$

Shu yechim to‘plamlarni berilgan tenglama yechimi deb kifoyalansak, bu to‘g‘ri emas, chunki shakl almashtirishda berilgan tenglamaning aniqlanish sohasi toraydi.

Berilgan tenglamaning ba’zi ildizlari yo‘qolgan bo‘lishi mumkin. Bu $\sin x = 0$ tenglamaning $x = k\pi, k \in Z$ yechimlari ichida bo‘lishi mumkin. Bu umumiy ko‘rinishdagi yechimni berilgan tenglamaga qo‘yib tekshiramiz.

$x = x_k = k\pi$ da $\sin^2 k\pi = \sin k\pi \cdot \cos k\pi$ yoki $0 = 0$. Demak $\sin x = 0$ tenglamaning barcha ildizlari berilgan tenglamaning yo‘qolgan ildizlaridan iborat ekan. Shunday qilib, bu tenglamaning yechimlar to‘plami

$x_k = \frac{\pi}{4} + k\pi$ va $x_n = n\pi,\ k,n \in Z$ ko‘rinishda bo‘ladi. ◄

4. Trigonometrik tenglamani yechishda bajariladigan shakl almashtirishlar jarayonida berilgan tenglamaning aniqlanish sohasi kengayganda, yoki berilgan tenglamaning har ikkala qismini juft darajaga ko‘tarish usulida yechim topilganda chet ildizlar paydo bo‘lishi mumkin.

Masalan:

$$\frac{\sin 2x}{\cos 3x}=\frac{\cos 2x}{\sin 3x}$$

► Tenglamaning aniqlanish sohasi, topilgan yechimlar $\frac{k\pi}{3}$ va $\frac{\pi}{6}(2n+1)$, $k,n\in Z$ lardan farqli sonlar to'plami bo'lish kerak.

Tenglamaning ikkala qismini cos3x · sin3x ko'paytirsak cos5x = 0, tenglama hosil bo'lib, bundan $x_m=\frac{\pi}{10}(2m+1), m\in Z$ qiymatni topamiz. Hosil bo'lgan tenglamaning aniqlanish sohasi (–∞; +∞) bo'lib, shakl almashtirishda berilgan tenglamaning aniqlanish sohasi kengaydi. Shuning uchun chet ildizlar paydo bo'lgan bo'lishi mumkin. Paydo bo'lgan chet ildizlarni topilgan qiymatlarni berilgan tenglamaning davri bo'yicha tekshiramiz. Berilgan tenglamaning davri 2π ga teng. Topilgan qiymatlardan (– π; π) oraliqqa tegishli $\left(-\pi/2\right)$ va $\left(\frac{\pi}{2}\right)$ qiymatlarni qo'ysak $\frac{\sin\pi}{\cos\frac{3\pi}{2}}=\frac{\cos\pi}{\sin\frac{3\pi}{2}}$ yoki $\frac{0}{0}=1$ bo'lib, bu qiymatlar yechim emas. Tekshirish natijasida ko'rinadiki $x=\pm\frac{\pi}{2}+2k\pi,\ k\in Z$ qiymatlar berilgan tenglamaning chet ildizlari ◄

Yana bitta misol:

$$\sin 3x-\cos 3x=1$$

► Tenglamaning ikkala qismini kvadratga ko'tarib soddalashtirsak sin6x = 1 tenglama hosil bo'ladi.

Bundan $x=\frac{n\pi}{6}$, $n\in Z$

Shakl almashtirish natijasida hosil bo'lgan tenglama bilan berilgan tenglama aniqlanish sohasi (–∞; +∞) bo'lib soha o'zgarmadi. Tenglamaning ikkala qismini juft darajaga ko'targanda (ratsional va irratsional tenglamalarda ham) teng kuchli tenglama hosil bo'lavermaydi. Shuning uchun topilgan qiymatlarni tekshirib ko'rish kerak. Berilgan tenglamaning davri $\frac{2\pi}{3}$ ga teng. Topilgan qiymatlardan $\left[-\pi/3;\pi/3\right]$ oraliqqa tegishli bo'lganlarini tekshirsak n = – 1 va n = 0 dagi $\left(-\pi/6\right)$

va 0 qiymatlar $\sin\left(-\frac{\pi}{2}\right)-\cos\left(-\frac{\pi}{2}\right)=-1-0=1$ va sin(0) – cos(0) = 0 – 1 = – 1 tenglamani qanoatlantirmaydi.

Demak topilgan qiymatlardan $\left(-\frac{\pi}{6}+\frac{2n\pi}{3}\right)$ va $\frac{2n\pi}{6}$ qiymatlar chet ildizlar bo'lib, tenglamaning ildizlari $x_n=\frac{\pi}{6}+\frac{2n\pi}{3}$ va $x_k=\frac{\pi}{3}+\frac{2n\pi}{3}$, $n,k\in Z$.◄

Trigonometrik tenglama yechimlarini topishni tenglamalarni punktlarga ajratib, yechimini topish metodlarini berishga harakat qilamiz.

17.1. Eng sodda (kanonik) trigonometrik tenglamalarning yechimlarini ifodalovchi formulalarni eslatib o'tamiz:

1. $\sin x = a,\ a\in R,\ 0 < a < 1$ ga yechim

$$x_n = (-1)^n \arcsin a + n\pi, n\in Z \qquad (1)$$

$-1 < a < 0$ da esa $x_n = (-1)^{n+1}\arcsin a + n\pi$,

$\sin^2 x = a,\ 0\le a\le 1$ da $x_n = \pm\arcsin\sqrt{a}+n\pi, n\in Z$

xususiy holda $\sin x = 0$ da $x_n = n\pi$ $\sin x = 1$ ga $x_n=\frac{\pi}{2}+2n\pi, n\in Z$

$\sin x = -1,\ x_n=-\frac{\pi}{2}+2n\pi.\ n\in Z$

$|a| > 1$ ga yechim mavjud emas, $x\in\varnothing$.

2. $\cos x = a,\ a\in R$

$$0 < a < 1 \text{ da } x_n = \pm\arccos a + 2n\pi,\ n\in Z \qquad (2)$$

$-1 < a < 0$ da $x_n = \pm(\pi - \arccos a) + 2n\pi$

$\cos^2 x = a,\ 0\le a\le 1$ da $x_n = \pm\arccos\sqrt{a}+n\pi\ \ n\in Z$

Xususiy holda

$\cos x = 0$ da $x_n = \frac{\pi}{2}+n\pi$

$\cos x = 1$ da $x_n = 2n\pi,\ n\in Z$

$\cos x = -1$ da $x_n = \pi + 2n\pi,\ n\in Z$

$|a| > 1$ da yechim mavjud emas.

3. $\mathrm{tg}\,x = b,\ b\in R$

$$b > 0 \text{ da } x_n = \mathrm{arc\,tg}\,b + n\pi,\ n\in Z \quad (3)$$

$b < 0$ da $x_n = \pi - \mathrm{arc\,ctg}\,b + n\pi$

$\mathrm{tg}^2 x = b\ (0\le b < +\infty)$ da $x_n = \pm arctg\sqrt{b}+n\pi\ \ n\in Z$

Xususiy hollarda:

$\mathrm{tg}\,x = 0$ da $x_n=\frac{\pi}{2}+n\pi$

$\mathrm{tg}\,x = 1$ da $x_n=\frac{\pi}{4}+n\pi.\ n\in Z$

$\mathrm{tg}x = -1$ da $x_n = -\dfrac{\pi}{4} + n\pi$

4. $\mathrm{ctg}x = b,\ b \in R$

$b > 0$ da $x_n = \mathrm{arcctg}b + n\pi$ (4)

$b < 0$ da $x_n = \pi - \mathrm{arcctg}b + n\pi$

$\mathrm{ctg}^2x = b\ (0 \le b < \infty)$ da $xn = \pm arcctg\sqrt{b} + n\pi, n \in Z$

Xususiy hollarda: $\mathrm{ctg}\,x = 0$ da $x_n = \dfrac{\pi}{2} + n\pi$

$\mathrm{ctg}\,x = 1$ da $x_n = \dfrac{\pi}{4} + n\pi,\ n \in Z$

$\mathrm{ctg}\,x = -1$ da $x_n = -\dfrac{\pi}{4} + n\pi$

⊕ Tenglamalar yechimi topilsin:

1. $\sin x = \dfrac{\sqrt{3}}{2}$

► $0 < \dfrac{\sqrt{3}}{2} < 1$ bo‘lganligi uchun, bu tenglama yechimi

$$x_n = (-1)^n \arcsin\frac{\sqrt{3}}{2} + n\pi = (-1)^n \frac{\pi}{3} + n\pi, n\in Z$$ ◄

2. $3\sin x = -\sqrt{15}$

► Tenglama yechimga ega emas. Chunki $\dfrac{-\sqrt{15}}{3} < -1$. ◄

3. $\cos x = -\dfrac{\sqrt{5}}{3}$

►Ko‘rsatilgan formulaga asosan tenglama yechimi

$$x_n = \pm\left(\pi - \arccos\frac{\sqrt{5}}{3}\right) + 2n\pi\ \ n\in Z$$ ◄

4. $tgx = -\dfrac{\sqrt{3}}{3}$

►Bu kanonik tenglama yechimi

$$x_n = -arctg\frac{\sqrt{3}}{3} + n\pi = -\frac{\pi}{6} + n\pi\ \ n\in Z$$ ◄

5. ctg$x$ = 0,75

► Tenglama yechimi $x_n = arcctg\frac{3}{4} + n\pi . n\in Z$

6. ctg$x$ = – 12

► $a = -12 < 0$ uchun $x_n = \pi - \text{arcctg}12 + n\pi,\ n \in Z$ ◄

17.2. Argumenti chiziqli ($ax + b$) ko'rinishda bo'lgan sodda trigonometrik tenglamalarda $ax + b = y$ almashtirish yordamida kanonik ko'rinishga keltirish mumkin.

⊕ Tenglamalar yechimi topilsin.

1. $2\cos\left(x - \frac{\pi}{4}\right) - \sqrt{3} = 0$,

► Tenglamani $\cos\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$ ko'rinishda yozib, $x - \frac{\pi}{4} = y$ almashtirish bajarsak, kanonik tenglama yechimidan foydalanish mumkin

$y_n = \pm\arccos\frac{\sqrt{3}}{2} + 2n\pi$ yoki $x_n = \frac{\pi}{4} \pm \frac{\pi}{6} + 2n\pi, n\in Z$. ◄

2. $\sin\frac{3x}{2} = -1$

► Tenglama yechimini (1) formulaga asosan $\left(\frac{3x}{2} = y \quad \text{deb}\right)$ yozish mumkin. $\frac{3x}{2} = -\frac{\pi}{2} + 2n\pi$ yoki $x_n = -\frac{\pi}{3} + \frac{4n\pi}{3}, n\in Z$ ◄

3. $ctg\left(-\frac{2x}{5}\right) = 4$

► (4) Formulaga asosan $-\frac{2x}{5} = arcctg4 + n\pi$ bo'lib, bundan

$x_n = -\frac{5arcctg4}{2} - \frac{5n\pi}{2}, n\in Z$ ◄

4. $\text{tg}^2(0{,}5x) = -4$

► Tenglama yechimi mavjud emas, chunki tenglikning chap tomoni musbat ifoda. ◄

5. $2\cos^2\left(0{,}5x + \frac{\pi}{8}\right) = 1{,}5$

Tenglamani $\cos^2\left(0{,}5x + \frac{\pi}{8}\right) = \frac{3}{4}$ ko'rinishda yozsak, u holda

$\frac{1}{2}x+\frac{\pi}{8}=\pm arc\cos\sqrt{\frac{3}{4}}+n\pi$ yoki $\frac{1}{2}x+\frac{1}{8}=\pm\frac{\pi}{6}+n\pi$ bundan

$x_n=\pm\frac{\pi}{3}-\frac{\pi}{4}+2n\pi, n\in Z$ ◄

6. $ctg^2\left(2x-\frac{\pi}{3}\right)=12$

►Ko'rsatilgan formuladan $2x-\frac{\pi}{3}=\pm arc\ ctg\sqrt{12}+n\pi$, yoki

$x_n=\pm\frac{arc\ ctg2\sqrt{3}}{2}+\frac{\pi}{6}+\frac{n\pi}{2}, n\in Z$ ◄

7. Ushbu $\cos 2x=\frac{3a-4}{5-a}$ tenglama a ning nechta butun qiymatida yechimga ega bo'ladi.

► $-1\le\cos\alpha x\le 1$ shartidan: $a=0$ da $\left(-\frac{4}{5}\right)$ yechim bor, $a=-1$ da $\left(-\frac{7}{6}\right)$ yechim mavjud emas, $a=1$ da $\left(-\frac{1}{4}\right)$ yechim bor, $a=-2$ da $\left(-\frac{10}{7}\right)$ yechim mavjud emas, $a=2$ da $\left(\frac{2}{3}\right)$ yechim bor. $a=-3$ da $\left(-\frac{13}{8}\right)$ yechim yo'q, $a=3$ da $\left(\frac{5}{2}\right)$ yechim mavjud emas, tekshirib ko'rish qiyin emas. a ning boshqa butun qiymatlarida yechim mavjud emas. Demak a ning uchta butun qiymatida yechim mavjud. ◄

17.3. Ikkita bir hil ismli har xil argumentli trigonometrik funksiyalar tengligi bilan berilgan tenglamalarda:

$\sin\alpha=\sin\beta$, yechimlari $\alpha-\beta=2n\pi$ va $\alpha+\beta=(2n+1)\pi$, $n\in Z$ (5)

$\cos\alpha=\cos\beta$, da $\alpha-\beta=2n\pi$, va $\alpha+\beta=2n\pi$, $n\in Z$ (6)

$tg\alpha=tg\beta\left(\alpha,\beta\neq\frac{\pi}{2}+n\pi\right)$ da $\alpha-\beta=n\pi$, $n\in Z$ (7)

$ctg\alpha=ctg\beta(\alpha\beta\ \beta\neq n\pi)$ esa $\alpha-\beta=n\pi$, $n\in Z$ (8)

Shu kabi $\sin\alpha=\cos\beta$ da $\alpha-\beta=\frac{\pi}{2}+2n\pi$ va

$\alpha+\beta=-\frac{\pi}{2}+2n\pi$ $n\in Z$ (9)

⊕ Tenglamalar yechimi topilsin

1. $\sin 12x = \sin 8x$

► (5) formulaga asosan $12x - 8x = 2n\pi$

$12x + 8x = \pi + 2n\pi$ dan $x_n = \dfrac{n\pi}{2}$ va $x_k = \dfrac{\pi}{20}(2k+1), n, k \in Z$. ◄

2) $ctg\left(2x + \dfrac{\pi}{3}\right) - ctg\left(x - \dfrac{\pi}{6}\right) = 0$

► (8) formulaga asosan $ctg\left(2x + \dfrac{\pi}{3}\right) = ctg\left(x - \dfrac{\pi}{6}\right)$ yechimi

$2x + \dfrac{\pi}{3} - \left(x - \dfrac{\pi}{6}\right) = n\pi$ dan $x_n = \dfrac{\pi}{4}(2n-1), n \in Z$ ◄

3) $\sin(3x - 35°) = \cos(0{,}5 + 75°)$

► Tenglama yechimi (9) formulaga asosan

$$3x - 35° - (0{,}5x + 75°) = \frac{\pi}{2} + 2n\pi$$

$3x - 35° + 0{,}5x + 75° = -\dfrac{\pi}{2} + 2n\pi$ berilgan tenglamaning yechimlari

$x_n = 80° + 144°n$, va $x_k = \dfrac{4k\pi}{7} - \dfrac{13\pi}{63}$ ga teng. n, $k \in Z$ ◄

17.4. Bir xil funksiyaga keltirilib yechiladigan tenglamalar. Bunda asosiy ayniyatlardan foydalanamiz.

⊕ Tenglamalar yechimi topilsin.

1. $tg^2 x - \left(1 + \sqrt{3}\right) tgx = -3$,

Tenglamaning aniqlanish sohasi $x \neq \dfrac{\pi}{2} + n\pi$, $n \in Z$. $tgx = y$ desak

$y^2 - \left(1 + \sqrt{3}\right) y + \sqrt{3} = 0$

Tenglama yechimlari $y_1 = \sqrt{3}$ va $y_2 = 1$ bo'lib, almashtirishdan

1) $tgx = \sqrt{3}$ dan $x_n = \dfrac{\pi}{3} + n\pi$ 2) $tgx = 1$ dan $x_k = \dfrac{\pi}{4} + k\pi$

Javob: $x_n = \dfrac{\pi}{3} + n\pi$ va $x_k = \dfrac{\pi}{4} + k\pi, n, k \in Z$. ◄

2. $8\cos^2 x + 6\sin x - 3 = 0$,

▶ Asosiy ayniyat $\cos^2 x + \sin^2 x = 1$ ga asosan tenglamani $8(1 - \sin^2 x) + 6\sin x - 3 = 0$ yoki $8\sin^2 x - 6\sin x - 5 = 0$, bo‘lib bu kvadrat tenglama yechimlari $\sin x = \frac{5}{4}$ va $\sin x = -\frac{1}{2}$ dan

1) $\sin x = \frac{5}{4}$ da $x \in \varnothing$,

2) $\sin x = -\frac{1}{2}$ da $x = (-1)^{n+1} \arcsin\frac{1}{2} + n\pi$

Javob: $x_n = (-1)^{n+1}\frac{\pi}{6} + n\pi, n \in Z$. ◀

3. $3\text{ctg}^4 x - 1\text{cosec}^2 x = -5$

$x \in (0; \pi)$ ga tegishli yechimlar yig‘indisi topilsin.

▶ $\text{cosec}^2 x = 1 + \text{ctg}^2 x$ ayniyatga asosan, tenglama $3\text{ctg}^4 x - 4\text{ctg}^2 + x + 1 = 0$, ctg$x$ ga nisbatan bikvadrat yenglama bo‘lib, bundan

$ctg^2 x = \frac{4 \pm 2}{6}$, u holda

1) $\text{ctg}^2 x = 1$ dan (4) formulaga asosan $(0; \pi)$ oraliqdagi yechimlar to‘plami $x_1 = \frac{\pi}{4}$ va $x_2 = \frac{3\pi}{4}$.

2) $ctg^2 x = \frac{1}{3}$ dan $ctgx = \pm\frac{1}{\sqrt{3}}$ bu tenglamaning $(0; \pi)$ yechimlari $x_3 = \frac{\pi}{3}$ va $x_4 = \frac{2\pi}{3}$.

Javob: yechimlar yig‘indisi $x_1 + x_2 + x_3 + x_4 = 2\pi$. ◀

4. $2\text{tg}x + \text{ctg}x = 3$

▶ Tenglamaning aniqlash sohasi $x \neq n\pi$ va $x \neq \frac{\pi}{2} + n\pi, n \in z$

$\text{tg}x\,\text{ctg}\alpha = 1$ ayniyatdan foydalanamiz $2tgx + \frac{1}{tgx} = 3$, yoki $2\text{tg}^2 x - 3\text{tg}x + 1 = 0$, tenglama yechimlari $(\text{tg}x)_1 = 1$, $(tgx)_2 = \frac{1}{2}$ bundan berilgan tenglama yechimlari

$x_n \frac{\pi}{4} + n\pi$, $x_k = arctg\frac{1}{2} + k\pi$ topamiz $k, n \in Z$ ◀

⊕ Quyidagi tenglamalarning yechimlari to‘plami va yechim bosh qiymati topilsin.

1. $\sin 4x = \frac{1}{2}$

► Tenglamaning yechimlar to‘plami

$4x = (-1)^n \frac{\pi}{6} + n\pi$ yoki $x_n = (-1)^n \frac{\pi}{24} + \frac{n\pi}{4}$, $n \in Z$

Bosh qiymati $\overset{\frown}{AB} = \frac{\pi}{24}$ yoyiga teng ◄

2. $\cos(0{,}5x) = -0{,}5$.

► (2) formulaga asosan tenglamaning yechimlar to‘plami $0{,}5x = \pm\left(\pi - \pi/3\right) + 2n\pi$ yoki $x_n = \pm\frac{4\pi}{3} + 4n\pi$, $n \in Z$ tenglama yechimining bosh qiymati $\overset{\frown}{AB} = \frac{4\pi}{3}$ yoyiga teng.

3. $tg\frac{3x}{2} = \sqrt{3}$

► Tenglamaning aniqlash sohasi $x \neq \frac{\pi}{3} + \frac{2n\pi}{3} n \in Z$.

(3) formula asosida tenglamaning yechimlar to‘plami

$x_n = \frac{2\pi}{9} + \frac{2n\pi}{3}$, $n \in Z$ bo‘lib, tenglamaning bosh yechimi (qiymati)

$\overset{\frown}{AB} = \frac{2\pi}{9}$, $\left]-\pi/3; \frac{\pi}{3}\right[$ oraliqda. ◄

17.5. Ko‘paytmaga keltirib yechiladigan tenglamalar. Bunda yuqorida ko‘rsatilgan metodlar, ifodadagi umumiy ko‘paytmani qavsdan tashqariga chiqarish, gruppalash, trigonometriyadagi asosiy munosabatlar, karrali argumentlar uchun, yarim argumentlar uchun shu kabi yig‘indini ko‘paytmaga keltirish formulalaridan foydalanish mumkin.

