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OLIY TA`LIM, FAN VA INNOVASIYALAR VAZIRLIGI

NAMANGAN DAVLAT UNIVERSITETI

Matematik analiz кafedrasi

“TRIGONOMETRIYADAN MASALALAR YECHISH”

fanidan

O’QUV– USLUBIY

MAJMUA

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Tuzuvchi:

M.Xolmurodov, NamDU dotsenti.

Taqrizchilar: A.Mashrabboev, NamDU dotsenti

R.Ibragimov-f.m.f.n., NamDU prof.

O'quv uslubiy majmua Namangan davlat universiteti Kengashining 2023 yil "28…" avgustidagi ".1.." – son yig'ilishida ko'rib chiqilgan va foydalanishga tavsiya etilgan.

Uslubiy majmuadada «Trigonometriyadan masalalar yechish» fanining «Matematika o‘qitishning umumiy metodikasi», «Matematika o‘qitishning maxsus metodikasi», «Matematika o‘qitishning aniq metodikasi», bo‘limlari bo‘yicha ta’lim texnologiyalari, ularni qo‘llash bo‘yicha uslubiy tavsiyalar bayon etilgan. Ushbu tavsiyalar didaktik tamoyillar, ma’ruza va amaliy mashg‘ulotlar texnologiyalarini ishlab chiqish usul va vositalari, ularning muhim belgilaridan iborat ta’limni texnologiyalash qoidalarini hisobga olgan holda loyihalashtirilgan.

Uslubiy qo‘llanma pedagogika oliy ta’lim muassasalari o‘qituvchilari va talabalari, matematika fanlarini o‘qitishda zamonaviy pedagogik va axborot texnologiyalarini qo‘llash masalasiga qiziquvchilar uchun mo‘ljallangan.

1-Mavzu: Trigonometrik funksiyalarning ta'riflari , xossalari, grafiklari, hamda ifodalarning qiymatlarini hisoblashda, soddalashtirishda, hamda trigonometrik tenglamalarni va tengsizliklarni yechishda qo'llaniladigan asosiy formulalar.

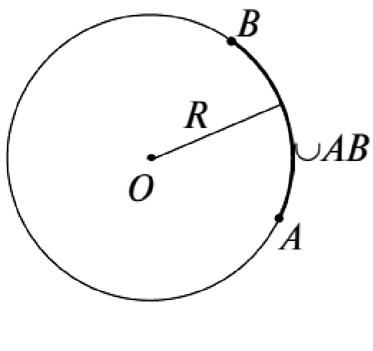
1. Burchaklar va yoylar. Markazi O nuqtada bo'lgan R radiusli aylanadagi A va B nuqtalar uni ikki qismga − *yoylar*ga ajratadi (I.1-rasm). Yoyning A va B nuqtalari *yoyning uchlari*, qolgan nuqtalari esa *yoyning ichki nuqtalari* deyiladi.

Uchlari A va B nuqtalar bo'lgan yoy uning faqat uchlarini ko'rsatish orqali $\cup AB$ ko'rinishda yoki uchlari A va B nuqtalar bo'lgan yoylarni bir-biridan farqlash uchun yoyning uchlari va yoyning biror ichki K nuqtasini ko'rsatish orqali $\cup AKB$ ko'rinishda belgilanadi (I.2-rasm).

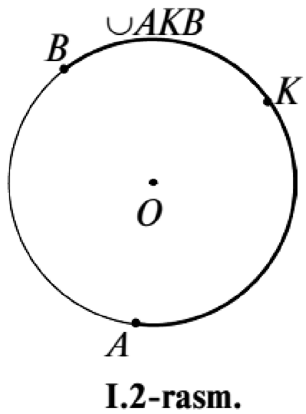
Agar AB kesma aylananing diametri bo'lsa, AB yoy *yarim aylana* deyiladi. Agar AB kesma aylananing diametri bo'lmasa va AB yoyning har qanday ichki nuqtasini aylananing markazi bilan tutashtiruvchi kesma AB kesmani kesib o'tsa (kesib o'tmasa), AB yoy *yarim aylanadan kichik* (mos ravishda *yarim aylanadan katta*) deyiladi.

Aylananing markazidan chiquvchi va berilgan yoyni kesib o'tuvchi barcha nurlardan tashkil topgan yassi burchakni *berilgan yoyga mos markaziy burchak*, berilgan yoyni esa shu *markaziy burchakka mos yoy* deb ataymiz (I.3-rasm).

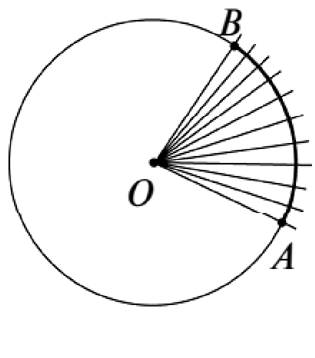
Yoy uzunligi metr (m) va uning ulushlarida, shuningdek, fut, duym, angstrem, mikronlarda ham o'lchanadi (1 fut = =12 duym $\approx$ 30,479 sm, 1 angstrem = $1 \cdot 10^{-8}$ sm, 1 mikron= $=1 \cdot 10^{-3}$ mm).



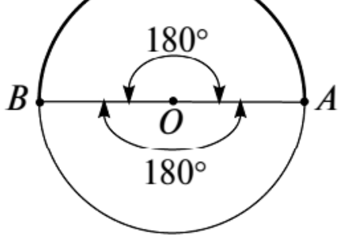
I.1-rasm.



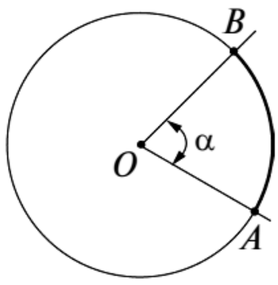
I.2-rasm.



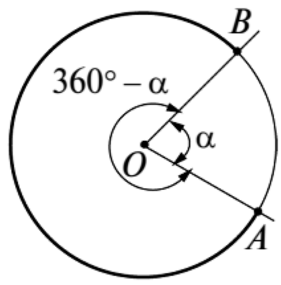
I.3-rasm.



I.4-rasm.



I.5-rasm.



I.6-rasm.

Geometriya kursidan ma'lumki, markaziy burchakning gradus o'lchovi va yoyning gradus o'lchovi quyidagicha aniqlanadi:

1) yarim aylanaga mos markaziy burchak 180° ga teng (I.4-rasm);

2) yarim aylanadan kichik AB yoyga mos markaziy burchakning gradus o'lchovi OA va OB nurlar hosil qilgan odatdagi burchakning gradus o'lchoviga teng (bu yerda O — aylana markazi, I.5-rasm);

3) yarim aylanadan katta yoyga mos markaziy burchakning gradus o'lchovi $360° - \alpha$ ga teng, bu yerda α — to'ldiruvchi burchak (yarim aylanadan kichik yoyga mos markaziy burchak)ning gradus o'lchovi (I.6-rasm).

4) yoyning gradus o'lchovi shu yoyga mos markaziy burchakning gradus o'lchoviga teng (I.7-rasm).

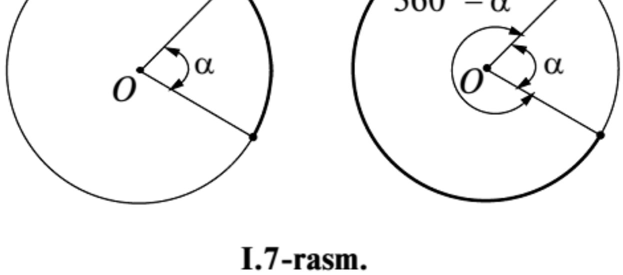
Burchak va yoylarning burchak kattaligini o'lchashda gradusning ulushlaridan ham foydalanishga to'g'ri keladi. Gradus va uning ayrim ulushlari orasidagi bog'lanishlarni keltiramiz:

$$1° = 60' \text{ (minut, daqiqa)}, \quad 1' = 60'' \text{ (sekund, soniya)}.$$

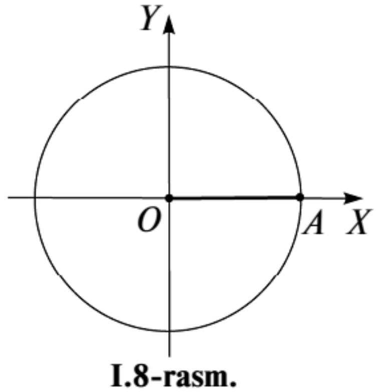
Buyuk o'zbek olimi Mirzo Ulug'bek o'z asarlarida sekundning $\frac{1}{60}$ ulushi solisa(tersiy)dan ham foydalangan. Uning asarlarida

$$1 \text{ daraja} = 60 \text{ daqiqa}, \quad 1 \text{ daqiqa} = 60 \text{ soniya},$$
$$1 \text{ soniya} = 60 \text{ solisa(tersiy)}$$

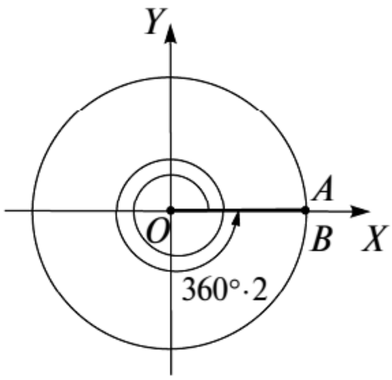
ekanligi keltiriladi.



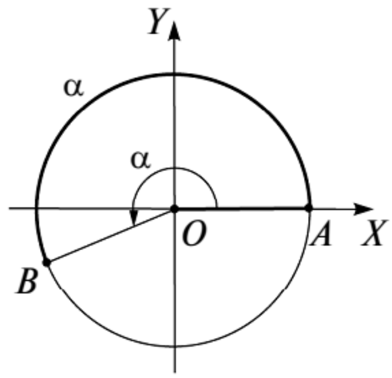
I.7-rasm.



I.8-rasm.



I.9-rasm.



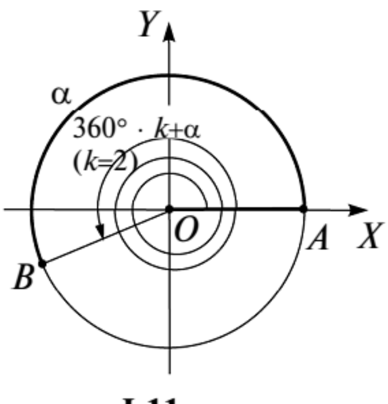
I.10-rasm.

Tekislikda to‘g‘ri burchakli *XOY* Dekart koordinatalari sistemasi kiritilgan bo‘lsin. Markazi koordinatalar boshida bo‘lgan *R* radiusli aylanani qaraymiz (I.8-rasm). Bu aylana *OX* o‘qning musbat yarim o‘qini *A* nuqtada kessin. *OA* radius *boshlang‘ich radius*, *A* nuqta esa *boshlang‘ich nuqta* deb ataladi.

OX o‘qning musbat yarim o‘qini koordinatalar boshi (qo‘zg‘almas nuqta) atrofida musbat yo‘nalish(soat strelkasining harakat yo‘nalishiga qarama-qarshi yo‘nalish)da va manfiy yo‘nalish (soat strelkasining harakat yo‘nalishi)da istalgancha uzluksiz siljitish (harakatlantirish) mumkin deb hisoblaymiz.

OX musbat yarim o‘q qo‘zg‘almas *O* nuqta atrofida musbat yo‘nalishda siljitilsa, *OA* radius biror *OB* radiusga o‘tadi. Agar *OA* va *OB* radiuslar ustma-ust tushsa (I.9-rasm), siljitish natijasida *A* nuqta aylanani bir yoki bir necha marta to‘liq aylanib chiqqan bo‘ladi. Bu holda biz boshlang‘ich tomoni *OA* va oxirgi tomoni *OB* bo‘lgan aylanish burchagiga ega bo‘lamiz. Uning gradus o‘lchovi $360° \cdot k$ ga teng, bu yerda k — aylanishlar soni.

Agar *OA* va *OB* radiuslar ustma-ust tushmasa, *A* nuqta aylanani to‘liq aylanib chiqmagan yoki aylanani bir yoki bir necha marta aylanib chiqib, yana *AB* yoyni bosib o‘tgan bo‘ladi. Bu holda boshlang‘ich tomoni *OA* va oxirgi tomoni *OB* bo‘lgan burish burchagiga ega bo‘lamiz. Bu burish burchagining gradus o‘lchovi quyidagicha aniqlanadi:



I.11-rasm.

1) *A* nuqta aylanani to‘liq aylanib chiqmagan bo‘lsa (I.10-rasm), burish burchagining gradus o‘lchovi *AB* yoyning gradus o‘lchoviga teng;

2. Burchak va yoylarning radian o‘lchovi. Koordinatali aylana. Burchak va yoylarning burchak kattaliklarini o‘lchashning yana bir sistemasi − *radian o‘lchovi sistemasi* bilan tanishamiz.

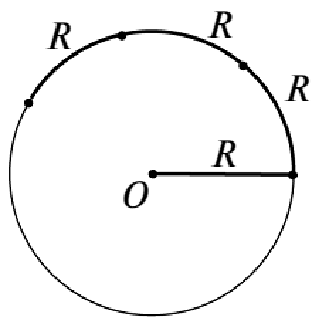
R radiusli aylanani qaraylik (I.14-rasm). Uzunligi $2\pi R$ bo‘lgan bu aylanada umumiy ichki nuqtaga ega bo‘lmagan va har birining uzunligi R ga teng bo‘lgan 2π ta yoy mavjud. Bu yoylardan har birining, shuningdek ularga mos har bir markaziy burchakning burchak kattaligi $\frac{360^\circ}{2\pi} = \frac{180^\circ}{\pi}$ ga tengdir. Demak, uzunligi aylana radiusiga teng yoyning va unga mos markaziy burchakning burchak kattaligi aylana radiusiga bog‘liq emas. Shu sababli, uzunligi aylana radiusiga teng bo‘lgan yoyning burchak kattaligini shu aylana yoylarini o‘lchashda o‘lchov birligi sifatida, unga mos markaziy burchak kattaligini esa burchaklarni o‘lchashda o‘lchov birligi sifatida olish mumkin.

Uzunligi aylana radiusiga teng yoy *1 radianli yoy*, unga mos markaziy burchak esa *1 radianli burchak* deyiladi (I.15-rasm).

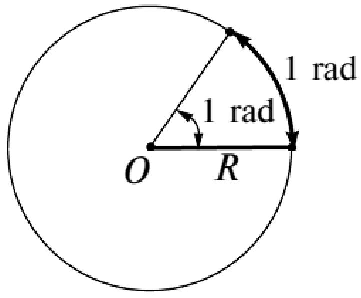
Yuqoridagi mulohazalardan quyidagi bog‘lanishlarni olamiz:

$$1 \text{ radian} = \frac{180^\circ}{\pi}, \qquad 1^\circ = \frac{\pi}{180} \text{ radian}.$$

Bu ikki tenglik yordamida radian o‘lchovidan gradus o‘lchoviga o‘tish va gradus o‘lchovidan radian o‘lchoviga o‘tish formulalari hosil bo‘ladi:



I.14-rasm.



I.15-rasm.

1-misol. 120° ni radianlarda, $\frac{3\pi}{4}$ va 5 (rad)larni esa graduslarda ifodalang.

Yechish. $\alpha^\circ = \frac{\pi \cdot \alpha}{180}$ (rad) formulaga ko‘ra

$$120^\circ = \frac{\pi \cdot 120}{180}(\text{rad}) = \frac{2\pi}{3}$$

tenglikni, $a_{\text{rad}} = \left(\frac{180a}{\pi}\right)^\circ$ formulaga ko‘ra

$$\frac{3\pi}{4} = \left(\frac{180 \cdot \frac{3\pi}{4}}{\pi}\right)^\circ = 135^\circ \quad \text{va} \quad 5 = \left(\frac{180 \cdot 5}{\pi}\right)^\circ = \left(\frac{900}{\pi}\right)^\circ$$

tengliklarni hosil qilamiz.

2-misol. Radiusi $R = 5$ (uzun. birl.) bo‘lgan aylananing uzunligi $l = 10$ (uzun. birl.)ga teng yoyini graduslarda va radianlarda ifodalang.

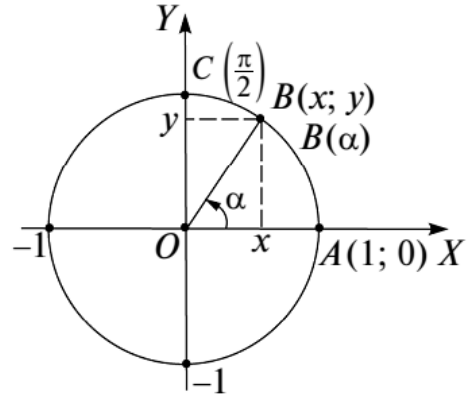
Yechish. $1 = 10$ (uzun. birl.) $= \frac{10}{5}$ (rad) $= 2$ (rad) va

$l = 2\ (\text{rad}) = \left(\frac{180 \cdot 2}{\pi}\right)^\circ = \left(\frac{360}{\pi}\right)^\circ \approx 114^\circ 19' 52''$ tengliklarga egamiz.

3-misol. Agar radiusi $R = 4$ bo‘lgan doiraviy sektorning yoyi 3 rad ga teng bo‘lsa, shu sektorning S yuzini toping.

Yechish. Yoyi π rad ga teng doiraviy sektor (yarim doira)ning yuzi $\frac{\pi \cdot R^2}{2}$ (bu yerda R — radius) ga teng bo‘lgani uchun, yoyi 1 rad bo‘lgan doiraviy sektorning yuzi $\frac{R^2}{2}$ ga, yoyi a rad ga teng bo‘lgan doiraviy sektorning yuzi esa $a \cdot \frac{R^2}{2}$ ga teng. Shu sababli $S = 3 \cdot \frac{4^2}{2} = 24$ kv. birlik.

Eslatma. 5 ning graduslarda ifodalangan aniq qiymati $\left(\frac{900}{\pi}\right)^\circ$ ga teng. Uning graduslarda ifodalangan taqribiy qiymatini hosil qilish uchun π ni uning kerakli aniqlikdagi taqribiy qiymati bilan almashtirish kerak bo‘ladi. Masalan, $\pi \approx 3$ deb olinsa, $5 = \left(\frac{900}{\pi}\right)^\circ \approx \left(\frac{900}{3}\right)^\circ = 300^\circ$ ga ega bo‘lamiz. Xuddi shu kabi, $1^\circ = \frac{\pi}{180^\circ}$ rad $\approx 0{,}017$ (rad), $1\ (\text{rad}) = \left(\frac{180}{\pi}\right)^\circ \approx$ $\approx 57^\circ 17' 44''$ munosabatlar hosil qilinadi.



I.16-rasm.

Tekislikda XOY Dekart koordinatalari sistemasi kiritilgan bo‘lsin. Markazi koordinatalar boshi O (0; 0) da bo‘lgan $R = 1$ radiusli aylananing A (1;0) nuqtasini *boshlang‘ich nuqta*, OA radiusini esa *boshlang‘ich radius* deb ataymiz (I.16-rasm) va shu aylanada koordinatalar sistemasini quyidagi tartibda kiritamiz.

Boshlang‘ich nuqta A (1; 0) ni yangi koordinatalar sistemasining koordinatalar boshi (sanoq boshi) sifatida olamiz. Uning yangi koordinatalar sistemasidagi koordinatasi 0 ga teng. Boshlang‘ich radiusni O (0; 0) nuqta atrofida α radianli burchakka buramiz (bu yerda va bundan keyin aylanish burchagi burish burchagining xususiy holi sifatida qaraladi). Natijada A nuqta aylananing biror $B(x;\ y)$ nuqtasiga o‘tadi (I.16-rasm). $B(x;\ y)$ nuqtaning yangi koordinatasi (aylanadagi koordinatasi) α ga teng deb qabul qilamiz va $B(\alpha)$ ko‘rinishda belgilaymiz.

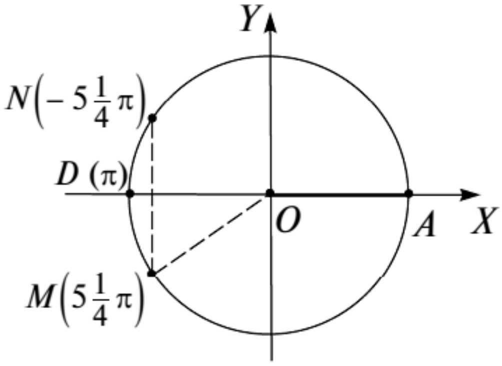
Masalan, C (0; 1) nuqta boshlang‘ich radiusni O (0; 0) nuqta atrofida $\frac{\pi}{2}$ rad burchakka burishdan hosil qilinadi. Shu sababli, uning yangi koordinatalar sistemasidagi koordinatasi $\frac{\pi}{2}$ ga tengdir (I.16-rasm).

Aylananing har bir nuqtasi aylanadagi koordinatalar sistemasida cheksiz ko‘p koordinatalarga ega, chunki boshlang‘ich radiusni O (0; 0) nuqta atrofida α, $\alpha \pm 2\pi$, $\alpha \pm 4\pi$, ..., ya’ni $\alpha \pm 2k\pi$, $k \in Z$ burchaklarga burish natijasida boshlang‘ich nuqta aylananing ayni bir B nuqtasiga o‘tadi va $\alpha \pm 2k\pi$, $k \in Z$ sonlarning har biri B nuqtaning koordinatasi (aylanadagi koordinatasi!) bo‘ladi.

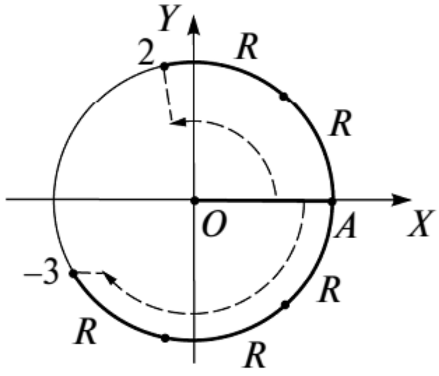
Yuqoridagi usul bilan koordinatalar sistemasi kiritilgan birlik aylana *koordinatali aylana* (yoki *koordinatalar aylanasi*) deb ataladi.

4 - m i s o l. Koordinatali aylanada $M\left(5\frac{1}{4}\pi\right)$, $N\left(-5\frac{1}{4}\pi\right)$ nuqtalarni belgilang.

Y e c h i s h. 1) $5\frac{1}{4}\pi = 2 \cdot 2\pi + \left(\pi + \frac{\pi}{4}\right)$ bo‘lgani uchun $M\left(5\frac{1}{4}\pi\right)$ va $M_1\left(\pi + \frac{\pi}{4}\right)$ nuqtalar koordinatali aylanada ustma-



I.17-rasm.



I.18-rasm.

ust tushadi. $D(\pi)$ nuqtani (I.17-rasm) musbat yo‘nalish bo‘yicha $\frac{\pi}{4} = 45^\circ$ burchakka burib, $M\left(5\frac{1}{4}\pi\right)$ nuqtani hosil qilamiz;

2) N va M nuqtalar AD diametrga nisbatan simmetrik nuqtalar bo‘lgani uchun M nuqtani shu diametrga nisbatan simmetrik almashtirib, $N\left(-5\frac{1}{4}\pi\right)$ nuqtani hosil qilamiz (I.17-rasm).

5 - m i s o l . Koordinatali aylanada 2 va –3 sonlarini belgilang.

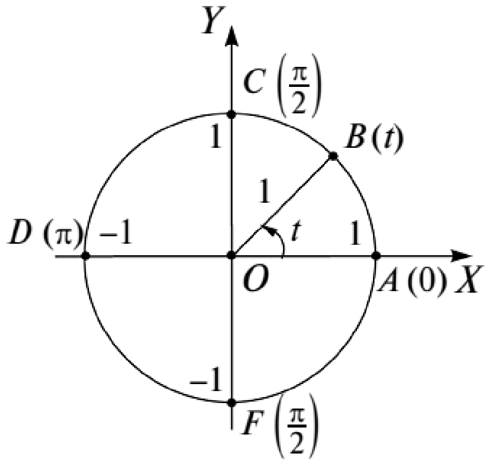
Y e c h i s h . 2 sonining koordinatali aylanadagi tasviri (koordinatasi 2 ga teng bo‘lgan nuqta)ni topish uchun uzunligi 1 radian (aylana radiusi)ga teng bo‘lgan yoyni boshlang‘ich A nuqtadan boshlab, musbat yo‘nalishda ketma-ket ikki marta qo‘yamiz (I.18-rasm).

–3 sonining koordinatali aylanadagi tasvirini topish uchun uzunligi 1 radianga teng bo‘lgan yoyni boshlang‘ich A nuqtadan boshlab, manfiy yo‘nalishda ketma-ket uch marta qo‘yish yetarli (I.18-rasm).

3. Sonli argumentning sinusi, kosinusi, tangensi va kotangensi. Tekislikda XOY Dekart koordinatalar sistemasi kiritilgan va t haqiqiy son berilgan bo‘lsin. t haqiqiy songa koordinatali aylananing koordinatasi t ga teng bo‘lgan $B(t)$ nuqtasini mos qo‘yamiz (I.19-rasm).

$B(t)$ nuqtaning abssissasi *t sonning kosinusi*, ordinatasi esa *t sonning sinusi* deyiladi va mos ravishda $\cos t$, $\sin t$ orqali belgilanadi.

$B(t)$ nuqta ordinatasining shu nuqta abssissasiga nisbati (agar bu nisbat mavjud bo‘lsa) *t sonning tangensi* deyiladi va tg$t$ orqali belgilanadi.



I.19-rasm.

$B(t)$ nuqta abssissasining shu nuqta ordinatasiga nisbati (agar bu nisbat mavjud bo‘lsa) *t sonning kotangensi* deyiladi va ctg$t$ orqali belgilanadi.

Sonning sinusi, kosinusi, tangensi va kotangensi tushunchalarining aniqlanishidan ko‘rinadiki,

$$\text{tg}\,t = \frac{\sin t}{\cos t} \quad (\cos t \neq 0), \qquad (1)$$

$$\text{ctg}\,t = \frac{\cos t}{\sin t} \quad (\sin t \neq 0) \qquad (2)$$

munosabatlar o‘rinli va koordinatali aylananing $B(t)$ nuqtasi XOY koordinatalar sistemasidagi $B(\cos t;\ \sin t)$ nuqta bilan ustma-ust tushadi. $B(\cos t;\ \sin t)$ nuqta birlik aylanada yotgani sababli, uning koordinatalari shu birlik aylana tenglamasi $x^2 + y^2 = 1$ ni qanoatlantiradi:

$$\cos^2 t + \sin^2 t = 1. \qquad (3)$$

Sonning sinusi va kosinusi tushunchalarining aniqlanishidan ko‘rinadiki, ixtiyoriy t haqiqiy son uchun $B(\cos t;\ \sin t)$ nuqta birlik aylanada yotadi. Shu sababli, (3) tenglik t ning har qanday haqiqiy qiymatida o‘rinli.

1 - m i s o l. $0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}$ sonlarining sinusi, kosinusi, tangensi va kotangensini toping.

Y e c h i s h. $0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}$ sonlariga koordinatali aylananing $A(0)$, $C\left(\frac{\pi}{2}\right)$, $D(\pi)$, $F\left(\frac{3\pi}{2}\right)$ nuqtalari mos keladi (I.19-rasm). Bu nuqtalar XOY koordinatalar sistemasida mos ravishda quyidagi koordinatalarga ega: $A\ (1;\ 0)$, $C\ (0;\ 1)$, $D\ (-1;\ 0)$, $F\ (0;\ -1)$.

Sonning sinusi, kosinusi, tangensi va kotangensi tushunchalarining aniqlanishiga ko‘ra, quyidagi tengliklarga ega bo‘lamiz:

$\cos 0 = 1$; $\cos\frac{\pi}{2} = 0$; $\cos \pi = -1$; $\cos\frac{3\pi}{2} = 0$;

$\sin 0 = 1$; $\sin\frac{\pi}{2} = 1$; $\sin \pi = 0$; $\sin\frac{3\pi}{2} = -1$;

$\mathrm{tg}\ 0 = 0$; $\mathrm{tg}\frac{\pi}{2}$ — mavjud emas; $\mathrm{tg}\ \pi = 0$; $\mathrm{tg}\frac{3\pi}{2}$ — mavjud emas;

$\mathrm{ctg}\ 0$ — mavjud emas; $\mathrm{ctg}\frac{\pi}{2} = 0$; $\mathrm{ctg}\ \pi$ — mavjud emas; $\mathrm{ctg}\frac{3\pi}{2} = 0$.

2 - m i s o l. $\sin\frac{\pi}{4}$, $\cos\frac{\pi}{4}$, $\mathrm{tg}\frac{\pi}{4}$, $\mathrm{ctg}\frac{\pi}{4}$ larni hisoblang.

Y e c h i s h. Koordinatali aylanada $B\left(\frac{\pi}{4}\right)$ nuqtani yasaymiz (I.20-rasm) va bu nuqtaning XOY koordinatalar tekisligidagi koordinatalarini aniqlaymiz.