⊕ Tenglamalarning umumiy yechimlar to‘plami topilsin

1. $\cos^2 x + \sin x \cdot \cos x - 1 = 0$

► $\cos^2 x + \sin^2 x = 1$ ayniyatdan foydalanib ko‘paytma ko‘rinishiga keltirish mumkin, $\sin x\cos x - (1 - \cos^2 x) = 0$ bundan $\sin x(\cos x - \sin x) = 0$,

Bu tenglama $\sin x = 0$ va $\cos x - \sin x = 0$ tenglamalarga teng kuchli bo‘lib:

1) $\sin x = 0$ da $x_n = n\pi$. Trigonometriyadagi $\sin x - \cos x = \sqrt{2}\sin\left(x - \frac{\pi}{n}\right)$

munosabatdan $0 = cosx - sinx = -\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$ bundan

$\sin\left(x - \frac{\pi}{4}\right) = 0$, yechimlarto'plami $x_k - \frac{\pi}{4} = k\pi$ yoki $x_k = \frac{\pi}{4} + k\pi$.

Javob: $x_n = n\pi$, $x_k = \frac{\pi}{4} + k\pi$ $k, n \in Z$. ◄

2. $\cos 5x - \cos 3x = 0$,

► Ayirish $cos\alpha - cos\beta = -2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$

Formulaga asosan $-2\sin\frac{5x+3x}{2}\cdot\sin\frac{5x-3x}{2} = 0$,

Bundan

1) $\sin 4x = 0$, dan $4x = n\pi$ yechimlar to'plami $x_n = \frac{n\pi}{4}$,

2) $\sin x = 0$, dan $x_k = k\pi$

Javob: $x_n = \frac{n\pi}{4}$ va $x_k = k\pi$, $k,n \in Z$. ◄

3. $\sin 3x - 2\sin 2x + \sin x = 0$,

► Qo'shish formulasi $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$ ga asosan $0 = (\sin 3x + \sin x) - 2\sin 2x = (2\cdot\sin 2x\cdot\cos x - 2\sin 2x) = 2\sin 2x(\cos x - 1)$ ko'paytmaga kelindi, bundan: 1) $\sin 2x = 0$ bo'lib $x_n = \frac{n\pi}{2}$,

2) $\cos x = 1$ dan $x_k = 2k\pi$

Javob: $x_n = \frac{n\pi}{2}$ va $x_k = 2k\pi$, $n,k, \in Z$. ◄

4. $1{,}5\cdot\sin 2x - 4\cos x\cdot\sin^3 x = 0$.

►Karrali argument formulalari

$\sin 2\alpha = 2\sin\alpha\cos\alpha$ va $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$ dan foydalanamiz

$0 = 1{,}5\cdot 2\sin x\cdot\cos x - 4\cos x\cdot\sin^3 x = \cos x(3\sin x - 4\sin^3 x)$ bundan

1) $\cos x = 0$ yechimlari $x_n = \frac{\pi}{2} + n\pi$ $0 = 3\sin x - 4\sin^3 x = \sin 3x$ dan

$3x_k = k\pi$ yoki $x_k = \frac{k\pi}{3}$

Javob: $x_n = \frac{\pi}{2} + n\pi$ va $x_k = \frac{k\pi}{3}$ $k,n \in Z$ ◀

5. $tg5x + tg3x = 0$

▶ Trigonometrik funksiyalardagi $tg\alpha + tg\beta = \frac{\sin(\alpha+\beta)}{cos\alpha \cdot \cos\beta}$ qo'shish formulasidan foydalanamiz. Tenglamaning aniqlanish sohasi.

$$x \neq \frac{\pi}{10} + \frac{n\pi}{5} \text{ va } x \neq \frac{\pi}{6} + \frac{n\pi}{3} n \in Z,$$

Tenglama qo'shish formulaga asosan $\frac{\sin 8x}{\cos 5x \cdot \cos 3x} = 0$ ko'rinishda bo'lib, aniqlanish sohada $\cos 5x \neq 0$ va $\cos 3x \neq 0$ bundan $\sin 8x = 0$ bo'lib, tenglamaning yechimlar to'plami $x_n = \frac{n\pi}{8}, n \in Z$

6. $2\sqrt{2} \cdot \sin x + 2\sin 2x = \sqrt{2} + 2\cos x$

▶Ikkilangan burchak formulasidan foydalanamiz $2\sqrt{2} \cdot \sin x + 4\sin \cdot \cos x = \sqrt{2} + 2\cos x$ Tenglikning chap tomondagi ifodalarni o'ng tomonga o'tkazib, gruppalasak ko'paytma ko'rinishga keladi.

$$2\cos x(1 - 2\sin x) + \sqrt{2}(1 - 2\sin x) = 0 \text{ dan}$$

$$(2cosx + \sqrt{2}) \cdot (1 - 2\sin x) = 0 \text{ bo'lib}$$

1) $2\cos x + \sqrt{2} = 0$ dan

$$x = \pm\left(\pi - arc\cos\frac{\sqrt{2}}{2}\right) + 2n\pi = \pm\frac{3\pi}{4} + 2n\pi$$

2) $1 - 2\sin x = 0$, yechimlar to'plami

$$x_k = (-1)^k \frac{\pi}{6} + k\pi$$

Javob: $x_n = \pm\frac{3\pi}{4} + 2n\pi$ va $x_k = (-1)^k \frac{\pi}{6} + k\pi, k, n \in Z$. ◀

7. $6\cos^2 x + 5\sin^2 x \cdot \cos x = 10\cos x - 6$

▶Ifodalarni tenglikning bir tomonida yozib, gruppalab olsak, ikkilangan formuladan foydalanish mumkin bo'ladi

$6(\cos^2 x + 1) + 5\cos x(\sin^2 x - 2) = 0$ yoki $6(\cos^2 x + 1) + 5\cos x(1 - \cos^2 x - 2) = 0$, bundan $(\cos^2 x + 1)(6 - 5\cos x) = 0$ bo'lib

1) $\cos^2 x + 1 = 0$ yechim mavjud emas,

2) $6 - 5\cos x = 0$ da ham yechim mavjud emas, chunki $\cos x = \frac{6}{5}$

Demak, berilgan tenglamaning yechimi mavjud emas, ya'ni $x \in \varnothing$ ◄

8. $tg^2 x + \cos^2 x = 1 + \sin^2 x$

► Tenglamaning aniqlanish sohasi $x \neq \frac{\pi}{2} + n\pi, n \in Z.$

Asosiy ayniyatlar $tg\alpha = \frac{sin\alpha}{cos\alpha}$ va $\sin^2 \alpha + \cos^2 \alpha = 1$ asosan tenglama ko'paytma ko'rinishga kelinadi $\sin^2 x\left(\frac{1}{\cos^2 x} - 2\right) = 0$ bundan

1) $\sin^2 x = 0$, yechimlar to'plami $x_n = n\pi, n \in Z$

2) $\frac{1}{\cos^2 x} - 2 = 0$ yoki $\cos^2 x = \frac{1}{2}$ dan $x = \pm\frac{\pi}{4} + 2k\pi$

Javob: $x_n = n\pi$ va $x_k = \pm\frac{\pi}{4} + 2k\pi$, $k,n \in Z$. ◄

9) $\cos\frac{x}{2} - \cos x = 1$

► Yarim burchak $\cos^2\frac{\alpha}{2} = \frac{1 + \cos\alpha}{2}$ formulaga asosan

$\cos\frac{x}{2} = 2\cos^2\frac{x}{2}$ yoki $\cos\frac{x}{2}\left(1 - 2\cos\frac{x}{2}\right) = 0$ ko'paytma ko'rinishga kelinib yechimlar to'plami:

1) $\cos\frac{x}{2} = 0, \quad x_n = \pi + 2n\pi,$

2) $\cos\frac{x}{2} = \frac{1}{2}$ da $x_k = \pm\frac{2\pi}{3} + 4k\pi, n, k \in Z$ ◄

10) $1 - \cos 3x = 0{,}5\sin 1{,}5x$

► Yarim burchak formulasi $\sin^2\frac{\alpha}{2} = \frac{1 - cos\alpha}{2}$ ga asosan tenglamani shakl almashtirib yozamiz $2\sin^2\frac{3x}{2} = \frac{1}{2}\sin\frac{3x}{2}$ dan $\sin\frac{3x}{2}\cdot\left(4\sin\frac{3x}{2} - 1\right) = 0$ bo'lib berilgan tenglama yechimlari

1) $\sin\frac{3x}{2}=0$ dan $x_n=\frac{2n\pi}{3}$, 2) $\sin\frac{3x}{2}=\frac{1}{4}$ da

$$x_k=2(-1)^k\frac{\arcsin\frac{1}{4}}{3}+\frac{2k\pi}{3},n,k\in Z$$ ◄

⊕ $\sin 4x=\cos(180°-2x)$

tenglamaning [– 30°;60°] oraliqdagi yechimlari topilsin.

► Keltirish formulasiga asosan, tenglamani $\sin 4x=-\cos 2x$ deb yozish mumkin, bundan $\cos 2x(2\sin 2x+1)=0$ u holda berilgan tenglama $\cos 2x=0$ va $\sin 2x=-0{,}5$ tenglamalarga teng kuchli bo‘lib, umumiy yechimlari

$x_n=\frac{\pi}{4}+\frac{n\pi}{2}$ va $x_k=(-1)^{k+1}\frac{\pi}{12}+\frac{k\pi}{2},k,n\in Z$ bo‘lib, berilgan oraliqdagi yechimlarini aniqlashda n va k ga butun qiymatlarni beramiz:

$n=0$ da $x=-45°$, $k=-1$ da $x=-75°$, $k=1$ da $x=105°$, $k=-2$ da $x=-195°$

Demak, tenglamaning berilgan oraliqdagi yechimlari $x_1=15°$ va $x_2=-45°$, $x_3=45°$ ◄

⊕ $\sin(0{,}5\pi\cos 6x)=1$

Tenglamaning, [0°;270°] oraliqdagi yechimlar yig‘indisi topilsin.

►Kanonik ko‘rinishdagi tenglama yechimi

$0{,}5\pi\cos 6x=\frac{\pi}{2}$, yoki $\cos 6x=1$, bu tenglama umumiy yechimi

$$x_k=\frac{k\pi}{3},k\in Z$$

Berilgan oraliqdagi yechimlar 60°,120°,180° va 240° bo‘lib, yechimlar yig‘indisi 600°. ◄

17.6. Sinus va Kosinusga nisbatan bir jinsli tenglamalarni yechish n darajali to‘liq bir jinsli tenglamada

$$a_0\sin^n x+a_1\sin^{n-1}x\cos x+\ldots+a_n\cos^n x=0$$

$$(a_0\neq 0 \text{ yoki } a_n\neq 0)$$

har bir hadini $\sin^n x$(yoki $\cos^n x$) ga bo‘lish mumkin, u holda tg$x$ (yoki ctg$x$) ga nisbatan algebraik tenglamaga kelinadi.

$a_0=0$ (yoki $a_n=0$) qolgan koeffisientlarning kamida bittasi noldan farqli bo‘lsa sin$x$ ning (yoki cos$x$) yuqori darajasiga bo‘lish mumkin emas, aks holda tenglama ildizlarning ba’zi ildizlari yo‘qolishi mumkin, shuning uchun avval sin$x$ ni (yoki cos$x$ni) qavsdan tashqariga chiqarib, keyin bo‘lishi amalini bajarish bilan yechish maqsadga muvofiqdir.

Tenglamalar yechimi topilsin:

1) $\sin^2 x+3\sin x\cos x-4\cos^2=0$

► Bu bir jinsli tenglamani $\cos^2 x \neq 0$ ga bo'lib, $\mathrm{tg}x$ ga nisbatan $\mathrm{tg}^2 x$ + $3\mathrm{tg}x - 4 = 0$

Kvadrat tenglama hosil bo'ladi, bu tenglama yechimi $(\mathrm{tg}x)_1 = 1$ va $(\mathrm{tg}x)_2 = -4$. U holda berilgan tenglama yechimlar to'plami:

1) $\mathrm{tg}x = 1$ da $x_n = \frac{\pi}{4} + n\pi$; 2) $\mathrm{tg}x = -4$ ga $x_k = -\mathrm{arctg}4 + k\pi$

$k,n \in Z$, ko'rinishda bo'ladi. ◄

2) $\cos^4 x = 5\sin^4 x - \sin^2 2x$

► Tenglikni ikkilangan burchak formulasidan foydalanib shakl almashtirib yozamiz

$\cos^4 x + 4\sin^2 x \cdot \cos^2 x - 5\sin^4 x = 0$ bu bir jinsli tenglama $\sin^4 x \neq 0$ ga bo'lsak $\mathrm{ctg}^4 x + 4\mathrm{ctg}^2 x - 5 = 0$ bikvadrat tenglama hosil qilamiz $\mathrm{ctg}^2 x = y$, almashtirishda $y^2 + 4y - 5 = 0$ hosil bo'lgan tenglama ildizlari 1 va (-5). U holda berilgan tenglama yechimlar to'plami:

1) $\mathrm{ctg}^2 x = 1$, (4) formulaga asosan

$$x_n = \pm\mathrm{arctg}1 + n\pi = \pm\frac{\pi}{4} + n\pi$$

2) $\mathrm{ctg}^2 x = -5$ da $x \in \varnothing$ javob: $x_n = \frac{\pi}{4} + n\pi$

3) $\sin^3 x \sin^2 x \cdot \cos x - 2\cos^2 x \cdot \sin x = 0$

► Bu to'liqsiz bir jinsli tenglama bo'lganligi uchun $\sin x$ ni qavsdan tashqariga chiqarib yozsak, tenglama ikkita

$$\sin x = 0 \text{ va } \sin^2 x - \sin x.\cos x - 2\cos^2 x = 0$$

Tenglamalarga teng kuchli bo'lib:

$\sin x = 0$ da $x_n = n\pi$, $n \in z$ $\sin^2 x - \sin x\cos x - 2\cos^2 x = 0$ ni $\cos^2 x \neq 0$ ga bo'lsak, $\mathrm{tg}^2 x - \mathrm{tg}x - 2 = 0$ bo'lib, bunda $(\mathrm{tg}x) = -1$ va $(\mathrm{tg}x) = 2$ u holda:

1) $x_n = -\frac{\pi}{4} + k\pi$; 2) $x_m = \mathrm{arctg}2 + m\pi$

javob: $x_n = n\pi$, $x_k = -\frac{\pi}{4} + k\pi$, $x_m = \mathrm{arctg}2 + m\pi$, $n,k,m \in Z$ ◄

4) $\sqrt{3}\sin 2x - 2\cos^2 x + 2 = \sin^2 x$

► $\sin 2x = 2\sin x\cos x$ va $\sin^2 x + \cos^2 x = 1$ formulalardan foydalanib, ko'paytma ko'rinishga keltiramiz

$2\sqrt{3}\sin x.\cos x + 2\left(1 - \cos^2 x\right) - \sin^2 x = 0$, bundan

$\sin x\left(2\sqrt{3}\cos x + \sin x\right) = 0$ u holda:

1) $\sin x = 0$ dan $x_n = n\pi$,

2) $2\sqrt{3}\cos x + \sin x = 0$ bir jinsli tenglama, tenglikning ikki

...tomonining $\cos x \neq 0$ ga bo'lamiz $ctgx = -2\sqrt{3}$ tenglama yechim to'plami

$x_k = \pi - arcctg\left(2\sqrt{3}\right) + k\pi$

Javob: $x_n = n\pi$ va $x_k = \pi - arctg\left(2\sqrt{3}\right) + k\pi, \quad k, n \in Z$ ◄

17.7. Ko'rib o'tilgan usullarni birgalikda ishlanib, yana qo'shish va ayirish formulalari, keltirish formulalari, ko'paytmani yig'indiga keltirish, darajani pasaytirish kabi formulalardan foydalanib, tenglama yechimini topish.

⊕ Tenglamalar yechimi topilsin.

1) $\quad \mathrm{ctg}2x - 0,(6)\mathrm{tg}4x = \mathrm{tg}2x$

► Tenglamaning aniqlanish sohasi

$x \neq 2n\pi$ va $x \neq \pi + 2n\pi$

Tenglamani asosiy ayniyatidan foydalanib, shakl almashtirib yozamiz

$\dfrac{1}{tg2x} - tg2x = \dfrac{2}{3}tg4x$; yoki $\dfrac{1 - tg^2 2x}{tg2x} = \dfrac{2}{3}tg4x$, ikkilangan burchak $tg2\alpha = \dfrac{2tg\alpha}{1 - tg^2\alpha}$ formulaga asosan $\dfrac{2}{tg4x} = \dfrac{2}{3}tg4x$ dan $\mathrm{tg}^2\,4x = 3$ bo'lib, (3) formulaga asosan teng kuchli tenglama yechimlar to'plami $4x_n = \pm arctg\sqrt{3} \pm n\pi$ yoki $x_n = \pm\dfrac{\pi}{12} + \dfrac{n\pi}{4}$, $n \in Z$ ga teng ◄

2) $\quad 3\cos(2\pi - x) + \sin\left(\dfrac{3\pi}{2} - x\right) = \sqrt{3}$,

► Tenglamani keltirish formulasiga asosan yozamiz

$3\cos x - \cos x = \sqrt{3}$ yoki $\cos x = \dfrac{\sqrt{3}}{2}$ bu tenglama yechimi

$$x_n = \pm 30° + 360°n, \; n \in Z ◄$$

3) $\quad 3\sin x + \cos x = 1 + 2\cos\dfrac{3x}{3}\cdot\cos\dfrac{x}{2}$

► Ko'paytmani yig'indiga keltirish formulasidan foydalanamiz

$$\cos\frac{3x}{2}\cdot\cos\frac{x}{2} = \frac{1}{2}\left[\cos\left(\frac{3x}{2} + \frac{x}{2}\right) + \cos\left(\frac{3x}{2} - \frac{x}{2}\right)\right] = \frac{1}{2}\left[\cos 2x + \cos x\right]$$

u holda $3\sin x = 1 + [\cos 2x + \cos x] - \cos x = 1 + \cos 2x$ asosiy ayniyatdan foydalanamiz $\cos 2x = \cos^2 x - \sin^2 x = (1 - \sin 2\, x) - \sin^2 x = 1 - 2\sin^2 x$ natijada tenglama sinusga nisbatan kvadrat tenglama bo'ladi.

$2\sin^2 x+3\sin x-2=0$ bundan $(sinx)_1=\frac{1}{2}$, $(\sin x)_2=-2$, Teng kuchli tenglama yechimlari

1) $sinx=\frac{1}{2}$ dan $x_n=(-1)^n\frac{\pi}{6}+n\pi, n\in Z$

2) $\sin x=-2$ da yechim mavjud emas, $x\in\varnothing$ ◄

Javob: $x_n=(-1)^n\frac{k}{6}+n\pi, n\in Z$

4) $tg\left(x+\frac{\pi}{4}\right)+tg\left(x-\frac{\pi}{4}\right)=2$

Tenglamaning aniqlanish sohasi

$$x\neq\frac{\pi}{4}+n\pi \text{ va } x\neq\frac{3\pi}{4}+n\pi$$

Yig'indini ko'paytmaga keltirish formulasiga asosan

$$\frac{\sin(x+\frac{\pi}{4}+x-\frac{\pi}{4})}{\cos\left(x+\frac{\pi}{4}\right)\cdot\cos\left(x-\frac{\pi}{4}\right)}=2$$

endi, ko'paytmani yig'indiga keltirish formulasidan

$$\cos\left(x+\frac{\pi}{4}\right)\cos\left(x-\frac{\pi}{4}\right)=\frac{1}{2}\left[\cos 2x+\cos\frac{\pi}{2}\right]=\frac{1}{2}\cos 2x$$

bo'lib tenglama $tg2x=1$ga teng kuchli bo'lib, berilgan tenglama yechimi $x_n=\frac{\pi}{8}+\frac{n\pi}{2}, n\in Z$ ga teng. ◄

5) $\sin(x+250°)+\cos600°+\cos(x-200°)=\sin870°$

► Har bir qo'shiluvchini keltirish formulasidan foydalanib soddalashtirib olamiz

$\sin(x+250°)=\sin[270°+(x-20°)]=-\cos(x-20°)$;

$\cos 600°=\cos(720°-120°)=\cos 120°=\cos(90°+30°)=-\sin 30°=$

$=-\frac{1}{2}$;

$\cos(x-200°)=\cos[180°-(x-20°)]=-\cos(x-20°)$;

$\sin 870°=\sin(720°-150°)=\sin 150°=\sin(180°-30°)=\sin 30°=\frac{1}{2}$;

ifodalarni o'rniga qo'ysak $-\cos(x-20°)-\frac{1}{2}-\cos(x-20°)=\frac{1}{2}$ yoki

$\cos(x-20°)=-\frac{1}{2}$ bo'lib $x_n-20°=\pm\left(\pi-\frac{\pi}{3}\right)+2n\pi$ berilgan tenglama yechimlar $x_n=\frac{7\pi}{9}+2n\pi$ va $x_k=\frac{5\pi}{3}+2n\pi.\ k,n\in Z$ ◄

6) $\text{tg}15°-\text{tg}x=\text{tg}15°\cdot\text{tg}x+1$

► ayirmani ko'paytma keltirish formulasi va asosiy ayniyatlardan $\frac{\sin(15°-x)}{\cos15°\cos x}=\frac{\sin15°\cdot\sin x+1}{\cos15°\cdot\cos x}$, deb yozish mumkin, yoki $\frac{\sin(15°-x)}{\cos15°\cdot\cos x}=\frac{\sin15°\sin x+\cos15°\cdot\cos x}{\cos15°\cdot\cos x}$, $\cos x\neq0$ deb, qo'shish formulasidan $\sin15°\cdot\sin x+\cos15°\cos x=\cos(15°-x)$ u holda, tenglama bir jinsli $\sin(15°-x)=\cos(15°-x)$ ko'rinishda bo'lib, bundan $\text{tg}(15°-x)=1$dan

$15°-x=\frac{\pi}{4}+k\pi$,berilgan tenglama yechimi $x_k=-\frac{\pi}{6}+k\pi, k\in Z$ ko'rinishda bo'ladi. ◄

7) $1-\frac{2\cos x}{\sin x+\cos x}=tg\left(\frac{\pi}{4}-x\right)$.

► Qo'shish formulasi $tg(\alpha-\beta)=\frac{tg\alpha-tg\beta}{1+tg\alpha\cdot tg\beta}$ ga asosan va $\cos x\neq0$ deb, tenglamani shakl almashtirib yozamiz $1-\frac{2}{tgx+1}=\frac{1-tgx}{1+tgx}$ endi $\text{tg}x+1\neq0$, yoki $x\neq-\pi/4$ deb, umumiy maxraj berib soddalashtirsak $\text{tg}x=1$ bo'lib, berilgan tenglamaning aniqlanish sohadagi yechimi $x_n=\frac{\pi}{4}+n\pi, n\in Z$.◄

8) $\sin^2 2x+\sin^2 x=1$

► Yarim burchak formulasidan foydalanib, darajani pasaytiramiz $\frac{1-\cos4x}{2}+\frac{1-\cos2x}{2}=1$. yoki $\cos4x+\cos2x=0$ qo'shish formulasidan foydalanamiz $2\cos3x\cdot\cos x=0$ ko'paytma nol bo'lishidan

1) $\cos3x=0$ dan $x_m=\frac{\pi}{6}+\frac{m\pi}{3}m, k\in Z$

2) $\cos x = 0$ da $x_k = \frac{\pi}{2} + k\pi$

Bu yechimlarni bitta formulada yozish mumkin.

$x_n = \frac{\pi}{6}(1+2n), n \in Z$ ◄

9) $\cos^4 + \sin^2 x = 0,75$

► Darajani kamaytirish formulalaridan

$\frac{3+4\cos 2x + \cos 4x}{8} + \frac{1-\cos 2x}{2} = \frac{3}{4}$ dan $\frac{3}{8} + \frac{\cos 4x}{8} + \frac{1}{2} = \frac{3}{4}$ soddalashtirsak $\cos 4x = -1$ bo'lib, berilgan tenglamaning yechimlar to'plami $x_n = \frac{\pi}{4} + \frac{n\pi}{2}, n \in Z.$ ◄

10) $\sin^3 x - 0,75 \sin x = 0,25$

► Darajani kamaytirish formulasidan

$\frac{3 sinx - \sin 3x}{4} - \frac{3}{4} sinx = \frac{1}{4}$ Yoki $\sin 3x = -1$, bu ekvavalent tenglama yechimlar to'plami $x_{n=} \frac{2n\pi}{3} - \frac{\pi}{6}, n \in Z$ ◄

⊕ Agar $\cos\alpha$, $\cos 2\alpha$ va $\cos 3\alpha$ $(0 < \alpha < 90°)$ lar arifmetik progressiya tashkil etsa, α burchakning qiymati topilsin.

► Arifmetik progressiya hadlari xossasidan

$\frac{\cos\alpha + \cos 3\alpha}{2} = \cos 2\alpha$ bundan $\frac{1}{2}[2\cos 2\alpha \cdot \cos\alpha] = \cos 2\alpha$ bu tenglama $\cos 2\alpha = 0$ va $\cos\alpha = 1$ tenglamalarga teng kuchli bo'lib, umumiy yechimlari $x_k = \frac{\pi}{4} + \frac{k\pi}{2}$ va $x_n = 2n\pi$ $n.k \in Z$

U holda shartli qanoatlantiruvchi yechim. $x = 45°$ ◄

⊕ $\cos\left(\frac{8\pi}{x}\right) = 1$ tenglamaning nechta butun yechimlari bor.

► Sodda ko'rinishdagi tenglamaning umumiy yechimi $\frac{8\pi}{x} = 2k\pi$ yoki $(x \neq 0)\, x = \frac{4}{k}, k \in Z$

K ga butun qiymatlar ± 1, ± 2; ± 4 bersak, tenglama mos ravishda ± 4, ± 2; ± 1 yani oltita butun qiymat qabul qiladi.

⊕ $\cos x + \cos 2x + \cos 3x + \cos 4x + \cos 5x = 0$,
tenglamaning (0;90°) kesmaga tegishli ildizlari yig'indisi topilsin.

▶ Trigonometriyadagi ba'zi bir munosabatlar formulasiga asosan [1],

$$\cos x + \cos 2x + \cos 3x + \cos 4x + \cos 5x = \frac{\cos 3x}{\sin\frac{x}{2}} \cdot \sin\frac{5x}{2}$$ u holda

berilgan tenglama $\cos 3x = 0$ va $\sin\frac{5x}{2} = 0$

Tenglamalarga teng kuchli bo'lib, berilgan tenglamaning yechimlar to'plami $x_k = \frac{\pi}{6} + \frac{k\pi}{3}$ va $x_n = \frac{2n\pi}{5}$ $n, k \in Z$ faqat $k = 0$ va $n = 1$ ga berilgan oraliqdagi yechimlarini olamiz, bu yechimlar yig'indisi 30° + 72° = = 102° ◀

17.8. Yordamchi burchak kiritish usulida, trigonometrik tenglama yechimini topish.

$$a\sin x + b\cos x = c \ (a^2 + b^2 \neq 0)$$

Tenglamaning ikki tomonini $r = \sqrt{a^2 + b^2}$ ga bo'lsak $\frac{a}{r}sinx + \frac{b}{r}cosx = \frac{c}{r}$, $\left|\frac{c}{r}\right| \leq 1$ da $\sin(x + y) = \frac{c}{r}$ ko'rinishga keladi,

bunda $y = arc\ tg\frac{b}{a}$, yoki $\sin y = \frac{b}{r}$, $\cos y = \frac{a}{r}$

⊕ Tenglamalar yechimi topilsin

1) $sinx - \sqrt{3}cosx = 1$

▶ Tenglamani $r = \sqrt{1^2 + \left(-\sqrt{3}\right)^2} = 2$ ga bo'lamiz

$\frac{1}{2}sinx - \frac{\sqrt{3}}{2}cosx = \frac{1}{2}$, bunda $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ va $\cos\frac{\pi}{3} = \frac{1}{2}$ ni hisob olsak

$sinx \cdot \cos\frac{\pi}{3} - cosxsin\frac{\pi}{3} = \frac{1}{2}$ ikki burchak ayirmasi formulasiga asosan

$\sin\left(x - \frac{\pi}{3}\right) = \frac{1}{2}$ bo'lib, tenglama yechimi $x_n = (-1)^n\frac{\pi}{6} + \frac{\pi}{3}k\pi, n \in Z$ ◀

2) $4\sin x + 3\cos x = 5$

▶ Yordamchi burchak kiritish usulidan foydalanamiz

$r=\sqrt{4^2+3^2}=5$, $y=arctg\frac{b}{a}=arctg\frac{3}{4}$ u holda tenglamaga teng kuchli bo'lgan $\sin\left(x+arctg\frac{3}{4}\right)=\frac{5}{5}$ tenglama yechimi

$x_n=\frac{\pi}{2}+2n\pi-arctg\frac{3}{4}, n\in Z$ ◄

3) $\sin x+\cos x=1$ Tenglamaning $(-360°);0°$ oraliqdagi yechimi topilsin.

► Tenglamani $r=\sqrt{1^2+1^2}=\sqrt{2}$ songa bo'lamiz.

$$\frac{1}{\sqrt{2}}\sin x+\cos x\cdot\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}$$

Tenglamani shakl almashtirib yozamiz

$$\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$$

Ikki burchak yig'indisi formulasiga asosan $\sin\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$ bo'lib, berilgan tenglamaning yechimi $x_n=(-1)^n\frac{\pi}{4}-\frac{\pi}{4}+n\pi,\ n\in Z$

Berilgan oraliqdagi yechim $n=-1$ da bo'ladi

$x=-\frac{\pi}{4}-\frac{\pi}{4}-\pi=-270°$ javob $-270°$ ◄

17.9. Agar tenglama bir xil argumentli sinus va kosinus funksiyalarga nisbatan ratsional ko'rinishdagi tenglama bo'lsa, u holda $\sin\alpha=\frac{2tg\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$, $\cos\alpha=\frac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$ (*) universal almashtirish usuldan foydalaniladi.

⊕ Tenglamalar yechimi topilsin

1) $3\sin x-\cos x=1$

► Tenglamani yechishda eng sodda usul universal (*) almashtirishdan foydalanamiz $3\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}}-\frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}=1$ tenglamani soddalashtirsak

$tg\frac{x}{2}=\frac{1}{3}$ bu tenglamaning yechimlar to‘plami $x_n=2arctg\frac{1}{3}+2n\pi,\ n\in Z$

Bu almashtirishda ildizlar yo‘qolgan bo‘lishi mumkin, sababi berilgan tenglamaning aniqlanish sohasi $(-\infty;+\infty)$ bo‘lib, almashtirish natijasida tenglama aniqlanish sohasi $x_k\neq\pi+2k\pi$, bu yo‘qolgan yechimlarni $tg\frac{x}{2}=0$ ning yechimlarida qidiramiz, tekshirish qiyin emas $x_k=\pi+2k\pi$ ham yechim bo‘ladi.

Javob: $x_n=2arctg\frac{1}{3}+2n\pi$ va $x_k=\pi+2k\pi,\ n,k,\in Z$◄

2) $5-2tg^2x=2\cos 2x$

► Universal (*) almashtirishdan foydalanamiz

$$5-2tg^2x=2\frac{1-tg^2x}{1+tg^2x}$$

Bu tenglama va berilgan tenglamaning aniqlanish sohasi bir xil $x\neq\frac{\pi}{2}+n\pi$

Hosil bo‘lgan tenglamada $tg^2x=y$ deb almashtirish bajarsak $2y^2-5y-3=0$

Bu tenglama yechimlari $y=3$ va $y=-\frac{1}{2}$. U holda almashtirishdan:

1) $tg^2x=3$ da (3) formulaga asosan $x=\pm\frac{\pi}{3}+n\pi$

2) $tg^2x=-\frac{1}{2}$ bo‘sh to‘plam. Berilgan tenglama yechimi $x_n=\frac{\pi}{3}+n\pi$ va $x_k=-\frac{\pi}{3}+k\pi\ n,k\in Z$ ◄

3) $2\cos^2x+\sin x=2+0{,}5\sin^2x$

► Tenglamaning aniqlanish sohasi $(-\infty;+\infty)$

Universal almashtirish bajaramiz, yozishning soddaligi uchun $tg\frac{x}{2}=y$ deb belgilasak, tenglama

$$2\left(\frac{1-y^2}{1+y^2}\right)^2+\frac{2y}{1+y^2}=2+\frac{2y}{1+y^2}\cdot\frac{1-y^2}{1+y^2}$$

Ko'rinishga keladi, bu tenglamaning aniqlanish sohasi $x_n \neq \pi + 2n\pi$. hosil bo'lgan tenglamani soddalashtirsak

$(1-y^2)^2 + y(1+y^2) = (1+y^2)^2 + y(1-y^2)$ yoki $y^3 - 2y^2 = 0$, dan $y^2 = 0$, $y = 2$ bo'lib, belgilangan

1) $tg^2 \frac{x}{2} = 0$, da $x_m = 2n\pi$, 2) $tg \frac{x}{2} = 2$ da $x_k = \text{arctg}2 + 2k\pi$

Universal almashtirishda aniqlanish soha toraygan natijada ba'zi ildizlar yo'qolgan bo'lishi mumkin. Buni $\cos\frac{x}{2} = 0$ tenglama yechimlari $x_n = \pi + 2n\pi$ da qidiramiz $x = \pi$ ni tenglamaga qo'yib tekshiramiz $2[\cos(\pi)]^2 + \sin\pi = 2 + 0{,}5 \cdot \sin2 \cdot \pi$ dan $2 = 2$ demak $x_n = \pi + 2n\pi$ ham tenglamaningyechimi ekan. Javob:

$x_n = n\pi$, va $x_k = 2\text{arctg}2 + 2k\pi$, $k,n \in Z$ ◄

17.10. Yuqorida ko'rib o'tilgan metodlar va trigonometrik funksiyalarning ba'zi burchaklardagi qiymatlari, trigonometrik shakl almashtirishlar, asosiy munosabatlardan foydalanish, ya'ni umumlashgan usul.

⊕ Tenglamalarning yechimi topilsin.

1) $\sin 5x + \cos 5x = \sqrt{2}\sin 3x$,

► Qo'shish formulasi $sin\alpha + cos\alpha = \sqrt{2}\sin\left(\alpha + \frac{\pi}{4}\right)$ ga asosan

$$\sqrt{2}\sin\left(5x + \frac{\pi}{4}\right) = \sqrt{2}\sin 3x$$

17.5. Punktdagi (5) formulaga asosan $5x + \frac{\pi}{4} - 3x = 2k\pi$

$$5x + \frac{\pi}{4} + 3x = \pi + 2k\pi$$

Berilgan tenglama yechimlari

$$x_n = n\pi - \frac{\pi}{8} \text{ va } x_k = \frac{3\pi}{32} + \frac{k\pi}{4}, k,n \in Z$$ ◄

2) $(sinx + cosx)^2 = \frac{1+tgx}{1-tgx}$

► Tenglamaning aniqlanish sohasi $x \neq \frac{\pi}{4} + k\pi, x \neq \frac{\pi}{2} + k\pi$ asosiy ayniyatlardan foydalanıb, tenglamani $(sinx + \cos x)^2 - \frac{sinx + cosx}{cosx - sinx} = 0$

bu tenglikdan $x \neq \frac{\pi}{2} + k\pi$ shartida soddalashtirsak (sin$x$ + cos$x$) · [(cos^2 x – sin^2 x) – 1] = 0 bo'lib, bunga ekvivant tenglamalar cos$x$ + sin$x$ = 0 va sin^2 x = 0 bo'lib bundan berilgan tenglamaning yechimlar to'plami

$x_k = -\frac{\pi}{4} + k\pi$, va $x_n = n\pi$, $n,k \in Z$ ◄

3) cos3x = – 2cos$x$ tenglamaning (50°;120°) intervalga tegishli yechimi topilsin.

► Uchlangan burchak formulasiga asosan 4cos^3 x – 3cos$x$ + 2cos$x$ = 0, bo'lib, bu tenglama cos$x$ = 0 va 4cos^2 x – 1 = 0 teng kuchli bo'lib, yechimlar to'plami $x_k = \frac{\pi}{2} + k\pi$ va $x = \pm\frac{\pi}{3} + k\pi$ $k, n \in Z$ tenglamaning berilgan intervaldagi yechimlari $x = \frac{\pi}{2}$ va $x = {}^{\pi}/_{3}$ ◄

4) tg(2x + 1) ctg (x + 1) = 1

► Tenglama $x \neq \frac{\pi}{4}(2k+1)$ va $x \neq n\pi - 1$ da mavjud $n,k \in Z$ tgα · ctgα = 1 ayniyatdan tg(2x + 1) = tg(x + 1) bu tenglama (7) formulaga asosan (2x + 1) – (x + 1) = $k\pi$ yoki $x_k = k\pi, k \in Z$ yechimga ega. ◄

5) $\cos(cosx) = \frac{\sqrt{3}}{2}$

►(2) formulaga asosan $cosx = \pm\frac{\pi}{6} + 2k\pi, k = 1$ da |cos$x$|≤1 shart bajariladi, u holda $cosx = \pm\frac{\pi}{6}$ tenglamaning yechimlar to'plami $\cos^2 x = \frac{\pi^2}{36}$ tenglama yechimlar to'plamiga ekvivalent bo'ladi, (2) formulaga asosan $x_k = \pm\arccos\frac{\pi}{6} + k\pi, k \in Z$ yechimlar to'plami. ◄

6) 5(1 – sin$x$ + cos$x$) = 2sin2x

► Asosiy ayniyatlardan foydalanamiz 5(sin^2 x + cos^2 x – sin$x$ + cos$x$) – – 4sin$x$.cos$x$ = 0 shakl almashtirib yozsak 2(sin^2 x + cos^2 x – 2sin$x$ · cos$x$) – – 5(sin$x$ – cos$x$) + 3 = 0

Agar y = sin$x$ – cos$x$ belgilash kiritsak $2y^2 - 5y + 3 = 0$ bo'lib bundan $y = 1$ va $y = \frac{3}{2}$ belgilashdan:

1) $\sin x - \cos x = 1$, asosiy munosabatlardan $\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=1$ b

tenglama yechimi

$$x_k = (-1)^k\frac{\pi}{4}+k\pi+\frac{\pi}{4}$$

2) $sinx - cosx = \frac{3}{2}$ yani asosiy tenglikdan

$\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=\frac{3}{2}$ bo‘lib , yechim mavjud emas.

Javob: $x_n = (-1)^n\frac{\pi}{4}+\frac{\pi}{4}(4n\pi+1), n\in Z$

7) $$\frac{\sin 2x}{1+\cos 2x}\cdot\frac{cosx}{1+cosx}=\frac{1}{\sqrt{3}}$$

► Tenglamaning aniqlanish sohasi

$$x\neq\pi+2n\pi, x\neq\frac{\pi}{2}+2n\pi, x\neq\frac{\pi}{2}+n\pi$$

Yarim burchak $tg\frac{\alpha}{2}=\frac{sin\alpha}{1+cos\alpha}$ formulasiga asosan va asosiy ayniyatdan foydalanib yozish mumkin

$$\frac{cosx}{1+cosx}\cdot tgx=\frac{1}{\sqrt{3}} \text{ yoki } \frac{cosx}{1+cosx}\cdot\frac{sinx}{cosx}=\frac{1}{\sqrt{3}}$$

Yana yarim burchak formulasidan foydalansak $tg\frac{x}{2}=\frac{1}{\sqrt{3}}$ bu ekvivalent tenglama yechimi $x_n=\frac{\pi}{3}+2n\pi, n\in Z$ ◄

8) $$\frac{sinx+\sin 2x+\sin 3x}{cosx+\cos 2x+\cos 3x}=ctg2x$$

tenglamaning $\left(-\pi/2;\pi/2\right)$ intervaldagi yechimi topilsin.

► Tenglamaning aniqlanish sohasi $x\neq\frac{\pi}{4}+\frac{k\pi}{2}, k\in Z$ quyidagi tenglikdan foydalanamiz

$$sin\alpha + \sin 2\alpha + \sin 3\alpha + \ldots + \sin n\alpha = \frac{\sin\frac{n+1}{2}\alpha}{\sin\frac{\alpha}{2}} \cdot \sin\frac{n\alpha}{2}$$

$$cos\alpha + \cos 2\alpha + \cos 4\alpha + \ldots + \cos n\alpha = \frac{\cos\frac{n+1}{2}\alpha}{\sin\frac{\alpha}{2}} \cdot \sin\frac{n\alpha}{2}$$

$$\frac{\frac{sin2x}{\sin\frac{x}{2}} \cdot \sin\frac{3x}{2}}{\frac{\cos 2x}{\sin\frac{x}{2}} \cdot \sin\frac{3x}{2}} = ctg2x$$

$x \neq 2k\pi$ va $x \neq \frac{2k\pi}{3}$ da soddalashtirilsa $\text{ctg}^2 2x = 1$ tenglamaga kelinadi,

bu tenglama yechimi (4) formulaga asosan $x_k = \pm\frac{\pi}{8} + \frac{k\pi}{4}, k \in Z$ Shartni hisobga olsak

Javob $x = \pm\frac{\pi}{8}$ va $x = \pm\frac{3\pi}{8}$ ◀

9) $\sin 6x + \cos 6x = 1 - 2\sin 3x$

▶ Tenglamaning aniqlanish sohasi $(-\infty; +\infty)$ formulalardan foydalanish uchun shakl almashtirib yozib olamiz $\sin 2 \cdot 3x + 2\sin 3x - (1 - \cos 6x) = 0$

Ikkilangan va yarim burchak formulalaridan foydalanamiz $2\sin 3x \cdot \cos 3x + 2\sin 3x - 2\sin^2 3x = 0$ soddalashtirsak berilgan tenglama yechimlari $\sin 3x = 0$ va $\cos 3x - \sin 3x + 1 = 0$ tenglama yechimlariga teng kuchli bo'ladi.

10) $(3\sin\pi x + \pi) \cdot (2\sin\pi x - 1) = 0$, tenglamaning eng kichik musbat ildizi topilsin.

▶ ko'paytma nolligidan:

1) $sin\pi x = -\frac{\pi}{3}$ da yechim mavjud emas, chunki $\left(-\frac{\pi}{3}\right) < -1$ dan,

2) $sin\pi x = \frac{1}{2}$ da $x = (-1)^n \frac{1}{6} + n$ bo'lib, eng kichik musbat yechim $\frac{1}{6}$ ga teng ◀

11) $\frac{2tgx}{1-cosx}=0$ tenglamaning $[-\pi; 3\pi]$ oraliqda nechta ildizi bor.