OBC teng yonli to‘g‘ri burchakli uchburchakda $OB^2 = OC^2 + BC^2 = 2BC^2$ bo‘lgani uchun $2BC^2 = 1$ yoki $BC = \frac{\sqrt{2}}{2}$ ga ega bo‘lamiz. $B\left(\frac{\pi}{4}\right)$ nuqtaning abssissasi ham, ordinatasi ham musbatdir. Demak, $B\left(\frac{\pi}{4}\right)$ nuqta $B\left(\frac{\sqrt{2}}{2};\ \frac{\sqrt{2}}{2}\right)$ nuqta bilan ustma-ust tushadi. $\sin\alpha$, $\cos\alpha$, $\mathrm{tg}\alpha$, $\mathrm{ctg}\alpha$ larning aniqlanishiga ko‘ra,

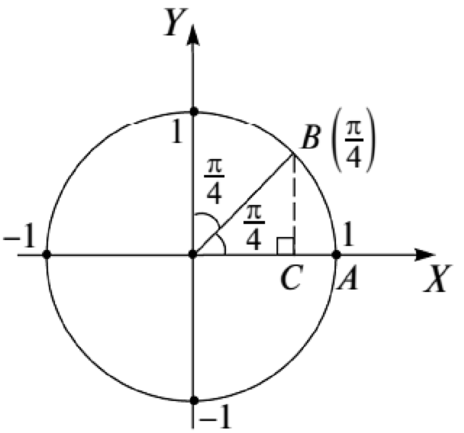
$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2},\quad \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2},\quad \mathrm{tg}\frac{\pi}{4} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1,\quad \mathrm{ctg}\frac{\pi}{4} = 1$$

tengliklarga ega bo‘lamiz.

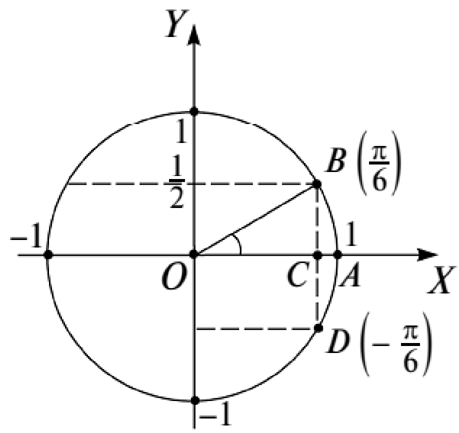
3 - m i s o l. $\frac{\pi}{6}$ va $-\frac{\pi}{6}$ ning sinusi, kosinusi, tangensi va kotangensini toping.

Y e c h i s h. $B\left(\frac{\pi}{6}\right)$ nuqtani yasaymiz (I.21-rasm) va bu nuqtaning dekart koordinatalarini topamiz. $B\left(\frac{\pi}{6}\right)$ nuqtaning dekart koordinatalari musbat sonlardir. OBC to‘g‘ri burchakli uchburchakda $BC = \frac{1}{2}OB = \frac{1}{2} \cdot 1 = \frac{1}{2}$ bo‘lgani uchun Pifagor teoremasiga ko‘ra $OC = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$ bo‘ladi. Demak, $B\left(\frac{\pi}{6}\right)$ nuqta $B\left(\frac{\sqrt{3}}{2};\ \frac{1}{2}\right)$ nuqta bilan ustma-ust tushadi. Son argumentning sinusi, kosinusi, tangensi va kotangensining aniqlanishiga ko‘ra

$$\sin\frac{\pi}{6} = \frac{1}{2}, \quad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}, \quad \operatorname{tg}\frac{\pi}{6} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}, \quad \operatorname{ctg}\frac{\pi}{6} = \sqrt{3}.$$



I.20-rasm.



I.21-rasm.

$D\left(-\frac{\pi}{6}\right)$ va $B\left(\frac{\pi}{6}\right)$ nuqtalar OX o‘qqa nisbatan simmetrik bo‘lgani uchun $D\left(-\frac{\pi}{6}\right)$ nuqta $D\left(\frac{\sqrt{3}}{2};\ -\frac{1}{2}\right)$ nuqta bilan ustma-ust tushadi. Shu sababli

$$\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}, \qquad \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2},$$

$$\operatorname{tg}\left(-\frac{\pi}{6}\right) = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}, \qquad \operatorname{ctg}\left(-\frac{\pi}{6}\right) = -\sqrt{3}.$$

$y = \sin t$, $y = \cos t$, $y = \operatorname{tg} t$ va $y = \operatorname{ctg} t$ formulalar bilan aniqlangan funksiyalar *asosiy trigonometrik funksiyalar* deyiladi. Ularning ayrim asosiy xossalarini keltiramiz.

1°. *$y = \sin t$ funksiya chegaralangan funksiya va barcha $t \in R$ lar uchun $|\sin t| \le 1$ munosabat o'rinli.*

I s b o t . Biror $t \in R$ uchun $|\sin t| > 1$ bo'lsin. U holda $|\sin^2 t| = |\sin t|^2 > 1$ bo'lgani uchun $\sin^2 t + \cos^2 t = |\sin t|^2 + |\cos t|^2 \ge |\sin t|^2 + 0 = |\sin t|^2 > 1$, ya'ni $\sin^2 t + \cos^2 t > 1$ tengsizlikka ega bo'lamiz. Bu esa (3) ga ziddir.

Demak, barcha $t \in R$ sonlar uchun $|\sin t| \le 1$ munosabat o'rinli va $\sin t$ funksiya chegaralangan funksiyadir.

2°. *$y = \cos t$ funksiya chegaralangan va barcha $t \in R$ lar uchun $|\cos t| \le 1$ munosabat o'rinli.*

α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	–1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	–1	0	–1
$\mathrm{tg}\alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Mavjud emas	0	Mavjud emas	0
$\mathrm{ctg}\alpha$	Mavjud emas	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Mavjud emas	0	Mavjud emas

I s b o t . $\forall t \in R$ da $0 \le \sin^2 t \le 1$ bo'lgani uchun $\cos^2 t = 1 - \sin^2 t \le 1$ bo'ladi. Oxirgi tengsizlikdan, $\forall t \in R$ da $|\cos t| \le 1$ ekani ko'rinadi. Demak, $\cos t$ funksiya chegaralangan funksiya va $\forall t \in R$ da $|\cos t| \le 1$.

1°, 2°- xossalardan, $y = \sin x$ va $y = \cos x$ funksiyalardan har birining qiymatlar sohasi $[-1; 1]$ kesmadan iborat ekanligi kelib chiqadi.

Trigonometrik funksiyalarning ayrim burchaklardagi qiymatlari jadvalini keltiramiz:

Mashqlar

1.9. Ushbu sonli qiymatlarga kosinus teng bo‘la oladimi? Sinus-chi?

1) $0,562$; 2) $-0,562$; 3) $1,002$; 4) $-1,002$;

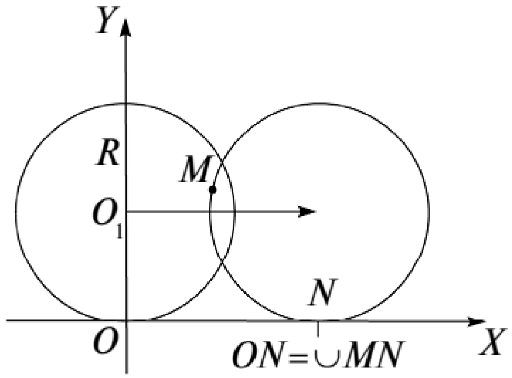
5) $\frac{a}{\sqrt{a^2-b^2}}$; 6) $\frac{a}{\sqrt{a^2+b^2}}$, $a > 0$, $b > 1$; 7) $\frac{\sqrt[3]{3}}{\sqrt{3}}$; 8) π;

9) $\frac{3\frac{1}{7}}{\pi}$; 10) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{3}-1}$; 11) $\sqrt{8}-\sqrt{2}$; 12) $\frac{1}{2}\left(a+\frac{1}{a}\right)$, $a > 1$.

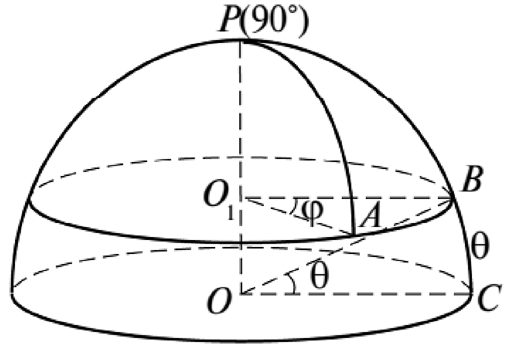
1.10. 1) Agar $\alpha = 0°$; $90°$; π; $\frac{\pi}{4}$; $1,5\pi$ bo‘lsa, $\sin\alpha + \cos\alpha$ va $2(1 - \cos\alpha)$ ni toping;

2) $y = \sin^2\alpha + 2\cos^2\alpha$ ifoda qabul qiladigan eng katta qiymatni toping.

1.11. *R* radiusli halqa *OX* o‘qining musbat yo‘nalishi bo‘yicha yumalab bormoqda (I.22-rasm). O‘qning 1 birlik kesma uzunligi *R* ga teng. Harakat boshida aylananing *M* nuqtasi *O* nuqtada turgan bo‘lsin.



I.22-rasm.



I.23-rasm.

1) Agar *M* nuqta α rad ga burilsa, aylananing O_1 markazi qanchaga siljiydi?

2) O_1 markaz ($x = 3$; $y = 1$) nuqtaga kelishi uchun *M* nuqta qancha burilishi kerak?

3) O_1 nuqta 5 birlik/s tezlik bilan siljimoqda. *M* nuqtaning burchak tezligini toping.

4) O_1 nuqta sekundiga *R* masofaga siljisa, *M* nuqtaning *t* momentdagi o‘rnining koordinatalarini toping.

1.12. Geografik kengligi θ ga teng bo‘lgan parallelda geografik uzunliklarining farqi φ ga teng bo‘lgan ikki A va B nuqta olingan (I.23-rasm). Yer shari radiusi R ga teng. $\cup AB = l$ ni toping.

1.13. A nuqtaga 120° burchak ostida qo‘yilgan 10 N va 12 N kattalikdagi ikki kuchning teng ta’sir etuvchisini toping.

1.14. Daryo qirg‘og‘idagi tepalikdan shu qirg‘oq gorizontal yo‘nalishga nisbatan 30°, narigi qirg‘oq 15° burchak ostida ko‘rinadi. Daryoning kengligi 100 m. Tepalikning balandligi va uning uchidan daryo qirg‘og‘igacha bo‘lgan masofani toping.

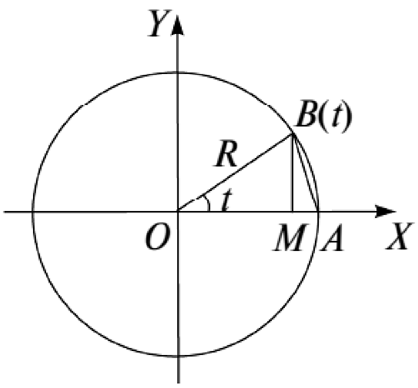
1.15. Yer radiusi R ga teng. Shimoliy yarim sharda geografik uzunligi λ ga, kengligi φ ga teng bo‘lgan B nuqta olingan. Toping:

1) B nuqtadan ekvator tekisligigacha bo‘lgan masofa;

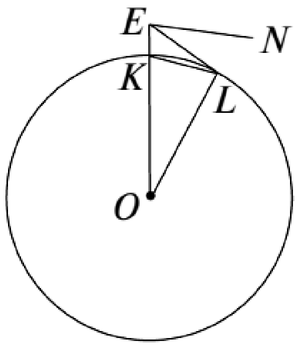
2) B nuqtaning ekvator tekisligidagi proyeksiyasining koordinatalari (abssissalar o‘qi ekvator bilan nolinchi meridian kesishuvidagi nuqta ustidan o‘tadi). Hisoblashlarni $R = 6367$ km, $\lambda = 30°$ va $\varphi = 60°$ uchun ham bajaring.

1.16. ΔBOA da $OA = OB = R$, $MA = R(1 - \cos t)$, $BM = R\sin t$, $OM = R\cos t$ (I.24-rasm). Isbot qiling:

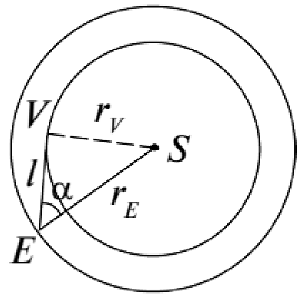
$$\sin t = \pm\sqrt{(1-\cos t)(1+\cos t)}; \quad AB = R\sqrt{2(1-\cos t)}.$$



I.24-rasm.



I.25-rasm.



I.26-rasm.

1.17. Teng yonli AOB uchburchak yuzi 64, $MA = 8$ (I.24-rasm). AB —?

1.18. Yer sathidan $EK = h$ (I.25-rasm) balandlikda joylashgan E kuzatuv punktidan gorizont chizig‘idagi L nuqta gorizontal yo‘nalishga nisbatan $\angle NEL = \alpha$ burchak ostida ko‘rinadi (Abu Rayhon Beruniyning «Qonuni Ma’sudiy» asaridan). Agar $h \approx 3$ km va $R \approx 6367$ km bo‘lsa, α ni toping.

1.19. I.26-rasmda V Venera va E Yer orbitalari aylana ko‘rinishida tasvirlangan, Yerning S Quyoshdan uzoqligi $r_E =$ $=149500000$ km.

Oddiy kuzatishda Venera Quyoshga nisbatan $\alpha \approx 46°$ burchak ostida chetlashgan ko‘rinadi. Bu chetlanish ko‘pi bilan qancha bo‘lishi mumkin?

1) Veneraning Quyoshdan r_V uzoqligini hisoblang.

2) Venera sutkalik harakati davomida Quyoshdan α qadar ortda qolishi mumkin. U holda u kechasi ko‘rinadi. Aksincha, α qadar oldin o‘tgan bo‘lsa, ertalab, Quyosh chiqmasdan oldin ko‘rinadi. Nima uchun, tushuntiring.

1.20. I.24-rasmda tasvirlangan koordinatali aylanada $\cup AB = t$.

1) t, $360° + t$, $360° - t$, $-360° + t$, $2\pi k + t$, $k \in Z$ yoylarga mos nuqtalar ustma-ust tushadimi? Agar ular ustma-ust tushsa, bu nuqtalarga mos trigonometrik funksiyalar o‘rtasida qanday bog‘lanishlar mavjud bo‘ladi? Misollar keltiring. Shu ishni $\pi k + t$, $k \in Z$ va $\frac{\pi}{2} + t$, $k \in Z$ nuqtalar uchun takrorlang;

2) yuqoridagi ishni $B(t)$ nuqtaga O markazga nisbatan simmetrik bo‘lgan $E(\pi + t)$ nuqtaga nisbatan ham bajaring.

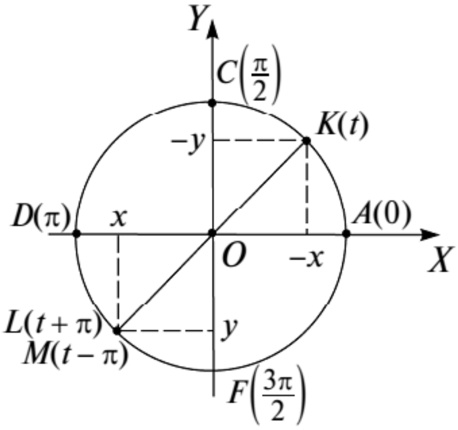
4. Trigonometrik funksiyalarning davriyligi. Trigonometrik funksiyalarning davriyligi haqidagi teoremalarni keltiramiz.

1 - t e o r e m a . cos*t* *va* sin*t* *funksiyalarning har biri davriy funksiya va ularning asosiy davri* 2π *ga teng.*

I s b o t . Ixtiyoriy $t \in R$ son uchun $K(t)$, $L(t + 2\pi)$, $M(t - 2\pi)$ nuqtalar koordinatali aylanada ustma-ust tushadi. Shu sababli ularning Dekart koordinatalari bir xil:

$$x = \cos t = \cos(t - 2\pi) = \cos(t + 2\pi),$$
$$y = \sin t = \sin(t - 2\pi) = \sin(t + 2\pi). \qquad (1)$$

Demak, $\cos t$ va $\sin t$ funksiyalar davriy funksiyalar va 2π soni ulardan har birining biror davridir. 2π soni ulardan har biri uchun asosiy davr bo'lishligini ko'rsatamiz.



I.27-rasm.

$0 < T_1 < 2\pi$ soni $\cos t$ ning davri deb faraz qilaylik. U holda, masalan, $t = 0$ da $\cos 0 = \cos(0 + T_1) = 1$, ya'ni $\cos T_1 = 1$ bo'lishi kerak. Koordinatali aylanada abssissasi 1 ga teng bo'lgan faqat bitta (1; 0) nuqta mavjud va unga $t = 2\pi k$, $k \in Z$ sonlari mos keladi. T_1 son esa bu sonlar orasida mavjud emas. Demak, farazimiz noto'g'ri, kosinus funksiyaning asosiy davri 2π sonidan iborat. Shu kabi, masalan, $t = \frac{\pi}{2}$ da $\sin\frac{\pi}{2} = \sin\left(\frac{\pi}{2} + T_1\right) = 1$ tenglikni qanoatlantiradigan va 2π dan kichik bo'lgan T_1 musbat son yo'q. Demak, $T = 2\pi$ soni sinus funksiyaning asosiy davri.

2-teorema. $\mathrm{tg}\,t$ *davriy funksiya va uning asosiy davri* π *ga teng.*

Isbot. $t \neq \frac{\pi}{2} + \pi k$, $k \in Z$ bo'lsin. $K(t)$, $L(t + \pi)$, $M(t - \pi)$ nuqtalarni qaraymiz. $L(t + \pi)$, $M(t - \pi)$ nuqtalar ayni bir xil Dekart koordinatalariga ega, ya'ni ular ustma-ust tushadi. Shu nuqtalarning umumiy abssissasi x, umumiy ordinatasi esa y bo'lsin (I.27-rasm). U holda, $\mathrm{tg}(t + \pi) = \mathrm{tg}(t - \pi) = \frac{y}{x}$ bo'ladi. $K(t)$ va $L(t + \pi)$ nuqtalar diametral qarama-qarshi nuqtalar bo'lgani uchun $K(t)$ nuqtaning abssissasi $-x$ ga, ordinatasi esa $-y$ ga tengdir (I.27-rasm). Shu sababli,

$$\mathrm{tg}\,t = \frac{-y}{-x} = \frac{y}{x} = \mathrm{tg}(t + \pi) = \mathrm{tg}(t - \pi).$$

Demak, $\mathrm{tg}\,t$ funksiya davriy funksiya va $t = \pi$ soni uning biror davridir. Bu son $\mathrm{tg}\,t$ ning asosiy davri ekanini ko'rsatamiz. T son $\mathrm{tg}\,t$ ning asosiy davri, ya'ni barcha $t \neq \frac{\pi}{2} + \pi k$, $k \in Z$ sonlari uchun $\mathrm{tg}(t + T) = \mathrm{tg}\,t$ tenglik o'rinli bo'lsin. Oxirgi tenglik $t = 0$ da ham bajariladi: $\mathrm{tg}\,T = 0$. Bu yerdan $T = \pi k$, $k \in Z$ ekanini ko'ramiz. Shunday qilib, $\mathrm{tg}\,t$ ning asosiy davri πk, $k \in Z$ sonlari orasidagi eng kichik musbat son, ya'ni π sonidir. Demak, $T = \pi$.

Mashqlar

1.21. $y = 1 - \cos t$ funksiyaning davriyligini isbot qiling va asosiy davrini toping.

1.22. 1) $f(x) = \sin x$, $g(x) = \frac{1}{x}$, $x \neq 0$ bo‘lsa, $g \circ f$ kompozitsiya davriy funksiya bo‘la oladimi? $f \circ g$ -chi? Agar shunday bo‘lsa, davrini toping.

2) Agar $\frac{5}{3}$ va $\frac{2}{7}$ sonlari f funksiyaning davrlari bo‘lsa, $\frac{17}{21}$ soni ham uning davri bo‘lishini isbot qiling.

3) $y = \cos(\alpha x)$ ning asosiy davrini toping.

4) $y = \sqrt{\cos x}$ ning aniqlanish sohasini toping va davriylikka tekshiring.

5) Quyidagi funksiyalarning davriy emasligini isbot qiling:

$$y = \cos x^2; \quad y = \sin\sqrt{|x|}; \quad y = \sin x^3;$$

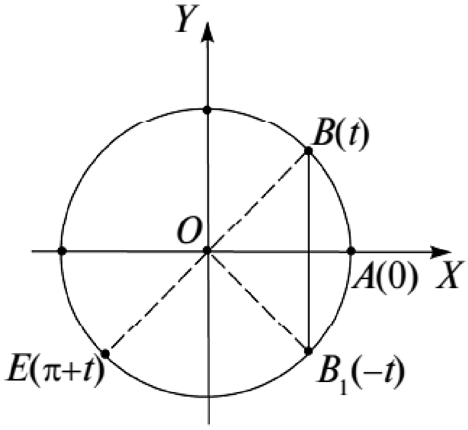
$$y = \sin x + \cos(x\sqrt{3}).$$

1.23. Funksiyalarning davrini toping:

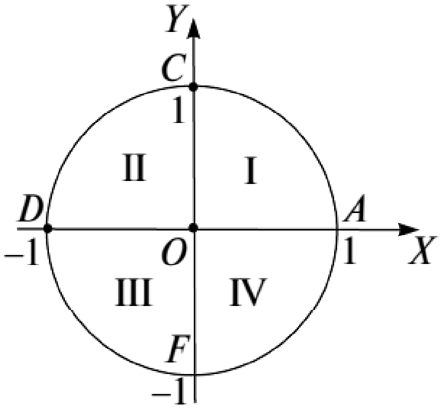
1) $y = \sin 5x + \cos 4x$; 2) $y = \cos x + 2\sin 4{,}9x$;

3) $y = \sqrt{\sin 4{,}3x - \sin 10x + 3}$.

5. Sinus va kosinus funksiyalarning xossalari. Sinus va kosinus funksiyalarning xossalari bilan tanishishni davom ettiramiz.



I.28-rasm.



I.29-rasm.

1) sin$t$ funksiya argumentning $t = \pi k,\ k \in Z$ qiymatlaridagina, cos$t$ funksiya esa argumentning $t = \frac{\pi}{2} + \pi k,\ k \in Z$ qiymatlaridagina nolga aylanadi. Haqiqatan, koordinatali aylanada faqat ikki $A(0) = A(1;\ 0)$ va $D(\pi) = D(-1;\ 0)$ nuqtaning ordinatasi nolga teng, ya'ni $y = \sin t = 0$ (I.27-rasm). Bu nuqtalarga $2\pi k,\ k \in Z$ va $\pi + 2\pi k,\ k \in Z$ sonlar to'plamlari mos. Bu ikkala to'plamni bitta $\{\pi k,\ k \in Z\}$ to'plamga birlashtirib yozamiz. Shu kabi koordinatali aylanada faqat ikki $C\left(\frac{\pi}{2}\right) = C(0;\ 1)$ va $F\left(\frac{3\pi}{2}\right) = F(0;\ -1)$ nuqta abssissasi nolga teng (I.27-rasm), ya'ni $x = \cos t = 0$. Bu nuqtalarga $\frac{\pi}{2} + 2\pi k$, $\frac{3\pi}{2} + 2\pi k = \frac{\pi}{2} + (2k+1)\pi$, $k \in Z$ sonlar to'plamlari yoki $\left\{\frac{\pi}{2} + \pi,\ k \in Z\right\}$ to'plam mos;

2) cos$t$ – juft funksiya, sin$t$ – toq funksiya. Haqiqatan, $B(t)$ va $B_1(-t)$ nuqtalar abssissalar o'qiga nisbatan simmetrik joylashganligidan (I.28-rasm) ularning abssissalari teng, ordinatalari esa faqat ishoralari bilan farq qiladi. Demak, $\cos(-t) = \cos t$, ya'ni cos$t$ juft funksiya, $\sin(-t) = -\sin t$, ya'ni sin t toq funksiya;

3) agar $B(t)$ nuqta koordinatali aylana bo'ylab π qadar siljitilsa, cos$t$ va sin$t$ funksiyalar o'z ishoralarini o'zgartiradi:

$$\cos(t + \pi) = -\cos t; \tag{1}$$

$$\sin(t + \pi) = -\sin t. \tag{2}$$

Haqiqatan, $B(t)$ va $E(\pi + t)$ nuqtalar koordinatalar boshiga nisbatan simmetrik joylashganligidan (I.28-rasm) ularning koordinatalari qarama-qarshi ishorali bo'ladi;

4) A (1; 0), C (0; 1), D (–1; 0), F (0; –1) nuqtalar koordinatali aylanani to'rt chorakka ajratadi (I.29-rasm). Agar $A(0)$ nuqta A dan C gacha siljitilsa, A nuqta abssissasi 1 dan 0 gacha kamayadi, ordinatasi esa 0 dan 1 gacha o'sadi. Demak, $0 \le t \le \frac{\pi}{2}$ oraliqda (I chorakda) sin$t$ funksiya nomanfiy va 0 dan 1 gacha o'sadi, cos$t$ ham nomanfiy, lekin 1 dan 0 gacha kamayadi. Qolgan choraklarda ham shu kabi ma'lumotlarni to'plab, quyidagi jadvalni tuzamiz:

Funksiya	$0 < t < \frac{\pi}{2}$	$\frac{\pi}{2} < t < \pi$	$\pi < t < \frac{3\pi}{2}$	$\frac{3\pi}{2} < t < 2\pi$
sin$t$	musbat, 0 dan 1 gacha o‘sadi	musbat, 1 dan 0 gacha kamayadi	musbat, 0 dan –1 gacha kamayadi	musbat, –1 dan 0 gacha o‘sadi
cos$t$	musbat, 0 dan 1 gacha o‘sadi	musbat, 1 dan 0 gacha kamayadi	musbat, 0 dan –1 gacha kamayadi	musbat, –1 dan 0 gacha o‘sadi

Mashqlar

1.24. $1 - \cos t$ funksiya (Mirzo Ulug‘bek bu funksiyani sahm t funksiya deb atagan) ishoralarining saqlanish oraliqlarini, nollarini, juft-toqligini aniqlang, $180° \pm \alpha$ yoy sahmining $1 + \cos\alpha$ ga tengligini tekshiring, α yoy sahmi $1 - \cos\alpha$ ga teng.

1.25. sin$t$, cos$t$ va $1 - \cos t$ mos ravishda:

1) $\frac{2\sqrt{10}}{7}$; $\frac{3}{7}$; $\frac{4}{7}$ ga teng bo‘lishi mumkinmi?

2) $-\frac{a}{\sqrt{a^2+b^2}}$; $\frac{b}{\sqrt{a^2+b^2}}$; $1-\frac{b}{\sqrt{a^2+b^2}}$ ga-chi ?

1.26. Quyida t ning qiymatlari ko‘rsatilgan. $B(t)$ nuqta qaysi chorakda joylashgan, sin$t$, cos t lar qanday ishoraga ega bo‘ladi?

1) $\frac{5}{4}\pi$; 2) $\frac{4}{5}\pi$; 3) $\frac{\pi}{7}$; 4) $\frac{2\pi}{3}$; 5) 3; 6) 3,13; 7) 1,7π; 8) 1,78π; 9) –1,78π; 10) –2,8; 11) –4; 12) –1,31; 13) 49600; 14) 356°; 15) 247°36′42″; 16) 34680°;17) –674°; 18) –107°13′55″.

1.27. Ayniyatlarni isbot qiling:

1) $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{1}{\cos x \sin x}$, bunda $\sin x \neq 0$, $\cos x \neq 0$;

2) $\frac{\sin 30°}{\cos 30°} + \frac{\cos 30°}{\sin 30°} = \frac{4}{\sqrt{3}}$; 3) $\sin^2 x \cdot \cos^2 x + \cos^2 x + \sin^4 x = 1$;

4) $\sin^2 x - \cos^2 x = \sin^4 x - \cos^4 x$;

5) $\sin^2 x - \sin^2 x \cos^2 x - \cos^4 x = 1 - 2\cos^2 x$;

6) $\cos^2 x + \sin^2 x \cdot \cos^4 x - \sin^6 x = 1 - 2\sin^4 x$;

7) $6(\sin^4 x + \cos^4 x) - 4(\cos^6 x + \sin^6 x) = 2$.

1.28. $\sin x - \cos x = m$ bo‘lsin. $\sin x$ va $\cos x$ ni hisoblamay, quyidagilarni toping:

1) $\sin^3 x - \cos^3 x$; 2) $\sin^4 x - \cos^4 x$.

1.29. Tenglamalarni yeching:

1) $\sin 10x = 0$; 2) $\cos 5x = 0$; 3) $\sin \frac{x}{3} = 0$; 4) $\cos \frac{x}{5} = 0$;

5) $\sin\left(2x - \frac{\pi}{6}\right) = 0$; 6) $\sin\left(6x + \frac{\pi}{4}\right) = 0$; 7) $\cos\left(\frac{x}{6} - \frac{\pi}{2}\right) = 0$;

8) $\sin\left(\frac{x}{4} + \frac{\pi}{8}\right) = 0$.

1.30. Ifodalarni soddalashtiring:

1) $2\cos(\pi + x) + 3\cos(-x) + \cos(\pi - x)$;

2) $\sin(\pi + x) - 2\sin(\pi - x) - 3\sin(-x)$;

3) $4\cos(-x) + 5\sin(\pi + x) - 2\sin(\pi - x) - 6\cos(\pi + x)$.

1.31. a) Quyidagi funksiyalarni juft-toqlikka tekshiring:

1) $\sin^9 x$; 2) $\cos^9 x$; 3) $\sin^8 x$; 4) $5\cos^5 x + 6\cos^4 x$;

5) $3\sin^3 x - 2\sin^2 x$; 6) $3\sin^3 x + 4\cos^5 x$; 7) $\frac{4\sin^2 x + \cos^5 x + 2}{\sin^5 x}$.

b) Ifodalarning ishoralarini aniqlang:

1) $\sin \frac{6}{5}\pi \cdot \cos \frac{7}{4}\pi$; 2) $\sin \frac{3}{4}\pi \cdot \sin \frac{7}{3}\pi \cdot \cos \frac{7}{4}\pi$;

3) $\sin\ 0{,}9\ \cos(-1)\ \cos\ 4$.

1.32. Sinus va kosinus funksiyalar qaysi choraklarda bir xil ishoraga ega?

1.33. Agar:

1) $\cos t = 3\sin t$; 2) $\cos t = \sin^2 t$; 3) $\sin t = 2\cos^3 t$; 4) $\cos t = \sin^4 t$ bo‘lsa, t qaysi chorakka tegishli bo‘ladi?

1.34. Agar: 1) t burchak ikkinchi chorakka tegishli va $\cos t = -\frac{2}{3}$ bo‘lsa, $\sin t$ nimaga teng bo‘ladi?

2) t burchak uchinchi chorakka tegishli va $\sin t = -\frac{4}{5}$ bo‘lsa, $\cos t$ nimaga teng bo‘ladi?

1.35. Qaysi biri katta:

1) $\sin 45°$ yoki $\sin \frac{\pi}{3}$; 2) $\cos 45°$ yoki $\cos \frac{\pi}{3}$; 3) $\sin 50°$ yoki $\cos 50°$?

1.36. Ifodalarning qiymatlarini hisoblang:

1) $\sin 240°$; 2) $\cos 240°$; 3) $\sin \frac{9\pi}{6}$; 4) $\sin\left(-\frac{11\pi}{3}\right)$;

5) $\cos\left(-\frac{7\pi}{3}\right)$; 6) $\cos^2\left(-\frac{13\pi}{3}\right)+\sin^2\left(-\frac{11\pi}{6}\right)$;

7) $\cos\pi\cdot\cos 90^{o}-\frac{3\cos(-180)^{o}}{\cos 180^{o}}$;

8) $\cos\pi\cdot\sin\frac{3\pi}{2}+\frac{\sin\frac{5\pi}{2}}{\cos 0}-\frac{1}{\cos\left(-\frac{5\pi}{4}\right)}$; 9) $\frac{\cos^2 30^{o}}{\sin^2 30^{o}}-\frac{\sin^2 45^{o}}{\sin^2 45^{o}}$.

1.37. $f(x) = 6\sin 4x - 3\cos 4x$ funksiyaning $f(0)$, $f\left(\frac{\pi}{2}\right)$, $f\left(-\frac{\pi}{4}\right)$ qiymatlarini hisoblang.

1.38. Ayirmalarning ishoralarini aniqlang:

1) $\sin 38^\circ - \sin 40^\circ$; 2) $\cos 51^\circ -$ sos21°; 3) $\sin\frac{\pi}{9}-\sin\frac{\pi}{4}$;

4) $\sin 48^\circ - \sin 52^\circ$; 5) $\cos\frac{\pi}{9}-\cos\frac{\pi}{4}$; 6) $\sin 132^\circ - \sin 152^\circ$;

7) $\cos\frac{\pi}{10}-\cos\frac{\pi}{20}$; 8) $\sin\frac{\pi}{10}-\sin\frac{\pi}{20}$; 9) $\sin 12^\circ - \cos 732^\circ$.

1.39. Funksiyalarning o‘sish va kamayish oraliqlarini toping:

1) $y = \sin\frac{x}{3}$; 2) $y = \cos\frac{x}{3}$; 3) $y = \sin 5x$; 4) $y = \cos 5x$;

5) $y=\sin\left(x+\frac{\pi}{3}\right)$; 6) $y=\cos\left(x+\frac{\pi}{3}\right)$; 7) $y=4\sin\left(x-\frac{\pi}{3}\right)$;

8) $y = \sin\left(\frac{x}{3}+3\right)$; 9) $y = \cos^2 x$; 10) $y=\sin^2\frac{x}{2}$;

11) $y=-3\cos^4\frac{x}{3}$; 12) $y=\cos(5x+60^\circ)$; 13) $y=\sin(2x-60^\circ)$.

1.40. Funksiyalarning o‘sish va kamayish oraliqlarini toping:

1) $y = 2\sin 3x - 3\cos 2x$; 2) $y=\frac{1}{2\cos x}$; 3) $y=\frac{1}{\cos(2x+1)}$;

4) $y=\sqrt{|\sin x|}$; 5) $y=\sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x$;

6) $y = \cos 2x + \sin^2 x$; 7) $y = \cos x + \sin 2x$;

8) $y=\sqrt{3}\sin\left(\frac{1}{2}x+\frac{\pi}{4}\right)$.

1.41. Funksiyalarni o‘sish va kamayishga tekshiring:

1) $y = \sin^2 x + 1$; 2) $y = \cos 3x - \cos 5x + 6x$;

3) $y = (\cos 2x + 3)(1 - 4\sin x)$; 4) $y=\cos\frac{1}{2}x-\sin\frac{1}{2}x+3x-7$.

1.42. Funksiya x ning qaysi qiymatlarida aniqlanmagan?

1) $y=\frac{1}{\sin x}$; 2) $y=\frac{1}{\cos x}$; 3) $y=\frac{\cos x}{1-\sin x}$;

4) $y = 0{,}8\sin^2 x - 0{,}75$; 5) $y=\frac{2\sin x+1}{2\cos x-1}$.

1.43. Quyidagi funksiyalarning eng kichik musbat davrini toping:

1) $y = \cos 4x$; 2) $y = 10\cos 0{,}5x$;

3) $y=2\cos\left(3x-\frac{\pi}{6}\right)$; 4) $y = 9\cos(\omega t+\varphi)$, $\omega\neq 0$;

5) $y=-30\sin\left(5x+\frac{\pi}{4}\right)$; 6) $y=\frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{6}-2x\right)$;

7) $y=\frac{5}{6}\sin\left(4\pi x+\frac{\pi}{3}\right)$; 8) $y = A\sin(\omega t+\varphi)$, $A\in R$, $\omega\neq 0$;

9) $y=10\cos 4x-8\sin\left(5x+\frac{\pi}{4}\right)$; 10) $y=-10\cos\frac{x}{2}+4\sin\frac{x}{2}$;

11) $y = 5\cos x\sin 2x$; 12) $y=\frac{\sin x-2}{\cos x+2}$.

1.44. Funksiyalarning aniqlanish sohasini toping, maksimal va minimal qiymatlarini hisoblang; agar x ning qiymati $\pi/6$ dan $\pi/3$ gacha ortsa, funksiya qanday o‘zgaradi?

1) $y=\sin\left(\frac{\pi}{6}-x\right)$; 2) $y = \cos(45^\circ - x)$.

6. Tangens va kotangens funksiyalarning xossalari.

1) *Tangens va kotangens davriy funksiyalardir va ularning asosiy davri* $T = \pi$ (4- band, 2, 3- teoremalar);

2) $\mathrm{tg}\,t$ va $\mathrm{ctg}\,t$ — toq funksiyalar. Haqiqatan, $\cos\alpha\neq 0$ da

$\mathrm{tg}(-t)=\frac{\sin(-t)}{\cos(-t)}=\frac{-\sin t}{\cos t}=-\mathrm{tg}\,t$ ga, $\sin t\neq 0$ da $\mathrm{ctg}(-t)=-\mathrm{ctg}\,t$ ga ega bo‘lamiz. Tangens va kotangenslarning davri π ga teng va ular toq funksiyalar bo‘lgani uchun $\mathrm{tg}(\pi - t) = \mathrm{tg}(\pi + (-t)) = \mathrm{tg}(-t) = -\mathrm{tg}\,t$, $\mathrm{ctg}(\pi + (-t)) = -\mathrm{ctg}\,t$, ya’ni

$$\mathrm{tg}(\pi - t) = -\mathrm{tg}\,t, \quad (1)$$

$$\mathrm{ctg}(\pi - t) = -\mathrm{ctg}\,t. \quad (2)$$

tengliklar o‘rinli bo‘ladi;

3) $\left(0;\ \frac{\pi}{2}\right)$ *oraliqda* $\mathrm{tg}\,t$ *funksiya* 0 *dan* $+\infty$ *gacha o‘sadi*, $\mathrm{ctg}\,t$ *esa* $+\infty$ *dan 0 gacha kamayadi.*

Haqiqatan, $\left(0;\ \frac{\pi}{2}\right)$ da $\sin t$ va $\cos t$ musbat, sinus o‘suvchi, kosinus kamayuvchi, demak, $\mathrm{tg}\,t$ o‘suvchi, xususan, $t = 0$ da

$\mathrm{tg}\,0=\frac{\sin t}{\cos t}=\frac{0}{1}=0$ bo‘ladi, t burchak $\frac{\pi}{2}$ ga yaqinlashganda sin$t$ qiymati 1 gacha o‘sadi, cos$t$ esa 0 gacha kamayadi, natijada $\frac{\sin t}{\cos t}=\mathrm{tg}t$ funksiya $+\infty$ gacha o‘sadi, aksincha $\frac{\cos t}{\sin t}=\mathrm{ctg}t$ funksiya 0 gacha kamayadi.

Olingan xulosalar hamda tangens va kotangensning toq funksiyaligidan foydalanib, $\left(-\frac{\pi}{2};\ 0\right)$ oraliqda ularning manfiy ekanligini, tg$t$ funksiyaning $-\infty$ dan 0 gacha o‘sishini hamda ctg t ning 0 dan $-\infty$ gacha kamayishini aniqlaymiz. Uchinchi va to‘rtinchi choraklardagi holatlarini aniqlashda ularning xossalari $T=\pi$ davr bilan takrorlanishidan foydalanamiz. Xususan, $\left(\pi;\ \frac{3\pi}{2}\right)$ dagi holat $\left(0;\ \frac{\pi}{2}\right)$ dagiga, $\left(\frac{\pi}{2};\ \pi\right)$ dagi holat $\left(-\frac{\pi}{2};\ 0\right)$ dagiga o‘xshash.

4) t ning tg$t$, ctg$t$ *funksiyalar aniqlangan qiymatlarida quyidagi ayniyatlar o‘rinli:*

$$\mathrm{tg}t\,\mathrm{ctg}t=1,\ t\neq\frac{k\pi}{2},\ k\in Z, \qquad (3)$$

$$1+\mathrm{tg}^2t=\frac{1}{\cos^2 t},\ t\neq\frac{\pi}{2}+k\pi,\ k\in Z, \qquad (4)$$

$$1+\mathrm{ctg}^2t=\frac{1}{\sin^2 t},\ t\neq k\pi,\ k\in Z. \qquad (5)$$

(3) ayniyat $\mathrm{tg}t=\frac{\sin t}{\cos t}$ va $\mathrm{ctg}t=\frac{\cos t}{\sin t}$ tengliklarni ko‘paytirish orqali, (4) va (5) ayniyatlar esa $\sin^2t+\cos^2t=1$ tenglikning har ikkala qismini avval $\cos^2t$ ga, so‘ng $\sin^2t$ ga bo‘lish orqali hosil bo‘ladi.

M i s o l. Agar $\mathrm{tg}t=-\frac{2}{3}$ va $t\in\left(\frac{\pi}{2};\ \pi\right)$ bo‘lsa, sin$t$, cos$t$, ctg$t$ ning qiymatini topamiz.

Y e c h i s h . II chorakda $\sin t>0$, $\cos t<0$, u holda $\mathrm{ctg}t<0$. (3) ayniyat bo‘yicha $\mathrm{ctg}t=-\frac{3}{2}$; (4) ayniyat bo‘yicha:

$\cos^2t=\frac{1}{1+\left(-\frac{2}{3}\right)^2}=\frac{9}{13}$, $\cos t=-\frac{3}{\sqrt{13}}=-\frac{3\sqrt{13}}{13}$. Shu kabi (5) bo‘yicha $\sin t=\frac{2\sqrt{13}}{13}$ ni topamiz.

Mashqlar

1.45. Ifodalarning qiymatini toping:

1) $(a\text{tg}30°)^2 - (b\text{ctg}45°)^2$; 2) $\left(a\text{ctg}\frac{\pi}{4}\right)^3 - \left(b\text{ctg}\frac{\pi}{6}\right)^3$;

3) $a^2\text{ctg}^2\frac{\pi}{4} + b^2\text{ctg}^2\frac{\pi}{6} - c^2\text{ctg}^2\frac{\pi}{3}$;

4) $a\sin\pi\text{tg}1,2\pi - b\cos1,5\pi + c\text{tg}\pi\sin1,8\pi$;

5) $a^2\left(1-\cos\frac{\pi}{2}\right)^2 + b^2\cos^2(-31,1\pi)\text{ctg}\frac{\pi}{2}$; 6) $a^2\text{tg}^3\frac{\pi}{3} - b^2\text{ctg}^2\frac{\pi}{3}$.

1.46. Quyidagi qiymatni asosiy trigonometrik funksiyalarning qaysi biri qabul qila oladi:

1) $\frac{a^2+1}{2a},\ a > 0$; 2) $\frac{2a^2+1}{3a},\ a > 0$; 3) $\frac{a^4+2a^2+1}{4a^2},\ a > 0$;

4) $\frac{(a+b)^2}{4ab},\ a > 0,\ b > 0,\ a \neq b$; 5) $\frac{2\sqrt{ab}}{a+b},\ a > 0,\ b > 0,\ a \neq b$.

1.47. Ayniyatlarni isbot qiling:

1) $\frac{\cos x}{\text{ctg}x} = \sin x$; 2) $\cos x\text{tg}x = \sin x$; 3) $1 = \sin^2 x + \frac{\sin^2 x}{\text{tg}^2 x}$;

4) $\frac{\cos^2 x}{\text{ctg}^2 x} + \cos^2 x = 1$; 5) $(1 - \cos^2 x)(1 + \text{ctg}^2 x) = 1$;

6) $\cos^2 x\ \text{tg}^2 x + \cos^2 x = 1$; 7) $(1 + \text{tg}^2 x)(1 - \sin^2 x) = 1$;

8) $(\text{tg}x + 1)^2 + (\text{tg}x - 1)^2 = \frac{2}{\cos^2 x}$; 9) $\frac{1}{1+\text{ctg}^2 x}\cdot\frac{1+\text{tg}^2 x}{\text{tg}^2 x} = 1$;

10) $(1 - \cos\alpha + \sin\alpha)^2 = 2(1 - \cos\alpha)(1 + \sin\alpha)$;

11) $\frac{1}{\sin^2\alpha\sin^2\beta} - \frac{\text{ctg}^2\alpha}{\sin^2\beta} - \text{ctg}^2\beta = 1$;

12) $\text{ctg}^2\alpha\text{ctg}^2\beta + \text{ctg}^2\alpha + \frac{1}{\sin^2\beta} = \frac{1}{\sin^2\alpha\sin^2\beta}$.

1.48. $\sin\alpha = \frac{2ab}{a^2+b^2},\ a > 0,\ b > 0,\ 0 \le \alpha \le \frac{\pi}{2}$ bo‘lsa, $\cos\alpha$ va $\text{tg}\alpha$ ni toping.

1.49. $\text{ctg}\alpha = -1$ ekani ma’lum. $\frac{8\sin\alpha - 6\cos\alpha}{4\cos\alpha - 3\sin\alpha}$ kasrning qiymatini toping.

1.50. $\cos\alpha = -0{,}5,\ 90° \le \alpha \le 180°$ bo'lsa, $\sin\alpha$, $\mathrm{tg}\alpha$ va $\mathrm{ctg}\alpha$ ni toping.

1.51. $\sin\alpha = -\frac{2}{3},\ \frac{3\pi}{2} < \alpha < 2\pi$ bo'lsa, $\cos\alpha$, $\mathrm{tg}\alpha$, $\mathrm{ctg}\alpha$ ni toping.

1.52. Ifodalarni soddalashtiring:

1) $\mathrm{ctg}^2\alpha - \cos^2\alpha + \cos^2\alpha\,\mathrm{ctg}^2\alpha$; 2) $\cos^2\alpha + \sin^2\alpha\,\mathrm{tg}^2\alpha - \mathrm{tg}^2\alpha$;

3) $\sin^2\alpha - \frac{1}{1+\mathrm{ctg}^2\alpha}$; 4) $\cos^2\alpha - \frac{1}{1+\mathrm{tg}^2\alpha}$.

1.53. Barcha trigonometrik funksiyalarni
1) $\sin\alpha$; 2) $\cos\alpha$; 3) $\mathrm{tg}\alpha$; 4) $\mathrm{ctg}\alpha$ orqali ifodalang.

1.54. Isbot qiling:

1) $\mathrm{tg}\alpha + \mathrm{ctg}\alpha \ge 2$, bunda $\mathrm{tg}\alpha > 0$;

2) $\frac{(\sin x - \cos x)^2 - 1}{\mathrm{tg}x - \sin x\cos x} = -2\mathrm{ctg}^2 x$;

3) $\frac{\sin^2 x}{\mathrm{tg}^3 x} - \frac{\cos^2 x}{\mathrm{ctg}^3 x} + \frac{1}{\sin x\cos x} = 2\mathrm{tg}x$; 4) $\sin^3 x - \sin^6 x \le \frac{1}{4}$.

1.55. Agar $\sin x + \cos x = 1{,}5$ bo'lsa, quyidagilarni hisoblang:
1) $\mathrm{tg}^2 x + \mathrm{ctg}^2 x$; 2) $\mathrm{tg}^3 x + \mathrm{ctg}^3 x$; 3) $\mathrm{tg}^4 x + \mathrm{ctg}^4 x$.

1.56. Agar $\cos^6 x + \sin^6 x = q$ bo'lsa, $\cos^4 x + \sin^4 x$ ni toping.

1.57. $y = \frac{\sin x}{x}$ funksiya $\left(0;\ \frac{\pi}{2}\right)$ oraliqda kamayuvchi funksiya ekanligini isbot qiling.

1.58. $y = \frac{\mathrm{tg}x}{x}$ funksiya $\left(0;\ \frac{\pi}{2}\right)$ oraliqda o'suvchi funksiya ekanligini isbot qiling.

1.59. Ayirmalar ishorasini aniqlang:

1) $\mathrm{tg}164° - \mathrm{tg}165°$; 2) $\mathrm{tg}379° - \mathrm{tg}10°$; 3) $\mathrm{ctg}187° - \mathrm{ctg}6°$;

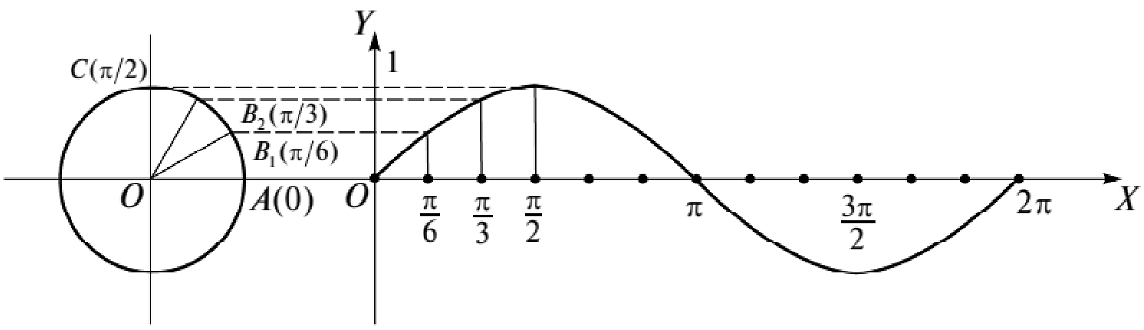
4) $\mathrm{tg}\left(5\frac{1}{7}\pi\right) - \mathrm{tg}\left(5\frac{1}{6}\pi\right)$; 5) $\mathrm{ctg}\frac{\pi}{6} - \mathrm{tg}\frac{\pi}{6}$.

1.60. $x \in (\pi;\ 2\pi)$ oraliqda quyidagi funksiyalarning monoton o'sish va monoton kamayish oraliqlarini aniqlang:

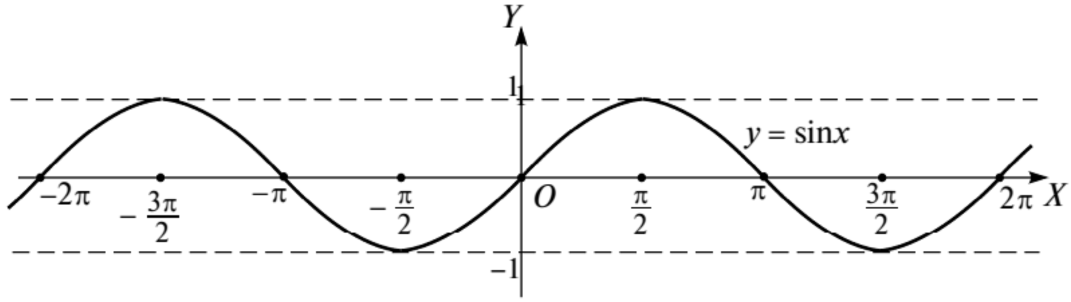
1) $y = \mathrm{tg}x$; 2) $y = \mathrm{ctg}x$; 3) $y = \frac{1}{1+\mathrm{ctg}^2 x}$; 4) $y = \frac{1}{1+\mathrm{tg}^2 x}$; 5) $y = \mathrm{ctg}^4 x$.

1. Sinus va kosinus funksiyalarning grafigi. $y = \sin x$ funksiya grafigi *sinusoida*, $y = \cos x$ funksiyaning grafigi esa *kosinusoida* deb ataladi. Ularni yasashda trigonometrik funksiyalarning xossalaridan foydalanamiz.

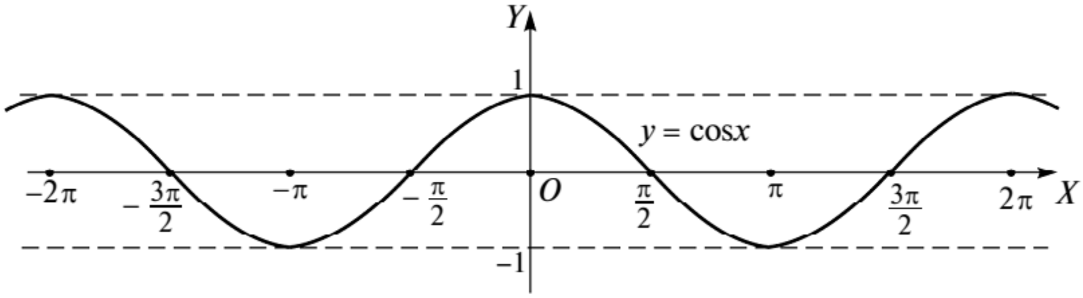
$\sin x$ − davriy funksiya va uning asosiy davri $T = 2\pi$ bo‘lgani uchun, OX o‘qida uzunligi 2π ga teng bo‘lgan biror oraliqni, masalan, $[-\pi; \pi]$ oraliqni ajratamiz (I.30-rasm) va unda grafikning mos qismini yasaymiz. Agar sinusning toq funksiya ekani e’tiborga olinsa, $[-\pi; \pi]$ oraliqning yarmi $[0; \pi]$ bilan chegaralanish, $\sin x = \sin(\pi - x)$ ekani, ya’ni x va $\pi - x$ nuqtalar $\frac{\pi}{2}$ ga nisbatan simmetrik joylashganliklari ham nazarda tutilsa, $[0; \frac{\pi}{2}]$ oraliq bilan chegaralanish yetarli. Shu oraliqda yasalgan qismi $x = \frac{\pi}{2}$ to‘g‘ri chiziqqa nisbatan simmetrik akslantirilsa, grafikning $[\frac{\pi}{2}; \pi]$ dagi qismi hosil qilinadi, natijada grafikning $[0; \pi]$ dagi qismi chizilgan bo‘ladi. Bu qism (0; 0) koordinatalar boshiga nisbatan simmetrik akslantirilsa, $[-\pi; \pi]$ oraliqdagi qismi hosil bo‘ladi. Endi uni 2π davr bilan son o‘qi bo‘yicha davom ettirish qoldi. Grafikni $[0; \frac{\pi}{2}]$ oraliqda geometrik yasash uchun koordinatali aylananing I choragini (AC yoyni, I.30-rasm)



I.30-rasm.



I.31-rasm.



I.32-rasm.

B_1, B_2, ... nuqtalar bilan teng bo‘laklarga ajratamiz. *OX* o‘qining shu oralig‘i ham shuncha teng bo‘lakka ajratiladi. Agar aylanadagi bo‘linish nuqtalaridan *OX* o‘qiga parallel va *OX* o‘qidagi bo‘linish nuqtalardan *OY* o‘qiga parallel to‘g‘ri chiziqlar o‘tkazsak, ularning kesishish nuqtalari izlanayotgan sinusoidada yotgan bo‘ladi. Nuqtalar ustidan uzluksiz chiziq chizamiz. U sinusoidaning eskizi bo‘ladi.

$y = \cos x$ kosinusoidani ham yuqorida ko‘rsatilgan tartibda yasash mumkin. Funksiyaning asosiy davri $T = 2\pi$. Demak, grafikni uzunligi 2π ga teng biror oraliqda, masalan, $[-\pi; \pi]$ oraliqda yasash, so‘ng uni son o‘qi bo‘yicha 2π davr bilan ikki tomonga davom ettirish kerak. $\cos x$ juft funksiya bo‘lganidan bu oraliqning $[0; \pi]$ qismini, $\cos(\pi - x) = -\cos x$ munosabatga ko‘ra esa yanada kichik $[0; \frac{\pi}{2}]$ oraliqni tanlaymiz. Unda yasalgan grafik *Ox* o‘qidagi $x = \frac{\pi}{2}$ nuqtaga nisbatan simmetrik almashtirilsa, grafikning $x = \pi$ gacha qismi hosil bo‘ladi. Bu qism ordinatalar o‘qiga nisbatan simmetrik almashtirilsa, grafikning $[-\pi; \pi]$ dagi qismi hosil qilinadi. Grafikning $[0; \frac{\pi}{2}]$ dagi qismi yuqorida sinusoidani yasashda ko‘rsatilgandek hosil qilinadi. Lekin bunda grafikdagi nuqta ordinatasi koordinatali aylanada unga mos nuqta abssissasiga teng bo‘lishi kerak.

Kosinusoidani yasashning boshqa yo‘li sinusoidani $\frac{\pi}{2}$ qadar chapga parallel ko‘chirishdan iborat.

I.31, I.32-rasmlarda mos ravishda sinusoida va kosinusoida tasvirlangan.

Mashqlar

1.62. $y = \cos x$ funksiya grafigi (I.32-rasm) bo‘yicha shu funksiyaning monotonlik oraliqlarini ko‘rsating.

1.63. Funksiyalarning grafiklarini yasang:

1) $y = 3\cos x$; 2) $y = -2\sin x$; 3) $y = |\cos x|$;

4) $y = \cos|x|$; 5) $y = \sin\left(x - \frac{\pi}{6}\right)$; 6) $y = \sin\left|x - \frac{\pi}{6}\right|$;

7) $y = [\cos x]$; 8) $y = \{\cos x\}$; 9) $y = 3\sin\left\{x + \frac{\pi}{4}\right\}$.

1.64. Grafiklarni chizing:

1) $|y| = \cos x$; 2) $|y| = \cos|x|$; 3) $|y| = |\sin x|$;

4) $|y| = \left|\cos\left|x - \frac{\pi}{6}\right|\right|$; 5) $y = 1 - \cos|x|$.

1.65. Funksiyalarning aniqlanish sohasini, qiymatlari sohalarini, o‘sish, kamayish va ishoralarining saqlanish oraliqlarini, maksimum va minimum nuqtalarini ko‘rsating, grafiklarini yasang:

1) $y = \cos x + 3$; 2) $y = 2\sin\left(\frac{\pi}{2} - 2x\right)$;

3) $y = 3\cos(\pi - 2x)$; 4) $y = \left|\cos x + \frac{3}{2}\right|$; 5) $y = -(1 - \cos x)$.

1.66. 1) $y = \sin x$ funksiyaning grafigini yasang. Sinusoidani siljitish yordamida kosinusoidani hosil qiling;

2) sinusoidani 2 birlik yuqoriga va 1 birlik chapga surish natijasida qanday funksiyaning grafigi hosil bo‘ladi? Shu funksiyaning xossalarini uning grafigi bo‘yicha aniqlang.

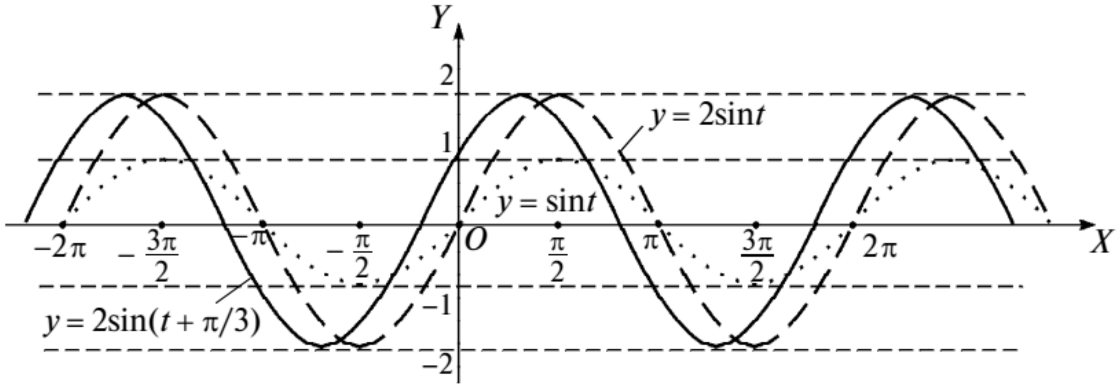
1.67. Quyidagi funksiyalarni juft-toqlikka, davriylikka tekshiring va grafiklarini yasang:

1) $y = \sin 3x$; 2) $y = \cos 3x$; 3) $y = \frac{1}{3}\sin x$;

4) $y = \frac{1}{3}\cos x$; 5) $y = 3\sin x$; 6) $y = 3\cos x$;

7) $y = \sin x + 3$; 8) $y = \cos x - 3$; 9) $y = 3\sin\ 0{,}5x + 1$;

10) $y = 2\cos 3x - 1$.