► Tenglamaning aniqlanish sohasi $x \neq 2k\pi$ va $x \neq \frac{\pi}{2}+k\pi$

Aniqlanish sohada tenglamaga tg = 0 teng kuchli tenglama bo'lib yechimlar to'plami $x_k = k\pi$ lekin $x_k \neq 2n\pi$ shartni hisobga olsak, berilgan oraliqda tenglama ildizlar soni (k ning 1,(– 1)va 3 dagi qiymatida) 3 ta. ◄

12) $\sin^4\frac{x}{3}+\cos^4\frac{x}{3}=0,5$

► Tenglamaning chap va o'ng tominiga $2\sin^2\frac{x}{3}\cdot\cos^2\frac{x}{3}$ ni qo'shsak $\left(\sin^2\frac{x}{3}+\cos^2\frac{x}{3}\right)^2=\frac{1}{2}+2\sin^2\frac{x}{3}\cdot\cos^2\frac{x}{3}$ yoki $1=\frac{1}{2}+2\sin^2\frac{x}{3}\cdot\cos^2\frac{x}{3}$

Shakl almashtirib yozsak $\frac{1}{2}=2\cdot\frac{1}{4}\left(2\sin\frac{x}{3}\cdot\cos\frac{x}{3}\right)^2$

bundan $\sin^2\frac{2x}{3}=1$

(1) Formulaga asosan tenglama yechimi $x_n=\frac{3\pi}{4}(2n\pm1)$ ◄

13) To'g'ri burchakli uchburchakning α va β o'tkir burchaklari uchun $\sin(\alpha-\beta)-\cos\alpha=0$ tenglik o'rinli bo'lsa o'tkir burchaklarning kattasi topilsin.

► Qo'shish va keltirish formulalaridan foydalanamiz $\sin\alpha\cdot\cos\beta-\sin\beta\cos\alpha-\cos(90°-\beta)=0$ ($\alpha+\beta=90°$ edi) Yana keltirish formulalaridan foydalanamiz $\sin(90°-\beta)\cos\beta-\sin\beta\cos(90°-\beta)-\sin\beta=0$ yoki $\cos^2\beta-\sin^2\beta-\sin\beta=0$ asosiy ayniyatdan foydalansak $2\sin^2\beta+\sin\beta-1=0$ bu tenglama ildizlari $(\sin\beta)_1=-1$ va $(sin\beta)_2=\frac{1}{2}$ yechim bosh qiymatlaridan $\beta_1=-\frac{\pi}{2}$ (chet ildiz) $\beta_2=30°$ U holda $\alpha=90°-\beta=90°-30°=60°$ Katta o'tkir burchak 60° ekan. ◄

14) $\sin3x+\sin5x=\sin4x$ tenglamaning nechta ildizi $|x|\leq\frac{\pi}{2}$ tengsizlikni qanoatlantiradi.

► yig'indini ko'paytmaga keltirish formulasidan foydalanamiz $2\sin x\cos x = \sin 4x$ bu tenglama $\sin 4x = 0$ va $\cos x = \frac{1}{2}$ tenglamalarga teng kuchli bo'lib, tenglamalarning umumiy yechimlari $x_k = \frac{k\pi}{4}$ va

$$x_n = \pm\frac{\pi}{3} + 2n\pi,\ n \in Z$$

Bu yechimlarda $|x| \le \frac{\pi}{2}$ shartni qanoatlantiradigan yechimlarini aniqlashda k ga qiymatlar berib aniqlaymiz:

$k = 0$ da $x = 0$ va $x = \pm\frac{\pi}{3}$, $k = -1$ da $x = \frac{-5\pi}{3}$ va $x = \frac{-7\pi}{3}$,

$k = 1$da $x = \frac{\pi}{4}$, $x = \frac{7\pi}{3}$ va $x = \frac{5\pi}{3}$,

$k = -2$ da $-\frac{\pi}{2}$, $\frac{-n\pi}{3}$ va $\frac{-13\pi}{3}$

$k = 2$ da $\frac{\pi}{2}$, $\frac{13\pi}{3}$ va $\frac{n\pi}{3}$ bo'lib $|x| \le \frac{\pi}{2}$ shartni qanoatlantiradigan yechimlar soni 7 ta. ◄

15) $\mathrm{tg}(\pi x^2 + 2\pi x) = \mathrm{tg}(\pi x^2)$ tenglamaning eng katta manfiy ildizini toping

► (7) formulaga asosan $(\pi x^2 + 2\pi x) - (\pi x^2) = k\pi,\ k \in Z$ yoki umumiy yechim $x = \frac{k}{2}$ bo'lib, eng katta manfiy yechim $x = -0{,}5$. ◄

16) $(3x + 6)(x - 2)\mathrm{tg}x\pi = 0$ tenglamaning $[-3;2]$ oraliqdagi manfiy yechimlar yig'indisini toping

► Tenglamaning aniqlanish sohasi $x \ne \frac{1}{2} + k, k \in Z$ ko'paytma ifodada:

1) $3x + 6 = 0, x = -2$ 2) $x - 2 = 0, x = 2$

2) $\mathrm{tg}x\pi = 0, x = k$

Bu berilgan tenglamaga teng kuchli bo'lganligi uchun yechimlar to'plami $x = -2, x = 2$ va $x = k$ $[-3;2]$ oraliqdagi manfiy yechimlar $x = -2$, $x = -1$ va $x = -2$ bo'lib yechimlar yig'indisi -5 ga teng ◄

17.11. Transendeniy ko'rinishdagi funksiyalar qatnashgan trigonometrik tenglamalar yechimini aniqlashda, tenglamada qatnashgan funksiyalarning xossalari, asosiy munosabatlar, tegishli formulalardan foydalanib yechish usullarini misollarda oydinlashtiramiz

⊕ Tenglamalar yechimi topilsin.

1) $2\cos\frac{x}{4} = 2^x + 2^{-x}$

► Bu tenglamada $\cos\frac{x}{4} \le 1$ va $2^x + 2^{-x} \ge 2$, tenglik o'rinli bo'lishi uchun $\cos\frac{x}{4} = 1$ va $2^x + 2^{-x} = 2$ bo'lishi kerak, oxirgi tenglamani $2^{2x} - 2^{x+1} + 1 = 0$ deb yozish mumkin. Bu tenglama yechimlari karrali $2^x = 1$ yechimga ega bundan $x = 0$ bu son birinchi tenglamaning ham yechimi bo'ladi. Berilgan tenglama bu ikki tenglamaga teng kuchli bo'lganligi uchun $x = 0$ tenglama yechimi ◄

2) $(0,5)^{\cos 2x} - \frac{1}{4^{\sin^2 x}} = 0,5$

► Tenglama aniqlanish sohasi $(-\infty; +\infty)$ tenglamani shakl almashtirib yozamiz $\left(\frac{1}{2}\right)^{\cos 2x} - \frac{1}{2^{2\sin^2 x}} = \frac{1}{2}$ tenglamani $2^{1+\cos 2x+2\sin^2 x}$ ga ko'paytirsak $2\cdot 2^{2\sin^2 x} - 2\cdot 2^{\cos 2x} = 2^{\cos 2x}\cdot 2^{2\sin^2} x$

Endi $\cos 2x = 1 - 2\sin^2 x$ ayniyatni hisobga olsak

$$2^{4\sin^2 x} - 2^{2\sin^2 x} - 2 = 0$$

$2\cdot 2^{\sin^2 x} - 2\cdot 2^{1-2\sin^2 x} = 2^{2.\sin^2 x}\cdot 2^{1-2\sin^2 x}$ yoki bu kvadrat tenglama yechimlari $\left(2^{2\sin^2 x}\right)_1 = 2$ va $\left(2^{2\sin^2 x}\right)_2 = -1$ hosil qilamiz bundan:

a) $2^{2\sin^2 x} = 2$ bo'lib, $\sin^2 x = \frac{1}{2}$ bu tenglama yechimlari

(1) formulaga asosan $x_n = \pm\frac{\pi}{4} + \frac{n\pi}{\alpha}, \quad n \in Z$

b) $2^{2\sin^2 x} = -1$ tenglama yechimi mavjud emas, berilgan tenglama yechimi $x_n = \pm\frac{\pi}{4} + \frac{n\pi}{2}, n \in Z$ ◄

3) $2\log_a \cos x = \log_a \cos\frac{3x}{2} + \log_a \cos\frac{x}{2} \quad (a > 0, a \ne 0)$

► Tenglamaning aniqlanish sohasi

$$-\frac{-\pi}{2}+2k\pi < x < \frac{3\pi}{2}+2k\pi$$

logarifm xossasidan $\log_a \cos^2 x = \log_a \cos\frac{3x}{2}\cdot\cos\frac{x}{2}$ yoki

$\cos^2 x = \cos\frac{3x}{2}\cdot\cos\frac{x}{2}$ ko'paytmani yig'indiga keltirish formulasidan

$\cos^2 x = \frac{1}{2}[\cos 2x + cosx]\cos 2x = 2\cos^2 x - 1$ asosiy ayniyatga asosan tenglama $\cos x - 1 = 0$ bo'lib bu tenglama yechimi $x_k = 2k\pi$. Berilgan tenglama yechimi $x_k = 4k\pi$ $(k = 2n)$ $n \in Z$.◀

4) $3^{\log_3(\sqrt{3}cosx)} + 3^{\log_3\sqrt{6}} = 9^{log_9(3sinx)}$

▶ Tenglamaning aniqlanish sohasi $\cos x > 0$ va $\sin x > 0$ (burchak I chorakda o'zgarish kerak). Logarifm xossasidan foydalanib yozamiz.

Yordamchi $\sqrt{3}cosx + \sqrt{6} = 3sinx$ burchak kiritish usulidan foydalanamiz. Tenglamani $r = \sqrt{(\sqrt{3})^2 + (-3)^2} = \sqrt{12}$ ga bo'lamiz

$sinx\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}cosx = \frac{\sqrt{2}}{2}$, Trigonometrk funksiyaning 30° dagi qiymatini hisobga olsak $sinx\cdot\cos 30° - cosx\cdot\sin 30° = \frac{\sqrt{2}}{2}$, endi qo'shish formulasidan foydalanamiz $sin(x - 30°) = \frac{\sqrt{2}}{2}$ bu kanonik tenglama berilgan teng kuchli bo'lib yechimlar to'plami

$$x_n = \frac{5\pi}{12} + 2n\pi, n \in Z\ .◀$$

17.12. Teskari trigonometrik funksiya qatnashgan trigonometrik tenglamalarda, teskari funksiya ta'rifi, asosiy tengliklardan foydalanamiz.

⊕ Tenglamaning yechimi topilsin.

1) $\arcsin(2x + 0,5) = -\frac{\pi}{6}$

▶Teskari funksiya ta'rifidan:

$2x + 0,5 = \sin\left(-\frac{\pi}{6}\right)$ dan $2x + \frac{1}{2} = -\frac{1}{2}$ bo'lib, berilgan tenglama yechimi $x = -0,5$ ◀

2) $arctg(0,4x)=\frac{5\pi}{6}$

► Teskari funksiya ta'rifidan

$0,4x=tg\frac{5\pi}{6}$, keltirish formulasidan foydalanamiz

$0,4x=tg\left(\pi-\frac{\pi}{6}\right)=-tg\frac{\pi}{6}=-\frac{1}{\sqrt{3}}$ u holda $0,4x=-\frac{1}{\sqrt{3}}$ bo'lib,

berilgan tenglama yechimi $x=-\frac{5}{2\sqrt{3}}$ ◄

3) $\arcsin\left(2\sin^{x}\right)=\frac{\pi}{2}$ tenglamaning eng kichik ildizi topilsin.

► Teskari funksiya ta'rifidan $2sinx=\sin\frac{\pi}{2}$, bu tenglama yechimi

$x_n=(-1)^n\frac{\pi}{6}+n\pi, n\in Z$

Berilgan tenglamaning eng kichik musbat ildizi $\frac{\pi}{6}$ ga teng. ◄

4) $2\text{arcctg}(x^2-4x+4)-0,5\pi=0$

Tenglama ildizlar yig'indisi topilsin.

► Tenglamani $arcctg\left(x^2-4x+4\right)=\frac{\pi}{4}$, deb yozish mumkin. Tenglama $x^2-4x+4=1$ tenglamaga teng kuchli bo'lib ildizlari $x_1=3$ va $x_2=1$ U holda berilgan tenglamaning ildizlar yig'indisi $x_1+x_2=4$. ◄

5) $2\arcsin x\cdot\arccos x=-3\pi^2$

► Agar arc $\cos x=y$ almashtirib bajarsak $|y|\le\frac{\pi}{2}$ da

$arcsinx+arccosx=\frac{\pi}{2}$ tenglikni asosan $y^2-\frac{\pi}{2}y-\frac{3\pi^2}{2}=0$

kvadrat tenglamaga kelamiz. Bu tenglama ildizlari $y_1=\frac{3\pi}{2}$ va

$y_2=-\pi$ bo'lib, bu sonlarning absolyut qiymati jihatdan $\frac{\pi}{2}$ dan katta

bo'lganligi uchun,berilgan tenglamaning yechimi mavjud emas. ◄

6) $arctg(x+3)-arctg(x+2)=\frac{\pi}{4}$ tenglamaning eng kichik ildizi topilsin.

► Tenglamaning aniqlanish soha (– ∞; + ∞) ga tangens tengligida yozamiz.

$tg\left[arctg(x+3)-arctg(x+2)\right]=tg\frac{\pi}{4}$ bundan, qo'shish formulasi $tg(\alpha-\beta)=\frac{tg\alpha-tg\beta}{1+tg\alpha\cdot tg\beta}$ ga asosan

$$\frac{tg\left[arctg(x+3)\right]-tg\left[arctg(x+2)\right]}{1+tg\left[arctg(x+3)tg\left[arctg(x+2)\right]\right]}=1$$

teskari trigonometrik funksiyadagi tg(arctgα) = α tenglikka asosan $\frac{x+3-(x+2)}{1+(x+3)(x+2)}=1$ $\frac{1}{x^2+5x+7}=1$ bo'lib (x^2+5x+7) ifoda nolga teng emas. Ifoda soddalashtirilsa $x^2+5x+6=0$ bo'lib ildizlari (– 2) va (– 3). U holda berilgan tenglamaning kichik yechimi $x=-3$ ◄

7) arcsin2x = 3arcsin$x$ tenglama yechimi topilsin.

► Tenglamaning aniqlanish sohasi $|x|\leq 1$ va $|2x|\leq 1$ yoki umumlashtirsak $|x|\leq\frac{1}{2}$.

Endi aniqlanish sohada tenglikdan sinuslar tenglikka o'tamiz sin(arcsin2x)= sin(3arcsin$x$) yoki 2x = sin(3arcsin$x$) karrali argument sin3x= = 3sinα – 4sin$^3\alpha$ formulasiga asosan 2x = 3sin(arcsin$x$) – 4sin^3 (arcsin$x$) bu tenglikdan $2x=3x-4x^3$ bu tenglama ildizlari $x_1=-0{,}5$, $x_2=0$ va $x_3=0{,}5$. Bu qiymatlar berilgan tenglamaning ham yechimlari ekanligini o'rniga qo'yib ishonish hosil qilish qiyin emas.◄

8) $arctg(2+cosx)-arctg(1+cosx)=\frac{\pi}{4}$ tenglama yechimi topilsin.

► Tenglamaning aniqlanish sohasida tangens tenglikka o'tamiz.

$$tg\left[arctg(2+cosx)-arctg(1+cosx)\right]=tg\frac{\pi}{4}$$

6-Misoldagi kabi amal bajaramiz $\frac{(2+cosx)-(1+cosx)}{1+(2+cosx)(1+cosx)}=1$

Soddalashtirsak, kosinusga nisbatan $\cos^2 x+3\cos x+2=0$, kvadrat tenglama bo'lib, ildizlari $(\cos x)_1 = -1$ va $(\cos x)_2 = -2$ bundan berilgan tenglama yechimlari topiladi

1) $\cos x = -1$ da $x_n = \pi + 2n\pi$, 2) $\cos x = -2$ da yechim mavjud emas.

Javob: $x_n = \pi + 2n\pi$. $n \in z$ ◄

9) $\arccos x - \arccos(-x) = \arccos(x - 1) - \pi$ tenglama yechimi topilsin.

► Tenglama aniqlanish sohasi $0 \le x \le 1$. Teskari trigonometrik funksiyadagi [1] $\arccos(-x) = \pi - \arccos x$ tenglikka asosan

$\arccos x - (\pi - \arccos x) = \arccos(x - 1)$ yoki $2\arccos x = \arccos(x - 1)$

Ikkilangan formula $2\arccos x = \arccos(2x^2 - 1)$ $0 \le x \le 1$ ga asosan, $\arccos(2x^2 - 1) = \arccos(x - 1)$ tenglikka kelamiz. Bundan $2x^2 - 1 = x - 1$ bo'lib, yechimlari $x = 0$ va $x = 0,5$ ◄

17.13. Tenglamalar sistemasi yechimini topishda har bir tenglamani yechish usullari, o'rniga qo'yish kabi usullardan foydalanganda, tenglamada qatnashgan funksiyalarga tegishli ma'lumotlarning hammasini hisobga olish kerak bo'ladi.

⊕ Tenglamalar sistemasi yechimi topilsin.

1) $\begin{cases} tgx + tgy = 1, \\ x + y = \dfrac{\pi}{4} \end{cases}$

► Sistema aniqlanish sohasi $x \ne \dfrac{\pi}{2} = k\pi$ va $y \ne \dfrac{\pi}{2} + k\pi,\ k \in Z$.

Sistema yechimini o'rniga qo'yish usulidan foydalanamiz $y = \dfrac{\pi}{4} - x$ ni sistemadagi birinchi tenglamaga qo'yib, qo'shish formulasidan foydalanib yozamiz $tgx + \dfrac{tg45° - tgx}{1 + tg45° tgx} = 1$, $\operatorname{tg} x \ne -1$ deb soddalashtirsak $\operatorname{tg}^2 x - \operatorname{tg} x = 0$ bu tenglama berilgan tenglamaga teng kuchli bo'lib yechimlari $x_k = k\pi$ va $x_n = \dfrac{\pi}{4} + k\pi$ almashtirishga asosan sistema yechimlari $x_n = k\pi, x_n = \pi/4 + n\pi$ va $y_n = \dfrac{\pi}{4} - k\pi$ va $y_n = -n\pi$, $n,k \in Z$. ◄

2) $\begin{cases} 2^{\sin x + \cos y} = 1, \\ 16^{\sin^2 x + \cos^2 y} = 4 \end{cases}$

► Ko'rsatkichli funksiya xossalaridan

$$\begin{cases} \sin^2 x + \cos^2 y = 0,5, \\ \sin x + \cos y = 0 \end{cases}$$

Ikkinchi tenglamadan $\sin^2 x = \cos^2 y$, bu tenglikni sistemadagi birinchi tenglamaga qo'ysak $\cos^2 y = \frac{1}{4}$ bo'lib (2) formulaga asosan

$y_k = \frac{\pi}{4} + k\pi, k \in Z$ Shu kabi o'zgaruvchi x topiladi.

$x_k = \pm\frac{\pi}{4} + k\pi . k \in Z$ ◄

3) $\begin{cases} \cos x \cdot \cos y = \frac{1}{2} \\ \operatorname{tg} x \cdot \operatorname{tg} y = \frac{1}{4} \end{cases}$ bo'lsa $\cos(x+y)$ nimaga teng

► Sistema aniqlanish sohasi $x \neq \frac{\pi}{2} + k\pi, y \neq \frac{\pi}{2} + k\pi$ asosiy ayniyatdan foydalanamiz

$$\begin{cases} \cos x \cdot \cos y = \frac{1}{2} \\ \frac{\sin x \cdot \sin y}{\cos x \cdot \cos y} = \frac{1}{4} \end{cases}$$

Soddalashtirilsa $\sin x \cdot \sin y = \frac{1}{8}$ qo'shish formulasiga asosan

$\cos(x+y) = \cos x \cdot \cos y - \sin x \cdot \sin y$ $\cos(x+y) = \frac{1}{2} - \frac{1}{8} = \frac{3}{8} = 0,375.$ ◄

3) $\begin{cases} \cos x \cdot \cos y = \frac{\sqrt{3}}{4} \\ \sin x \cdot \sin y = \frac{\sqrt{3}}{4} \end{cases}$

Sistema yechimi topilsin.

► Sistemadagi tenglamalarni mos ravishda qo'shib, keyin yana ayirish amalini bajarsak $\begin{cases} \cos x \cdot \cos y + \sin x \cdot \sin y = \frac{\sqrt{3}}{2} \\ \cos x \cdot \cos y - \sin x \cdot \sin y = 0 \end{cases}$, qo'shish formulasiga asosan $\begin{cases} \cos(x-y) = \frac{\sqrt{3}}{2} \\ \cos(x+y) = 0 \end{cases}$, bundan $\begin{cases} x-y=\pm\dfrac{\pi}{6}+2k\pi \\ x+y=\dfrac{\pi}{2}+n\pi \end{cases}$ $k,n\in Z$ bo'lib sistema yechimlari topiladi. ◄

MASHQLAR

x nimaga teng.

329. 1) $0,5arctg\dfrac{x}{2}=-\dfrac{\pi}{8}$, 2) $\arccos(x-1)=\dfrac{\pi}{2}$.

330. 1) $\arcsin(0,2x)=\dfrac{5\pi}{6}$, 2) arcctg$(2x+3)=45°$.