I.33-rasm.

2. Sinusoidal tebranishlar. Tebranma harakat trigonometrik funksiyalar orqali ifodalanadi. Matematik mayatnikning harakat tenglamasi, o‘zgaruvchan elektr toki kuchi yoki kuchlanishining o‘zgarish qonuniyatlari bunga misol bo‘la oladi. Eng sodda tebranma harakat *sinusoidal* (yoki *garmonik*) tebranishlardir.

Biror nuqta radiusi A ga teng aylana bo‘yicha ω rad/s burchak tezlik bilan harakat qilayotgan bo‘lsin. Nuqta t s da ωt radianga teng yoy chizadi. Agar aylananing markazi koordinatalar boshida joylashtirilgan va $t = 0$ vaqt momentida nuqta biror $B_0(\alpha)$ nuqtada turgan bo‘lsa, t vaqtdan so‘ng u $B(\omega t + \alpha)$ ga keladi. B nuqta koordinatalari:

$$x = A\cos(\omega t + \alpha) \tag{1}$$

va

$$u = A\sin(\omega t + \alpha). \tag{2}$$

Bunga qaraganda B nuqtaning t ga bog‘liq ravishda harakati davomida uning x va y koordinatalari OX va OY o‘qlari bo‘yicha ko‘pi bilan $|A|$ qadar oldinga-keyinga siljiydi, tebranadi va o‘tilgan masofa (1) va (2) munosabatlardagi sinus va kosinus qiymatiga bog‘liq bo‘ladi. Bu harakat sinusoidal tebranishdir. (1) va (2) tenglikdagi A son tebranishning qulochini ifodalaydi va tebranish *amplitudasi* deyiladi, ω esa 2π vaqt birligi ichidagi to‘liq tebranishlar soni bo‘lib, *burchak chastotasi* deyiladi. α son nuqtaning aylanadagi boshlang‘ich o‘rni, ya’ni *boshlang‘ich faza.* (1) va (2) funksiyalarning asosiy davri $T = \frac{2\pi}{\omega}$ (isbot qiling!).

(1) yoki (2) garmonik tebranishlar grafigini sinusoidadan foydalanib yasash maqsadida (2) funksiya ifodasini $y = A\sin\omega\left(t + \frac{\alpha}{\omega}\right)$ ko‘rinishda yozamiz. Bunga qaraganda grafikni yasash uchun sin$t$

sinusoidani OX o'qi bo'yicha A koeffitsiyent bilan cho'zish, OY o'qi bo'yicha ω koeffitsiyent bilan qisish va koordinatalar boshini $L\left(-\frac{\alpha}{\omega};\ 0\right)$ nuqtaga akslantiruvchi parallel ko'chirishni bajarish kerak.

M i s o l. $y = 2\sin\left(t + \frac{\pi}{3}\right)$ funksiya grafigini yasaymiz.

Y e c h i s h. $y = \sin t$ sinusoidani OY o'qi yo'nalishida 2 marta cho'zishni va Ot o'qi bo'yicha $\frac{\pi}{3}$ qadar chapga parallel ko'chirishni bajaramiz (I.33-rasm).

Mashqlar

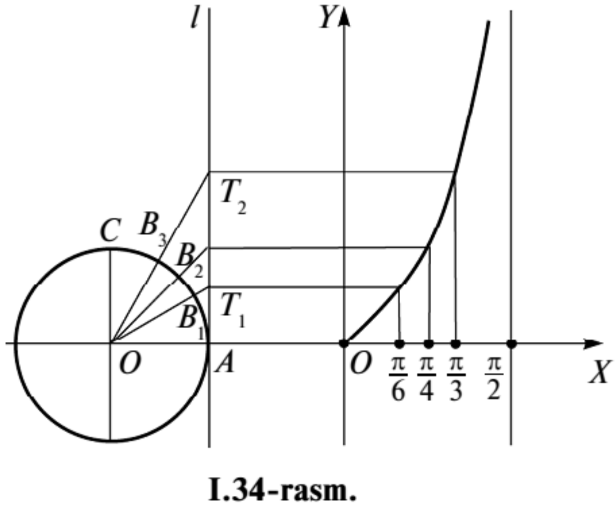
1.68. Formulalar bilan berilgan garmonik tebranishlarning amplitudasi, davri, boshlang'ich fazasini toping va grafigini yasang:

1) $y = 3\sin\left(2t + \frac{\pi}{2}\right)$; 2) $y = 0{,}8\sin\pi\left(t + \frac{\pi}{3}\right)$; 3) $y = 2{,}5\sin(0{,}5t + 1)$;

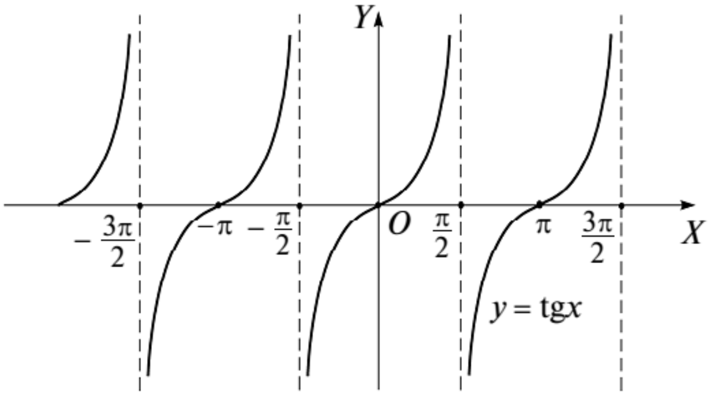
4) $y = 3{,}5\sin\left(0{,}5\pi t - \frac{\pi}{6}\right)$; 5) $y = 2\pi\sin 4t$; 6) $y = 3\sin(2t - 1)$.

3. Tangens va kotangens funksiyalarning grafigi. tg$x$ toq funksiya, davri $T = \pi$ bo'lganidan uning grafigini $\left[0;\ \frac{\pi}{2}\right]$ oraliqda yasash, so'ng uni koordinatalar boshiga nisbatan simmetrik akslantirish va abssissalar o'qi bo'yicha πk, $k \in Z$ lar qadar surish kerak bo'ladi.

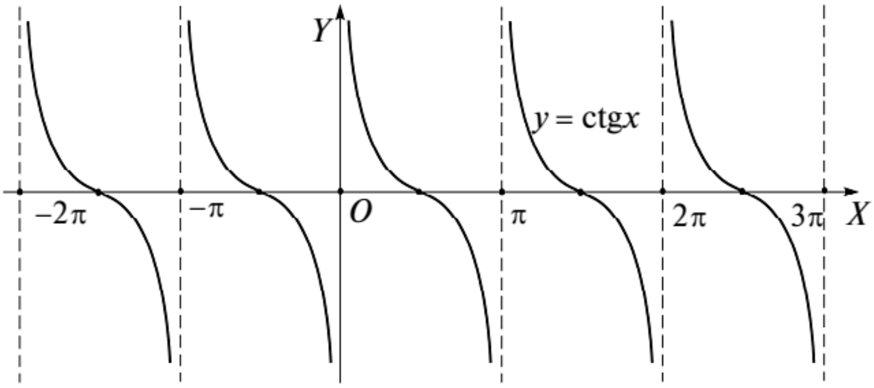
Markazi O_1 $(-1;\ 0)$ nuqtada bo'lgan birlik aylananing $\cup AC = \frac{\pi}{2}$ yoyi teng uzoqlikda olingan B_1, B_2,... nuqtalar bilan bir necha teng bo'lakka bo'lingan bo'lsin (I.34-rasm). Bu nuqtalar va O_1 nuqtadan o'tkazilgan O_1B_1, O_1B_2, ... to'g'ri chiziqlar Al tangenslar chizig'i bilan T_1, T_2, ...



I.34-rasm.



I.35-rasm.



I.36-rasm.

nuqtalarda kesishsin. Chizmaga qaraganda $AT_1 = \mathrm{tg}\frac{\pi}{6}$, $AT_2 = \mathrm{tg}\frac{\pi}{3}$ va hokazo. T_1, T_2, ... nuqtalardan OX o'qiga parallel va $x = \frac{\pi}{6}$, $\frac{\pi}{3}$, ... nuqtalardan OY o'qiga parallel o'tkazilgan to'g'ri chiziqlarning kesishish nuqtalari belgilanadi. Ular ustidan o'tadigan egri chiziq tg$x$ funksiyaning grafigi (*tangensoida*) bo'ladi.

Grafik $x = \frac{\pi}{2}$ to'g'ri chiziqqa tomon yaqinlashganida yuqoriga cheksiz ko'tariladi. Endi koordinatalar boshiga nisbatan markaziy simmetriya, so'ng abssissalar o'qi bo'yicha πk, $k \in Z$ davrlar bilan parallel ko'chirishlarni bajarish grafikning kattaroq oraliqdagi davomini beradi (I.35-rasm).

ctg$x$ funksiyaning grafigi (*kotangensoida*) ham shu kabi yasaladi (I.36-rasm).

Mashqlar

1.69. Quyidagi funksiyalarning xossalarini tekshiring va grafiklarini yasang:

1) $y = \mathrm{ctg}2x$; 2) $y = \mathrm{ctg}\left(2x - \frac{\pi}{3}\right)$; 3) $y = \mathrm{ctg}\frac{x}{2}$;

2-Mavzu: Trigonometrik ifodalarning qiymatini hisoblash va soddalashtirishga doir masalalar yechish.

1. Ikki burchak ayirmasining va yig‘indisining kosinusi va sinusi. Chizmada (I.37- rasm) $\angle BOA = \alpha$, $\angle COA = \beta$, $\varphi = \beta - \alpha$, $BD \perp CE$, $CQ \perp OB$, $DE = BF$, $DB = EF = OF - OE = \cos\alpha - \cos\beta$, $QB = OB - OQ = 1 - \cos\varphi$, $CQ = \sin\varphi$, $CD = CE - BF = \sin\beta - \sin\alpha$, CDB va CQB to‘g‘ri burchakli uchburchaklar umumiy CB gipotenuzaga ega. Pifagor teoremasi bo‘yicha:

$$BC^2 = CQ^2 + QB^2 = CD^2 + DB^2$$

yoki

$$\sin^2\varphi + (1 - \cos\varphi)^2 = (\sin\beta - \sin\alpha)^2 + (\cos\alpha - \cos\beta)^2,$$

$$\sin^2\varphi + 1 - 2\cos\varphi + \cos^2\varphi =$$

$$= \sin^2\beta - 2\sin\beta\sin\alpha + \sin^2\alpha + \cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta,$$

$$(\sin^2\varphi + \cos^2\varphi) + 1 - 2\cos\varphi = (\sin^2\beta + \cos^2\beta) + (\cos^2\alpha + \sin^2\alpha) -$$

$$- 2(\sin\alpha\sin\beta - \cos\alpha\cos\beta),\ 2 - 2\mathrm{sos}\varphi = 2 - 2(\sin\alpha\sin\beta -$$

$$-\cos\alpha\cos\beta)$$

yoki

$$\cos(\beta - \alpha) = \cos\alpha\cos\beta + \sin\alpha\sin\beta. \qquad (1)$$

(1) munosabat bo‘yicha va funksiyalarning xossalaridan foydalanib, yana boshqa formulalarni topish mumkin:

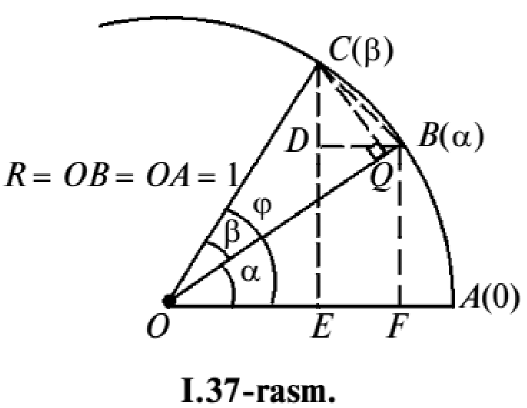
$$\cos(\alpha + \beta) = \cos(\alpha - (-\beta)) =$$

$$= \cos\alpha\cos(-\beta) + \sin\alpha\sin(-\beta),$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta. \quad (2)$$

Xususan:

a) $\cos\left(\frac{\pi}{2} - \alpha\right) = \cos\frac{\pi}{2}\cos\alpha + \sin\frac{\pi}{2}\sin\alpha = 0 \cdot \cos\alpha + 1 \cdot \sin\alpha = \sin\alpha$,



I.37-rasm.

$$\cos\left(\frac{\pi}{2}+\alpha\right)=\cos\frac{\pi}{2}\cos\alpha-\sin\frac{\pi}{2}\sin\alpha=0\cdot\cos\alpha-1\cdot\sin\alpha=-\sin\alpha;$$

$$\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha; \qquad (3)$$

$$\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha. \qquad (4)$$

b) $\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\left(\frac{\pi}{2}-\left(\frac{\pi}{2}-\alpha\right)\right)=\cos\alpha$,

$\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\left(\frac{\pi}{2}-\left(\frac{\pi}{2}+\alpha\right)\right)=\cos(-\alpha)=\cos\alpha$.

Demak,

$$\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha; \qquad (5)$$

$$\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha. \qquad (6)$$

$\alpha\pm\beta$ burchak sinusi uchun formulalar yuqorida topilgan formulalardan foydalanib chiqariladi:

$$\sin(\alpha+\beta)=\cos\left(\frac{\pi}{2}-(\alpha+\beta)\right)=\cos\left(\left(\frac{\pi}{2}-\alpha\right)-\beta\right)=$$

$$=\cos(\frac{\pi}{2}-\alpha)\cos\beta+\sin\left(\frac{\pi}{2}-\alpha\right)\sin\beta=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

yoki

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta. \qquad (7)$$

Agar (7) formuladagi β o‘rniga $-\beta$ qo‘yilsa, natijada:

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta. \qquad (8)$$

M i s o l. $\cos150^\circ$ va $\sin150^\circ$ ni topamiz.

Y e c h i s h. (4) va (6) formulalar bo‘yicha:

$$\cos150^\circ=\cos(90^\circ+60^\circ)=-\sin60^\circ=-\frac{\sqrt{3}}{2};$$

$$\sin150^\circ=\sin(90^\circ+60^\circ)=\cos60^\circ=\frac{1}{2}.$$

Mashqlar

1.70. Hisoblang: 1) $\sin\frac{\pi}{12}$; 2) $\cos\frac{4\pi}{3}$; 3) $\cos\frac{5\pi}{4}$; 4) $\sin\frac{4\pi}{3}$.

1.71. Agar $\sin x = \frac{3}{8}$; $\sin t = \frac{4}{9}$; $0 < x < \frac{\pi}{2}$; $\frac{\pi}{2} < t < \pi$ bo‘lsa, quyidagilarni toping:

1) $\sin(x - t)$; 2) $\sin(x + t)$; 3) $\cos(x - t)$; 4) $\cos(x + t)$.

1.72. Agar $\cos x = -0{,}8$; $\sin y = 0{,}4$; $\pi < x < \frac{3\pi}{2}$; $\frac{\pi}{2} < y < \pi$ bo‘lsa, quyidagilarni toping:

1) $\cos(x + y)$; 2) $\cos(x - y)$; 3) $\sin(x + y)$; 4) $\sin(x - y)$.

1.73. Ifodalarni soddalashtiring:

1) $\cos(x + t)\sin(x - t) + \sin(x + t)\cos(x - t)$;

2) $\cos(\alpha + \beta)\cos(\alpha - \beta) - \sin(\alpha + \beta)\sin(\alpha - \beta)$;

3) $\cos(45^\circ + \alpha)\cos(45^\circ - \alpha) + \sin(45^\circ + \alpha)\sin(45^\circ - \alpha)$;

4) $\frac{\cos(\alpha-\beta)-\sin\alpha\sin\beta}{\sin(\alpha-\beta)+\sin\alpha\cos\beta}$; 5) $\frac{\sin(\beta+\alpha)-2\sin\alpha\cos\beta}{\cos(\alpha+\beta)+2\sin\alpha\sin\beta}$;

6) $\frac{\cos(\alpha+\beta)+\sin\alpha\sin\beta}{\cos(\alpha+\beta)-\cos\alpha\cos\beta}$; 7) $\frac{\cos(\alpha-\beta)-\sin\alpha\sin\beta}{\cos(\alpha-\beta)-\cos\alpha\cos\beta}$.

1.74. Ayniyatlarni isbot qiling:

1) $\cos(45^\circ - \alpha) = \frac{\sqrt{2}}{2}(\cos\alpha + \sin\alpha)$; 2) $\frac{\cos(\alpha-\beta)}{\sin\alpha\sin\beta} = \operatorname{ctg}\alpha\operatorname{ctg}\beta + 1$;

3) $\sin(\alpha - \beta)\sin(\alpha + \beta) = \cos^2\beta - \cos^2\alpha$;

4) $\sin 2x \cos x + \cos 2x \sin x = \sin 3x$;

5) $\sin(\alpha + \beta) + \cos(\alpha - \beta) = (\sin\alpha + \cos\alpha)(\sin\beta + \cos\beta)$;

6) $\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta} + \frac{\sin(\beta-\gamma)}{\cos\beta\cos\gamma} + \frac{\sin(\gamma-\alpha)}{\cos\gamma\cos\alpha} = 0$.

1.75. Funksiyalarning juft-toqligi va davriyligini tekshiring hamda grafiklarini yasang:

1) $y = \cos x \cos\frac{x}{2} + \sin x \sin\frac{x}{2}$; 2) $y = \sin x \cos\frac{x}{3} - \sin\frac{x}{3}\cos x$.

2. Ikki burchak yig‘indisi va ayirmasining tangensi va kotangensi. 1-banddagi formulalardan foydalanamiz. Buning uchun $\cos(\alpha + \beta) \neq 0$, ya’ni $\alpha + \beta \neq \frac{\pi}{2} + \pi k$, $k \in Z$ va $\cos\alpha \neq 0$, $\cos\beta \neq 0$ bo‘lishi, ya’ni α va β lar $\frac{\pi}{2} + \pi k$, $k \in Z$ ga teng bo‘lmasligi kerak. Shu shartlardan quyidagilarga ega bo‘lamiz:

$$\operatorname{tg}(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}=\frac{\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta}}{1-\frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin\beta}{\cos\beta}}=\frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{1-\operatorname{tg}\alpha\operatorname{tg}\beta}.$$

Bundan:

$$\operatorname{tg}(\alpha+\beta)=\frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{1-\operatorname{tg}\alpha\operatorname{tg}\beta}. \qquad (1)$$

Xuddi shunday,

$$\operatorname{tg}(\alpha-\beta)=\frac{\operatorname{tg}\alpha-\operatorname{tg}\beta}{1+\operatorname{tg}\alpha\operatorname{tg}\beta}. \qquad (2)$$

Quyidagi formulalar ham shu kabi hosil qilinadi:

$$\operatorname{ctg}(\alpha+\beta)=\frac{\operatorname{ctg}\alpha\operatorname{ctg}\beta-1}{\operatorname{ctg}\alpha+\operatorname{ctg}\beta}, \qquad (3)$$

$$\operatorname{ctg}(\alpha-\beta)=\frac{\operatorname{ctg}\alpha\operatorname{ctg}\beta+1}{\operatorname{ctg}\alpha-\operatorname{ctg}\beta}. \qquad (4)$$

M i s o l. $\operatorname{ctg}\frac{7\pi}{12}$ ni hisoblaymiz.

Y e c h i s h. $\operatorname{ctg}\frac{7\pi}{12}=\operatorname{ctg}\left(\frac{\pi}{3}+\frac{\pi}{4}\right)=\frac{\operatorname{ctg}\frac{\pi}{3}\cdot\operatorname{ctg}\frac{\pi}{4}-1}{\operatorname{ctg}\frac{\pi}{3}+\operatorname{ctg}\frac{\pi}{4}}=\frac{\frac{\sqrt{3}}{3}\cdot1-1}{\frac{\sqrt{3}}{3}+1}=\frac{\sqrt{3}-3}{\sqrt{3}+3}=\frac{1+\sqrt{3}}{1-\sqrt{3}}.$

Mashqlar

1.76. Agar $\sin(2\alpha+\beta)=2\sin\beta$ bo‘lsa, $\operatorname{tg}(\alpha+\beta)=3\operatorname{tg}\alpha$ bo‘lishini isbot qiling, bunda $\beta\neq k\pi$, $k\in Z$.

1.77. (1)−(4) formulalarning chap qismlarini uning o‘ng qismlaridan hosil qiling.

1.78. Hisoblang:

1) $\operatorname{tg}75^\circ$; 2) $\operatorname{ctg}\frac{5\pi}{12}$; 3) $\operatorname{ctg}105^\circ$; 4) $\operatorname{tg}15^\circ$; 5) $\operatorname{ctg}15^\circ$.

1.79. Berilgan: 1) $\operatorname{tg}x=1{,}5$, $\operatorname{tg}y=-0{,}5$; 2) $\operatorname{ctg}x=1{,}5$, $\operatorname{ctg}y=-0{,}5$. Toping: 1) $\operatorname{tg}(x-y)$; 2) $\operatorname{tg}(x+y)$; 3) $\operatorname{ctg}(x-y)$; 4) $\operatorname{ctg}(x+y)$.

1.80. Berilgan: $\operatorname{tg}\alpha=\frac{1}{3}$, $\operatorname{ctg}\beta=\frac{4}{5}$, $\operatorname{tg}\gamma=1$. Hisoblang:

1) $\operatorname{tg}(\alpha+\beta+\gamma)$; 2) $\operatorname{ctg}(\alpha+\beta+\gamma)$; 3) $\operatorname{tg}(\alpha+\beta-\gamma)$.

3-Mavzu: Trigonometrik ayniyatlarni isbotlash.

Agar $\alpha + \beta$ burchak trigonometrik funksiyalari formulalarida $\alpha = \beta$ deyilsa, 2α burchak trigonometrik funksiyalari formulalari hosil qilinadi. Ular 2α argument funksiyasini α argument funksiyasi orqali ifodalashga imkon beradi:

$$\sin 2\alpha = 2\sin\alpha\cos\alpha; \quad (1) \qquad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha; \quad (2)$$

$$\operatorname{tg} 2\alpha = \frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}; \quad (3) \qquad \operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2\alpha - 1}{2\operatorname{ctg}\alpha}. \quad (4)$$

Aksincha, α argument funksiyasini 2α funksiyasi orqali ham berish mumkin. Chunonchi, $1 = \sin^2\alpha + \cos^2\alpha$ ayniyat va (2) formula bo‘yicha $1 + \cos 2\alpha = 2\cos^2\alpha$ va $1 - \cos 2\alpha = 2\sin^2\alpha$ yoki

$$\cos 2\alpha = 2\cos^2\alpha - 1 \quad (5)$$

va

$$\cos 2\alpha = 1 - 2\sin^2\alpha \quad (6)$$

hosil qilinadi. (5) va (6) formulalarni quyidagi ko‘rinishda ham yozish mumkin:

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}; \quad (7)$$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}. \quad (8)$$

Agar $\cos\alpha \neq 0$ bo‘lsa, (1) tenglikning o‘ng qismini $\sin^2\alpha + \cos^2\alpha$ ga, ya’ni 1 ga, so‘ng surat va maxrajni $\cos^2\alpha$ ga bo‘lsak, quyidagini hosil qilamiz:

$$\sin 2\alpha = \frac{2\sin\alpha\cos\alpha}{\sin^2\alpha + \cos^2\alpha} = \frac{2 \cdot \frac{\sin\alpha\cos\alpha}{\cos^2 a}}{\frac{\sin^2\alpha + \cos^2\alpha}{\cos^2 a}},$$

bundan:

$$\sin 2\alpha = \frac{2\operatorname{tg}\alpha}{1+\operatorname{tg}^2\alpha}. \quad (9)$$

Shu kabi:

$$\cos\alpha \neq 0 \text{ да } \cos 2\alpha = \frac{1-\mathrm{tg}^2\alpha}{1+\mathrm{tg}^2\alpha}. \qquad (10)$$

Shuningdek, $\mathrm{ctg}2\alpha = \frac{1}{\mathrm{tg}2\alpha}$ va (3) formula bo'yicha:

$$\mathrm{ctg}2\alpha = \frac{1-\mathrm{tg}^2\alpha}{2\mathrm{tg}\alpha},\ a \neq \frac{\pi k}{2},\ k \in Z. \qquad (11)$$

Uchlangan argument 3α ning trigonometrik funksiyalarini yuqorida topilgan formulalardan foydalanib topish mumkin. Masalan,

$$\sin 3\alpha = \sin(2\alpha + \alpha) = \sin 2\alpha\cos\alpha + \cos 2\alpha\sin\alpha =$$

$$= 2\sin\alpha\cos^2\alpha + (1 - 2\sin^2\alpha)\sin\alpha = \sin\alpha(2(\cos^2\alpha - \sin^2\alpha) + 1) =$$

$$= \sin\alpha(2(1 - 2\sin^2\alpha) + 1) = \sin\alpha(3 - 4\sin^2\alpha),$$

$$\sin 3\alpha = \sin\alpha(3 - 4\sin^2\alpha). \qquad (12)$$

Shu kabi: $\cos 3\alpha = \cos\alpha(4\cos^2\alpha - 3)$.

Mashqlar

1.90. $\sin\alpha = -0{,}83$, $\pi < \alpha < \frac{3\pi}{2}$ bo'yicha $\sin 2\alpha$, $\cos 2\alpha$, $\mathrm{tg}2\alpha$ ni toping.

1.91. $\cos\alpha = -0{,}4$, $\sin\alpha < 0$ bo'yicha $\sin 2\alpha$, $\cos 2\alpha$, $\mathrm{tg}2\alpha$ ni toping.

1.92. $\mathrm{tg}\alpha = -3$ bo'yicha $\mathrm{ctg}2\alpha$ ni toping.

1.93. Agar $0 < \alpha < \frac{\pi}{2}$ bo'lsa, $\sin 2\alpha < 2\sin\alpha$ bo'lishini isbot qiling.

1.94. $\mathrm{ctg}\alpha = -1{,}2$, $\frac{\pi}{2} < \alpha < \pi$ bo'yicha $\sin 3\alpha$, $\cos 3\alpha$, $\cos 4\alpha$, $\mathrm{tg}4\alpha$ ni toping.

1.95. Agar $\mathrm{tg}\alpha = 0{,}3$, $\mathrm{tg}\beta = 0{,}4$ bo'lsa, $\mathrm{tg}(2\alpha - \beta)$ ni toping.

1.96. Ayniyatlarni isbot qiling:

1) $\dfrac{\cos 2t+1-\cos^2 t}{\cos\left(\frac{5\pi}{2}+2t\right)} = -\dfrac{1}{2}\,\mathrm{ctg}t$; 2) $\dfrac{2\sin 2t-\sin 4t}{\sin 4t+2\sin 2t} = \mathrm{tg}^2 t$;

3) $\dfrac{\sin 2t+\mathrm{tg}2t}{\mathrm{tg}2t} = 2\cos^2 t$; 4) $\dfrac{1-2\cos^2 2t}{\frac{1}{2}\sin 4t} = \mathrm{tg}2t - \mathrm{ctg}t$;

5) $\dfrac{\cos t+\mathrm{ctg}t}{\mathrm{ctg}t} = 1 + \sin 2t$; 6) $\dfrac{2\cos^2 t-1}{2\mathrm{ctg}\left(\frac{\pi}{4}-t\right)\sin^2\left(\frac{\pi}{4}-t\right)} = 1$;

7) $1 + \cos t = \dfrac{\sin t+\mathrm{tg}t}{\mathrm{tg}t}$; 8) $\dfrac{2\sin 4t(1-\mathrm{tg}^2 2t)}{1+\mathrm{ctg}^2\left(\frac{\pi}{2}+2t\right)} = \sin 8t$;

9) $\dfrac{\cos 2t+5\cos 3t+\cos 4t}{\sin 2t+5\sin 3t+\sin 4t} = \mathrm{ctg}3t$; 10) $\mathrm{tg}55^\circ\mathrm{tg}65^\circ\mathrm{tg}75^\circ = \mathrm{tg}85^\circ$.

1.97. Ifodalarni soddalashtiring:

1) $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$;

2) $\sin\left(\alpha-\frac{5\pi}{2}\right)\cos\alpha-\sin^2(5\pi-\alpha)\sin^2\alpha-\cos^2(5\pi-\alpha)\cos^2\left(\frac{11\pi}{2}+\alpha\right)$;

3) $2\,\mathrm{ctg}\left(\frac{13\pi}{2}-2\alpha\right)+\dfrac{2\sin(17\pi-\alpha)}{\sin\left(\frac{15\pi}{2}+\alpha\right)+\mathrm{tg}\alpha\sin(-\alpha)}$;

4) $\dfrac{2\cos\alpha-\sin 2\alpha}{\sin^2 3\alpha-\sin\alpha+\cos^2 3\alpha}$; 5) $\dfrac{\sin^4 2t+2\cos 2t\sin 2t-\cos^4 2t}{2\cos^2 2t-1}$;

6) $1 + 2\cos 2\alpha + 2\cos 4\alpha + 2\cos 6\alpha$;

7) $\cos\left(\frac{7\pi}{2}-2\alpha\right)\mathrm{tg}(5\pi-\alpha)+\sin\left(\frac{17\pi}{2}+2\alpha\right)$; 8) $\dfrac{(1+\mathrm{tg}2\alpha)\cos\left(\frac{\pi}{4}+2\alpha\right)}{1-\mathrm{tg}2\alpha}$.