Tenglama yechimlari topilsin.

331. 1) $2\sin\left(\dfrac{x}{2}+\dfrac{\pi}{6}\right)=\sqrt{3}$, 2) $\sqrt{3}tg\left(\dfrac{4x-\pi}{2}\right)+1=0$.

332. 1) $ctg\left(2x-\dfrac{\pi}{12}\right)=-1$, 2) $\cos[3x-0,(3)]=0,5$.

333. 1) $2\cos^2\left(2x+\dfrac{\pi}{3}\right)=1$, 2) $ctg^2\left(0,5x-\dfrac{\pi}{8}\right)=-\sqrt{3}$.

334. 1) $3tg^2(\pi-2x)=1$, 2) $0,(3)\sin^2\left(\dfrac{3x-\pi}{2}\right)=0,25$

Taqqoslash usuli bilan tenglama yechimlari topilsin.

335. 1) $\cos(0,5x-360°)=\cos\left(3,5x-\frac{\pi}{3}\right)$,

2) $\mathrm{tg}(210° + 2x) - \mathrm{tg}(80° - 3x) = 0$.

336. 1) $ctg\left(2x+\frac{\pi}{9}\right)+tg(90°+x)=0$,

2) $\sin 2x-\cos\left(x+\frac{\pi}{6}\right)=0$.

Tenglama yechimlari topilsin.

337. $\sin 3x \cdot \cos 3x = \sin 2x$.

338. $1-\cos(\pi+x)=\sin\frac{3\pi+x}{2}$.

339. $\cos 6x = 2\sin\left(\frac{3\pi}{2}+2x\right)$.

340. $\left(\sin^3 x\cdot cosx-\sin\cdot\cos^3 x\right)8=\sqrt{2}$.

341. $1 + \sin 2x = (\cos 3x + \sin 3x)^2$.

342. $\cos^4 x - \sin^4 x = \sin 2x$.

343. $9\cos^2 x - 9\sin x - 5 = 0$.

344. $\sin(2x - 30°) + \cos(2x + 30°) = 0$.

345. $\cos 5x + \cos 7x = \cos(\pi + 6x)$.

346. $\cos 2x + \cos 6x + 2\sin^2 x = 1$.

347. $\sin 3x \cdot \cos 11x = \sin x \cdot \cos 13x$.

348. $\sin 2x - \sqrt{3}\cos 2x = 1$.

349. Ushbu $4\cos^3 x = 3\cos x$ tenglamaning nechta ildizi $|x| \leq \frac{\pi}{2}$ tengsizlikni qanoatlantiradi.

350. $ctg(270° + \alpha) - tg^2\alpha = \frac{\cos 2x - 1}{\cos^2 x}$ tenglamaning (30°;90°) intervalga tegishli yechimi topilsin.

351. $4\sin^2 \cdot \cos x - 4\sin \cdot \cos^2 x + \cos^3 x = 0$.

352. $\sin^2 x + \cos^4 x = 1 + \sin^4 x$.

353. $\frac{tg36^\circ + tg2x}{tgx \cdot tg36^\circ - 1} = \sqrt{3}$.

354. $tg(x + 45^\circ) + tg(x - 45^\circ) = 2$.

355. $\sin\left(\frac{\pi}{2} + 2x\right) - tg(\pi - x)\sin 2x = tg\left(\frac{\pi}{2} - x\right)$.

356. $\sin 2x = tg^2 x(1 + \cos 2x)$.

357. $\cos(30^\circ - x) - 0{,}5sinx = \frac{\sqrt{3}}{2}cosx$.

358. $\sin\left(\pi cos\frac{x}{3}\right) = 1$ tenglama umumiy yechim topilsin.

359. Ushbu $\cos 6x + \cos 4x = \cos 5x$ tenglamaning $|x| \le \frac{\pi}{2}$ tengsizlikni qanoatlantiruvchi yechimlar yig'indisi topilsin.

360. $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ tenglamaning $(0;90^\circ]$ kesmaga tegishli yechimlar yig'indisi topilsin.

361. $(3cos\pi x - \pi)(2sin\pi x - \sqrt{3}) = 0$ tenglamaning eng kichik musbat ildizi topilsin.

Tenglama yechimlari topilsin.

362. $\sin 2x \cdot \mathrm{ctg}x - \cos 2x = \mathrm{tg}3x$.

363. $2cosx \cdot tg^2 \frac{x}{2} + 3 = 0$.

364. $2sinx \cdot \sin\frac{x}{2} + \frac{1}{\cos 0,5x} = 4\sin 0,5x$.

365. $\frac{7cosx - 3}{2\cos^2(x+\pi)} = 1$. Tenglamaning $30° < x < 90°$ intervalga tegishli yechimi topilsin.

366. $ctg\frac{x}{2} - tg\frac{x}{2} = 2$, tenglamaning $0 < x < 90°$ intervalga tegishli yechimi topilsin.

367. Uchburchakning α va β burchaklari orasida $\sin\alpha + sin\beta = \sqrt{2}\cos\frac{\alpha - \beta}{2}$ tenglik o'rnili bo'lsa, shu uchburchakning eng katta burchagini toping.

368. $3\sin 5x + 4\cos 5x = 6$, tenglamaning $[-\pi, 2\pi]$ oraliqdagi musbat ildizlarini ko'rsating.

369. $[0; 2\pi]$ oraliqda $\sin 2x = (\cos x - \sin x)^2$ tenglamaning nechta ildizi bor.

370. $ctg\left[\frac{\pi(x-1)}{2}\right]=0$, tenglamaning [– 1; 5] oraliqda nechta ildizi bor.

371. $3\sin^2 2x + 7\cos2x - 3 = 0$, tenglamaning (– 90°,180°) oraliqdagi ildizlar yig'indisi topilsin.

372. $(0{,}5\cos\pi x + \pi)\cdot(\sin\pi x + 1) = 0$, teglamaning [– 2; 6] oraliqda nechta ildiz bor.

373. $\frac{\sin^2 x + 2sinx}{\cos 2x} = 0$, tenglamaning [– π; 3π) da nechta ildiz bor.

374. $\sin\frac{6\pi}{x} = 0$ tenglamaning manfiy butun ildizlar yig'indisini toping.

375. $(2x + 1)(0{,}5x - 1)\sin2\pi x = 0$, tenglamaning [– 2;3) oraliqdagi natural yechimlar yig'indisi topilsin.

376. $3^{-1+\sin-\sin^2 x+\dots} = 0,(1)$, tenglamani yeching, eng kichik musbat ildizni toping.

377. $|tgx + ctgx| = \frac{4}{\sqrt{3}}$, tenglamaning eng katta manfiy ildizini toping.

378. $\sin\frac{6\pi}{x}=0$, tenglamaning nechta butun yechimi bor.

379. $\sin x+\sin 2x+\sin 3x+\sin 4x=\cos x+\cos 2x+\cos 3x+\cos 4x$ tenglamaning eng kichik musbat ildizi topilsin.

Tenglama yechimlari topilsin.

380. $2\cdot 4^{|x|}=2\cdot\cos\frac{x}{2}$,

381. $3^{|x|}=\sin x$.

382. $2^{|x|-1}=\sin\left(\pi\cdot\sqrt{x}\right)+1$.

383. $(\cos x)^{\sin^2 x-1,5\sin x+0,5}=1$.

384. $(\log_{\sin x}\cos x)^2=1$.

385. $(\sin x)^{-\sin x}-1=\mathrm{ctg}^2\,x$.

386. $\log_{\cos x}\sin x+\log_{\sin x}\cos x=2$.

387. $16^{\sin x}=\sqrt[\sin x]{4}$.

388. $5\cdot 5^{\sin^2 x+\cos 2x}=0,04$.

389. $5^{1+\log_5 cosx} = 2,5$.

390. $\log_2 (2^x + \sin x - 0,5) = x$, $0 < x < \pi$ da

391. $9^{\cos x} = 9^{\sin x} \cdot 3^{2\cos x}$.

Tenglamalar sistemasi yechimi topilsin.

392. $\begin{cases} sinx + siny = 1, \\ x + y = \dfrac{\pi}{3}. \end{cases}$

393. $\begin{cases} 4cosx \cdot cosy = 3, \\ 4sinx \cdot siny = -1. \end{cases}$

394. $\begin{cases} tgx - tgy = \dfrac{1}{3}, \\ x + y = \dfrac{\pi}{3}. \end{cases}$

395. $\begin{cases} cosx + cosy = \sqrt{3}, \\ x + y = 60°. \end{cases}$

396. $\begin{cases} sinx - 2siny = 0, \\ x - y = 300°. \end{cases}$

397. $\begin{cases} x - y = -\dfrac{1}{3}, \\ \cos^2 x - \sin^2 \pi y = 0,5. \end{cases}$

Tenglama yechimi topilsin.

398. $\operatorname{arctg}3^x - \operatorname{arctg}3^{-x} = 30°$.

399. $arcctg2x + arcctg3x = \dfrac{3\pi}{4}$.

400. $\arccos x - \arccos(-x) = \arccos(x - 1) - \pi$.

401. $\operatorname{arctg}x = 2\operatorname{arcctg}x$.

402. $\arccos\dfrac{x}{2} = 2arctg(x-1)$.

403. $arctg(x+3) - arctg(x+2) = \dfrac{\pi}{4}$.

404. $\arcsin 2x = 3\arcsin x$.

405. $arcsinx - arccosx = \dfrac{\pi}{3}$.

TEST TOPSHIRIQLARI

(O'zingizni sinab ko'ring)

1. a qanday bo'lganda $2,4a-1,(3)x=-3\frac{3}{4}a$, tenglama noldan farqli yechimi musbat bo'ladi?

A) $a = 16$ B) $a < 0$ C) $a = -12$ D) $a > 0$ E) $a = -1,5$

2. a va b qanday bo'lganda $4ax-1,3b=\frac{2}{3}b-1,2ax$, tenglama noldan farqli yechimi manfiy bo'ladi?

A) $a > 0, b > 0$ B) $a > 0, b = 0$ C) $ab < 0$ D) $a = -2, b = 3$ E) $ab > 0$

3. a va b qanday bo'lganda $1,2ax-4,(2)b=1\frac{3}{5}b$, tenglama noldan farqli yechimi musbat bo'ladi?

A) $ab > 0$ B) $a < 0, b > 0$ C) $a > 0, b < 0$
D) $a = 5, b = -10$ E) $a = -10, b = 5$

4. $\frac{2a+3}{2}-3b$ ifoda $b = 0,(3)$ va a ning biror qiymatida 0,25ga teng. Endi a ning shu qiymatida va $b = 2,1(6)$ da berilgan ifoda qiymati nimaga teng.

A) 1,25 B) – 2,25 C) 1,05 D) – 1,45 E) 3,05

5. a va b qanday bo'lganda $1\frac{1}{2}(2x-a)=0,5ax+1,2b$ tenglama yechimi mavjud emas?

A) $a = -6, b \neq 1{,}5$ B) $a = -2{,}5, b \neq 7$
C) $a = -2, b \neq 7$ D) $a \neq -6, b = -4$ E) $a = -6, b \neq -7{,}5$
Tenglama yechimi topilsin.

6. $$\frac{x}{6} + \frac{x}{12} + \frac{x}{20} + \frac{x}{30} + \frac{x}{42} = \frac{5}{7}$$

A) 2,3 B) 2 C) – 0,25 D) 1,3 E) 2,5

7. $$\frac{3x}{0{,}2} = 2{,}5 : 3{,}(3)$$

A) 1,3 B) 1,5 C) 1,3 D) 1,3 E) 0,005

8. $5 - 3(x - 2[x - 2(x - 2)\,]\,) = 2$

A) 3 B) 2 C) 1,3 D) 1,2 E) 1

9. Tenglama yechimi topilsin

$$\frac{3}{2 - \dfrac{3}{2 - \dfrac{3}{2 - x}}} = \frac{21}{8}$$

A) 1 B) – 2 C) – 3 D) $\frac{3}{8}$ E) 4

10. *a* qanday bo‘lganda $0,(2)y = 1,1(6)x + 0,(9)$ to‘q‘ri chiziq $(-6;\ 1)$ nuqtadan o‘tadi.

A) 0,5 B) 1,1 C) 0,6 D) 0,(1) E) 0,(3)

Tenglama yechimi topilsin.

11. $$\frac{\sqrt[6]{64}\sqrt[3]{2}}{8x} = 2^{\frac{1}{3}}$$

A) 1,4 B) 0,8 C) 0,25 D) 2,5 E) 1,5

12. $\dfrac{0,(2)(4x-1)}{2x+1}=0,(4)$

A) ∅ B) 0,3 C) 1,4 D) – 0,5 E) – 1,6

13. Tenglama yechimlar yig'indisi topilsin.

$\left(0,(5)x-0,(1)\right)\left(0,5x+1,5\right)\left(2\dfrac{1}{4}-0,25x\right)=0$

A) 1,2 B) 1,3 C) 2,3 D) 6,2 E) 4,4

Tenglama yechimi topilsin

14. 0,(7) : (157 – 900 : x) = 0,(1)

A) 2,7 B) 6 C) – 4,6 D) 3,3 E) – 2,6

15. $2,8:x=1,(6):2\dfrac{6}{7}$

A) 1,8 B) $1\dfrac{5}{6}$ C) 3,4 D) $1\dfrac{3}{7}$ E) 4,8

16. $3x+\dfrac{x}{2}+\dfrac{x}{4}+\dfrac{x}{8}+\dfrac{x}{16}=31,5$

A) 8 B) 4,5 C) – 2,5 D)1,5 E) – 2,4

17. $0,(1)[(0,5)^{-1}+1][(0,5)^{-2}+1][(0,5)^{-4}+1]=0$

A) 6 B) 3,5 C) 9 D) 2,5 E) 8

18. Tenglama yechimi topilsin

$0,(3)[-0,5+2x^2-(x+2)(2x-4)]=0,1(6)x+1,3(3)$

A) 4,6 B) 7 C) 3, 2 D) – 4,6 E) 3,6

19. Tenglama yechimi topilsin

$3,5x+2x+2,5x+3x+\cdots+5,5x=6,3$

A) 0,5 B) 2,2 C) – 1,5 D) 1,2 E) 0,2

20. *a* qanday bo‘lganda

$0,1(1)x+0,(1)=0,1(6)$ tenglama yechimi 2 bo‘ladi?

A) 0,6 B) 1,2 C) – 1,4 D) 0,25 E) 1,(3)

21. $1\le x<3$ da $|3x-2|-|x+1|+|x-3|-2x+4$ ifodani modulsiz yozing

A) $x-2$ B) $1+x$ C) $2-x$ D) $2x-1$ E) $1-2x$

22. Agar $a>b>c>d>0$ bo‘lsa $|d+b|-|a-c|+|b-a|+|c-d|+$ $+b-c$ ifodani soddalashtiring

A) $b-c$ B) $a-b$ C) $c+b$ D) $d+a$ E) $c-d$

23. Agar $a<b<c<d<0$ bo‘lsa $|a-c|-|a+b|-|c-b|+a-b$ ifodani soddalashtiring

A) $c-b$ B) $a-c$ C) $b+c$ D) $c-a$ E) $a+b$

24. $|2x-0,(2)|=1,3(3)$

Tenglama yechimlar yig‘indisi topilsin

A) 0,5 B) 0,(2) C) – 0,(7) D) 1,(3) E) – 0,2

25. Tenglama yechimlari topilsin $|x+5|-8=|x-3|$

A) $[3;+\infty)$ B) -5 C) -3 va 5 D) -1 va 3 E) 3

26. Tenglama eng katta manfiy va eng kichik musbot butun yechimlar yig'indisi topilsin $||6x|-|6x-3||=3$

A) $-0,5$ B) 2 C) 1,5 D) 0 E) $-2,5$

27. $|2-|1-|x|||=1$ tenglama nechta yechima ega

A) 4 B) 1 C) 3 D) 2 E) 5

28. $|x-2|+|4-x|=2$

Tenglamaning katta kichik yechimlar yig'indisi topilsin.

A)0 B) 6 C) 2 D) -1 E) 3

29. $\frac{5}{3-|x-1|}=|x|+2$ tenglamaning haqiqiy yechimlar yig'indisi topilsin.

A)1,5 B) $\sqrt{5}$ C) $1+\sqrt{5}$ D) $2-\sqrt{5}$ E) $2\cdot\sqrt{5}$

30. $|x-3|+|x+2|-|x-4|=3$ tenglama katta ildizi kichik ildizidan qanchaga ortiq.

A)2 B)1 C)5 D)8 E)4

Tenglama yechimi topilsin

31. $|2-x|+|x|+|x+2|=1+x$

A) 0,5 B) $\varnothing$ C) $-0,5$ D) $\frac{2}{3}$ E) $-\frac{2}{3}$

32. $\dfrac{4\left(x^2-a\right)}{2ax-a+1-2x}+\dfrac{2x}{2x-1}=\dfrac{2x}{a-1}(a\neq 1, a\neq 0)$

A) 1,2 B) 1 C) – 1,2 D) 2 E) 2,2

33. a va b qanday bo‘lganda $\dfrac{4x-2b+1}{6x(x+4)}=\dfrac{a}{3(x+4)}+\dfrac{3}{2x^2+8^x}$ tenglik o‘rinli bo‘ladi.

A) $a=2, b=-4$ B) $a=-2, b=2$ C) $a=1, b=-4$
D) $a=-2, b=1$ E) $a=-1, b=-4$.

34. $\dfrac{6}{x+1}+\dfrac{1+x}{2-2x^2}=\dfrac{2x-1}{x^2-1}-\dfrac{1}{2-2x}$ tenglama yechimlar yig‘indisi topilsin.

A) 0 B) 3 C) ∅ D) - 2 E) 2

35. a qanday bo‘lganda $\begin{vmatrix} 3 & 2 & 2 \\ 1 & -5 & a \\ 4 & 2 & 1 \end{vmatrix}$ determinant qiymati 11 ga teng bo‘ladi.

A) 4 B) 0 C)3 D) – 4 E) – 8

36. b ning qanday qiymatida $\begin{vmatrix} 1 & 2 & -4 \\ 2 & -1 & 2 \\ b & -3 & -6 \end{vmatrix}=0$, tenglik o‘rinli

A) 3 B) $b\in R$ C) 5 D) $b\in(-3,5)$ E) – 5

37. Tenglama yechimi topilsin. $\begin{vmatrix} 3 & 2 & 0 \\ 1 & x & -3 \\ -2 & 1 & 0 \end{vmatrix} - 3\begin{vmatrix} 3x & 1 \\ -4 & 2 \end{vmatrix} = 0$

A) 0,5 B) 1,5 C) 0 D) – 0,5 E) 2

38. a qanday bo'lganda $\begin{cases} 3x+4y=18, \\ 2x+5y=a \end{cases}$ sistema yechimida $y = 0$ bo'ladi.

A) – 5 B)2 C) 12 D) – 8 E) 10

39. Sistema yechimlar yig'indisi topilsin $\begin{cases} 5x+y-3z=-z, \\ 4x+3y+2z=10, \\ 2x-3y+z=17. \end{cases}$

A) 4 B) 6 C) – 1 D) – 4 E)3

40. $\begin{cases} x+3y+2z=0, \\ x+y-3z=0, \\ 4x+2y+z=0. \end{cases}$ Sistema yechimlari uchun $(2x+y-z)$nimaga teng

A) 2 B) – 1 C) – 3 D) 0 E) – 2

41. b qanday bo'lganda $\begin{cases} 2x+by+z=5 \\ 4x+3y+2z=10 \\ 5x+y-3z=3. \end{cases}$ sistema yagona yechimga ega bo'ladi.

A) 2 B) – 1 C) – 3 D) 4 E) 0

42. *a* qanday bo'lganda $4y + 3x - 25 = 0$ to'g'ri chiziq $5x + ay - 7 = 0$ to'g'ri chiziq bilan kelishadi.

A) -2 B) 1 C) 3 D) 2 E) -3

43. *a* qanday bo'lganda $\begin{cases} 2x + ay = 8 \\ 3x + y = 1 \end{cases}$ sistema yechimlari uchun $5x - y = 7$ tenglik o'rinli.

A) 2 B) -1 C) -2 D) -3 E) 4

44. $\begin{cases} 0,(3)(2x-6) - 0,5(x-2) = 2y, \\ 0,5(3x-6+y) = x \end{cases}$ sistema yechimlari $2x - 3y$ nimaga teng.

A) 8 B) 4 C) 12 D) -8 E) 0

45. *b* qanday bo'lganda $bx + 3y - 13 = 0$ va $5x - y - 7 = 0$ to'g'ri chiziqlar (2;3) nuqtada kesishadi.

A) $+3$ B) 2 C) 1 D) 3 E) -4

46. *b* qanday bo'lganda $\begin{cases} 4x - 2y + z = 0, \\ x + y + bz = 0, \\ x + 3y + 2z = 0 \end{cases}$ sistema yagona yechimga ega.

A) 2 B) 1 C) -4 D) -2 E) -3

47. $1,3(3)\,x^2 - 0,75 = 0$, tenglama katta ildizi kichik ildizidan qanchaga ortiq

A) 1,5 B) 2 C) 3,5 D) 1,(3) E) 0,7

48. Qaysi tenglamalar a) ratsional ildizlarga; b) irratsional ildizlarga ega:

$3x^2 - 13x + 4 = 0$, (1); $3x^2 - 18x + 7 = 0$, (2);

$2x^2 - 10x + 15 = 0$, (3); $45x^2 - 108x + 63 = 0$, (4); $0,5x^2 - 4 = 0$,(5); $17x^2 + 58x + 41 = 0$, (6),

ga

A) (2), (3), (5) B) (1), (2), (5) C) (1) (4) (6)
D) (1) (5) (3) E) (2) (5) (6)

ga

A) (1), (5), (3) B) (2) (3) C) (2) (4) (5)
D) (2) (5) E) (5) (3) (6)

49. x_1 va x_2 sonlar $3x^2 - 2x - 6 = 0$, tenglama yechimlari bo'lsa $\frac{1}{x_1} + \frac{1}{x_2}$ nimaga teng

A) 0,5 B) – 0,(3) C) 1 D) 1,3 E) – 1,5

50. x_1 va x_2 sonlar $x^2 + x - 5 = 0$ tenglama yechimlari bo'lsa $x_1^2 + x_2^2$ nimaga teng.

A) 9 B) 3,2 C) 1,7 D) – 10 E) 11

51. Tenglama yechimi topilsin $\left(\frac{1}{x} + x\right) + \frac{x}{1+x^2} = -2,5$

A) – 1,2 B) 0,5 C) – 1,5 D) – 1 E) 2,1

52. Yechimlari (– 3) va $(2 \pm i)$ bo'lgan tenglama topilsin

A) $x^3 - 7x^2 + x + 15 = 0$, B) $x^3 - x^2 - 7x + 15 = 0$
C) $x^3 + 2x^2 - 3x + 15 = 0$ D) $x^3 - 3x^2 + 15 = 0$, E) $2x^3 + x^2 - 7x + 5 = 0$

53. Yechimlari 0, – 1,1va 3 bo'lgan tenglama topilsin

A) $x^4-3x^3-x^2+3x=0$ B) $x^4+x^3-x^2-5x=0$

C) $x^4-2x^3+3x^2-x=0$ D) $x^4+2x^3-x^2+5x=0$ E) $2x^4-x^3+x^2+3=0$

Tenglama haqiqiy yechimlari topilsin

54. $x^3-6x^2+12x-10=0$,

A) $\sqrt[3]{2},2$ B) $1-\sqrt[3]{2}$ C) $2+\sqrt[3]{2}$ D) $\sqrt[3]{2};-0,5$ E) $-\sqrt[3]{2},1,5$

55. $\dfrac{x-4}{x+5}+\dfrac{x+5}{x-4}=2$

A) 1,3 B) – 1,6 C) $\varnothing$ D)2,3 E) 2,6

56. $4(x-1)^4-5(x-1)^2-6=0$

A) $\pm2\sqrt{2},0,5$ B) $1\pm2\sqrt{2}$ C) $2\pm\sqrt{2}$ D) $1,2,\pm3\sqrt{2}$ E) $2\sqrt{3},-\sqrt{2}$

57. $(x^2-5x+4)(x^2-5x+6)=120$.

Tenglama ildizlar yig‘indisi topilsin.

A) 5 B) 6,5 C)5,2 D) 4 E) 4,5

58. Ikki hadli tenglamaning $2x^3+16=0$, barcha ildizlari topilsin.

A) – 2,1,6; – 1,2 B) $-2,\pm\sqrt{3}i$

C) $2\pm\sqrt{2}i,-2$ D) $-2;1\pm\sqrt{3}i$ E) $-2,\pm\sqrt{2}i$

Tenglama yechimlari topilsin.

59. $x^4-3x^3+4x^2-3x+1=0$

A) 1; – 2,1;3,4 B) – 2; 1,4; – 3,1 C) 2;1; 2,4 D) 1;0,6: – 2,1 E) 1

Tenglama ildizlari yig'indisi topilsin

60. $(x+1)^4+(x+3)^4=16$

A) 3 B) – 4 C)5 D) – 3 E)12

61. Tenglama barcha ildizlari topilsin $x^2-x+1=\dfrac{1}{x-1}$

A) 2; $\pm 3i$ B) $-0,5\pm\sqrt{3}$ D) $0;\pm\sqrt{2}$ E) $0;2\pm\sqrt{3}$

62. $x^8-17x^4+16=0$ tenglamaning nechta butun yechimi bor

A) 4 B) 3 C) 6 D) 2 E) 8

63. $1,(3)\,x^2-2,4(4)x-3,5(5)=0$

Tenglama ildizlar ishoralarini aniqlang

A) $x_1>0, x_2>0$ B) $x_1<0, x_2<0$ C) $|x^+|>|x^-|$

D) $|x_1^+|=|(x_2^-)|$ E) $|x^+|<|x^-|$.

64. a qanday bo'lganda $16x^2+ax+1=0$ tenglama ildizlari karrali va musbat bo'ladi.

A) 4 B) – 8 C) 8 D) ± 2 E) 0

65. a va b qanday bo'lganda $0,5x^2+bx+a=0$ tenglama bitta ildizi nol ikkinchi ildizi manfiy bo'ladi.

A) $b=0, a<0$ B) $b=0, a<0$ C) $a=0, b<0$

D) $a>0, b>0$ E) $a=0, b<0$

66. x_1 va x_2 sonlar $45x^2-108x+63=0$,

Tenglama yechimlari bo'lsa $|x_2-x_1|$ — nimaga teng

A) 0,4 B) 1,5 C) 2,4 D) 3,5 E)3

67. a son qanaday bo'lganda $x^2 + 2ax + a^2 = 1$ tenglama ildizlari $[-1;3]$ oraliqda bo'ladi.

A) 1,2 B) [1,3] C) [– 2,0] D) [1,2] E) 2,3

68. a va b qanday bo'lganda $ax^2 + bx + 16 = 0$ tenglama ildizlari karrali 4 ga teng bo'ladi.

A) $a = 2, b = -4$ B) $a = 1, b = -3$ C) $a = -2$ $b = 8$
D) $a = -1, b = 4$ E) $a = 1, b = +8$

69. Ildizlari $(2i - 4)$bo'lgan ratsional koeffisientli kvadrat tenglama topilsin.

A) $2x^2 + 8x + 17 = 0$, B) $x^2 - 8x + 14 = 0$
C) $x^2 + 6x - 13 = 0$, D) $x^2 + 8x + 20 = 0$, E) $x^2 + 8x - 23 = 0$

70. $x(x + 1)(x + 2)(x + 3) = 24$ tenglama haqiqiy yechimlar yig'indisi topilsin.

A) 4 B) 1 C) 1,6 D) – 3 E) – 5,4

71. x ning qaysi qiymatida $\dfrac{2x-9}{2x-5}$ kasr $\dfrac{3x}{2-3x}$ kasrdan ikkitaga ortiq bo'ladi.

A) 0,5 B) – 0,25 C) 0,45 D) – 0,5 E) 1,25

72. a qanday bo'lganda $\begin{cases} ax + 2y = a, \\ 8x + ay = 2a \end{cases}$ sistema cheksiz ko'p yechimga ega.

A) – 2 B) 3 C) 4 D) – 3 E) 1

73. x va y o‘zgaruvchilar $\begin{cases}(1-2y)0,2-\dfrac{x}{5}-2y=4,\\ 2(1-y)-x=1\end{cases}$ sistema yechimi bo‘lsa $(x+2y)$ nimaga teng

A) 1 B) + 3 C) 2 D) – 2 E) 4

74. Tenglamalar sistemasi yechimi topilsin.

$$\frac{1}{1+2x}+\frac{1}{1-2y}=1,\quad \frac{2}{1+2x}-\frac{3}{1-2y}=-\frac{4}{3}$$

A) $x = 1{,}2; y = -0{,}2$ B) $x = 1, y = 2{,}25$

C) $x = -1{,}2; y = 0{,}4$ D) $x = 0{,}8, y = 1{,}2$ E) $x = 1; y = -0{,}25$

75. $x^4 - 4x^3 - x^2 + 16x - 12 = 0$ tenglamaning butun yechimlari nechta

A) 2 B) 1 C) 3 D) 4 E) mavjud emas.

76. a va b qanday bo‘lganda $a(3x^2 - 2x - 1) + b(5x^2 - x + 5) = 0$, tenglama qarama-qarshi ishorali ildizlarga ega bo‘ladi.

A) $3a \neq 0{,}5b \neq 0$ B) $2a + b = 0$ C) $3a + 5b \neq 0{,}2a + b = 0$

D) $5b \neq 0$ $3a = 0,$ E) $2a - b = 0{,}3a - 5b = 0$

77. $(7x - 20x^2 + 6)$ uch hadni chiziqli ko‘paytma ko‘rinishida yozing

A) $20(2 + 5x)(4 - 3x)$ B) $(2x - 5)(3 - 4x)$

C) $20(5x + 2)(3x - 4)$ D) $(5x + 2)(3 - 4x)$ E) $20(5 + 2x)(3 - 4x)$

78. a qanday bo‘lganda $2x^2+\sqrt{24}x+a=0$ tenglama o‘zaro teng haqiqiy ildizlarga ega bo‘ladi.

A) $2\sqrt{3}$ B) $3\sqrt{2}$ C) 2 D) $\sqrt{6}$ E) 3

79. Qaysi tenglamalar R-to'plamda o'zaro teng kuchli?

$2x-3=6-x$ va $2x-3+\sqrt{1-x}=6+x+\sqrt{1-x}$ (1)

$5x-2x^2-7=0$ va $0{,}5x^2+2=0$,(2)

$$\frac{x^3-1}{x^2+4}=0 \text{ va } 2\left[0{,}1(6)x-\frac{1}{6}\right]=0{,}(3)$$

$$(x-3)(x-2)=0 \text{ va } \frac{(x-2)^2(x-3)}{(x-2)}=0 \quad (4)$$

$$3(x-2)+\frac{2}{x^2+1}=2-x+\frac{2}{x^2+1} \text{ va } 2(x-2)=2-x \ (5)$$

A) (1) (3) (4) B) (2) (3) (5) C) (1) (2) (5) D) (2) (3) (4) E) (1) (4) (5)

80. a qanday bo'lganda $4a(x^2+x)=a-2{,}5$ va $x(x-1)+a-1{,}25=0$ Tenglamalar bir xil sondagi yechimga ega bo'ladi.

A) $a>1{,}5$ B) $a=0{,}5$ C) $a<0$ D) $a=-1{,}5$ E) $a=0$

81. Kasrni qisqartiring $\dfrac{0{,}5(3x^2+16x-12)}{(10-13x-3x^2)(2x+10)^{-1}}$

A) $x+6$ B) $2(x+5)$ C) $\dfrac{2}{x+6}$ D) $\dfrac{1}{2(x+5)}$ E) $\dfrac{4}{x+5}$

82. Tenglama yechimlari topilsin

$$\frac{27}{2x^2+7x-4}+\frac{2x}{x+4}=\frac{6}{2x-1}-1$$

A) $-0{,}5$; 0,2 B) $-0{,}(3)$ C) 0,5;1,2 D) 0,1(6) E) 0,(3), 1,2

83. $(x+0{,}5)(x^2-9)=(1+2x)(x+3)^2$

A) $-0{,}5;3;1{,}2$ B) $0{,}5;-9{,}1{,}2$

C) $-0{,}5;2;-3$ D) $-0{,}5;-3;-9$ E) $-0{,}5;-3;2;-1{,}2$

84. $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$

A) 1;2; – 1,5;4 B) 1; – 2,5;3; – 1,5 C) 1;3; – 2,4;4
D) 1,2, – 5;3; – 3,5 E) 1;2;3;4

85. Tenglamaning butun yechimlari nechta? $x^4 - x^3 + 2x^2 - x + 1 = 0$

A) 2 B) 3 C) mavjud emas D) 4 E)1

R to'plamda berilgan funksiyalar aniqlanish sohasining umumiy qismini yozing

86. $\sqrt{x-2},\ \sqrt[3]{\dfrac{2x+3}{x+2}},\ \dfrac{\sqrt[4]{2x-1}}{x^2+4}$

A) [2;3) B) [2; + ∞) C) (– 2;3) D) $\left[\dfrac{1}{2};2\right)$ E) (– 4;2)

87. $\sqrt[4]{0,5x-2},\ \sqrt{4-x},\ \sqrt[3]{\dfrac{2x}{x-4}}$

A) (4; + ∞) B) $x \neq 4$ C) $\left[\dfrac{1}{2};4\right)$ D) ∅ E) [0;4)

88. $\sqrt[3]{\dfrac{x-1}{x-2}},\ \sqrt{x^2-9},\ \sqrt{\dfrac{x^2+1}{x-1}}$

A) (1; 3], B) (2; + ∞) C) (1;2) ∪ (2; + ∞)
D)(1;2) ∪ (2,3) E) (– 3;2) ∪ (2;3)

89. $\sqrt{(0,5x)^2-1},\ \sqrt[4]{4-x^2},\ \sqrt[3]{\dfrac{3-x}{x-2}}$

A) -2 B) $(-2;2)$ C) $(0;2)$ D) $\left(\frac{1}{2};2\right)$ E) $\left(-\frac{1}{2};\frac{1}{2}\right)$.

Mantiqiy mulohazalar yuritib qaysi tenglamalar yechimiga ega emasligini ko'rsating.

90. $\sqrt{5+\sqrt{x-\sqrt{2}}}=\sqrt{3}$ (1)

$\sqrt[4]{x-4}+\sqrt{-0,5x+2}=4-x,$ (2)

$\sqrt{0,2x-5}+\sqrt{2-0,5x}=4$ (3)

$\sqrt[3]{1-2x}+\sqrt[3]{x-1}=-1,$ (4)

$2\sqrt{x^2-9}+\sqrt{4-x^2}=1$ (5)

$2x^2-4x+\sqrt{2x^2-4x+12}+18=0,(6)$

A) (2)(3)(6) B) (1)(4)(6) C) (2) (5) (6) D) (2) (6) E) (1) (3) (5) (6)

Tenglama yechimlari topilsin

91. $\sqrt[4]{2x-3}+2=-\sqrt{0,1(6)+0,(3)x}$

A) 0,(6) B) $\varnothing$ C) 0,1(1) D) 0,3(3) E) 1(2)

92. $\sqrt{\frac{x+1}{x-1}}-\sqrt{\frac{x-1}{x+1}}=\frac{3}{2}$

A) $\varnothing$ B) 1,04 C) 1,(6) D) 1,75 E) 1,2(1)

93. $\sqrt{5x+7}=\sqrt{3x+1}+\sqrt{x+3}$

A) – 0,(09) B) – 0,(6) C) 1,2 D) $1\frac{3}{11}$ E) $\frac{5}{11}$

94. $\left(2-0,5x^2\right)\sqrt{0,(3)x-9}=0,$

A) 2;3 B) – 2;2 C) – 2;3 D) 9 E) 2;9

95. $\sqrt[3]{8x+4}-\sqrt[3]{8x-4}=2$ tenglama ildizlar yig‘indisi topilsin

A) 1,5 B) 2,5 C) 0 D) – 0,4 E) 1

96. $\sqrt[3]{x^3-2x-3}=x-1$ tenglama ildizlar yig‘indisi topilsin

A) $1\frac{1}{3}$ B) 1,(6) C) – 0,6 D) 1,2 E) 0,(6)

97. tenglama yechimi topilsin $\sqrt[3]{\sqrt[x-1]{3^{3x-1}}}-\sqrt[3x-7]{27^{x-3}}=0,$

A) 1,3 B) 0,65 C) 1,6 D) 2,1 E) 1,(6)

98. $(x+4)(x+1)-6=3\sqrt{x^2+5x+2}$ tenglamalar yechimlar yig'indisi topilsin

A) – 5 B) 2 C) 4 D)7 E) – 3

99. $\sqrt{x\cdot\sqrt[5]{x}}-\sqrt[5]{x\cdot\sqrt{x}}=56$ tenglama nechta ildizga ega.

A) 2 B) ∅ C)1 D) 3 E) 4

Tenglama yechimi topilsin.

100. $\sqrt[4]{(3x-1)^{0,6}\sqrt[4]{(3x-1)^{0,6}\sqrt[4]{(3x-1)^{0,6}\ldots}}}=2$

A) 1 B) 11 C) 4,3 D) 1,3 E) 10

101. $4^x-3^{x-\frac{1}{2}}=3^{x+0,5}-2^{2x-1}$

A) 0,5 B) 2,5 C) 1,2 D) 1,5 E) 0,8

102. $0,25^{|\sin x|}=2\cdot\sqrt{2}$

A) ∅ B) 0 C) $\frac{\pi}{6}$ D) 0,5 E) $\frac{3\pi}{8}$

103. $\sqrt[x]{5^{5\sqrt{x}}}=5^{\sqrt{x}-4}$

A) 12 B) 20 C) 16 D) 25 E) 18

104. $\sqrt[x^2-1]{32}\cdot\sqrt[x-1]{4}-\sqrt[x+1]{8}=0$

A) 4 B) $\varnothing$ C) 10 D) 6 E) 8

105. x quyidagi tenglama yechimi bo‘lsa $6\cdot15^{2x+4}=2\cdot3^{3x+1}\cdot5^{4x-4}$, $(2x+1)$ nimaga teng

A) 4 B) 9 C) 5 D) 3 E) 2,4

106. $2^{3x}\cdot7^{x-2}=(0,25)^{-x-1}$ tenglama yechimi x bo‘lsa, $(3x-2)$ nimaga teng.

A) 0 B) 3 C) 1 D) – 2 E) 4

107. x quyidagi tenglama $\sqrt[x-2]{5^{5-3x}}=0,04$ yechimi bo‘lsa, $(2x-3)$ nimaga teng.

A) – 1 B) 0 C) 2 D) 1,4 E) – 1,4

108. Tenglama yechimi topilsin $16\cdot3^{\sqrt{x-8}}=0,25\cdot\sqrt[\sqrt{x-8}]{6^{x-8}}$

A) 12 B) 24 C) 9 D) 44 E) 28

109. $\left(\frac{4}{9}\right)^x\cdot(3,375)^{x-1}=\left[0,(4)\right]^{-1}$

A) 3,4 B) 2,5 C) 5 D) – 3 E) 1,5

110. $4\cdot3^x-9\cdot2^x=5\cdot(6)^{\frac{x}{2}}$

A) 2 B) 4 C) 2,5 D) 4,5 E) – 1,5

111. Tenglama yechimlar yig'indisi topilsin

$$|2-x|^{\sqrt{x^2-x-2}}=|2-x|^2$$

A) 2 B) – 1 C) 3 D) 1 E) 5

Tenglama yechimi topilsin

112. $\log_x \sqrt[3]{0,(1)}=-0,(6)$

A) 9 B) $\frac{1}{27}$ C) 81 D) 15 E) $\frac{1}{9}$

113. $x(\lg 5-1)=\lg(2^x+1)-\lg 6$

A) 4 B) 10 C) 8 D) 1 E) 2

114. $0,5\left[\lg 5+lgx+\lg\sqrt{0,(1)}\right]=\lg\sqrt{15}$

A) 9 B) 3 C) 10 D) 5 E) $\frac{1}{9}$

115. $\log_4\left\{2\log_3\left[1+\log_2\left(1+3\log_2 x\right)\right]\right\}=0,5$

A) 4 B) 9 C) 0,5 D) $\frac{1}{4}$ E) 2

116. $\log_x 256=4\left(2-\sqrt{2}+\frac{2-\sqrt{2}}{\sqrt{2}}+\frac{2-\sqrt{2}}{2}+\ldots\right)$

A) $\frac{1}{4}$ B) 2 C) 6 D) 16 E) $\frac{1}{2}$

117. $\lg(3^x + x - 17) = x\lg 30 - x$

A) 10 B) 9 C) 17 D) 27 E) 4

118. Tenglama yechimlar ko‘paytmasi topilsin

$\log_3^2 6 - \log_3^2 2 = \left(lg^2 x - 3\right)\log_3 12$

A) 4 B) 5 C) 3 D) 1 E) 6

119. Tenglama yechimlar ko‘paytmasi topilsin $x^{3-4\log_2 x} = 0,5$

A) $\sqrt[4]{8}$ B) 4 C) 1 D) $\sqrt[4]{4}$ E) $\sqrt[4]{0,5}$

120. Tenglama yechimi topilsin $\log_4 x + \log_4^2 x + \log_4^3 x + \ldots = 1$

A) 8 B) $\frac{1}{4}$ C) 2 D) $\frac{1}{8}$ E) 4

121. Quyidagi tenglama $\log_2^2 x - \log_2 x^2 = \lg^2 3 - 1$ yechimlar yig‘indisi topilsin

A) 30 B) 33,(3) C) 16 D) 16,(3) E) 3,(3)

122. Tenglama yechimlar ko‘paytmasi topilsin

$$\frac{1-\log_2^2 x^2}{\log_2 x - 2\log_2^2 x} = \log_2 x^4 + 5\left(x > 0, x \neq 1\right)$$

A) 8 B) $\sqrt[4]{4}$ C) 0,5 D) $\sqrt[4]{\frac{1}{8}}$ E) $\sqrt[4]{2}$

Tenglama yechimi topilsin

123. $\sin x \cdot \mathrm{ctg}x + \cos x \cdot \mathrm{tg}x = 0,$

A) $k\pi$ B) $\frac{\pi}{4}+2k\pi$ C) $(-1)\frac{\pi}{4}+k\pi$ D) $2k\pi$ E) $k\pi-\frac{\pi}{4}$