5. Yarim argumentning trigonometrik funksiyalari. Bu formulalar oldingi bandda berilgan (4)−(11) formulalardagi α oʻrniga $\frac{\alpha}{2}$ ni qoʻyish orqali hosil qilinadi. Jumladan,(7),(8) formulalar boʻyicha $\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$, $\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$ yoki

$$\left|\cos\frac{\alpha}{2}\right| = \sqrt{\frac{1+\cos\alpha}{2}}\,; \qquad (1)$$

$$\left|\sin\frac{\alpha}{2}\right| = \sqrt{\frac{1-\cos\alpha}{2}}\,. \qquad (2)$$

Agar (2) tenglik (1) ga hadma-had bo‘linsa:

$$\left|\operatorname{tg}\frac{\alpha}{2}\right| = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} \qquad (3)$$

tenglik hosil bo‘ladi. $\operatorname{ctg}\alpha = \frac{1}{\operatorname{tg}\alpha}$ bo‘lgani uchun

$$\left|\operatorname{ctg}\frac{\alpha}{2}\right| = \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}\,. \qquad (4)$$

tenglik ham o‘rinlidir.

(1)−(4) formulalar trigonometrik funksiyalar qiymatlarining modulini topishga imkon beradi. Ularning ishoralari esa $\frac{\alpha}{2}$ argumentning qaysi chorakka tegishli ekaniga bog‘liq.

M i s o l. $\sin\alpha = \frac{\sqrt{5}}{3}$, $\frac{\pi}{2} \le \alpha \le \pi$ ekani ma’lum. $\sin\frac{\alpha}{2}$, $\cos\frac{\alpha}{2}$, $\operatorname{tg}\frac{\alpha}{2}$ ni topamiz.

Y e c h i s h. Shartdan foydalanib $\frac{\pi}{4} \le \frac{\alpha}{2} \le \frac{\pi}{2}$ bo‘lishini aniqlaymiz. Bu oraliqda barcha trigonometrik funksiyalar musbat. Yuqorida topilgan formulalardan foydalanamiz. Oldin $\cos\alpha$ ni topaylik:

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = -\sqrt{1-\frac{5}{9}} = -\frac{2}{3}.$$

U holda:

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\left(-\frac{2}{3}\right)}{2}} = \frac{1}{\sqrt{6}},\ \ \sin\frac{\alpha}{2} = \sqrt{\frac{1-\left(-\frac{2}{3}\right)}{2}} = \sqrt{\frac{5}{6}},\ \ \operatorname{tg}\frac{\alpha}{2} = \sqrt{5}\,.$$

Yarim argumentning tangensi uchun yana bir formula hosil qilish maqsadida $\operatorname{tg}\frac{\alpha}{2} = \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}$ tenglikning o‘ng qismidagi kasr surat va maxrajini $2\sin\frac{\alpha}{2}$ ga ko‘paytiramiz:

$$\operatorname{tg}\frac{\alpha}{2}=\frac{2\sin^2\frac{\alpha}{2}}{2\cos\frac{\alpha}{2}\sin\frac{\alpha}{2}}=\frac{2\cdot\frac{1-\cos\alpha}{2}}{\sin\left(2\cdot\frac{\alpha}{2}\right)}=\frac{1-\cos\alpha}{\sin\alpha},$$

$$\operatorname{tg}\frac{\alpha}{2}=\frac{1-\cos\alpha}{\sin\alpha}. \qquad (5)$$

Agar surat va maxraj $2\cos\frac{\alpha}{2}$ ga ko‘paytirilsa,

$$\operatorname{tg}\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}. \qquad (6)$$

(5) va (6) formulalar bo‘yicha:

$$\operatorname{ctg}\frac{\alpha}{2}=\frac{\sin\alpha}{1-\cos\alpha}=\frac{1+\cos\alpha}{\sin\alpha}. \qquad (7)$$

Mashqlar

1.98. Berilgan: 1) $\alpha=\frac{\pi}{6}$; 2) $\alpha=\frac{\pi}{4}$; 3) $\cos\alpha=-0{,}4$, $\frac{\pi}{2}<\alpha<\pi$; 4) $\operatorname{ctg}\alpha=4$, $\pi<\alpha<\frac{3}{2}\pi$; 5) $\sin\alpha=0{,}8$; $450^\circ<\alpha<540^\circ$.

Toping: $\sin\frac{\alpha}{2}$; $\cos\frac{\alpha}{2}$; $\operatorname{tg}\frac{\alpha}{2}$.

1.99. $\sin 15^\circ$, $\cos 15^\circ$, $\sin 18^\circ$, $\cos 18^\circ$, $\sin 12^\circ$, $\cos 12^\circ$ lar hisoblansin.

1.100. Ifodalarni soddalashtiring:

1) $\sqrt{\frac{1-\cos 6\alpha}{2}}$; 2) $\sqrt{1+\cos 10x}$; 3) $2\cos^2\frac{\alpha}{2}-\cos\alpha$;

4) $\frac{1+\cos 4\alpha}{\sin 4\alpha}$; 5) $\sqrt{\frac{\sin\left(\frac{7\pi}{2}-\alpha\right)+1}{\sin\left(\frac{5\pi}{2}+\alpha\right)+1}}$.

1.101. Ayniyatlarni isbot qiling:

1) $1-\cos\left(\frac{\pi}{2}+\alpha\right)=2\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$;

2) $1+\cos\left(\frac{5\pi}{2}+\alpha\right)=2\sin^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$; 3) $\dfrac{\mathrm{tg}\left(\pi-\frac{\alpha}{2}\right)-\mathrm{ctg}\frac{\alpha}{2}}{\mathrm{tg}\frac{\alpha}{2}+\mathrm{ctg}\left(\pi-\frac{\alpha}{2}\right)}=-\cos\alpha$;

4) $\mathrm{tg}\left(\pi-\frac{\alpha}{2}\right)+\mathrm{ctg}\frac{\alpha}{2}=2\mathrm{ctg}\alpha$; 5) $\sin^4\alpha+\cos^4\alpha=\frac{3+\cos 4\alpha}{4}$;

6) $\cos^4\alpha-\sin^4\alpha=\cos 2\alpha$.

6. Trigonometrik funksiyalarni yarim argument tangensi orqali ifodalash. $\sin\alpha$ va $\cos\alpha$ ni $\mathrm{tg}\frac{\alpha}{2}$ orqali ifodalashda $\sin\alpha=$ $=2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}$, $\cos\alpha=\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}$ va $\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}=1$

formulalardan foydalanamiz. $\sin\alpha=\dfrac{2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}}$ tenglikka egamiz. Bu tenglikdagi kasrning surat va maxrajini $\cos^2\frac{\alpha}{2}\neq 0$ ga bo‘lib,

$$\sin\alpha=\frac{2\,\mathrm{tg}\frac{\alpha}{2}}{1+\mathrm{tg}^2\frac{\alpha}{2}} \qquad (1)$$

tenglikni hosil qilamiz. Xuddi shu kabi, $\cos\alpha=\dfrac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}}$ tenglik yordamida quyidagi tenglik hosil qilinadi:

$$\cos\alpha=\frac{1-\mathrm{tg}^2\frac{\alpha}{2}}{1+\mathrm{tg}^2\frac{\alpha}{2}}. \qquad (2)$$

$\mathrm{tg}\alpha$ va $\mathrm{ctg}\alpha$ ni $\mathrm{tg}\frac{\alpha}{2}$ orqali ifodalash uchun (1) ni (2) ga va aksincha, (2) ni (1) ga hadma-had bo‘lish yetarli. Natijada quyidagi tengliklarga ega bo‘lamiz:

$$\mathrm{tg}\alpha=\frac{2\,\mathrm{tg}\frac{\alpha}{2}}{1-\mathrm{tg}^2\frac{\alpha}{2}}, \qquad (3)$$

$$\mathrm{ctg}\alpha=\frac{1-\mathrm{tg}^2\frac{\alpha}{2}}{2\,\mathrm{tg}\frac{\alpha}{2}}. \qquad (4)$$

M i s o l. Agar $\mathrm{tg}\frac{\alpha}{2}=-\frac{2}{3}$ bo‘lsa, $\frac{2+3\cos\alpha}{4-5\sin\alpha}$ ni hisoblang.

Y e c h i s h . (1) va (2) formulalarga ko‘ra,

$$\sin\alpha = \frac{2\cdot\left(-\frac{2}{3}\right)}{1+\left(-\frac{2}{3}\right)^2} = -\frac{12}{13}, \quad \cos\alpha = \frac{1-\frac{4}{9}}{1+\frac{4}{9}} = \frac{5}{13}.$$

Bundan $\dfrac{2+3\cos\alpha}{4-5\sin\alpha} = \dfrac{2+\frac{15}{13}}{4+\frac{60}{13}} = \dfrac{41}{112}$.

Mashqlar

1.102. Ifodani soddalashtiring:

1) $\dfrac{\mathrm{tg}\alpha\cdot\cos 2\alpha}{1+\mathrm{tg}^2\alpha}$; 2) $\dfrac{\cos 2\alpha-\cos 2\alpha\cdot\mathrm{tg}^2\alpha}{1+\mathrm{tg}^2\alpha}$.

1.103. $\mathrm{tg}\,\frac{x}{2}=\frac{1}{2}$ bo‘lsa, $\sin x$, $\cos x$, $\mathrm{tg}x$, $\mathrm{ctg}x$ ni toping.

1.104. $\sin x+\cos x=\frac{1}{5}$ bo‘lsa, $\mathrm{tg}\,\frac{x}{2}$ ni toping.

1.105. Ayniyatni isbotlang:

1) $\dfrac{\cos\alpha}{1-\sin\alpha} = \dfrac{1+\mathrm{tg}\frac{\alpha}{2}}{1-\mathrm{tg}\frac{\alpha}{2}}$; 2) $\dfrac{\cos\alpha}{1+\sin\alpha} = \dfrac{\mathrm{ctg}\frac{\alpha}{2}-1}{\mathrm{ctg}\frac{\alpha}{2}+1}$;

3) $\left(\dfrac{\mathrm{tg}\frac{\alpha}{2}+1}{\mathrm{tg}\frac{\alpha}{2}-1}\right)^2 = \dfrac{1+\sin\alpha}{1-\sin\alpha}$; 4) $\dfrac{1+\sin\alpha+\cos\alpha}{1+\sin\alpha-\cos\alpha} = \mathrm{ctg}\,\frac{\alpha}{2}$;

5) $\cos^2\frac{\alpha}{2}\left(1+\mathrm{tg}\,\frac{\alpha}{2}\right)^2 = 1+\sin\alpha$; 6) $\sin^2\frac{\alpha}{2}\left(\mathrm{ctg}\,\frac{\alpha}{2}-1\right)^2 = 1-\sin\alpha$.

7. Trigonometrik funksiyalar yig‘indisini ko‘paytmaga va ko‘paytmasini yig‘indiga aylantirish. Ikki burchak yig‘indisi va ayirmasi sinusi munosabatlarini hadma-had qo‘shaylik:

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\overline{\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta,}$$

bundan:

$$\sin\alpha\cos\beta = \frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right). \quad (1)$$

Shu kabi ikki burchak kosinusi yig‘indisi va ayirmasi munosabatlarini hadma-had qo‘shsak va ayirsak, quyidagi formulalar hosil bo‘ladi:

$$\cos\alpha\cos\beta = \frac{1}{2}\left(\cos(\alpha+\beta)+\cos(\alpha-\beta)\right), \quad (2)$$

$$\sin\alpha\sin\beta = \frac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right). \quad (3)$$

Trigonometrik funksiyalar ko‘paytmasini yig‘indi yoki ayirma ko‘rinishiga keltirish maqsadida $\alpha+\beta=u$, $\alpha-\beta=\upsilon$ deb olamiz. Bulardan $\alpha=\frac{u+\upsilon}{2}$, $\beta=\frac{u-\upsilon}{2}$ larni topib, (1) formulaga qo‘ysak, natijada:

$$\sin u+\sin\upsilon = 2\sin\frac{u+\upsilon}{2}\cos\frac{u-\upsilon}{2}. \quad (4)$$

(4) formulada υ ni $-\upsilon$ ga almashtirsak,

$$\sin u-\sin\upsilon = 2\sin\frac{u-\upsilon}{2}\cos\frac{u+\upsilon}{2}. \quad (5)$$

(2) va (3) formulalar bo‘yicha quyidagi tengliklar hosil bo‘ladi:

$$\cos u+\cos\upsilon = 2\cos\frac{u+\upsilon}{2}\cos\frac{u-\upsilon}{2}, \quad (6)$$

$$\cos u-\cos\upsilon = -2\sin\frac{u+\upsilon}{2}\sin\frac{u-\upsilon}{2}. \quad (7)$$

1 - m i s o l. $\cos 45^\circ+\cos 15^\circ$ ni hisoblaymiz.

Y e c h i s h. (6) formula bo‘yicha:

$$\cos 45^\circ+\cos 15^\circ = 2\cos\frac{45^\circ+15^\circ}{2}\cos\frac{45^\circ-15^\circ}{2} =$$

$$= 2\cos 60^\circ\cos 30^\circ = 2\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}.$$

Tangens va kotangensga taalluqli formulalarni chiqaraylik:

$$\mathrm{tg}u+\mathrm{tg}\upsilon = \frac{\sin u}{\cos u}+\frac{\sin\upsilon}{\cos\upsilon} = \frac{\sin u\cos\upsilon+\cos u\sin\upsilon}{\cos u\cos\upsilon} = \frac{\sin(u+\upsilon)}{\cos u\cos\upsilon},$$

bundan

$$\mathrm{tg}u+\mathrm{tg}\upsilon = \frac{\sin(u+\upsilon)}{\cos u\cos\upsilon},\ u,\ \upsilon\neq\frac{\pi}{2}+\pi k,\ k\in Z. \quad (8)$$

Quyidagi formulalar ham shu tartibda keltirib chiqariladi:

$$\mathrm{tg}u-\mathrm{tg}\upsilon = \frac{\sin(u-\upsilon)}{\cos u\cos\upsilon},\ u,\ \upsilon\neq\frac{\pi}{2}+\pi k,\ k\in Z, \quad (9)$$

$$\operatorname{ctg}u + \operatorname{ctg}v = \frac{\sin(u+v)}{\sin u \sin v},\ u,\ v \neq \pi k,\ k \in Z, \qquad (10)$$

$$\operatorname{ctg}u - \operatorname{ctg}v = \frac{\sin(u-v)}{\sin u \sin v},\ u,\ v \neq \pi k,\ k \in Z. \qquad (11)$$

2 – m i s o l. Agar $u + v + w = \pi$ bo‘lsa, $\operatorname{ctg}u + \operatorname{ctg}v - \operatorname{tg}w = -\operatorname{ctg}u\,\operatorname{ctg}v\,\operatorname{tg}w$ bo‘lishini isbot qilamiz.

Y e c h i s h.

$$\operatorname{ctg}u + \operatorname{ctg}v - \operatorname{tg}w = \operatorname{ctg}u + \operatorname{ctg}v - \operatorname{tg}(\pi - (u+v)) = \operatorname{ctg}u + \operatorname{ctg}v + \operatorname{tg}(u+v) =$$

$$= \frac{\sin(u+v)}{\sin u \sin v} + \frac{\sin(u+v)}{\cos(u+v)} = \frac{\sin(u+v)(\cos(u+v) + \sin u \sin v)}{\sin u \sin v \cos(u+v)} = \frac{\sin(u+v)\cos u \cos v}{\sin u \sin v \cos(u+v)} =$$

$$= \operatorname{ctg}u\,\operatorname{ctg}v\,\operatorname{tg}(u+v) = \operatorname{ctg}u\,\operatorname{ctg}v\,\operatorname{tg}(\pi - w) = -\operatorname{ctg}u\,\operatorname{ctg}v\,\operatorname{tg}w$$

Mashqlar

1.106. Trigonometrik funksiyalar yig‘indisi ko‘rinishida yozing:

1) $2\sin 22^\circ \cos 12^\circ$; 2) $\sin x \sin(x-1)$;

3) $4\sin 35^\circ \cos 25^\circ \sin 15^\circ$; 4) $8\cos 3^\circ \cos 6^\circ \cos 12^\circ \cos 24^\circ$.

1.107. Hisoblang:

1) $\cos 80^\circ \cos 40^\circ \cos 20^\circ$; 2) $\operatorname{tg}35^\circ \operatorname{tg}55^\circ$.

3) $\frac{2\cos 80^\circ - \cos 20^\circ}{\sin 10^\circ}$; 4) $\cos 20^\circ \cos 40^\circ \cos 80^\circ$;

5) $\cos 9^\circ \cos 27^\circ \cos 63^\circ \cos 81^\circ$; 6) $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$.

1.108. Ko‘paytma yoki ko‘paytmalar ko‘rinishida tasvirlang:

1) $\cos 43^\circ + \cos 37^\circ$; 2) $\sin 76^\circ - \sin 26^\circ$;

3) $\sin 57^\circ - \cos 64^\circ$; 4) $\sin 18^\circ + \cos 15^\circ$;

5) $\sin\frac{\pi}{5} - \cos\frac{\pi}{7}$; 6) $\cos 6x + \cos 4x$;

7) $\cos^2\alpha - \cos^2\beta$; 8) $\cos\alpha - \cos\beta$;

9) $\operatorname{ctg}^2\alpha - \operatorname{ctg}^2\beta$; 10) $\cos x + \cos 3x + \cos 5x + \cos 7x$;

11) $\sin x + \sin 2x + \sin 3x + \sin 4x$;

12) $\sin 20^\circ + \sin 10^\circ + \sin 30^\circ$;

13) $\cos\alpha + \cos\beta + \sin\frac{\alpha+\beta}{2}$; 14) $\sin 2x - \cos x - \sin 5x$;

15) $\cos\left(\frac{\pi}{10} + x\right) - \cos\left(\frac{\pi}{10} - x\right) - \sin x$;

16) $\sin(5\alpha + \beta) + \sin(3\alpha + \beta) + \sin 2\alpha$;

17) $\sqrt{\text{ctg}\alpha + \cos\alpha} + \sqrt{\text{ctg}\alpha - \cos\alpha},\ 0 < \alpha < \frac{\pi}{2}$;

18) $1 - \text{ctg}\alpha$;

19) $1 + \text{ctg}\alpha$;

20) $1 - \text{tg}\alpha$;

21) $1 - \sin(\pi - x) + \cos(\pi - x)$;

22) $\text{ctg}\alpha + \text{ctg}2\alpha - \text{tg}3\alpha$.

1.109. Ayniyatni isbot qiling:

1) $\frac{\sin(t+s)-\sin(t-s)}{\sin(t+s)+\sin(t-s)} = \text{ctg}t\,\text{ctg}s$;

2) $2(1+\sin t\sin s+\cos t\cos s) = 4\cos^2\frac{t-s}{2}$.

3) $(\sin 3t + \sin 5t)^2 + (\cos 3t + \cos 5t)^2 = 4\cos^2 t$;

4) $\text{tg}20° + \text{tg}40° + \text{tg}80° - \text{tg}60° = 8\cos 50°$;

5) $\text{ctg}^6 20° - 9\text{ctg}^4 20° + 11\text{ctg}^2 20° = \frac{1}{3}$;

6) $\cos 9°\cos 27°\cos 63°\cos 81° + \cos 12°\cos 24°\cos 48°\cos 96° = 0$;

7) $\sin\alpha + \sin 2\alpha + \sin 3\alpha + \ldots + \sin n\alpha = \frac{\sin\frac{n\alpha}{2}\sin\frac{(n+1)\alpha}{2}}{\sin\frac{\alpha}{2}}$;

8) $\cos\alpha + \cos 2\alpha + \cos 3\alpha + \ldots + \cos n\alpha = \frac{\sin\frac{n\alpha}{2}\cos\frac{(n+1)\alpha}{2}}{\sin\frac{\alpha}{2}}$;

9) Agar $\sin\alpha + \sin\beta = 2\sin(\alpha + \beta)$ va $\frac{\alpha+\beta}{2} \neq k\pi$ bo‘lsa, $\text{tg}\frac{\alpha}{2} + \text{tg}\frac{\beta}{2} = \frac{1}{3}$ bo‘ladi;

10) Agar $0 \le x \le \frac{\pi}{2}$ bo‘lsa, $\sqrt{1+\sin x} - \sqrt{1-\sin x} = 2\sin\frac{x}{2}$ bo‘ladi.

1.110. Yig‘indini hisoblang:

1) $\cos\alpha + 2\cos 2\alpha + 3\cos 3\alpha + \ldots + n\cos n\alpha$;

2) $\frac{1}{\cos\alpha\cos 2\alpha} + \frac{1}{\cos 2\alpha\cos 3\alpha} + \ldots + \frac{1}{\cos 9\alpha\cos 10\alpha}$;

3) $\sin\alpha - \sin 2\alpha + \sin 3\alpha - \ldots + (-1)^n \sin n\alpha$.

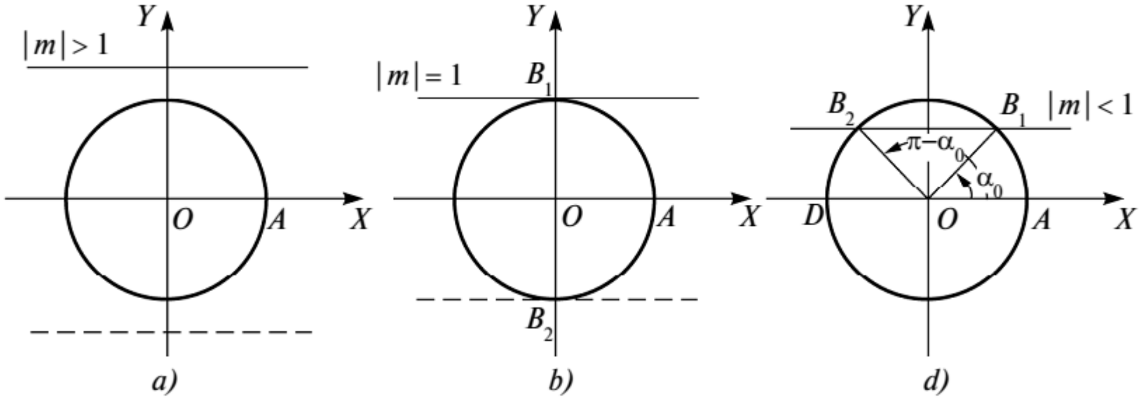
4-Mavzu: Trigonometrik tenglamalar, ularning turlari va yechish usullari.

1. $\sin\alpha = m$ ko'rinishdagi eng sodda tenglama. Arksinus. $\sin\alpha = m$ tenglamani yechish birlik aylanadagi shunday $B(\alpha)$ nuqtani topishdan iboratki, uning $y = \sin\alpha$ ordinatasi m ga teng bo'lishi kerak. Buning uchun gorizontal diametrga parallel bo'lgan $y = m$ to'g'ri chiziq bilan birlik aylananing kesishish nuqtalarini topish kerak. Uch hol bo'lishi mumkin:

a) agar $|m| > 1$ bo'lsa, $y = m$ to'g'ri chiziq aylanani kesmay, undan yuqori yoki quyidan o'tadi (I.39-*a* rasm). Demak, bu holda tenglama yechimga ega emas;

b) agar $|m| = 1$ bo'lsa, to'g'ri chiziq aylanaga yo yuqoridagi $B_1\left(\frac{\pi}{2}\right)$ nuqtada yoki quyidagi $B_2\left(-\frac{\pi}{2}\right)$ nuqtada urinib o'tadi (I.39-*b* rasm). Bu holda tenglama yagona ildizga ega: $\alpha = \frac{\pi}{2}$ yoki $\alpha = -\frac{\pi}{2}$. Agar funksiyaning $T = 2\pi$ asosiy davri ham e'tiborga olinsa, yechimni $\alpha = \frac{\pi}{2} + 2\pi k,\ k \in Z$ $\left(\alpha = -\frac{\pi}{2} + 2\pi k,\ k \in Z\right)$ ko'rinishda yozish mumkin;

d) $|m| < 1$ bo'lsa, $y = m$ to'g'ri chiziq aylanani $B_1(\alpha_0)$ va $B_2(\pi - \alpha_0)$ nuqtalarda kesadi(1.39-*d* rasm). Demak, tenglamaning yechimi shu nuqtalarning koordinatalari bo'lgan barcha sonlar to'plamlarining birlashmasi bo'ladi:



$$\{\alpha_0 + 2k\pi,\ k\in Z\}\cup\{\pi - \alpha_0 + 2k\pi,\ k\in Z\}.$$

Yechimni $x = \alpha_0 + 2k\pi,\ k\in Z;\ x = \pi - \alpha_0 + 2k\pi,\ k\in Z$ ko‘rinishda ham yozish mumkin.

Yechimning geometrik tahlilida $y = m$ to‘g‘ri chiziq bilan sinusoidaning kesishish nuqtasi haqida ham gapirilishi mumkin.

1 - m i s o l. $\sin\alpha = \frac{\sqrt{3}}{2}$ tenglamani yechamiz.

Y e c h i s h. $y = \frac{\sqrt{3}}{2}$ $(y < 1)$ to‘g‘ri chiziq koordinatali aylanani $B_1\left(\frac{\pi}{3}\right)$ va $B_2\left(\frac{2\pi}{3}\right)$ nuqtalarda kesadi (I.39-*d* rasm). B_1 nuqta barcha $\frac{\pi}{3} + 2k\pi,\ k\in Z$ sonlar to‘plamiga, B_2 nuqta esa barcha $\frac{2\pi}{3} + 2k\pi,\ k\in Z$ ko‘rinishdagi sonlar to‘plamiga mos. Barcha yechimlar to‘plamini $\alpha = \frac{\pi}{3} + 2k\pi,\ k\in Z;\ \ \alpha = \frac{2\pi}{3} + 2k\pi,\ k\in Z$ yoki $\left\{\frac{\pi}{3} + 2k\pi,\ k\in Z\right\}\cup\left\{\frac{2\pi}{3} + 2k\pi,\ k\in Z\right\}$ ko‘rinishda yozish mumkin.

2 - m i s o l. a) $\sin\alpha = 1$; b) $\sin\alpha = -1$; d) $\sin\alpha = 0$ tenglamalarni yechamiz.

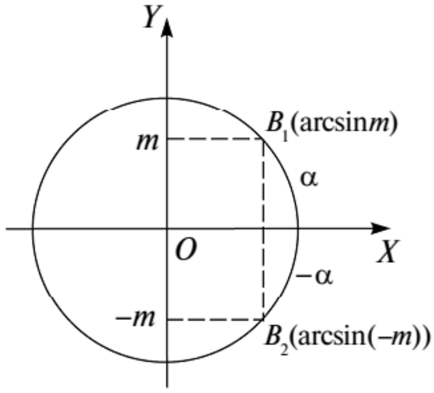
Y e c h i s h. a) Koordinatali aylanada faqat bitta $B_1\left(\frac{\pi}{2}\right)$ nuqtaning ordinatasi 1 ga teng (I.39-*b* rasm). Y e c h i m: $\alpha = \frac{\pi}{2} + 2\pi k,\ k\in Z$;

b) $B_2\left(-\frac{\pi}{2}\right) = B_2(0;\ -1)$ nuqta bo‘yicha $\alpha = -\frac{\pi}{2} + 2\pi k,\ k\in Z$;

d) ordinatasi 0 bo‘lgan nuqta ikkita: $A(0)$ va $D(\pi)$ (I.39-*d* rasm). A nuqtaga $2k\pi,\ k\in Z$, D nuqtaga esa $\pi + 2k\pi,\ k\in Z$ sonlar mos keladi.

J a v o b: $\alpha = 2k\pi,\ k\in Z;\ \alpha = \pi + 2k\pi,\ k\in Z.$

$|m| \le 1$ da $y = m$ to‘g‘ri chiziq va o‘ng yarim birlik aylana yagona umumiy nuqtaga ega bo‘ladi. Shu sababli $\sin\alpha = m$ $(|\ m\ | \le 1)$ tenglama $\left[-\frac{\pi}{2};\ \frac{\pi}{2}\right]$ oraliqqa tegishli bo‘lgan yagona x_0 yechimga ega. $\sin\alpha = m$ tenglamani qanoatlantiruvchi $\alpha_0 \in \left[-\frac{\pi}{2};\ \frac{\pi}{2}\right]$ soni m sonning *arksinusi* deyiladi va arcsin$m$ orqali belgilanadi. Ta’rifga ko‘ra



I.40-rasm.

$$\sin(\arcsin m) = m \qquad (1)$$

va

$$-\frac{\pi}{2} \le \arcsin m \le \frac{\pi}{2} \qquad (2)$$

bo‘ladi. Aksincha, $\sin\alpha = m$ va $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$ bo‘lsa, $\alpha = \arcsin m$ bo‘ladi.

3 - m i s o l. a) $\arcsin\frac{\sqrt{3}}{2}$; b) $\arcsin\left(-\frac{1}{2}\right)$; d) $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$ ifodalarni hisoblaymiz.

Y e c h i s h . a) $\sin x = \frac{\sqrt{3}}{2}$ bo‘yicha $x_1 = \frac{\pi}{3}$, $x_2 = \frac{2\pi}{3}$. Arksinusning ta’rifi bo‘yicha $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ bo‘lishi kerak. Bu shartga $x_1 = \frac{\pi}{3}$ to‘g‘ri keladi. Demak, $\arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3}$.

b) $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$, $-\frac{\pi}{2} \le -\frac{\pi}{6} \le \frac{\pi}{2}$ bo‘lgani uchun $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ bo‘ladi.

d) $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$, $-\frac{\pi}{2} \le -\frac{\pi}{3} \le \frac{\pi}{2}$. Demak, $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$.