124. $\sin x \cdot \cos x = \sin 31°$

A) $\varnothing$ B) $k\pi$ C) $\frac{\pi}{3}+k\pi$ D) $(-1)^k\frac{\pi}{3}+k\pi$ E) $\frac{\pi}{6}+2k\pi$

125. $\mathrm{Sin}(45° - x) \cdot \mathrm{Sin}(45° + x) = 0{,}5$

A) $(-1)^k\frac{\pi}{6}+k\pi$ B) $2k\pi$ C) $k\pi$ D) $\pm\frac{\pi}{6}+2k\pi$ E) $\frac{\pi}{3}+k\pi$

126. $\frac{\sqrt{3}+tgx}{1-\sqrt{3}tgx}=1,$

A) $\frac{\pi}{6}+2k\pi$ B) $180°k - 15°$

C) $45° + 180°k$ D) $(-1)\frac{k\pi}{4}+k\pi$ E) $30° + 2k\pi$

127. $\mathrm{Cos}1{,}5x = \mathrm{Cos}0{,}5x \cdot \mathrm{Cos}x$

A) $\pm\frac{\pi}{4}+2k\pi$ B) $(-1)^k\frac{\pi}{4}+k\pi$ C) $\frac{\pi}{6}+k\pi$ D) $k\pi$ E) $\varnothing$

128. $2\text{ctg}(180° - x) - \text{ctg}405° - \text{ctg}(360° - x) = 0,$

A) $180°k$ B) $\frac{\pi}{4} + k\pi$ C) $(-1)^k \frac{\pi}{4} + k\pi$ D) $2k\pi$ E) $135° + 180°k$

129. $1 - cosx = sinx \cdot \sin\frac{x}{2},$

A) $k\pi$ B) $(-1)^k \frac{\pi}{3} + k\pi$ C) $2k\pi$ D) $\pm\frac{\pi}{3} + 2k\pi$ E) $(-1)^k \frac{\pi}{6} + 2k\pi$

130. $\sin\left(x + \frac{\pi}{3}\right) \cdot \cos\left(x - \frac{\pi}{6}\right) = 1$

A) $90°k - 15°$ B) $90°k + 60°$ C) $(-1)^k \frac{\pi}{3} + k\pi$ D) $\pm\frac{\pi}{3} + 2k\pi$ E) $k\pi$

131. $\text{tg}5x + \text{tg}3x = 0$

A) $\frac{\pi}{3} + k\pi$ B) $\frac{k\pi}{3}$ C) $\frac{\pi}{6} + k\pi$ D) $\frac{k\pi}{8}$ E) $k\pi$

132. $\cos4x \cdot \cos2x = \cos5x \cdot \cos x$

A) $\pm\frac{\pi}{4} + k\pi$ B) $\frac{k\pi}{3}$ C) $(-1)^k \frac{\pi}{4} + k\pi$ D) $\frac{k\pi}{3}$ E) $k\pi$

133. $\sin^2 2x + \sin^2 x = 1$

A) $(-1)^k \frac{\pi}{3} + k\pi$ B) $\pm\frac{\pi}{6} + k\pi$ C) $\frac{\pi}{6} + \frac{k\pi}{3}$

D) $(-1)^k \frac{\pi}{6} + k\pi$ E) $k\pi$

134. $sinx - \sqrt{3}cosx = 1$

A) $(-1)^n \frac{\pi}{3} + n\pi$ B) $\pm\frac{\pi}{6} + k\pi$ C) $2n\pi$ D) $n\pi$ E) $(-1)^n \frac{\pi}{6} + \frac{\pi}{3}(1-3n)$

135. $tgx + tg\left(\frac{\pi}{4} + x\right) = -2$

A) $\frac{\pi}{4} + k\pi$ B) $(-1)^k \frac{\pi}{4} + k\pi$ C) $\pm\frac{\pi}{6} + 2k\pi$ D) $\pm\frac{\pi}{3} + k\pi$ E) $k\pi$

136. $\sin x \cdot \sin 7x = \sin 3x \cdot \sin 5x$

A) $\frac{\pi}{12} + k\pi$ B) $(-1)^k \frac{\pi}{4} + k\pi$ C) $\frac{k\pi}{4}$ D) $\frac{\pi}{4} + 2k\pi$ E) $\frac{2k\pi}{3}$

Tenglamalarning ko‘rsatilgan oraliqqa tegishli yechimlarini toping.

137. $\sin x = \sin 3x, x \in (0°;90°)$

A) $\frac{\pi}{3}$ B) $\frac{\pi}{12}$ C) $\frac{3\pi}{10}$ D) $\frac{5\pi}{12}$ E) $\frac{\pi}{4}$

138. $\cos 6x + \sin^2 3x = 0,\ x \in (90°;180°)$

A) $\frac{3\pi}{4}$ B) $\frac{5\pi}{6}$ C) $\frac{2\pi}{3}$ D) $\frac{7\pi}{10}$ E) $\frac{4\pi}{5}$

139. $4\sin^2 x + 4\sin x - 3 = 0,\ x \in (90°;180°)$

A) $\frac{\pi}{4}$ B) $\frac{4\pi}{5}$ C) $\frac{2\pi}{3}$ D) $\frac{5\pi}{6}$ E) $\frac{3\pi}{4}$

140. $\sin 6x + \cos 4x = 0, x \in (45°;90)$

A) 63° B) 40° C) 65° D) 50° E) 75°

141. $2tg\left(\frac{\pi}{4} - x\right) + tg\left(\frac{3\pi}{4} + x\right) = 1, \qquad x \in (0°, 200°]$

A) 75° B) 120° C) 180° D) 105° E) 135°

142. Sistema yechimlar yig'indisi $(x + y)$ nimaga teng

$$\begin{cases}(x-1)^2 - (x+2)^2 = 9y,\\ (y-3)^2 - (y+2)^2 = 5x\end{cases}$$

A) 4 B) -2^{-2} C) -5 D) 2 E) 5

143. Sistemada $\begin{cases}3^x + 3^y = 12,\\ 5^{x+y} = 125\end{cases}$ $(x + y)$ ning eng katta qiymatini toping

A) 4 B) -3 C) 5 D) 3 E) 7

144. $\begin{cases} \log_y x - 3\log_x y = 2, \\ \log_2 x = 4 - \log_2 y \end{cases}$

Sistema yechimlar ko'paytmasi nimaga teng

A) 16 B) 8 C) 3 D) 4 E) 5

145. Agar

$\begin{cases} \dfrac{7}{\sqrt{x-7}} - \dfrac{4}{\sqrt{y+6}} = 1,(6), \\ \dfrac{5}{\sqrt{x-7}} + \dfrac{3}{\sqrt{y+6}} = 2,1(6), \end{cases}$ bo'lsa, $(y - x)$ nimaga teng

A) – 4 B) 10 C) – 2 D) 8 E) 14

146. Sistema yechimlari x va y bo'lsa $\begin{cases} \sqrt[x]{x+y} = 2, \\ (x+y)\cdot 5^x = 100 \end{cases}$ $(3x - y)$ nimaga teng

A) 2 B) 4 C) – 3 D) 1 E) – 2

147. Sistemada $\begin{cases} \sqrt[x]{y} = 2, \\ y^x = 16. \end{cases}$ Kichik yechimlar ko'paytmasi nimaga teng.

A) 1,5 B) 3 C) – 0,5 D) 4 E) – 1,5

148. $\begin{cases} x+y+xy=11, \\ xy^2+yx^2=30 \end{cases}$ sistema uchun $(x+y)$ning katta qiymatini toping

A) 4 B) 12 C) 2 D) 6 E) – 6

149. Sistema yechimi topilsin $\begin{cases} sinx+siny=1 \\ x+y=\dfrac{\pi}{3} \end{cases}$

A) $x=\dfrac{\pi}{3}+n\pi, y=-\pi/3-n\pi$ B) $x=-\pi/3+2n\pi, y=n\pi$

C) $x=\pi/6+n\pi, y=\pi/3-2n\pi$ D) $x=\dfrac{\pi}{3}+2n\pi, y=\pi/6-2n\pi$

E) $x=\pi/6(1+12n), y=\pi/6(1-12n)$

150. Sistema $\begin{cases} (x-y)^{2y-x}=125, \\ \lg 2(x-y)=1 \end{cases}$ yechimida $(x+y)$ nimaga teng

A) 18 B) 21 C) 5 D) 12 E) 14

151. Agar $\begin{cases} cosx \cdot cosy=\dfrac{1}{6} \\ tgx \cdot tgy=2 \end{cases}$ bo'lsa $(x-y)$ nimaga teng

A) $(-1)^n \pi/3 + 2k\pi$ B) $\frac{\pi}{6} + k\pi$ C) $\pm\frac{\pi}{3} + 2k\pi$

D) $(-1)^k \frac{\pi}{6} + 2k\pi$ E) $2k\pi$

Tenglama yechimlar yig'indisi topilsin

152. $27x^2 + 99x - 126 = 0,$

A) – 3,(6) B) – 3,(2) C) – 3,6 D) 3,4 E) 3,65

153. $37x^2 + 138x + 101 = 0,$

A) – 3,27 B) $-3\frac{7}{37}$ C) – 3,7 D) $-3\frac{27}{37}$ E) $-3\frac{7}{27}$

154. $0,(9)\,x^2 - 1,(3)x + 0,1(6) = 0$

Tenglama ildizlari qiymatiga teskari qiymatli ildizlarga ega bo'lgan tenglama topilsin.

A) $9x^2 - 13x + 16 = 0$ B) $x^2 - 8x + 6 = 0,$

C) $x^2 - 1,3x + 16 = 0$ D) $9x^2 - 13x + 6 = 0$ E) $x^2 + 1,3x - 6 = 0$

155. $0,(6)\,x^2 - 0,5x - 0,(3) = 0$

Tenglama ildizlariga qarama-qarshi ishorali bo'lgan tenglama topilsin.

A) $6x^2 - 3x + 2 = 0$ B) $2x^2 - 3x + 2 = 0$

C) $4x^2 + 3x - 2 = 0$ D) $4x^2 - 5x - 3 = 0$ E) $2x^2 - 5x - 2 = 0$

156. Ildizlari $x_1 = 0,1(6)$ va x_2 bo'lgan $6x^2 + ax - 2 = 0$ tenglamada $(a + x_2)$ nimaga teng.

A) 9 B) – 13 C) – 8 D) 13 E) 4

157. Tenglama yechimi topilsin. 2,(6);1,2x) = – 3,(3)

A) 0,6 B) – 0,4 C) 0,(3) D) – 1,(3) E) – 0,(6)

158. a sonining qiymati qanday bo'lganda $3x^2-2x+a=0$. tenglamaning ildizlari musbat ishorali bo'ladi.

A) $a<3$ B) $0<a<\frac{1}{3}$ C) $\frac{1}{3}<a<3$ D) $a>\frac{1}{3}$ E) $0<a<3$

159. a sonining qiymati qanday bo'lganda $2x^2+4x+a=0$ tenglamaning ildizlari manfiy ishorali bo'ladi

A) $-3<a<2$ B) $a<2$ C) $a<-2$ D) $0<a<2$ E) $a<3$

160. Agar x berilgan $\log_{\frac{1}{\sqrt{3}}}(2x+5)=3^{\log_9 4}$ tenglama yechimi bo'lsa (x + 3) nimaga teng.

A) 2,(3) B) 1,6 C) 0,(6) D) – 0,(3) E) 1,3

161. $\log_2(5-2x)=5^{\frac{2}{\log_2 5}}$ tenglama yechimi x bo'lsa (6 + x)nimaga teng.

A) 0,5 B) – 2,5 C) – 3,5 D) 1,5 E) – 1,5

162. $\sqrt{2}\sin^2 x+Cosx=0$ tenglamaning (0;2π) oraliqdagi yechimlar yig'indisi topilsin

A) $\frac{11\pi}{4}$ B) 4π C) $\frac{5\pi}{4}$ D) $\frac{3\pi}{2}$ E) $\frac{7\pi}{3}$

163. (45°;150°) oraliqda $\sin\left(x-\frac{\pi}{3}\right)\cdot\cos(7x+30°)=0$ tenglamaning nechta ildizi bor.

A) 3 ta B) $\varnothing$ C) 1 ta D) 2 ta E) 5 ta

164. $3ctg\left(2x+\frac{\pi}{6}\right)=\sqrt{3}$ tenglamaning [–90°;150°) oraliqdagi yechimlar yig'indisi topilsin

A) 0 B) $\frac{4\pi}{9}$ C) $\frac{2\pi}{3}$ D) $\frac{5\pi}{12}$ E) $\frac{13\pi}{12}$

165. $\log_2\left(2^x+cosx-\frac{\sqrt{3}}{2}\right)=x$, tenglamaning $\left[-\pi/2;\frac{\pi}{2}\right]$ oraliqdagi yechimlari topilsin

A) $\pi/6;\frac{\pi}{8}$ B) $-\frac{\pi}{8};\frac{\pi}{4}$ C) $-\pi/6;\frac{\pi}{8}$ D) $-\frac{\pi}{6}$ E) $\pm\frac{\pi}{6}$

166. $tg^2\left(2x+\frac{\pi}{4}\right)=\frac{1}{3}$, tenglamaning [– 30°;75°) oraliqdagi yechimlar yig'indisi topilsin.

A) $\frac{\pi}{4}$ B) $-\frac{\pi}{12}$ C) $\frac{\pi}{3}$ D) $-\frac{\pi}{9}$ E) $-\frac{7\pi}{18}$

Tenglama yechimlar to'plamining bosh qiymati topilsin.

167. $0,5Sin0,75x=\sqrt{2}$

A) $\frac{2\pi}{5}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{3}$ D) $\frac{3\pi}{5}$ E) $\frac{2\pi}{3}$

168. $ctg(3x+30°)=-\sqrt{3}$

A) $\frac{2\pi}{7}$ B) $\frac{\pi}{3}$ C) $\frac{3\pi}{2}$ D) $\frac{2\pi}{9}$ E) $\frac{4\pi}{9}$

169. [– 30°;300°) oraliqdagi $1-tg(45°-x)=\frac{2cosx}{Sinx+cosx}$ tenglama yechimlar yig'indisi topilsin

A) 75° B) 270° C) 135° D) 105° E) 225°

170. $cosx-\sqrt{3}sinx=0$ tenglama $\left(\pi;\frac{3\pi}{2}\right)$ oraliqda nechta yechimga ega

A) 2 ta B) $\varnothing$ C) 1 ta D) 3ta E) 5 ta

171. Agar $\arccos(3x-2)=\frac{4\pi}{3}$

Tenglama yechimi x bo'lsa $(2-x)$nimaga teng.

A) – 0,5 B) $\frac{3}{4}$ C) 0,5 D) – 1,3 E)1,5

Tenglama yechimi topilsin

172. $arctg(1+x)+arctg(1-x)=\pi/4$

A) $x_1=\sqrt{2}, x_2=-\sqrt{2}$ B) $\sqrt{3}$ C) $-\sqrt{3}$

D) $x_1=\sqrt{3}, x_2=-\sqrt{2}$ E) $x_1=-\sqrt{3}, x_2=\sqrt{2}$

173. $\arccos\frac{x}{2}=2arctg(x-1)$

A) $\sqrt{3}$ B) $\sqrt{2}$ C) $\sqrt{3}-\sqrt{2}$ D) 0,5 E) $-\sqrt{2}$

174. $arctgx+\frac{1}{2}raccos\frac{1}{5x}=\frac{\pi}{4}$

A) 0,5 B) $\frac{1}{4}$ C) $\pm\frac{1}{2}$ D) $\pm\frac{1}{3}$ E) $\frac{2}{3}$

175. $arctg(2+cosx)-arctg(1+cosx)=\frac{\pi}{4}$

A) $\frac{\pi}{2}+k\pi$ B) $\pi(k+1)\ k\in z$ C) $\pi+2k\pi, k\in Z$

D) $\pm\frac{\pi}{3}+k\pi$ E) $\pm\frac{\pi}{6}+2k\pi,\ k\in Z$

176. $tg5x=tgx$,

A) $\frac{\pi}{4}+k\pi,\ k\in Z$ B) $\frac{2k\pi}{3}\ k\in Z$ C) $\frac{\pi}{3}+k\pi,\ k\in Z$

D) $\frac{3k\pi}{4},\ k\in Z$ E) $\frac{k\pi}{4},\ k\in Z$

177. $\begin{cases} 4^{3x}-3^{y}=-26, \\ 4^{x}-3^{\frac{y}{3}}=-2, \end{cases}$ Sistema yechimi x va y bo‘lsa $(2x + 3y)$ nimaga teng

A) 9 B) 3 C) – 3 D) 6 E) – 4

178. $\begin{cases} 2^{1+2\log_2(y-x)}=32, \\ 2\log_5(2y-x-12)=\log_5(y-x)+\log_5(x+y); \end{cases}$

Sistema yechimlar yig‘indisi topilsin

A) 20 B) 36 C) 16 D) 18 E) 22

179. $\cos 8x + 3\cos 4x = 1$, tenglama eng kichik musbat gradus yechimi topilsin.

A) 35° B) 20° C) 65° D) 15° E) 45°

180. $\sqrt{2^{x^2-2x-10}}=\sqrt{33+\sqrt{128}}-1$ tenglama ildizlari yig‘indisi topilsin

A) 5 B) – 3 C) 2 D) 4 E) – 1

181. $\cos 2x = x$ tenglama nechta haqiqiy yechimga ega.

A) 1ta B) 3ta C) $\varnothing$ D) 2ta E) 5 ta

182. $x + |x| = 4x + 5$ tenglama yechimi x bo'lsa $(0,5 - x)$ nimaga teng.

A) 3 B) – 1,5 C) 4,2 D) – 3,5 E) 3,2

183. $\arccos x = \operatorname{arctg} x$, tenglama ildizlar ko'paytmasi topilsin.

A) $\frac{\sqrt{5}}{2}$ B) 2 C) $\frac{\sqrt{5}-1}{2}$ D) 1 E) $\sqrt{5}+1$

184. $\frac{\cos 2x}{0,5\sqrt{2}+\sin x}=0$, tenglamaning $[0,\ 4\pi]$ oraliqda nechta yechimi bor

A) 1 ta B) 4 ta C) ∅ D) 3ta E) 2ta

185. $\begin{cases} \cos x+\cos y=-1,6, \\ x+y=120° \end{cases}$ tenglamalar sistemasi $(-\pi,\ 3\pi)$ oraliqda nechta yechimga ega

A) 2ta B) 1 ta C) ∅ D) 3ta E) 5ta

186. $\begin{cases} 2^x+3^y=12, \\ 2^y\cdot 3^x=18 \end{cases}$ sistema yechimi x va y bo'lsa $(2x + y)$ nimaga teng?

A) 3 B) 8 C) 6 D) 2 E) 5

187. $\sin\frac{10\pi}{x}=0$, tenglamaning nechta butun yechimi bor

A) 8 ta B) 2 ta C) 1 ta D) 3 ta E) ∅

188. $arc\cos^2 x - \frac{2\pi}{3} arccosx + \frac{\pi^2}{12} = 0$, tenglama yechimi x bo‘lsa $(2x - 1)$ nimaga teng ?

A) $\frac{1}{\sqrt{2}}$ B) $\frac{\sqrt{3}}{3}$ C) $\sqrt{3}-1$ D) 0,5 E) $\sqrt{3}$

189. Tenglama yechimi topilsin $\cos x^2 = 1$

A) $k\pi$ B) $\pm\sqrt{2\pi|k|}$, $k \in Z$ C) $\frac{k\pi}{2}$, $k \in Z$ D) $\pm\sqrt{\pi|k|}, k \in Z$ E) $\varnothing$

190. $0,3x^2 - 5\sqrt{3}x + 0,8(3) = 0$,

Tenglama ildizlarining o‘rta proporsional qiymati topilsin.

A) 1,6 B) $1\frac{2}{3}$; C) 1,4 D) 1,(6) E) 1,(3).

191. $x^2 - 2ax - c(2a + c) = 0, (a > 0, c > 0)$ tenglama ildizlar ayirmasi nimaga teng?

A) $2(a + c)$; B) $2a + c$; C) $0,5(a + c)$ D) $a - c$ E) $a - 2c$

192. Ildizi $[- 2(1 + 2i)]$ bo‘lgan haqiqiy koeffitsientli kvadrat tenglama topilsin

A) $2x^2 + x + 10 = 0$; B) $x^2 + 4x + 20 = 0$;
C) $2x^2 - 3x - 10 = 0$; D) $2x^2 - 5x - 4 = 0$; E) $x^2 - 5x - 10 = 0$

193. Agar $\left[0,(4)x-0,4\right]\cdot\left[0,5x-1\frac{3}{4}\right]=0$ bo'lsa $\left(0,5x-1\frac{3}{4}\right)$ ifoda qabul qiladigan qiymatlar yig'indisi topilsin.

A) 2,7; B) – 1,3; C) 2,2 D) 3,5; E) – 4,8

194. Agar $ax^2 + 3x - 2 = 0$,

Tenglama bitta yechimi 0,5 bo'lsa, tenglama koeffisientlar ko'paytmasi topilsin.

A) 6; B) – 6; C) 8; D) – 8; E) – 12

195. a va b qanday bo'lganda $0,5x^2 + 2ax + b = 0$, tenglama bitta yechimi nol , ikkinchi yechimi musbat bo'ladi.

A) $b < 0, a > 0$; B) $b > 0, a = 0$

C) $b = 4, a > 0$ D) $b = 0, a < 0$, E) $b = -4, a = 0$

196. a va b qanday bo'lganda $2ax - 0,4x^2 + 5b = 0$, tenglama ildizlarida $|x_1^-| = |x_2^+|$ bajariladi

A) $a < 0, b = 0$; B) $a = 5; b = 10$; C) $a = 0, b < 0$

D) $a > 0, b < 0$, E) $a < 0, b = 0$

197. Tenglama yechimi topilsin $\sqrt{1+x}+\sqrt{1-x}=1$

A) 0,45; B) $\varnothing$; C) – 0,35; D) 0,25; E) 0,035

198. $\log_2 (3x^2 - x) = 1$ tenglamaning $y=\sqrt{2-5x}$ funksiyaning aniqlanish sohasidagi yechimi topilsin.

A) 1; B) 0,6; C) 0,75; D) – 0,(6) E) – 0,45

199. $16^{\sin^2 x}+16^{\cos^2 x}=10$, tenglamaning barcha ildizlari topilsin.

A) $x=\pm\frac{\pi}{6}+\frac{k\pi}{2}$; B) $x=\pm\frac{\pi}{3}+k\pi$; C) $x=\frac{\pi}{6}+k\pi$

D) $x=\frac{\pi}{3}+k\pi$; E) $x=\pm\frac{\pi}{4}+k\pi$

200. $x^{\log_2 x^3-\log_2^2 x-3}=\frac{1}{x}$, Tenglama ildizlar yig'indisi topilsin

A) 6; B) 4; C) 7; D) 3; E) 2

201. $14^{\log_7 2}\cdot x^{\log_7 4x+1}=1$

A) $\frac{5}{14}$; B) $\frac{4}{7}$; C) $\frac{3}{7}$; D) $\frac{9}{14}$; E) $\frac{5}{7}$

202. Tenglama yechimi topilsin $\left(\sqrt{7+4\sqrt{3}}\right)^x+\left(\sqrt{7-4\sqrt{3}}\right)^x=14$

A) $x=2, x=-3$; B) $x=3; x=-2$; C) $x=\pm 3$ D) $x=1, x=3$ E) $x=\pm 2$

203. $\sin x+\sin 2x+\sin 3x+\sin 4x+\sin 5x=0$, tenglamaning $[0;\pi]$ kesmaga tegishli ildizlar yig'indisini toping

A) 386° B) 426° C) 396° D) 576° E) 528°

204. Ushbu Sin3x – Sin4x + Sin5x = 0 tenglamaning nechta ildizi $|x| \le \frac{\pi}{2}$ tengsizlikni qanoatlantiradi

A) 7 ta B) 4 ta C) 6 ta D) 3 ta E) 5 ta

205. $\cos 6x - Sin\left(\frac{\pi}{2} + 4x\right) = 0$, tenglamaning $\left[0; \frac{\pi}{2}\right]$ kesmaga tegishli ildizlar yig'indisi topilsin

A) 126° B) 108° C) 216° D) 98° E) 164°

206. $Sin\left(\frac{\pi}{0,3x}\right) = 0$ tenglamaning nechta ildizi [1;3] kesmaga tegishli.

A) 2 ta B) ∅ C) 3 ta D) 1 ta E) 4 ta

207. $\cos^3 x = 0,75 \cos x$

Tenglamaning (0;90°)dagi eng katta yechim topilsin

A) 50° B) 80° C) 35° D) 65° E) 70°

208. Ushbu $(0,5Sin\ \pi x - \pi) \cdot \left(2cos\pi x - \sqrt{3}\right) = 0$

Tenglamaning eng kichik musbat ildizi topilsin

A) 0,3 B) 0,6 C) 0,25 D) 0,1(6) E) 0,(3)

TENGLAMADAGI TEST TOPSHIRIQLARI JAVOBLARI

1	2	3	4	5	6	7	8	9	10
D	C	A	B	E	B	C	A	E	D
11	12	13	14	15	16	17	18	19	20
C	A	D	B	E	A	C	B	E	D
21	22	23	24	25	26	27	28	29	30
A	C	E	B	A	D	E	B	C	B
31	32	33	34	35	36	37	38	39	40
B	D	A	C	E	B	A	C	B	D
41	42	43	44	45	46	47	48	49	50
C	A	D	C	B	E	A	a) C	B	E
							b) D		
51	52	53	54	55	56	57	58	59	60
D	B	A	C	C	B	A	D	E	B
61	62	63	64	65	66	67	68	69	70
D	A	C	B	E	A	C	E	D	D
71	72	73	74	75	76	77	78	79	80
B	C	A	E	D	C	D	E	B	C
81	82	83	84	85	86	87	88	89	90
A	B	D	E	C	B	D	C	A	E
91	92	93	94	95	96	97	98	99	100
B	C	A	D	C	B	E	A	C	B

101	102	103	104	105	106	107	108	109	110
D	A	D	C	B	E	A	D	C	B
111	112	113	114	115	116	117	118	119	120
D	C	D	A	E	B	C	D	A	C
121	122	123	124	125	126	127	128	129	130
B	D	E	A	C	B	D	E	C	A
131	132	133	134	135	136	137	138	139	140
D	B	C	E	D	C	E	B	D	A
141	142	143	144	145	146	147	148	149	150
C	B	D	A	E	B	C	D	E	B
151	152	153	154	155	156	157	158	159	160
C	A	D	B	C	A	E	B	D	C
161	162	163	164	165	166	167	168	169	170
A	B	D	C	E	A	C	D	B	C
171	172	173	174	175	176	177	178	179	180
E	A	B	D	C	E	A	B	D	C
181	182	183	184	185	186	187	188	189	190
B	A	D	B	C	E	A	C	B	D
191	192	193	194	195	196	197	198	199	200
A	B	C	E	D	C	B	D		
201	202	203	204	205	206	207	208		
	E	D	A	B	C	E	D		

MASHQLAR JAVOBLARI

2.1) yo‘q; 2) ha; 3) yo‘q. **4.** 1) yo‘q; 2) ha; 3) yo‘q. **6.** 1) – 0,5; 2) 0.25. **8.** 1) yo‘q; 2) ha. **10.** 1) 0,6; 2) $\frac{15}{16}$. **12.** 1) $\frac{1}{3}$; 2) 0,75; 3) – 4. **14.** 1) – 2,25; 2) 1,(6); 3) 0. **16.** 1) $\varnothing$; 2) 180; 3) 3. **18.** 1) chek ko‘p; 2) 6,(6); 3) – 17,8(3). **20.** 1) $\varnothing$; 2) 8. **22.** 1) – 1; 2) 4,5; 3) chek ko‘p. **24.** 1) $\varnothing$; 2) 1; 3) 0,25; 4) 0,2. **26.** 1) 3; 2) 0,8(3). **28.** 1) 2 va (– 3); 2) 0. **30.** 1) $x = 3$ yoki $y = -3$; 2) $x = y = 2$. **32.** 1) $\frac{a+5}{3a-1}$; 2) – 0,5. **34.** 1) $a \neq 3$ da $x = \frac{3a+b}{3-a}$, $a = 3, b \neq -9$ da $\varnothing$, $a = 3$, $b = -9$ da $x \in R$; 2) $a \neq \pm 2$ da $x = \frac{1}{a-2}$; $a = 2$ da $\varnothing$; $a = -2$ da $x \in R$. **36.** $x < -2$ da $3x - 10$; $-2 < x < 0$ da; $0 < x < 24$ da $-x-6$; $x > 4$ da $2 - 3x$. **38.** 3. **40.** 1) yo‘q; 2) ha. **42.** 1) ha; 2) yo‘q. **44.** 1) ± 3; 2) ± 2,5. 3) ± 0,(1). **46.** 1) – 14 va 2; 2) – 1 va 2. **48.** 1) – 8 va 28; 2) – 15 va 1; 3) $\frac{5}{12}$ va $1\frac{11}{12}$. **50.** 1) $\varnothing$; 2) 0,625. **52.** 1) $\varnothing$; 2) $x \in (-\infty; 0,6]$; 3) – 4 va 4. **54.** 1) – 2 va 1,(6); 2) – 1. **56.** $2(a - b)$. **58.** $-\varphi - \alpha$. **60.** $a > b > 0$ da $a - 2b$; $b > a$ da $-a$; $b < 0$ da a. **62.** 1) 3; 2) 1,(4). **64.** $\varnothing$. **66.** 1) 0,5; 2) $\varnothing$. **68.** 1) 43; 2) 0; 3) $7a^3 + 9a^2 - a^4$. **70.** – 6. **72.** $5\frac{2}{13}$. **74.** 1. **76.** 1) $x = 2$; $y = 3$; 2) $\varnothing$; 3) $x = k, y = \frac{3k}{5}$, $x \in R$. **78.** 1) $x = 2$, $y = -1$; $z = 3$. 2) $z = k$, $x = \frac{5-k}{7}$, $y = \frac{3+5k}{7}$. 3) $\varnothing$ **80.** 1) $x = y = 0$; 2) $x = k$, $y = -1{,}5k$. **82.** 1) $x = y = z = 0$; 2) $x = 3k$, $y = -k$, $z = -5k$ $k \in z$. **84.** 1) $x = 10k$, $y = 8k$, $z = 7k$ $k \in z$; 2) $z = 2(x + 2y)$. **86.** 1) $\varnothing$; 2) $x = 1, y = -2$. **88.** 1) $a \neq 2{,}5$ da $x = \frac{b}{5-8a}$, $a = 2{,}5$, $b \neq 0$ da $\varnothing$; $a = 2{,}5$, $b = 0$ da chek.ko‘p.yech. 2) $a \neq 9$ da $x = \frac{b+2a}{9-a}$, $a = 9$, $b \neq -18$ da $\varnothing$, = $a = 9$ va $b = -18$ da chek. ko‘p.yech. **90.** 23,(3). **92.** $a \neq -1{,}(3)$, $b \in R$. **94.** $a = -8$ da $x = 2$, $y = 1$. **96.** 2. **98.** $a \neq -4$. **100.** – 6. **102** . $a = -1$ da. **104.** 1) 0,(3); 2) – 0,5; 3) $2i$; 4) $1 - 2i$; 5) 1. **106.** yo‘q **108.** 1) yo‘q; 2) yo‘q. **110.** 1) rats. ildizga ega; 2) kompleks ildizi; 3) irratsional ildizga ega. **112.** 1) ha; 2) yo‘q; 3) ha; 4) yo‘q. **114**. 1) ± 6; 2) ± 0,25; 3) $\varnothing$; 4) 0 va 2,8; 5) ± 5; 6) 0 va (– 5). **116.** 1) $x_1 = x_2 = -1{,}(3)$; 2) (– 2,8(3)) va 3; 3) $\sqrt{2}$ va $4\sqrt{2}$; 4) $-3{,}8$; 5) 0, -0 va -68; 6) 3 va 4,(3). **118.** 1) $a-b$, $a+b$; 2) $\frac{a}{b}, \frac{b}{a}$; 3) a, b. **120.** 1) $2 \pm i\sqrt{2}$; 2) $0{,}5 \pm i$; 3) $3 \pm \sqrt{2}i$; 4) $2i, 2+i$; **122.** 1) 6; 2) 14; 3) 1 va ($-2{,}8$), 4) 1 va ($-2{,}(27)$); 5) -1 va ($-3{,}(7)$); 6) 1 va $\frac{1}{390}$. **124.** 1) 28, 13,75; 2) $4\sqrt{3}$, -7; 3) 3,(3) va 2,(6). **126.** 1) $(x-2)(5-2x)$; 2) $(x-a+1)(x+3+2a)$; 3) $\left(x-3-\sqrt{2}i\right)\left(x-3+\sqrt{2}i\right)$; 4) $\left(x-4+\sqrt{3}\right)\left(x-4-\sqrt{3}\right)$.

128. 1) $|x^-| > |x^+|$; 2) $|x^+| > |x^-|$; 3) x_1^-, x_2^-; 4) x_1^+, x_2^+. **130.** 1) $x=0$ va $x=1$; 2) $[-2;1]$; 3) $x \in R$; 4) $x=0$ va $x=4$. **132.** 1) $24x^2+38x+15=0$; 2) $x^2-2ax+a^2-b^2=0$; 3) $x^2-6x+10=0$; 4) $x^2-4x+6=0$; **134.** 1) $x^2-\left(3-\sqrt{3}\right)x+2\left(1-\sqrt{3}\right)=0$;

2) $x^2-\sqrt{2}\left(1+\sqrt{3}\right)x+2\sqrt{3}=0$; 3) $x^2-(2-2i)x+4(2+i)=0$.

136. 1) $+4$ va 1; 2) $2 \pm \sqrt{3}i$. **138.** 3 va 4 da. **140.** 2. **142.** $a=6$, $b=-72$. **144.** 1) $\frac{x+5}{2x+1}$; **148.** 1) $-0{,}(6)$; 2) 0,5 **150.** 1) -2; 2) 4; 3) 5; 4) -6 **152.** $3x^2+5x-2=0$ **154.** 2 va ($-0{,}3$) **156.** $b<0$ **158.** $a>0$, $b \in R$ **160.** 5. **162.** 1) $x^4+3x^3-6x^2-8x=0$; 2) $x^4-3x^3-x^2+13x-10=0$; 3) $2x^3-5x^2+6x-2=0$ **164.** 1) $2{,}2\left(\cos\frac{2\pi}{5} \pm iSin\frac{2\pi}{5}\right)$; 2) $\left(-\cos\frac{\pi}{5} \pm iSin\frac{\pi}{5}\right)$;

2) $\pm\sqrt[4]{2}\left(\cos 7°30^1 + iSin7°30^1\right)$, $\sqrt[4]{2}\left(Sin7°30^1 icos7°30^1\right)$

3) $-0{,}5, 0{,}25\left(1 \pm i\sqrt{3}\right)$. **166.** 1) $\pm 5, \pm 3$; 2) $\pm 2, \pm 3i$ **168.** 1) $(x-2)(x+2)$ $(2x-1)(2x+1)$; 2) $(x-2)(x+2)(3x-2i)(3x+2i)$

170. 1) $\pm 1, \pm 2, \pm i, \pm 2i$ 2) $0, \pm\sqrt{2{,}5}i$; 3) 0,3 va $\frac{3 \pm i\sqrt{7}}{2}$; 4) $-2{,}1$ va $\frac{-1 \pm i\sqrt{23}}{2}$ **172.** 1) $-1{,}5, 2 \pm \sqrt{2}$; 2) $x_1 = x_2 = -4$, $x_{3,4} = -4 \pm \sqrt{6}i$

174. $2, 3, \frac{3 \pm i\sqrt{3}}{2}$ **176.** 1) $\pm 2{,}5$ va $\pm 2{,}5i$; 2) $-0{,}4$ va $0{,}2\left(1 \pm i\sqrt{3}\right)$

178. 1) 1, -2 va $0{,}5\left(1 \pm \sqrt{5}\right)$; 2) 1 va $0{,}5\left(-1 \pm \sqrt{13}\right)$ **180.** 0,5, -1 va

$0,5\pm\sqrt{7}i$ **182.** ± 1 va $\frac{3\pm\sqrt{13}}{2}$ **184.** 1) 0,5,2 va $\left(-2\pm\sqrt{3}\right)$
2) 1,2; 2,5 va 5 **186.** 1) mavjud emas; 2) 0,5 va (– 1,5) **188.** 1) 0,5; 2 va $0,5\left(-3\pm\sqrt{5}\right)$; 2) – 6 va $-1\pm i\sqrt{2}$ **190.** 1) 0; – 3; – 0,5;0,(3) va 2
2) – 1; 0,5 va $0,5\left(-1\pm i\sqrt{7}\right)$ **192.** 1) $\left(x+i\sqrt{7}\right)\left(x-i\sqrt{7}\right)\left(x^2+i\sqrt{3}\right)\cdot\left(x^2-i\sqrt{3}\right)$; 2) $(x+2)(x-2)\left(x\pm i\sqrt{2}\right)\left(x-i\sqrt{2}\right)$ **194.** $\varnothing$
196. $0,5\left(-5\pm\sqrt{13}\right)$, $0,5\left(3\pm\sqrt{3i}\right)$ **198.** $0,5\left(1\pm\sqrt{5}\right)$ va $\left(2\pm\sqrt{5}\right)$
200. ± 3 **202.** – 4 va 9 **204.** 1) $\varnothing$; 2) $x\neq 1$, $x\neq 2$; 3) $-\frac{1}{3}\leq x<9$;
4) $x>-1$ **206.** 1) $\varnothing$; 2) $\varnothing$; 3) $\varnothing$ **208.** 1) (– 1) va 2; 2) 28; 3) 0 va 5
4) (– 1) va 4; **210.** 1) 1; 2) 8 **212.** 6,(1) **214.** 2 **216.** 6 **218.** 19 va 84
220. 1 va (– 4). **222.** 1) (1;3), (3;1); 2) (2;1) (3;3). **224.** (4;1).
226. (3;2) va (– 2; – 3) **228.** (1;0) **230.** (9;4) **232.** (1;3), (– 3; – 1).
234. $\left(\pm 3\sqrt{3};\pm\sqrt{3}\right)$, (± 4; ± 5) **236.** (5;1), $\left(-11;-\frac{11}{5}\right)$ **238.** (2; ± 3),
$\left(9;\pm\sqrt{2}\right)$ **240.** $\left(\frac{44}{7};-\frac{39}{7}\right)$ **242.** (+ 2; ± 3), (± 3; ± 2) **244.** 1) $\pm\frac{\sqrt{30}}{3}$;
2) ±0,3 3) $\varnothing$ **246.** 1) 4, 2) 8 va 27 **248.** 2 **250.** 3 **252.** 4 **254.** $\frac{5}{7}$ **256.** 2 **258.** – 1
260. 2 **262.** 1 **264.** $x=\log_2 3-1$ **266.** $x_1=0$, $x_2=\log_5 3$ **268.** ± 2 **270.** 6
272. 1) $2<x<3$ 2) (3; + ∞) **274.** 1) $\left(\frac{1}{2};\frac{1}{\sqrt{2}}\right)$ 2) $x>0, x\neq 1, x\neq\sqrt{10}$
276. 8,8 **278.** 75 **280.** 1) $3^{\frac{1}{2}}$; 2) $\sqrt{2}-1$ **282.** $a=0,5(b+1)$ **284.** 0 **286.** 3
288. $x=1$ va $x=2$ **290.** $\frac{1}{2}$ va $\frac{1}{4}$ **292.** 10 va 0,1 **294.** $x=2\frac{\lg 3}{\lg 6}$ **296.** $\frac{1}{9}$
va 9 **298.** 0,01 va $\sqrt[3]{10}$ **300.** 10 va 3 **302.** 3 va $\frac{1}{3}$ **304.** $\frac{1}{14}$ va $\frac{1}{2}$ **306.** 2
308. $\varnothing$ **310.** 0,04 **312.** (1;2) **314.** (7;3) **316.** (4;1) **318.** (13;8) **320.** (3;9)
322. 4;2 **324.** (8;10) **326.** $\left(\sqrt[3]{2};\frac{1}{\sqrt{2}}\right)$ **328.** (2;6) **330.** 1) 2,5; 2) – 1

332. 1) $75° + 90°k, k \in Z$; 2) $\frac{1}{9}(\pm\pi + 6k\pi + 1), k \in Z$

234. 1) $\frac{\pi}{2}\left(+\frac{1}{6}+1-k\right)k \in Z$; 2) $\frac{\pi}{3}\left(\pm\frac{2}{3}+2k+1\right)k \in Z$

336. 1) $k\pi - 30°k \in Z$; 2) $40°(3k-1), \frac{2\pi}{3}(1+3k), k \in Z$

338. $\pi(2k+1), k \in Z, \frac{4\pi}{3}(\pm1+3k), k \in Z$ **340.** $\frac{k\pi}{16}\left(4+(-1)^{k+1}\right), k \in Z$

342. $\frac{\pi}{8}(4k+1), k \in Z$ **344.** $\frac{\pi}{8}(4k-1), k \in Z$ **346.** $\frac{\pi}{12}(2k+1), k \in Z$

348. $\frac{\pi}{4}(8k+1); k \in Z$ **350.** 45° **352.** $k\pi,\ k \in Z$ **354.** $\frac{\pi}{8}(4k+1), k \in Z$

356. $k\pi, \frac{\pi}{4}+k\pi, k \in Z$ 358. $\pi(6k \pm 1),\ k \in Z$ **360.** 129°

362. $\frac{\pi}{12}(4k+1), k \in Z$ **364.** $\frac{\pi}{2}(4k+1), k \in Z$ **366.** 45° **368.** $\varnothing$

370. 3 ta **372.** 4 ta **374.** (– 12) **376.** 30° **378.** 8 **380.** $x = 0$ **382.** 1

384. $\frac{\pi}{4}+2k\pi, k \in Z$ **386.** $\frac{\pi}{4}+k\pi, k \in Z$ **388.** $\varnothing$ **390.** $\frac{\pi}{6}$ va $\frac{5\pi}{6}$

392. $x = \frac{\pi}{6}+2k\pi, y = \frac{\pi}{6}-2k\pi,\ k \in Z$

394. $x = \pi/6 + k\pi, y = \frac{\pi}{6}-k\pi, k \in Z$

396. $x = \frac{\pi}{2}+k\pi, y = k\pi - \frac{7\pi}{6}, k \in Z$ **398.** 0,5 **400.** $x = 0$ va $x = 0{,}5$

402. $\sqrt{2}$ **404.** $x = 0, x = \frac{1}{2}$ va $x = -\frac{1}{2}$

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