I.40-rasmdan $y = m$ va $\alpha = \arcsin m$ sonlari orasidagi bog‘lanish ayon bo‘ladi. Chizmada $\alpha = \arcsin m$ va $-\alpha = \arcsin(-m)$. Demak,

$$\arcsin(-m) = -\arcsin m. \qquad (3)$$

Shunday qilib, $|m| \le 1$ bo‘lgan holda $\sin\alpha = m$ tenglamaning α yechimi $\{\arcsin m + 2k\pi,\ k\in Z\} \cup \{\pi - \arcsin m + 2k\pi,\ k\in Z\}$ to‘plamlar birlashmasi ko‘rinishida yoki $\alpha = \arcsin m + 2k\pi$, $k\in Z$; $\alpha = \pi - \arcsin m + 2k\pi$, $k\in Z$ ko‘rinishda yoki bu keyingi ikki formulani birlashtirib,

$$\alpha = (-1)^k \arcsin m + k\pi,\ k\in Z \qquad (4)$$

ko‘rinishda yozish mumkin.

4 - m i s o l . a) $\sin\alpha = \frac{1}{7}$; b) $\sin\alpha = -\frac{1}{9}$ tenglamalarni yechamiz.

Y e c h i s h . a) $\sin\alpha = \frac{1}{7}$ tenglama yechimini (4) formula bo‘yicha $\alpha = (-1)^k \arcsin\frac{1}{7} + k\pi$, $k \in Z$ ko‘rinishda yozamiz;

b) (3) munosabatga ko‘ra $\arcsin\left(-\frac{1}{9}\right) = \arcsin\frac{1}{9}$.

Y e c h i m : $\left\{-\arcsin\frac{1}{9} + 2k\pi,\ k \in Z\right\} \cup \left\{\pi + \arcsin\frac{1}{9} + 2k\pi,\ k \in Z\right\}$ yoki $\alpha = (-1)^{k+1}\arcsin\frac{1}{9} + k\pi,\ k \in Z$ ekanligi kelib chiqadi.

Mashqlar

1.116. Tenglamalarni yeching va grafik yordamida tushuntiring:

1) $\sin x = -0{,}5$; 2) $\sin x = -0{,}75$; 3) $\sin x = 0{,}2$; 4) $\sin x = \frac{7}{8}$;
5) $\sin x = \frac{\sqrt{2}}{2}$; 6) $\sqrt{3}\sin x + \frac{3}{2} = 0$; 7) $5\sin x - 7 = 0$; 8) $6\sin x - 2 = 0$.

1.117. Tenglamalarni yeching:

1) $4\sin^2 x - 1 = 0$; 2) $-2\sin^2 x + \sin x + 1 = 0$;
3) $3\sin^2 x - 4\sin x - 0{,}75 = 0$; 4) $\sqrt{3}\sin x - 2\sin^2 x = 0$.

1.118. Qiymatini toping:

1) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$; 2) $\arcsin\left(\frac{\sqrt{2}}{2}\right)$; 3) $\arcsin 0{,}5$.

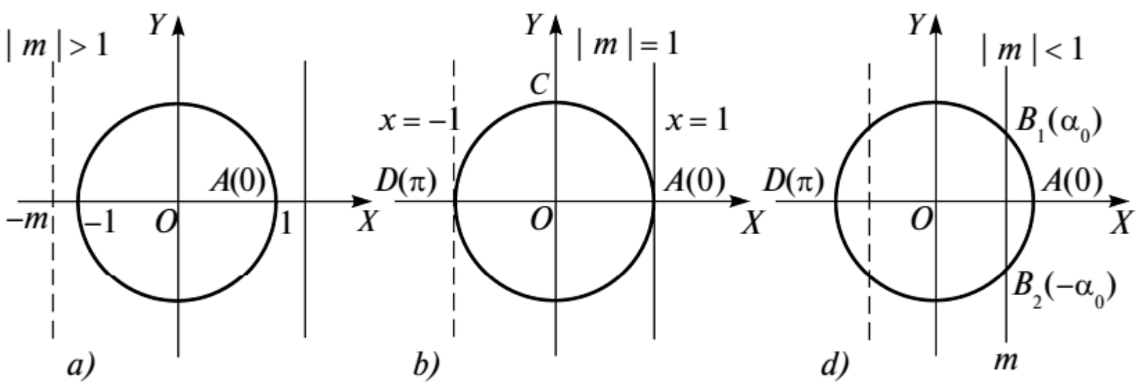
1.119. Hisoblang:

1) $\arcsin(\sin 30°)$; 2) $\arcsin\left(\sin\frac{\pi}{12}\right)$;
3) $\arcsin(\sin 2)$; 4) $\arcsin(\sin 10)$.

1.120. $\arcsin\alpha$ quyidagi qiymatlarni qabul qila oladimi?

1) $\frac{\pi}{3}$; 2) -3π; 3) $-\frac{\pi}{6}$; 4) $\frac{\pi}{4}$; 5) $-\frac{\pi}{4}$; 6) $\sqrt{3}$; 7) $-\pi$; 8) $4\sqrt{5}$.

2. $\cos\alpha = m$ ko‘rinishdagi eng sodda tenglama. Arkkosinus. Koordinatali aylanada olingan har qaysi $B(\alpha)$ nuqtaning



abssissasi $x = \cos\alpha$ ga teng. Shunga ko‘ra berilgan m bo‘yicha $\cos\alpha = m$ tenglamani yechish nuqtaning $x = m$ abssissasi bo‘yicha unga mos $\alpha = \alpha_0$ yoy kattaligini topishdan iborat. Uch holni qaraymiz:

1-hol. $|m| > 1$ da $x = m$ vertikal to‘g‘ri chiziq aylanani kesmaydi (I.41-*a* rasm). Bu holda tenglama yechimga ega emas. Masalan, $\cos\alpha = 2{,}8$ tenglama yechimga ega emas, chunki $m = 2{,}8 > 1$.

2-hol. Agar $|m| = 1$ bo‘lsa, to‘g‘ri chiziq aylanani faqat bir nuqtada, ya’ni yo $A(1;\ 0)$ nuqtada, yoki $D(-1;\ 0)$ nuqtada kesadi (I.41-*b* rasm). A nuqtaning aylana bo‘yicha koordinatasi $\alpha = 2\pi k$, $k \in Z$. Shunga ko‘ra $\cos\alpha = 1$ ning yechimi $\alpha = 2\pi k$, $k \in Z$ sonlar to‘plami bo‘ladi. $D(-1;\ 0) = D(\pi + 2\pi k)$ ekani e’tiborga olinsa, $\cos\alpha = -1$ ning yechimi $\alpha = \pi + 2\pi k$ sonlar to‘plami bo‘ladi.

3-hol. $|m| < 1$ bo‘lsa, $x = m$ to‘g‘ri chiziq aylanani ikki nuqtada kesadi (I.41-*d* rasm). Ulardan biri $B_1(\alpha_0)$ nuqta $0 \le \alpha_0 \le \pi$ yuqori yarim aylanada joylashadi. α_0 son m sonning *arkkosinusi* deyiladi va $\alpha_0 = \arccos m$ orqali belgilanadi. Ta’rifga ko‘ra $\cos\alpha = \cos(\arccos m) = m$ va $0 \le \arccos m \le \pi$ bo‘ladi.

Shu kabi $B_2(-\alpha_0)$ nuqta uchun: $\cos(-\alpha_0) = \cos\alpha_0 = m$. Bundan $-\alpha_0 = \arccos m$ yoki $\alpha_0 = -\arccos m$. Demak, $|m| < 1$, $k \in Z$ da $\cos\alpha = m$ tenglamaning yechimi $\{\arccos m + 2\pi k,\ k \in Z\} \cup \{-\arccos m + 2\pi k,\ k \in Z\}$ sonlarto‘plamlari birlashmasi bo‘ladi. Uni

$$\{\pm\arccos m + 2\pi k,\ k \in Z\} \tag{1}$$

yoki

$$\pm\arccos m + 2\pi k,\ k \in Z \tag{2}$$

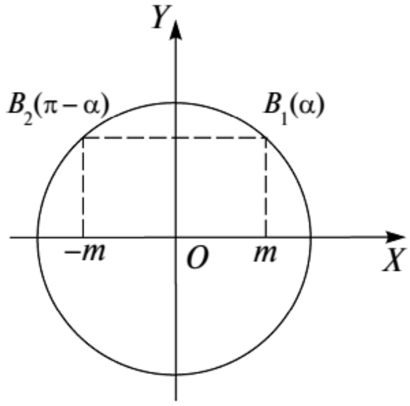
ko‘rinishda ham yozish mumkin. I.42-rasmdan, OY o‘qiga nisbatan simmetrik joylashgan $B_1(\arccos m) = B_1(\alpha)$ va $B_2(\arccos(-m)) = B_2(\pi - \alpha)$ nuqtalar bo‘yicha $\alpha = \arccos m$ va $\pi - \alpha = \arccos(-m)$ bo‘lishini aniqlaymiz. Undan:

$$\arccos(-m) = \pi - \arccos m \tag{3}$$

hosil qilinadi, bunda $0 \le \alpha \le \pi$.

1-misol. $\cos\alpha = \frac{\sqrt{3}}{2}$ tenglamani yechamiz.

Yechish. $\cos\frac{\pi}{6} = \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ bo‘ladi. Demak, $x = \frac{\sqrt{3}}{2}$ to‘g‘ri chiziq koordinatali aylanani $B_1\left(\arccos\frac{\sqrt{3}}{2}\right) = B_1\left(\frac{\pi}{6}\right)$ nuqtada va abssissalar o‘qiga nisbatan B_1 ga simmetrik joylashgan



I.42-rasm.

$B_2\left(-\arccos\frac{\sqrt{3}}{2}\right)=B_2\left(-\frac{\pi}{6}\right)$ nuqtada kesadi. Yechim B_1 nuqta bo‘yicha $\frac{\pi}{6}+2\pi k,\ k\in Z$ sonlar to‘plami va B_2 nuqta bo‘yicha $-\frac{\pi}{6}+2\pi k,\ k\in Z$ sonlar to‘plami birlashmasi bo‘ladi:

$\cos\alpha=\frac{\sqrt{3}}{2};\ \left\{\frac{\pi}{6}+2k\pi,\ k\in Z\right\}\cup$

$\cup\left\{-\frac{\pi}{6}+2k\pi,\ k\in Z\right\}$ yoki

$$\alpha=\pm\frac{\pi}{6}+2k\pi,\ k\in Z.$$

2 - m i s o l. $\arccos\left(-\frac{1}{2}\right)$ ni hisoblang.

Y e c h i s h. (3) formulaga ko‘ra, quyidagini topamiz:

$$\arccos\left(-\frac{1}{2}\right)=\pi-\arccos\frac{1}{2}=\pi-\frac{\pi}{3}=\frac{2\pi}{3}.$$

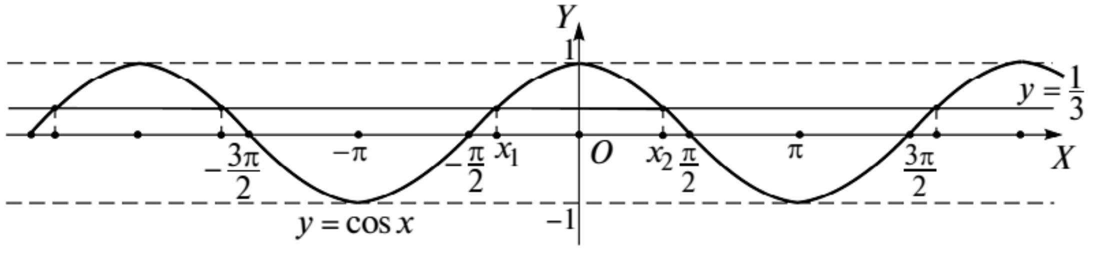
3 - m i s o l. $\cos x=-\frac{3}{7}$ tenglamani yeching.

Y e c h i s h. $x=\pm\arccos\left(-\frac{3}{7}\right)+2\pi k,\ k\in Z$ ga egamiz. (3) ga ko‘ra $x=\pm\left(\pi-\arccos\frac{3}{7}\right)+2\pi k,\ k\in Z$ bo‘ladi.

4 - m i s o l. $\cos x=\frac{1}{3}$ tenglamani $y=\cos x$ funksiya grafigi yordamida yeching.

Y e c h i s h. Ayni bir XOY koordinatalar sistemasida $y=\cos x$ va $y=\frac{1}{3}$ funksiyalar grafiklarini yasaymiz (I.43-rasm).

Bu grafiklar cheksiz ko‘p nuqtalarda kesishadi. $y=\cos x$ funksiya davri 2π bo‘lgan davriy funksiya bo‘lgani uchun berilgan



I.43-rasm.

tenglamaning $[-\pi;\ \pi]$ kesmadagi barcha yechimlarini topish va qolgan yechimlarni shu yechimlar orqali aniqlash mumkin.

$[-\pi;\ \pi]$ oraliqda $y = \cos x$ funksiya grafigi $y = \frac{1}{3}$ funksiya grafigi bilan ikkita kesishish nuqtasiga ega. Kesishish nuqtalarining $x_1 = -\arccos\frac{1}{3}$, $x_2 = \arccos\frac{1}{3}$ abssissalari berilgan tenglamaning $[-\pi;\ \pi]$ dagi barcha yechimlaridir. Shu sababli barcha yechimlar quyidagicha aniqlanadi: $x = \pm\arccos\frac{1}{3} + 2\pi k,\ k \in Z$.

5 - m i s o l. $\arccos(\cos 53°)$ ni toping.

Y e c h i s h. $\arccos(\cos m) = m$, $(0 \le m \le \pi)$ ayniyatdan foydalanamiz. $53^{\circ} = \frac{53\pi}{180}$ va $0 < \frac{53\pi}{180} < \pi$ bo‘lgani uchun bu ayniyatga ko‘ra

$$\arccos(\cos 53^{\circ}) = \arccos\left(\cos\frac{53\pi}{180}\right) = \frac{53\pi}{180}.$$

Mashqlar

1.121. Tenglamani $y = \cos x$ funksiya grafigi yordamida yeching:

1) $\cos x = 0$; 2) $\cos x = 0{,}5$; 3) $\cos x = -\frac{2}{9}$;

4) $\cos x = -\frac{\sqrt{2}}{2}$; 5) $\cos x = 2{,}4$; 6) $2\cos x + \sqrt{3} = 0$.

1.122. Ifodaning qiymatini toping:

1) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$; 2) $\arccos(-0{,}5)$; 3) $\arccos(\cos 30°)$;
4) $\arccos(\cos(-30°))$; 5) $\arccos(\sin 30°)$; 6) $\arccos(\cos 2)$;
7) $\arccos(\cos(-2))$; 8) $\arccos(\sin 2))$; 9) $\arccos(\sin(-2))$;
10) $\arccos(\cos 88)$; 11) $\arccos(\sin 86)$.

1.123. Tengliklarning to‘g‘riligini tekshiring:

1) $\arccos x = -\arcsin x$; 2) $-\arccos x = \pi + \arccos x$.

1.124. Ifodaning qiymatini toping:

1) $\cos\left(\arccos\frac{\sqrt{3}}{2} - \arcsin\frac{\sqrt{3}}{2}\right)$; 2) $\sin\left(\arccos\frac{1}{2} + \arcsin\frac{\sqrt{3}}{2}\right)$.

1.125. Tenglamani yeching:

1) $\cos^2 x - 3 = 0$; 2) $\cos 2x = \left(-\frac{\sqrt{2}}{2}\right)$; 3) $6\cos^2 x + 3 = 0$;

4) $3\cos^2 x - 5 = 0$; 5) $2\cos^2 x - 1 = 0$; 6) $4\cos^2 x - 1 = 0$.

1.126. Tenglamani yeching:

1) $\cos^2 x - 2\cos x = 0$; 2) $2\cos^2 x - \cos x = 0$;

3) $2\cos^2 x - \cos x - 1 = 0$; 4) $2\cos^2 x - 3\cos x + 1 = 0$.

3. tgα = *m* va ctgα = *m* ko'rinishdagi eng sodda tenglamalar. Arktangens va arkkotangens. Koordinatali aylananing har bir $B(\alpha)$ nuqtasi Dekart koordinatalar sistemasidagi biror $B\ (x,\ y)$ nuqta bilan ustma-ust tushishini va $x = \cos\alpha$, $y = \sin\alpha$ ekanini bilamiz. Shunga ko'ra, noma'lum α qatnashayotgan $\operatorname{tg}\alpha = m$ yoki $\frac{\sin\alpha}{\cos\alpha} = m$ tenglamaning barcha yechimlarini koordinatali aylana bilan $\frac{y}{x} = m$, ya'ni $y = mx$ to'g'ri chiziqning kesishish nuqtalari yordamida aniqlash mumkin. m ning har qanday qiymatida $y = mx$ to'g'ri chiziq aylanani $O\ (0;\ 0)$ nuqtaga nisbatan simmetrik bo'lgan B_1 va B_2 nuqtalarda kesadi (I.44-rasm). Ulardan biri $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ o'ng yarim aylanada yotadi. Bu nuqta $B_1(\alpha_0)$ bo'lsin. Ikkinchi nuqta $B_2(\alpha_0 + \pi)$ bo'ladi. Demak, $\operatorname{tg}\alpha = m$ tenglamaning barcha yechimlari to'plami $\alpha = \alpha_0 + 2k\pi,\ k \in Z$ va $\alpha = (\alpha_0 + \pi) + 2k\pi,\ k \in Z$ sonlar to'plamlari birlashmasidan iborat. Barcha yechimlar

$$\alpha = \alpha_0 + k\pi,\ k \in Z \tag{1}$$

formula bilan aniqlanadi.

m sonning *arktangensi* deb $\left(-\frac{\pi}{2};\ \frac{\pi}{2}\right)$ oraliqda yotadigan shunday α songa aytiladiki, uning uchun $\operatorname{tg}\alpha = m$ bo'ladi. m sonning arktangensi $\alpha = \operatorname{arctg} m$ orqali belgilanadi. Ta'rifga asosan, har qanday m son uchun quyidagi munosabatlar o'rinli bo'ladi:

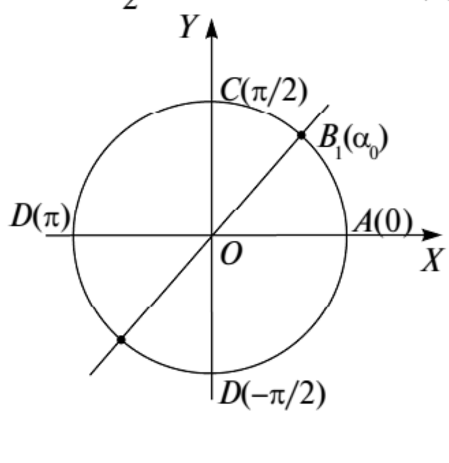
$$\operatorname{tg}(\operatorname{arctg} m) = m,\ -\frac{\pi}{2} < \operatorname{arctg} m < \frac{\pi}{2}. \tag{2}$$

Aksincha, $\operatorname{tg}\alpha = m$, $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ bo'lsa, $\alpha = \operatorname{arctg} m$ bo'ladi.

Yuqoridagi shartlardan va tangens toq funksiyaligidan $\operatorname{tg}(-\alpha) = -\operatorname{tg}\alpha = -m$ bo'lgani uchun quyidagi tenglik o'rinli bo'ladi:

$$\operatorname{arctg}(-m) = -\operatorname{arctg} m \tag{3}$$

Arkkotangens tushunchasi ham shu kabi kiritiladi.



I.44-rasm.

m sonning *arkkotangensi* deb $(0;\ \pi)$ oraliqda yotadigan shunday α songa aytiladiki, uning uchun $\mathrm{ctg}\alpha = m$ bo‘ladi. m sonning arkkotangensi $\alpha = \mathrm{arcctg}\, m$ orqali belgilanadi. Uning uchun quyidagi tenglik o‘rinli:

$$\mathrm{arcctg}(-m) = \pi - \mathrm{arcctg}\, m. \qquad (4)$$

1 - m i s o l. a) $\mathrm{tg}x = -\sqrt{3}$; b) $\mathrm{ctg}x = -\sqrt{3}$ tenglamalarni yechamiz.

Y e c h i s h. a) $\mathrm{tg}\left(-\frac{\pi}{3}\right) = -\sqrt{3}$, demak, $x = -\frac{\pi}{3} + \pi k,\ k \in Z$.

b) $\mathrm{ctg}\frac{5\pi}{6} = -\sqrt{3}$, demak, $x = \frac{5\pi}{6} + \pi k,\ k \in Z$.

2 - m i s o l. a) $\mathrm{arctg}\left(-\sqrt{3}\right)$; b) $\mathrm{arcctg}\left(-\sqrt{3}\right)$ sonlarni topamiz.

Y e c h i s h. a) (3) formula bo‘yicha $\mathrm{arctg}\left(-\sqrt{3}\right) =$
$= -\mathrm{arctg}\sqrt{3} = -\frac{\pi}{3}$;

b) (4) formula bo‘yicha $\mathrm{arcctg}\left(-\sqrt{3}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

Mashqlar

1.127. Tenglamani yeching (grafikdan ham foydalaning):

1) $\mathrm{tg}x = -\frac{\sqrt{3}}{2}$; 2) $\mathrm{ctg}x = -\sqrt{3}$; 3) $\mathrm{ctg}x = 0{,}2$.

1.128. Ifodaning qiymatini toping:

1) $\mathrm{arcctg}\left(-\frac{\sqrt{3}}{2}\right)$; 2) $\mathrm{arcctg}1$; 3) $\mathrm{arctg}(-1)$; 4) $\mathrm{arctg}0$;

5) $\mathrm{arcctg}0$.

1.129. Hisoblang:

1) $\mathrm{tg}\left(\arcsin\frac{\sqrt{3}}{2}\right)$; 2) $\mathrm{ctg}(\arcsin 0{,}5)$;

3) $\mathrm{tg}(\arccos 0{,}5)$; 4) $\mathrm{ctg}(\mathrm{arctg}(-1))$;

5) $\mathrm{tg}\left(\mathrm{arcctg}\left(-\frac{2}{3}\right)\right)$; 6) $\sin\left(\mathrm{arcctg}(\sqrt{3})\right)$;

7) $\cos\left(\mathrm{arctg}\left(-\frac{\sqrt{3}}{3}\right)\right)$; 8) $\cos(\mathrm{arcctg}(-0{,}8))$.

1.130. $\mathrm{arcctg}x = \frac{\pi}{2} - \mathrm{arctg}x$ tenglikning to‘g‘riligini tekshiring.

5-Mavzu: Trigonometrik tenglamalarni o’zgaruvchilarni almashtirish usuli bilan yechish.

Tenglamalarni yechishning asosiy usullari.

$$R(z) = a_0 z^n + a_1 z^{n-1} + \ldots + a_{n-1} z + a_n = 0 \qquad (1)$$

ko‘rinishdagi algebraik tenglamaga keltirilishi mumkin, bunda z orqali $\sin\lambda x$, $\cos\lambda x$, $\mathrm{tg}\lambda x$, $\mathrm{ctg}\lambda x$ funksiyalardan biri ifodalangan. Algebraik tenglama kabi (1) trigonometrik tenglamalarni yechishda yangi noma’lum kiritish, ko‘paytuvchilarga ajratish va hokazo usullar qo‘llaniladi. Jarayon eng sodda trigonometrik tenglamalardan birini yechishgacha boradi. Trigonometrik tenglamalarni yechishda asosan quyidagi hollar uchraydi:

1) $R(f(x)) = 0$ tenglamada R trigonometrik funksiya belgisi ostida x ga bog‘liq bo‘lgan $f(x)$ ifoda turibdi. $f(x) = z$ almashtirish orqali tenglama eng sodda $R(z) = 0$ trigonometrik tenglamalardan biriga keltirilishi mumkin. Uning $z = z_i$ ildizlari birma-bir $f(x) = z$ ga qo‘yiladi va x ning qiymatlari topiladi.

1 - misol. $\sin\left(10x + \frac{\pi}{8}\right) = \frac{\sqrt{3}}{2}$ tenglamani yechamiz.

Yechish. Misolimizda $f(x) = 10x + \frac{\pi}{8}$. Tenglamaga $10x + \frac{\pi}{8} = z$ almashtirish kiritsak, $\sin z = \frac{\sqrt{3}}{2}$ tenglama hosil bo‘ladi. Uning yechimi: $z = (-1)^k \frac{\pi}{3} + k\pi,\ k \in Z$. Bu $10x + \frac{\pi}{8} = z$ ga qo‘yiladi va javob topiladi:

$$x = \frac{1}{10}\left(-\frac{\pi}{8} + (-1)^k \frac{\pi}{3} + k\pi\right),\ k \in Z.$$

2 - misol. $\mathrm{tg}\left(x^2 + 6x + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$ tenglamani yechamiz.

Yechish. $z = x^2 + 6x + \frac{\pi}{6}$ almashtirish kiritamiz. Tenglama $\mathrm{tg}z = \frac{\sqrt{3}}{3}$ ko‘rinishga keladi. Undan $z = \frac{\pi}{6} + k\pi,\ k \in Z$ ni topamiz.

U holda $x^2+6x+\frac{\pi}{6}=\frac{\pi}{6}+k\pi,\ k\in Z$ yoki $x^2+6x-k\pi=0,\ k\in$Z. Kvadrat tenglamaning ildizlari $x=-3\pm\sqrt{9+k\pi},\ k\in Z$, bunda $9+k\pi\geq 0$ yoki $k\geq-\frac{9}{\pi}=-2,86...$, ya'ni $k=-2;\ -1;\ 0;\ 1;...$.

Javob: $x=-3\pm\sqrt{9+k\pi},\ k\in Z,\ k\geq -2$.

2) $\sin x=\sin\alpha$, $\cos x=\cos\alpha$ va $\mathrm{tg}x=\mathrm{tg}\alpha$ tenglamalar. Bu tenglamalar mos ravishda $x=(-1)^k\alpha+k\pi,\ k\in Z,\ x=\pm\alpha+2n\pi$, $n\in Z,\ x=\alpha+m\pi,\ m\in Z$ formulalar yordamida yechilishi mumkin.

3-misol. $\cos(5x-45°)=\cos(2x+60°)$ tenglamani yeching.

Yechish. $5x-45°=\pm(2x+60°)+360°k,\ k\in Z$ tenglamalarni yechamiz. $5x-45°=+(2x+60°)+360°k,\ k\in Z$ tenglikdan $x=35°+120°k,\ k\in Z$ yechimlar guruhini, $5x-45°=-(2x+60°)+360°k$, $k\in Z$ tenglikdan esa $x=\frac{1}{7}\left(-15^{\circ}+360^{\circ}k\right),\ k\in Z$ yechimlar guruhini topamiz.

Shunday qilib, $x=35°+120°k,\ k\in Z;\ x=\frac{1}{7}\left(-15^{\circ}+360^{\circ}k\right)$, $k\in Z$.

4-misol. $\sin x^2=\sin(6x-5)$ tenglamani yechamiz.

Yechish. $x^2=(-1)^k(6x-5)+k\pi,\ k\in Z$ tenglama hosil bo'ladi.

Agar k juft bo'lsa, ya'ni $k=2n,\ n\in Z$ da $x^2=6x-5+2n\pi,\ n\in Z$ kvadrat tenglama kelib chiqadi. Uning yechimi

$$x_{1,2}=3\pm\sqrt{9-(5-2n\pi)},\ n\in Z,\ n\geq\left[-\frac{2}{\pi}\right].$$

Agar k toq bo'lsa, ya'ni $k=2m+1,\ m\in Z$ da $x^2=-6x+5+(2m+1)\pi,\ m\in Z$ ko'rinishda bo'ladi va bundan

$$x_{1,2}=-3\pm\sqrt{9+(5+2(m+1)\pi)},\ m\in Z,\ m\geq\left[-\frac{14+\pi}{2\pi}\right].$$

3) $f(R(x))=0$ tenglamada R trigonometrik funksiya boshqa f funksiya belgisi ostida turadi. $R(x)=z$ almashtirish masalani $f(z)=0$ tenglamani yechishga keltiradi. Bu tenglamaning $z_1, z_2, ...$ ildizlari bo'yicha $R(x)=z_1,\ R(x)=z_2, ...$ tenglamalar majmuasini hosil qilamiz. Uni yechish bilan masala hal qilinadi.

5-misol. $\sin^2x+3\sin x+1,25=0$ tenglamani yechamiz.

Y e c h i s h . $\sin x = z$ almashtirish natijasida $z^2 + 3z + 1,25 = 0$ kvadrat tenglama hosil bo‘ladi. Uning ildizlari $z_1 = -5$, $z_2 = -1$. $\sin x = -5$ tenglama yechimga ega emas. $\sin x = -1$ tenglama $x = -90° + 360°k$, $k \in Z$ yechimlarga ega.

4) Ba’zan berilgan tenglamani *ko‘paytuvchilarga ajratish* usulidan trigonometrik funksiyalar yig‘indisini ko‘paytma ko‘rinishiga keltirishda foydalaniladi.

6 - m i s o l . $2\cos x - 2\sin 2x - 2\sqrt{2}\sin x + \sqrt{2} = 0$ tenglamani yechamiz.

Y e c h i s h . $\sin 2x = 2\sin x\cos x$ almashtirish tenglamani $2\cos x - 4\sin x\cos x - 2\sqrt{2}\sin x + \sqrt{2} = 0$ ko‘rinishga keltiradi. Uning chap qismini ko‘paytuvchilarga ajratamiz:

$$2\cos x(1 - 2\sin x) + \sqrt{2}(1 - 2\sin x) = 0,$$

$$(1 - 2\sin x)(2\cos x + \sqrt{2}) = 0,$$

bundan:
$$\begin{cases} 1 - 2\sin x = 0, \\ 2\cos x + \sqrt{2} = 0, \end{cases} \Rightarrow \begin{cases} \sin x = 0,5, \\ \cos x = \frac{-\sqrt{2}}{2}. \end{cases}$$

J a v o b : $\left\{(-1)^k 30^\circ + 180^\circ k,\ k \in Z\right\} \cup \left\{\pm 135^\circ + 360^\circ k,\ k \in Z\right\}$.

7 - m i s o l . $\sqrt{1 + \frac{1}{2}\sin x} = \cos x$ tenglamani yeching.

Y e c h i s h . Bu tenglama $\begin{cases} \cos x \geq 0, \\ 1 + \frac{1}{2}\sin x = \cos^2 x \end{cases}$ yoki $\begin{cases} \cos x \geq 0, \\ \sin x\left(\sin x + \frac{1}{2}\right) = 0 \end{cases}$ tenglamalar sistemasiga teng kuchlidir (VI bob, 7-§; 1-band). $\begin{cases} \cos x \geq 0, \\ \sin x\left(\sin x + \frac{1}{2}\right) = 0, \end{cases} \Rightarrow \left[\begin{array}{l} \begin{cases} \cos x \geq 0, \\ \sin x = 0; \end{cases} \\ \begin{cases} \cos x \geq 0, \\ \sin x = -\frac{1}{2} \end{cases} \end{array}\right.$ bo‘lgani uchun berilgan tenglamaning barcha yechimlari $x = 2k\pi$, $k \in Z$ va $x = -\frac{\pi}{6} + 2k\pi$, $k \in Z$ formulalar bilan aniqlanadi.

J a v o b: $\{2k\pi,\ k \in Z\} \cup \left\{-\frac{\pi}{6} + 2k\pi,\ k \in Z\right\}$.

Mashqlar

1.133. Tenglamani yeching:

1) $\sin 10x = -\frac{\sqrt{3}}{2}$;
2) $\cos 10x = \frac{\sqrt{3}}{2}$;
3) $\mathrm{tg}10x = \sqrt{3}$;
4) $\mathrm{ctg}10x = \frac{\sqrt{3}}{3}$;
5) $\sin(6x - 60^\circ) = -1$;
6) $\cos(4x + 30^\circ) = 0$;
7) $\mathrm{tg}(5x - 45^\circ) = 0$;
8) $\sin^2 \frac{3}{x} = \frac{3}{4}$;
9) $\cos^2 (2x - 45^\circ) = \frac{1}{4}$;
10) $\mathrm{tg}^2 \left(6x - \frac{\pi}{4}\right) = 3$;
11) $\sin^2 \left(7x - \frac{\pi}{6}\right) = 3$;
12) $\cos^2 \left(4x + \frac{\pi}{3}\right) = -\frac{1}{4}$;
13) $\mathrm{tg}^2 \left(5x - \frac{\pi}{3}\right) = -1$;
14) $\sin(4x^2) = 0{,}5$;
15) $\cos^2 6x^2 = 0{,}25$;
16) $\mathrm{tg}\frac{5}{x} = -1$.

1.134. Tenglamani yeching:

1) $\sin 4x \cos 3x\ \mathrm{tg}8x = 0$;
2) $\cos 4x = -\cos 5x$;
3) $\mathrm{tg}5x = -\mathrm{tg}\frac{x}{3}$;
4) $\sin 11x = -\sin 15x$;
5) $\cos 4x = \cos x$;
6) $\mathrm{tg}3x = -\mathrm{ctg}5x$;
7) $\sin \frac{x}{3} = \cos \frac{x}{4}$;
8) $\mathrm{tg}(5\pi - x) = -\mathrm{ctg}\left(2x + \frac{\pi}{6}\right)$;
9) $\sin \frac{x}{6} = \sin \frac{4}{x}$;
10) $\sin \left(x^2 - \frac{\pi}{4}\right) = -\cos x^3$;
11) $\mathrm{ctg}7 = -\mathrm{ctg}\frac{3}{x}$;
12) $\mathrm{ctg}\sqrt{x} = \mathrm{tg}2x$;
13) $\cos^2 2x + 3\cos 2x + 2 = 0$;
14) $\mathrm{tg}^2 5x - 3\mathrm{tg}5x - 4 = 0$;
15) $\sin x^2 = -\sin 3x^2$;
16) $\sin^2 x + \sin^2 2x + 2 = 0$;
17) $\sqrt{2}(\cos^3 x + \sin^3 x) = \sin 2x$.

5. Xususiy usullar. 1) Agar tenglama tarkibida har xil trigonometrik funksiyalar qatnashsa, ularni bir ismli funksiyaga keltirish, so‘ngra almashtirishlarni bajarish kerak.

1 - m i s o l. $3\sin^2x+4\sin x+2\cos^2x-7=0$ tenglamani yechamiz.

Y e c h i s h. $\cos^2x = 1 - \sin^2x$ almashtirish berilgan tenglamani $3\sin^2x + 4\sin x + 2 - 2\sin^2x - 7 = 0$ yoki $\sin^2x + 4\sin x - 5 = 0$ ko‘rinishga keltiradi. Oxirgi tenglamadan $\sin x = z$ almashtirish bajarsak, $z^2+4z-5=0$ kvadrat tenglama hosil bo‘ladi. Bu kvadrat tenglama $z_1=-5$, $z_2=1$ ildizlarga ega. $z=\sin x$ ekanligini e’tiborga olsak, $\sin x = -5$ va $\sin x =1$ tenglamalar hosil bo‘ladi. Ularning birinchisi yechimga ega emas, ikkinchisi esa $x=\frac{\pi}{2}+2k\pi,\ k\in Z$ yechimlarga ega.

2) Chap qismi $\sin x$ va $\cos x$ ga nisbatan ratsional funksiya bo‘lgan $R(\sin x, \cos x) = 0$ tenglama. Oldingi bandlarda ko‘rsatib o‘tilganidek, u va v ga nisbatan *ratsional funksiya* deb, qiymatlari u va v larni qo‘shish, ko‘paytirish va bo‘lish orqali hosil bo‘ladigan funksiyaga aytiladi. $R(\sin x, \cos x) = 0$ tenglamada:

a) agar $\sin x$ (yoki $\cos x$) faqat juft daraja bilan qatnashayotgan bo‘lsa, $\cos x = u$ (mos ravishda $\sin x = u$) almashtirish bajariladi;

b) agar bir vaqtda $\sin x$ ifoda $-\sin x$ ga, $\cos x$ esa $-\cos x$ ga almashtirilganda $R(\sin x; \cos x)$ funksiya o‘zgarmasa, ya’ni $R(\sin x; \cos x) = R(-\sin x; -\cos x)$ bo‘lsa, $\mathrm{tg}u = z$ almashtirish bajariladi.

2 - m i s o l. $\cos^4x + 3\sin x - \sin^4x - 2 = 0$ tenglamani yechamiz.

Y e c h i s h. $\cos x$ funksiya faqat juft daraja bilan qatnashmoqda. $\cos^4x = (1 - \sin^2x)^2 = 1 - 2\sin^2x + \sin^4x$ bo‘lganidan tenglama faqat sinusga bog‘liq: $2\sin^2x - 3\sin x + 1 = 0$; endi bu tenglama $\sin x = u$ almashtirish bilan $2u^2 - 3u + 1 = 0$ ko‘rinishga keladi. Buning ildizlari: $u_1 = \frac{1}{2}$, $u_2 = 1$. Shu tariqa masala $\sin x=\frac{1}{2}$ va $\sin x = 1$ eng sodda trigonometrik tenglamalarni yechishga keladi. Bu tenglamalar berilgan tenglamaning barcha yechimlarini beradi:

$$x=(-1)^k\frac{\pi}{6}+\pi k,\ k\in Z;\ x=\frac{\pi}{2}+2\pi k,\ k\in Z.$$

3 - m i s o l. $\sin^2x + 2\sin 2x + 5\cos^2x - 4 = 0$ tenglamani yechamiz.

Y e c h i s h. Tenglamani $\sin^2x + 4\sin x \cos x + 5\cos^2x - 4 = 0$ ko‘rinishda yozib olaylik. Bu tenglamani x ning $\cos x$ ni nolga aylantiradigan hech qanday qiymati qanoatlantirmaydi, chunki $\cos x = 0$ bo‘lganda $\sin^2x =1$ bo‘lib, tenglamadan $-3 = 0$ noto‘g‘ri tenglik hosil bo‘ladi. Bundan tashqari, $\sin x$ va $\cos x$ oldidagi ishoralar bir vaqtda o‘zgartirilganda, tenglikning chap qismi o‘zgarmaydi. Demak, $\mathrm{tg}x = u$ almashtirishni bajarish mumkin.

Tenglamaning ikkala qismini $\cos^2 x$ ga bo‘lamiz:

$$\mathrm{tg}^2 x + 4\,\mathrm{tg}x + 5 - \frac{4}{\cos^2 x} = 0.$$

$\frac{1}{\cos^2 x} = 1 + \mathrm{tg}^2 x$ bo‘lgani uchun

$$3\mathrm{tg}^2 x - 4\mathrm{tg}x - 1 = 0$$

tenglama hosil bo‘ladi. Bu tenglamada $\mathrm{tg}x = t$ almashtirish bajarsak, $3t^2 - 4t - 1 = 0$ tenglamaga ega bo‘lamiz. Bu kvadrat tenglama $\frac{2 \pm \sqrt{7}}{3}$ ildizlarga ega. Topilgan ildizlar yordamida berilgan tenglamaning barcha ildizlarini aniqlaymiz:

$$x = \mathrm{arctg}\,\frac{2-\sqrt{7}}{3} + \pi k,\ k \in Z;\ x = \mathrm{arctg}\,\frac{2+\sqrt{7}}{3} + \pi k,\ k \in Z.$$

3) $R(\sin x;\ \cos x) = 0$ tenglamaning chap qismi sinus va kosinusga nisbatan bir jinsli funksiya, ya’ni, agar $\sin x$ va $\cos x$ bir vaqtda biror λ ga ko‘paytirilsa, tenglamaning chap qismi λ^n ga ko‘paytirilgan bo‘ladi: $R(\lambda\sin x;\ \lambda\cos x) = \lambda^n R(\sin x;\ \cos x)$, bunda n — funksiyaning bir jinslilik darajasi, o‘zgarmas miqdor. Bu holda tenglikning ikkala qismi $\cos^n x$ ga bo‘linadi va $\mathrm{tg}x = u$ almashtirish bajariladi. Agar tenglikning barcha hadlari $\cos^m x$ ga bo‘linadigan bo‘lsa, u holda $\cos^m x$ qavsdan tashqari chiqarilsa, berilgan tenglama ikki tenglamaga ajraladi.

4 - m i s o l . $9\cos^6 x - 4\sin^3 x\cos^3 x = 0$ tenglamani yechamiz.

Y e c h i s h . Tenglamaning barcha hadlari $\cos^3 x$ ga bo‘linadi. $\cos^3 x$ ni qavsdan tashqariga chiqaramiz:

$$\cos^3 x(9\cos^3 x - 4\sin^3 x) = 0 \Rightarrow \left[\begin{array}{l}\cos^3 x = 0,\\ 9\cos^3 x - 4\sin^3 x = 0.\end{array}\right.$$

$\cos x = 0$ tenglama izlanayotgan yechimning bir turkumini beradi: $x = \frac{\pi}{2} + \pi k,\ k \in Z$. Ikkinchi tenglama $\cos x$ va $\sin x$ ga nisbatan bir jinsli. Uning ikkala qismini $\cos^3 x$ ga bo‘lamiz ($\cos x \neq 0$, ya’ni $x \neq \pm\frac{\pi}{2} + \pi k,\ k \in Z$ hol qaralyapti). Natijada: $9 - 4\mathrm{tg}^3 x = 0$ tenglamaga ega bo‘lamiz. Bu tenglama yechimning yana bir turkumini beradi: $x = \mathrm{arctg}\,\sqrt[3]{\frac{9}{4}} + \pi k,\ k \in Z$.

Ba’zan oddiy almashtirishlar tenglamani unga teng kuchli bir jinsli tenglamaga keltirishi mumkin. Masalan, $\cos^2 x - 6\sin x\cos x = 4$ ning o‘ng qismini $\sin^2 x + \cos^2 x$ ga (ya’ni 1 ga) ko‘paytirish

6-Mavzu: sin (ax)+sin (bx)=0, sin (ax)-sin (bx)=0, cos (ax)+cos (bx)=0, cos (ax)-cos (bx)=0 ko'rinishdagi tenglamalarni yechish.

2.2. Bir jinsli tenglamalar. $a\sin x + b\cos x = 0$; $a\sin^2 x + b\sin x\cos x + c\cos^2 x = 0$; $a\sin^3 x + b\sin^2 x\cos x + c\sin x\cos^2 x + d\cos^3 x = 0$, $a, b, c, d \in R$ kabi tenglamalar **sinx va cosx ga nisbatan bir jinsli tenglamalar** deyiladi. Bunday tenglamalarning barcha hadlarida sin$x$ va cos$x$ ning daraja ko'rsatkichlari yig'indisi bir xildir. Bu yig'indi bir jinsli tenglamaning **darajasi** deyiladi. Keltirilgan tenglamalar mos ravishda birinchi, ikkinchi, uchinchi darajali tenglamalardir. Bunday tenglamalar $\cos^n x \neq 0$ ga (n — tenglama darajasi) bo'lib yuborish natijasida tg$x$ ga nisbatan algebraik tenglamaga keltiriladi.

$a\sin^2 x + b\sin x\cos x + c\cos^2 x = d$ shaklidagi tenglama o'ng tomonini $\sin^2 x + \cos^2 x = 1$ ga ko'paytirish yordamida bir jinsli tenglama shakliga keltiriladi.

4-misol. $4\sin^2 x + 2\sin x\cos x = 3$ tenglamani yeching.

Yechilishi. $4\sin^2 x + 2\sin x\cos x = 3 \Leftrightarrow 4\sin^2 x + 2\sin x\cos x = 3(\sin^2 x + \cos^2 x) \Leftrightarrow \sin^2 x + 2\sin x\cos x - 3\cos^2 x = 0$ tenglamaning har ikkala tomonini $\cos^2 x \neq 0$ ga bo'lamiz:

$$\mathrm{tg}^2 x + 2\mathrm{tg}\, x - 3 = 0 \Rightarrow \left[\begin{array}{l}\mathrm{tg}x = -3\\ \mathrm{tg}x = 1\end{array}\right. \Rightarrow \left[\begin{array}{l}x = -\mathrm{arctg}3 + k\pi,\ \ k \in Z;\\ x = \frac{\pi}{4} + k\pi,\ \ k \in Z.\end{array}\right.$$

Javob: $x = -\mathrm{arctg}3 + k\pi,\ \frac{\pi}{4} + k\pi,\ k \in Z$.

5-misol. $\sin x - \sqrt{3}\cos x = 0$ tenglamani yeching.

Yechilishi. Bunday tenglamalarni yechishda tenglamaning ikkala qismi cos x ga bo'linadi. Tenglamani noma'lum miqdor tarkibida bo'lgan ifodaga bo'lganda ildizlar yo'qolishi mumkin. Shuning uchun cos$x$ = 0 tenglamaning ildizlari bo'lish-bo'lmasligini tekshirib ko'rish kerak. Agar cos$x$ = 0 bo'lsa, berilgan tenglamadan sin$x$ = 0 ekanligi kelib chiqadi. Lekin sin x va cos x lar bir vaqtda nolgа teng bo'la olmaydi. Demak, berilgan tenglamani $\cos x$ ga bo'lishda tenglama ildizlari yo'qolmaydi. Shunday qilib,

$$\sin x-\sqrt{3}\cos x=0 \Leftrightarrow \sin x=\sqrt{3}\cos x \Leftrightarrow \operatorname{tg}x=\sqrt{3} \Rightarrow x=\frac{\pi}{3}+k\pi,\ k\in Z.$$

Javob: $\frac{\pi}{3}+k\pi,\ k\in Z.$

Mustaqil ishlash uchun test topshiriqlari

1. $\sin\left(3x-\frac{\pi}{2}\right)=0$ tenglamani yeching.

A) $\frac{\pi}{6}+\frac{k\pi}{3}, k\in Z$; B) $\frac{\pi}{4}+\frac{k\pi}{2}, k\in Z$; C) $\frac{\pi}{3}+\frac{k\pi}{3}, k\in Z$;

D) $\frac{\pi}{2}+k\pi,\ k\in Z$; E) $k\pi, k\in Z$.

2. $4\cos^3 x+4=0$ tenglamani yeching.

A) $\pi+2k\pi, k\in Z$; B) $\frac{\pi}{3}+\frac{2k\pi}{3}, k\in Z$; C) $\frac{\pi}{2}+k\pi, k\in Z$;

D) $2k\pi, k\in Z$; E) $-\frac{\pi}{3}+\frac{k\pi}{3}, k\in Z$.

3. $4\cos^2\frac{x}{2}-3=0$ tenglamani yeching.

A) $\frac{\pi}{3}+2k\pi, k\in Z$; B) $\frac{2\pi}{3}+4k\pi, k\in Z$; C) $\pm\frac{\pi}{3}+2k\pi, k\in Z$;

D) $\frac{\pi}{3}+k\pi, k\in Z$; E) $\pm\frac{\pi}{6}+2k\pi, k\in Z$.

4. $4\sin^2 2x-1=0$ tenglamani yeching.

A) $\pm\frac{\pi}{24}+\frac{k\pi}{2}, k\in Z$; B) $\pm\frac{\pi}{3}+2k\pi, k\in Z$; C) $\pm\frac{\pi}{6}+k\pi, k\in Z$;

D) $\pm\frac{\pi}{12}+\frac{k\pi}{2}, k\in Z$; E) $(-1)^k-\frac{\pi}{12}+\frac{k\pi}{2}, k\in Z$.

5. $\sqrt{3}\operatorname{tg}\left(\frac{\pi}{6}-3x\right)=3$ tenglamani yeching.

A) $-\frac{\pi}{6}+\frac{k\pi}{3}, k\in Z$; B) $\frac{\pi}{6}+\frac{k\pi}{3}, k\in Z$; C) $\frac{\pi}{3}+\frac{k\pi}{2}, k\in Z$;

D) $-\frac{\pi}{3}+\frac{k\pi}{3}, k\in Z$; E) $-\frac{\pi}{18}+\frac{k\pi}{3}, k\in Z$.

6. $\sqrt{3}\,\mathrm{ctg}\left(\frac{\pi}{3}-2x\right)=3$ tenglamani yeching.

A) $\frac{\pi}{12}+\frac{k\pi}{2}, k\in Z$; B) $\frac{\pi}{12}-k\pi, k\in Z$; C) $\pm\frac{\pi}{12}+2k\pi, k\in Z$;

D) $\frac{k\pi}{2}, k\in Z$; E) $\frac{\pi}{6}+k\pi, k\in Z$.

7. $\sin^2 x+3\sin\left(\frac{\pi}{2}+x\right)=-3$ tenglamaning $[0;\ 2\pi]$ oraliqda nechta ildizi bor?

A) yo'q; B) 4; C) 3; D) 2; E) 1.

8. $\cos^2 x+3\cos\left(\frac{\pi}{2}-x\right)=-3$ tenglamani yeching.

A) $k\pi, k\in Z$; B) $\frac{\pi}{2}+k\pi, k\in Z$; C) $-\frac{\pi}{2}+2k\pi, k\in Z$;

D) $\frac{\pi}{2}+2k\pi, k\in Z$; E) $\varnothing$.

9. $\cos^2 x + 10\sin^2 x = 3\sin 2x$ tenglamani yeching.

A) $\varnothing$; B) $k\pi$; C) $\frac{\pi}{4}+2k\pi, k\in Z$; D) $\mathrm{arctg}\,\frac{7}{10}+k\pi, k\in Z$;

E) $-\mathrm{arctg}\,\frac{1}{10}+k\pi, k\in Z$.

10. $\sqrt{3}\cos x=\sin^2 x\cos x$ tenglamaning $[0;\ 360°]$ oraliqdagi ildizlari yig'indisini toping.

A) 90°; B) 60°; C) 360°; D) 300°; E) 150°.

11. $2\sin^2 2x - 5\cos 2x + 1 = 0$ tenglamaning $[\pi;\ 2\pi]$ oraliqqa tegishli ildizlarini ko'rsating.

A) $\frac{4\pi}{3}; \frac{5\pi}{3}$; B) $\frac{7\pi}{6}; \frac{11\pi}{6}$; C) $\frac{7\pi}{6}$;

D) $\frac{5\pi}{3}$; E) ko'rsatilgan oraliqda ildizlari yo'q.

12. $\sin x - \cos x = 1$ tenglamaning $[-2\pi;\ 2\pi]$ oraliqda nechta ildizi bor?

A) 1; B) 2; C) 3; D) 4; E) ko'rsatilgan oraliqda ildizlari yo'q.

13*. $\frac{\sin 2x}{\mathrm{ctg}\,x-1}=0$ tenglamani yeching.

A) $\frac{k\pi}{2}, k \in Z$; B) $\frac{\pi}{2} + k\pi, k \in Z$; C) $2k\pi, k \in Z$;

D) $\pi + 2k\pi, k \in Z$; E) $k\pi, k \in Z$.

14*. $\sin 3x = \cos x$ tenglamaning eng kichik musbat ildizini toping.

A) $\frac{3\pi}{8}$; B) $\frac{\pi}{4}$; C) $\frac{\pi}{8}$; D) $\frac{\pi}{12}$; E) $\frac{\pi}{24}$.

15*. $\cos 4x = \cos 6x$ tenglamaning $[0; 180°]$ oraliqdagi ildizlari yig'indisini toping.

A) 216°; B) 288°; C) 360°; D) 390°; E) 540°.

16*. $\mathrm{tg}\left(5x + \frac{\pi}{3}\right)\mathrm{ctg}3x = 1$ tenglamani yeching.

A) $-\frac{\pi}{6} + \frac{k\pi}{2}, k \in Z$; B) $-\frac{\pi}{6} + k\pi, k \in Z$; C) $-\frac{\pi}{6}$;

D) $\frac{\pi}{12} + k\pi, k \in Z$; E) $\varnothing$.

17. $2\mathrm{ctg}^2 x\cdot\cos^2 x+4\cos^2 x-\mathrm{ctg}^2 x-2=0$ tenglamani yeching.

A) $\frac{\pi}{2}+k\pi, k\in Z$; B) $\frac{\pi}{4}+k\pi, k\in Z$; C) $\frac{\pi}{4}+\frac{k\pi}{2}, k\in Z$;

D) $\frac{\pi}{2}+2k\pi, k\in Z$; E) $\frac{\pi}{4}+2k\pi, k\in Z$.

18. $\sin^2\frac{x}{2}-\cos\frac{x}{2}=1$ tenglama [0; 2π] oraliqda nechta ildizga ega?

A) 4; B) 3; C) 2; D) 1; E) yo‘q.

19*. $\mathrm{ctg}^2 x-\mathrm{tg}^2 x=8\cos 2x$ tenglamani yeching.

A) $\frac{\pi}{4}+2k\pi, k\in Z$; B) $-\frac{\pi}{4}+2k\pi, k\in Z$;

C) $\frac{\pi}{4}+2k\pi$; $\frac{\pi}{8}+2k\pi, k\in Z$; D) $\frac{\pi}{4}+\frac{k\pi}{2}$; $\frac{\pi}{8}+\frac{k\pi}{4}, k\in Z$;

E) $\frac{\pi}{8}+2k\pi, k\in Z$.

20. $\sin^2 x-\sin^3 x+\sin 8x=\cos\left(\frac{\pi}{2}-7x\right)$ tenglamaning [0; 90°]

7-Mavzu: Trigonometrik tenglamalarni ko’paytuvchilarga ajratib yechish.

Ko‘pgina trigonometrik tenglamalarni yechishda algebraik ifodalarni ko‘paytuvchilarga ajratishning umumiy ko‘paytuvchini qavsdan tashqariga chiqarish, guruhlash usuli bilan ko‘paytuvchilarga ajratish, qisqa ko‘paytirish formulalaridan foydalanib ko‘paytuvchilarga ajratish kabi usullardan foydalaniladi.

6-misol. $(1+\cos 4x)\sin 2x=\cos^2 2x$ tenglamani yeching.

Yechilishi. $\cos^2 2x$ ni tenglamaning chap qismiga o‘tkazib, darajani pasaytirish formulasidan foydalanamiz va hosil bo‘lgan ifodani ko‘paytuvchilarga ajratamiz:

$$(1+\cos 4x)\sin 2x = \cos^2 2x \Leftrightarrow (1+\cos 4x)\sin 2x - \frac{1+\cos 4x}{2} = 0 \Leftrightarrow$$

$$\Leftrightarrow (1+\cos 4x)(2\sin 2x - 1) = 0.$$

1) $(1+\cos 4x) = 0 \Leftrightarrow \cos 4x = -1 \Rightarrow 4x = \pi + 2k\pi \Rightarrow x = \frac{\pi}{4} + \frac{k\pi}{2}$; $k \in Z$.

2) $2\sin 2x - 1 = 0 \Rightarrow \sin 2x = \frac{1}{2} \Rightarrow 2x = (-1)^k \frac{\pi}{6} + k\pi \Rightarrow x =$

$= (-1)^k \frac{\pi}{12} + \frac{k\pi}{2}$, $k \in Z$.

Javob: $\frac{\pi}{4} + \frac{k\pi}{2}$, $(-1)^k \frac{\pi}{12} + \frac{k\pi}{2}$, $k \in Z$.

7-misol. $3(1 - \sin x) + \sin^4 x = 1 + \cos^4 x$ tenglamani yeching.

Yechilishi. $3(1 - \sin x) + \sin^4 x = 1 + \cos^4 x \Leftrightarrow 3(1 - \sin x) = 1 + \cos^4 x - \sin^4 x \Leftrightarrow 3(1 - \sin x) = 1 + (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) \Leftrightarrow \Leftrightarrow 3(1 - \sin x) = 1 + \cos^2 x \Leftrightarrow 3(1 - \sin x) = 2\cos^2 x \Leftrightarrow 3(1 - \sin x) = 2(1 - \sin^2 x) \Leftrightarrow 3(1 - \sin x) - 2(1 - \sin x)(1 + \sin x) = 0 \Leftrightarrow \Leftrightarrow (1 - \sin x)(3 - 2(1 + \sin x)) = 0 \Leftrightarrow (1 - \sin x)(1 - 2\sin x) = 0 \Rightarrow$

$$\Rightarrow \left[\begin{array}{l} \sin x = 1 \\ \sin x = \frac{1}{2} \end{array}\right. \Rightarrow \left[\begin{array}{l} x = \frac{\pi}{2} + 2k\pi, \quad k \in Z; \\ x = (-1)^k \frac{\pi}{6} + 2k\pi, \quad k \in Z. \end{array}\right.$$

Javob: $\frac{\pi}{2} + 2k\pi$, $(-1)^k \frac{\pi}{6} + k\pi$, $k \in Z$.

8-misol. $\sin 3x + \sin 5x = \sin 4x$ tenglamaning nechta ildizi $|x| \leq \frac{\pi}{2}$ tengsizlikni qanoatlantiradi?

Yechilishi. Tenglamani uning chap qismini ko'paytma shakliga keltirib yechamiz:

$$\sin 3x + \sin 5x = \sin 4x \Leftrightarrow 2\sin\frac{3x+5x}{2}\cos\frac{3x-5x}{2} = \sin 4x \Leftrightarrow$$

$$\Leftrightarrow 2\sin 4x \cos x - \sin 4x = 0 \Leftrightarrow \sin 4x(2\cos x - 1) = 0 \Rightarrow \left[\begin{array}{l} \sin 4x = 0, \\ \cos x = \frac{1}{2}, \end{array}\right. \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} 4x = \frac{k\pi}{4}, \quad k \in Z, \\ x = \pm\frac{\pi}{3} + 2k\pi, \quad k \in Z. \end{array}\right.$$

$k = 0; \pm 1; \pm 2$ qiymatlar uchun tenglamaning $|x| \leq \frac{\pi}{2}$ shartni qanoatlantiruvchi $x = \pm\frac{\pi}{2}$; $\pm\frac{\pi}{4}$; $\pm\frac{\pi}{3}$; 0 ildizlarini ko'rsatish mumkin.

Javob: 7 ta.

2.4. Bir ismli trigonometrik funksiyalarning tenglik shartlaridan foydalanib yechiladigan tenglamalar. Bunday tenglamalarni yechish bir ismli trigonometrik funksiyalarning tengligi shartlariga, ya'ni α va β burchaklarning $\sin\alpha = \sin\beta$, $\cos\alpha = \cos\beta$, $\text{tg}\alpha = \text{tg}\beta$ tengliklarni qanoatlantiruvchi shartlariga asoslanadi.

1-teorema. **Ikki burchakning sinuslari teng bo'lishi uchun quyidagi shartlardan birining bajarilishi zarur va yetarlidir: bu burchaklar ayirmasi π ni juft songa ko'paytirilganiga teng bo'lishi**

kerak yoki bu burchaklar yig'indisi π ni toq songa ko'paytirilganiga teng bo'lishi kerak, ya'ni $\alpha-\beta=2k\pi$ **yoki** $\alpha+\beta=(2k+1)\pi,\ k\in Z$ **bo'lsa,** $\sin\alpha=\sin\beta$ **bo'ladi.**

2-teorema. **Ikki burchakning kosinuslari teng bo'lishi uchun quyidagi shartlardan birining bajarilishi zarur va yetarlidir: bu burchaklar ayirmasi yoki yig'indisi π ni juft songa ko'paytirilganida teng bo'lishi kerak, ya'ni** $\alpha-\beta=2k\pi$ **yoki** $\alpha+\beta=2k\pi,\ k\in Z$ **bo'lsa,** $\cos\alpha=\cos\beta$ **bo'ladi.**

3-teorema. **Ikki burchakning tangenslari teng bo'lishi uchun quyidagi ikki shartning bir paytda bajarilishi zarur va yetarlidir: bu ikki burchakning tangenslari mavjud bo'lishi va bu burchaklar ayirmasi π ni butun songa ko'paytirilganiga teng bo'lishi kerak, ya'ni** $\alpha\neq\frac{\pi}{2}+k\pi,\ \beta\neq\frac{\pi}{2}+k\pi,\ \alpha-\beta=k\pi,\ k\in Z$ **bo'lsa,** $\mathrm{tg}\alpha=\mathrm{tg}\beta$.

Keltirilgan teoremalardan foydalanib yechiladigan tenglamalardan na'munalar keltiramiz.

9-misol. $\sin 2x=\sin 5x$ tenglamani yeching.

Yechilishi. 1-teoremaga ko'ra ikki burchak sinuslarining teng bo'lishi shartlarini yozamiz.

1) $5x-2x=2k\pi \Leftrightarrow 3x=2k\pi \Rightarrow x=\frac{2k\pi}{3},\ k\in Z$;

2) $5x+2x=(2k+1)\pi \Leftrightarrow 7x=2k\pi+\pi \Rightarrow x=\frac{\pi}{7}+\frac{2k\pi}{7},\ k\in Z.$

Javob: $\frac{2k\pi}{3};\ \frac{\pi}{7}+\frac{2k\pi}{7},\ k\in Z.$

10-misol. $\sin 5x=\cos 7x-\cos\frac{3\pi}{2}$ tenglamani yeching.

Yechilishi. $\sin 5x=\cos 7x-0 \Leftrightarrow \cos\left(\frac{\pi}{2}-5x\right)=\cos 7x.$

Ikki burchak kosinuslarining tenglik shartlaridan foydalanamiz:

1) $7x-\frac{\pi}{2}+5x=2k\pi \Leftrightarrow 12x=\frac{\pi}{2}+2k\pi \Rightarrow x=\frac{\pi}{24}+\frac{k\pi}{6},\ k\in Z.$

2) $7x+\frac{\pi}{2}-5x=2k\pi \Leftrightarrow 2x=-\frac{\pi}{2}+2k\pi \Rightarrow x=-\frac{\pi}{4}+k\pi,\ k\in Z.$

Javob: $-\frac{\pi}{24}+\frac{k\pi}{6};-\frac{\pi}{4}+k\pi,\ k\in Z.$

11-misol. $\mathrm{tg}(x+1)\,\mathrm{ctg}(2x+3)=1$ tenglamani yeching.

Yechilishi. $\mathrm{tg}(2x+3)\neq 0$ bo'lganligidan berilgan tenglamani

$$\operatorname{tg}(x+1)\cdot\frac{1}{\operatorname{tg}(2x+3)}=1 \Leftrightarrow \operatorname{tg}(x+1)=\operatorname{tg}(2x+3)$$

ko‘rinishga keltirib, ikki burchak tangenslari tengligi shartidan foydalanamiz:

$$2x+3-x-1=k\pi \Rightarrow x=k\pi-2,\ k\in Z.$$

x ning bu to‘plamdagi har qanday qiymatida ham $\operatorname{tg}(x+1)$ va $\operatorname{tg}(2x+3)$ aniqlangan.

Javob: $k\pi-2,\ k\in Z.$

2.5. $a\sin x+b\cos x=c$ **shaklidagi tenglamalar.** Bu yerda a, b, $c\in R$ tenglama koeffitsiyentlari. Agar $a=b=0$, $c\neq 0$ bo‘lsa, tenglama ma’noga ega bo‘lmaydi.

$a\sin x+b\cos x=c$ shaklidagi tenglamalarni yechishning bir necha usullari mavjud. Ulardan ayrimlarini keltirib o‘tamiz.

8-Mavzu: Trigonometrik tenglamalarni yordamchi burchak kiritish usuli bilan yechish.

Ma'lumki, agar $a^2 + b^2 = 1$ bo'lsa, u holda shunday φ burchak mavjudki, $a = \cos\varphi$, $b = \sin\varphi$ va aksincha. Shunga ko'ra

$$a\sin x + b\cos x = c \Leftrightarrow \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\sin x + \frac{b}{\sqrt{a^2+b^2}}\cos x\right) = c;$$

$$\left[\frac{a}{\sqrt{a^2+b^2}} = \cos\varphi; \quad \frac{b}{\sqrt{a^2+b^2}} = \sin\varphi\right] \Leftrightarrow \sqrt{a^2+b^2}(\cos\varphi\sin x +$$

$$+\sin\varphi\cos x) = c \Leftrightarrow \sin(x+\varphi) = \frac{c}{\sqrt{a^2+b^2}}.$$

Hosil bo'lgan tenglama $a^2 + b^2 \geq c^2$ bo'lsagina yechimga ega:

$$x = (-1)^k \arcsin\frac{c}{\sqrt{a^2+b^2}} + k\pi - \varphi,\ k \in Z.$$ Bunda $\varphi = \operatorname{arctg}\frac{b}{a}$.

b) **Ratsionallashtirish usuli.** Bu usulga ko'ra $x \neq \pi + 2k\pi$ da

$$\sin x = \frac{2\operatorname{tg}\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}},\ \cos x = \frac{1-\operatorname{tg}^2\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}},\ \operatorname{tg}x = \frac{2\operatorname{tg}\frac{x}{2}}{1-\operatorname{tg}^2\frac{x}{2}}$$

tengliklar o'rinli ekanligidan $a\sin x + b\operatorname{sos}x = \mathrm{c}$ tenglama $\operatorname{tg}\frac{x}{2} = t$ belgilash yordamida

$$(b + c)t^2 - 2at + (c - b) = 0$$

tenglamaga keltiriladi. Agar $b + c \neq 0$ bo'lib, $a^2 + b^2 \geq c^2$ bo'lsa, t ning qiymatlari haqiqiy bo'ladi:

$$t_{1,2} = \frac{a\pm\sqrt{a^2+b^2-c^2}}{b+c};$$

1) Agar $a^2 + b^2 < c^2$ bo'lsa, tenglama yechimga ega bo'lmaydi.

2) Agar $a^2 + b^2 \geq c^2$ bo'lib, $c \neq -b$ bo'lsa,

$$x = 2\operatorname{arctg}\frac{a\pm\sqrt{a^2+b^2-c^2}}{b+c} + 2k\pi,\ k \in Z.$$

3) Agar $c=-b$ bo'lsa, tenglama $x=\pi+2k\pi$ va $x=-2\operatorname{arctg}\frac{b}{a}+2k\pi,\ k\in Z$ yechimlarga ega bo'ladi.

d) **Yarim burchaklarga o'tish yo'li bilan bir jinsli tenglamaga keltirish usuli.**

Bu usulga ko'ra $a\sin x+b\cos x=c$ tenglama quyidagi ko'rinishga keltiriladi:

$$2a\sin\frac{x}{2}\cos\frac{x}{2}+b\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)=c\left(\sin^2\frac{x}{2}+\cos^2\frac{x}{2}\right)\Leftrightarrow$$

$$\Leftrightarrow (c+b)\sin^2\frac{x}{2}-2a\sin\frac{x}{2}\cos\frac{x}{2}+(c-b)\cos^2\frac{x}{2}=0.$$

Bunday bir jinsli tenglamalarning yechilishi oldingi bandda bayon qilinganidek amalga oshiriladi.

12-misol. $5\sin x-4\cos x=4$ tenglamani yeching.

Yechilishi. Berilgan tenglamada $a=5$, $b=-4$, $c=4$, bo'lib, $c=-b$. Ratsionallashtirish usulidan foydalanamiz:

$$\frac{10t}{1+t^2}-\frac{4(1-t^2)}{1+t^2}=4\Leftrightarrow 10t-4+4t^2=4+4t^2\Leftrightarrow 10t=8\Rightarrow$$

$$\Rightarrow\left[t=\frac{4}{5}\right]\Rightarrow \operatorname{tg}x=\frac{4}{5}\Rightarrow [x=2\operatorname{arctg}0,8+2k\pi,\ k\in Z.$$

$c=-b$ bo'lganligi uchun $x=\pi+2k\pi,\ k\in Z$ shaklidagi yechimlar to'plami ham mavjud.

Berilgan tenglamada quyidagi almashtirishlarni bajarish mumkin:

$$5\sin x-4\cos x=4\Leftrightarrow 5\sin x=4(1+\cos x)\Leftrightarrow 10\sin\frac{x}{2}\cos\frac{x}{2}=$$

$$=8\cos^2\frac{x}{2}\Leftrightarrow 2\cos\frac{x}{2}\left(5\sin\frac{x}{2}-4\cos\frac{x}{2}\right)=0\Leftrightarrow$$

$$\Leftrightarrow\left[\begin{array}{ll}2\cos\frac{x}{2}=0, & (a)\\ 5\sin\frac{x}{2}-4\cos\frac{x}{2}=0 & (b)\end{array}\right.$$

a) tenglamaning yechimi $x=\pi+2k\pi,\ k\in Z$;

b) tenglama yechimi $x=2\operatorname{arctg}0,8+2k\pi,\ k\in Z$.

Javob: $2\operatorname{arctg}0,8+2k\pi,\ \pi+2k\pi,\ k\in Z$.

9-Mavzu: Teskari trigonometrik funksiyalar qatnashgan tenglamalar.

Teskari trigonometrik funksiyalar qatnashgan tenglamalarni yechishda arksinus, arkkosinus, arktangenslarning ta'riflaridan va XII bob, 8.8-bandda keltirilgan ayniyatlardan foydalaniladi.

1-misol. $\arcsin^2 x - \frac{\pi}{2}\arcsin x + \frac{\pi^2}{18} = 0$ tenglamani yeching.

Yechilishi. Qulaylik uchun $\arcsin x = t\left(-\frac{\pi}{2} \le t \le \frac{\pi}{2}\right)$ belgilash kiritamiz. U holda

$$t^2 - \frac{\pi}{2}t + \frac{\pi^2}{18} = 0 \Rightarrow \left[\begin{array}{l} t_1 = \frac{\pi}{6}, \\ t_2 = \frac{\pi}{3}. \end{array}\right.$$

Bundan, $\arcsin x = \frac{\pi}{6} \Rightarrow x_1 = \frac{1}{2}$,

$$\arcsin x = \frac{\pi}{3} \Rightarrow x_2 = \frac{\sqrt{3}}{2}.$$

Javob: $\frac{1}{2}; \frac{\sqrt{3}}{2}$.

2-misol. $6\arcsin(x^2 - 6x + 8{,}5) = \pi$ tenglamani yeching.

Yechilishi. $6\arcsin(x^2 - 6x + 8{,}5) = \pi \Leftrightarrow \arcsin(x^2 - 6x +$

$$+8{,}5) = \frac{\pi}{6} \Leftrightarrow x^2 - 6x + 8{,}5 = 0{,}5 \Leftrightarrow x^2 - 6x + 8 = 0 \Rightarrow \left[\begin{array}{l} x_1 = 2, \\ x_2 = 4. \end{array}\right.$$

Javob: 2; 4.

3-misol. $\arcsin \frac{2}{3\sqrt{x}} - \arcsin\sqrt{1-x} = \arcsin\frac{1}{3}$ tenglamani yeching.

Yechilishi. Tenglama $0 < x \le 1$ oraliqda aniqlangan.

$\arcsin \frac{2}{3\sqrt{x}} = \alpha \left(-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}\right)$, $\arcsin\sqrt{1-x} = \beta \left(-\frac{\pi}{2} \le \beta \le \frac{\pi}{2}\right)$ belgilashlar kiritamiz. U holda

$$\sin\alpha = \frac{2}{3\sqrt{x}} \qquad (a), \qquad \sin\beta = \sqrt{1-x} \qquad (b)$$

$$\cos^2\alpha = 1 - \sin^2\alpha = 1 - \frac{4}{9x} = \frac{9x-4}{9x}; \ \cos\alpha > 0.$$

$$\cos\alpha = \frac{1}{3}\sqrt{\frac{9x-4}{x}},$$

$$\cos\beta = \sqrt{1-1+x} = \sqrt{x}.$$

Qabul qilingan belgilashlar va (a), (b), (d), (e) munosabatl inobatga olib, berilgan tenglamaning ildizini topamiz:

$$\arcsin\frac{2}{3\sqrt{x}} - \arcsin\sqrt{1-x} = \arcsin\frac{1}{3} \Rightarrow \alpha - \beta = \arcsin\frac{1}{3} \Leftrightarrow$$

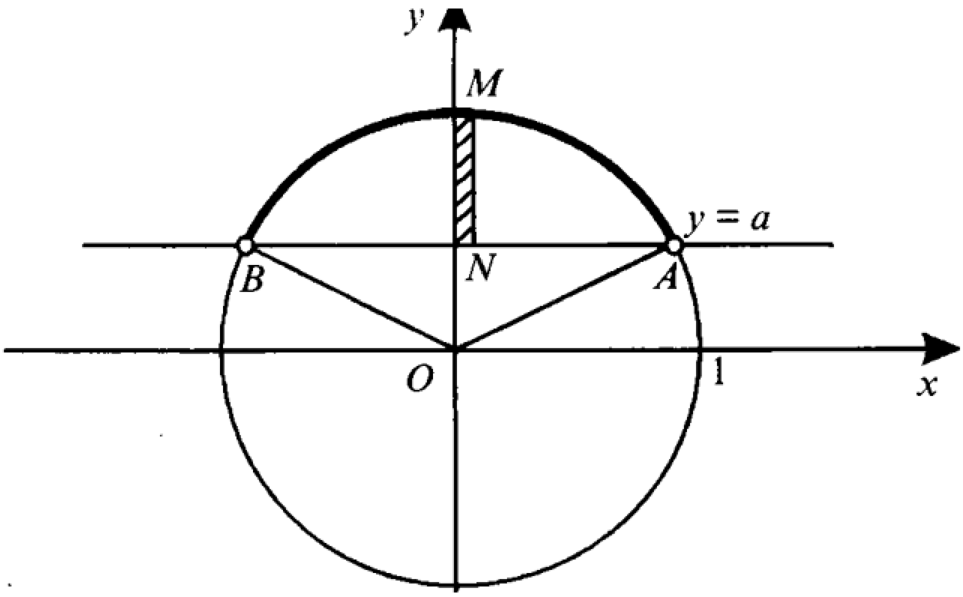
$$\Leftrightarrow \sin(\alpha-\beta) = \sin\left(\arcsin\frac{1}{3}\right) \Leftrightarrow \sin\alpha\cos\beta - \cos\alpha\sin\beta = \frac{1}{3} \Rightarrow$$

$$\Rightarrow \frac{2}{3\sqrt{x}}\cdot\sqrt{x} - \frac{1}{3}\sqrt{\frac{9x-4}{x}}\cdot\sqrt{1-x} = \frac{1}{3} \Leftrightarrow \sqrt{\frac{9x-4}{x}}\cdot\sqrt{1-x} = 1 \Leftrightarrow$$

$$\Leftrightarrow 9x^2 - 12x + 4 = 0 \Leftrightarrow (3x-2)^2 = 0 \Leftrightarrow 3x - 2 = 0 \Rightarrow [x = \frac{2}{3}.$$

Javob: $\frac{2}{3}$.

10-Mavzu: Trigonometrik tengsizliklarni yechish.



«>» yoki «<» tengsizlik belgilari bilan berilgan trigonometrik ifodalar **trigonometrik tengsizliklar** deyiladi. Trigonometrik tengsizliklarni yechish — bu tengsizlikdagi noma'lumlarning tengsizlikni qanoatlantiruvchi barcha qiymatlarini topish demakdir. Trigonometrik tengsizliklarni yechishda trigonometrik funksiyalarning monotonlik xossalaridan va davriyligidan foydalaniladi.

$\sin x > a$, $\sin x < a$, $\cos x > a$, $\cos x < a$, $\operatorname{tg} x > a$, $\operatorname{tg} x < a$, $\sin x \geq a$, $\operatorname{tg} x \geq a$, kabi tengsizliklar **eng sodda trigonometrik tengsizliklar** deyiladi. Faqat $\sin x$ yoki $\cos x$ qatnashgan tengsizliklarni yechish uchun bunday tengsizlikni uzunligi 2π bo'lgan biror oraliqda yechish yetarlidir. Barcha yechimlar to'plami kesmada topilgan yechimga $2k\pi$, $k \in Z$ sonni qo'shib qo'yish yo'li bilan topiladi. Trigonometrik tengsizliklarni yechishda $y = \sin x$, $y = \cos x$, $y = \operatorname{tg} x$, va $y = \operatorname{ctg} x$ funksiyalarining grafiklaridan yoki birlik aylanadan foydalaniladi.

4.1. $\sin x > a$, $\sin x < a$ tengsizliklarni yechish. $|\sin x| \leq 1$ bo'lganligi sababli quyidagi tasdiqlar o'rinlidir.

Agar:

$a \leq -1$ bo'lsa, $\sin x < a$; $a > 1$ bo'lsa, $\sin x \geq a$;

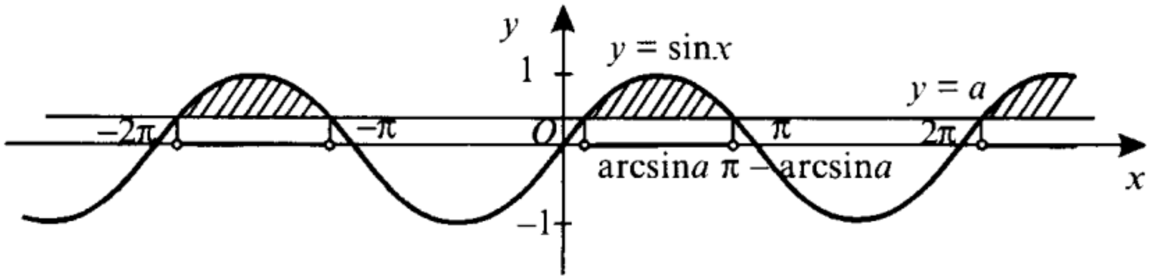
$a < -1$ bo'lsa, $\sin x \leq a$; $a \geq 1$ bo'lsa, $\sin x > a$

tengsizliklar yechimga ega emas.

Agar

$a > 1$ bo‘lsa, $\sin x < a$; $a \leq -1$ bo‘lsa, $\sin x \geq a$;

$a \geq 1$ bo‘lsa, $\sin x \leq a$; $a < -1$ bo‘lsa, $\sin x > a$;
tengsizliklar x ning har qanday qiymatida bajariladi.



138-rasm

1. $\sin x > a$ ($|a| < 1$) tengsizlikning yechilishi. Birlik aylanada abssissalar o‘qiga parallel $y = a$ to‘g‘ri chiziqni chizamiz. Bu to‘g‘ri chiziq birlik aylanani A va B nuqtalarda kesib o‘tadi (137-rasm). Rasmdagi chizmadan ko‘rinib turibdiki, NM oraliqda y ning barcha qiymatlari a dan katta; birlik aylana AMB yoyining barcha nuqtalari a dan katta ordinataga ega. Shuning uchun $\sin x > a$ tengsizlikning $[0; 2\pi]$ kesmadagi yechimlari $(\arcsin a; \pi - \arcsin a)$ oraliqqa tegishli barcha x sonlar bo‘ladi (138-rasm). $\sin x$ ning davriyligini hisobga olsak, tengsizlikning butun sonlar o‘qidagi yechimlari

$$\arcsin a + 2k\pi < x < \pi - \arcsin a + 2k\pi, k \in Z \qquad (1)$$

shaklda yoziladi.

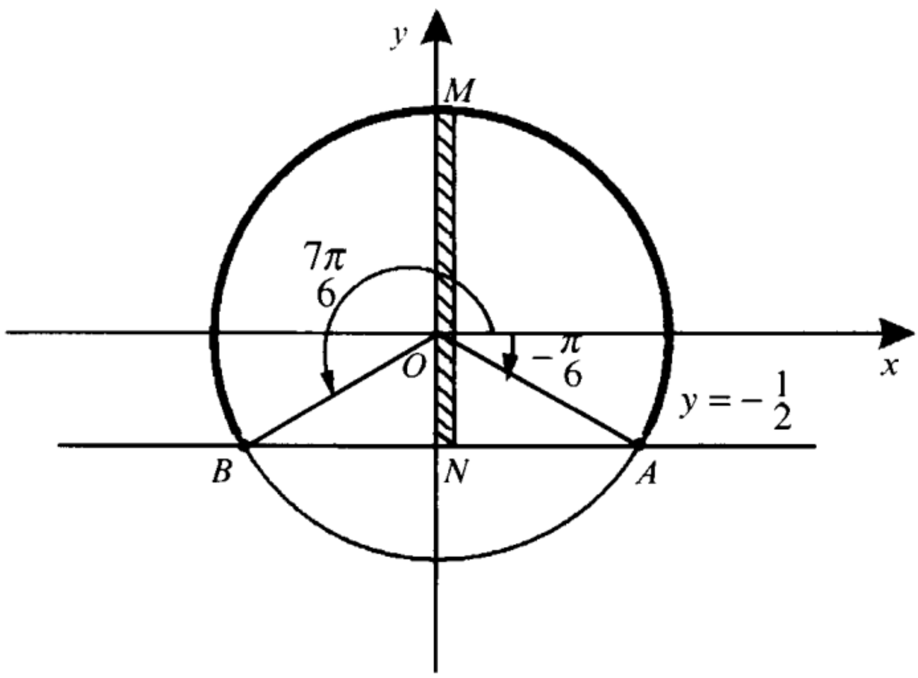
Bundan buyon eng sodda trigonometrik tengsizliklarning yechimlarini topishga doir misollarda tengsizlik yechimlarini tasvirlovchi chizmalarni sharhlarsiz keltiramiz. O‘quvchilarga ularni mustaqil tahlil qilish tavsiya qilinadi.

$\sin x > 0$ tengsizlikning yechimlar to‘plami $2k\pi < x < \pi + 2k\pi$, $k \in Z$.

$\sin x < 0$ tengsizlikning yechimlar to‘plami $2k\pi - \pi < x < 2k\pi$, $k \in Z$ ekanligini yodda tuting.

1-misol. $\sin x > \frac{\sqrt{3}}{2}$ tengsizlikni yeching.

Yechilishi. (1) formuladan foydalanib, berilgan tengsizlikning yechimlar to‘plamini aniqlaymiz:



139-rasm

$$\arcsin\frac{\sqrt{3}}{2}+2k\pi < x < \pi-\arcsin\frac{\sqrt{3}}{2}+2k\pi \Leftrightarrow \frac{\pi}{3}+2k\pi < x <$$

$$< \pi-\frac{\pi}{3}+2k\pi \Rightarrow \frac{\pi}{3}+2k\pi < x < \frac{2\pi}{3}+2k\pi, k\in Z.$$

Javob: $\left(\frac{\pi}{3}+2k\pi; \frac{2\pi}{3}+2k\pi\right), k\in Z.$

2-misol. $\sin x \geq -\frac{1}{2}$ tengsizlikni yeching.

Yechilishi. (1) formuladan foydalanamiz. Unga ko‘ra

$$\arcsin\left(-\frac{1}{2}\right)+2k\pi \leq x \leq \pi-\arcsin\left(-\frac{1}{2}\right)+2k\pi \Leftrightarrow -\arcsin\frac{1}{2}+$$

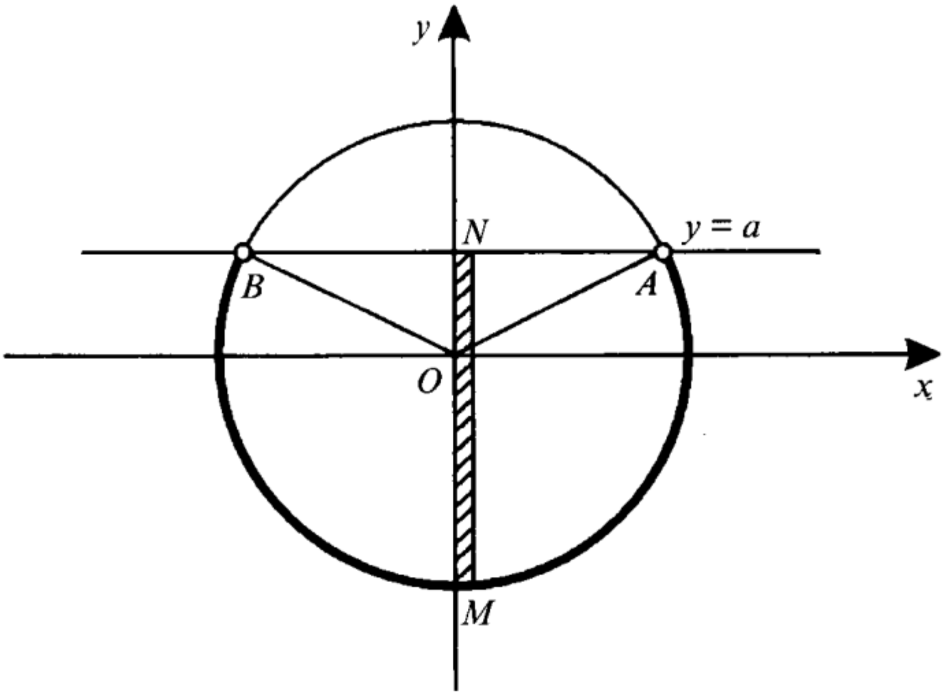
$$+2k\pi \leq x \leq \pi+\arcsin\frac{1}{2}+2k\pi \Leftrightarrow -\frac{\pi}{6}+2k\pi \leq x \leq \pi+\frac{\pi}{6}+2k\pi \Rightarrow$$

$$\Rightarrow -\frac{\pi}{6}+2k\pi \leq x \leq \frac{7\pi}{6}+2k\pi,\ k\in Z.$$

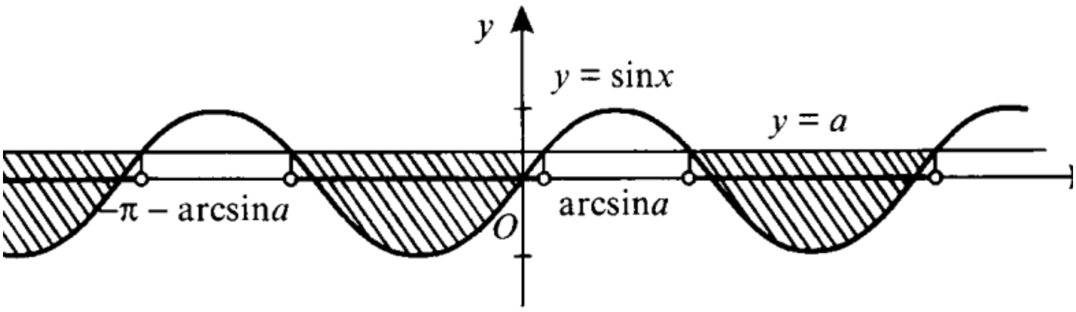
Tengsizlikning yechimi 139-rasmda tasvirlangan.

Javob: $\left[-\frac{\pi}{6}+2k\pi; \frac{7\pi}{6}+2k\pi,\ k\in Z\right].$

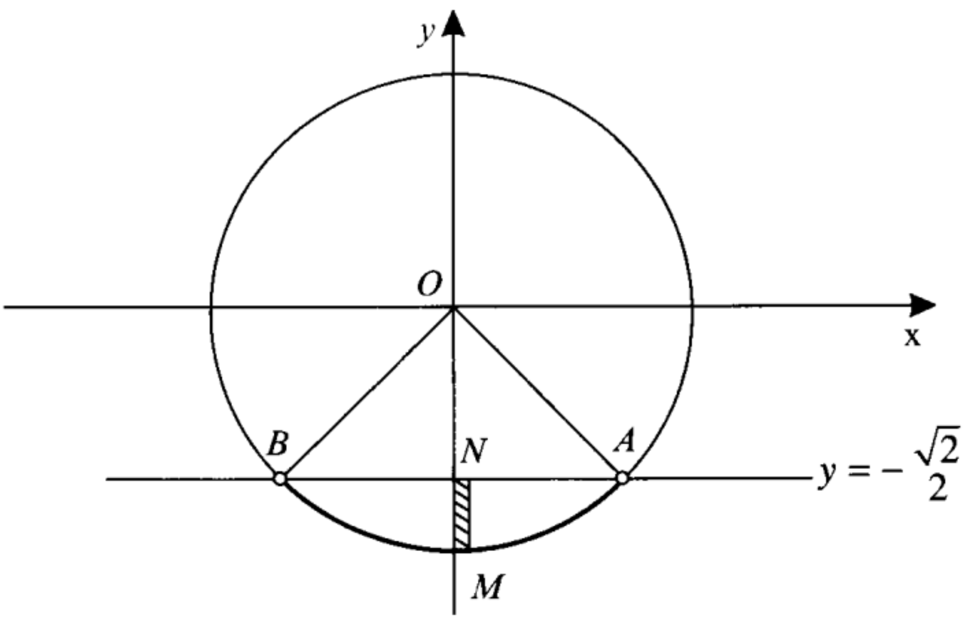
2. sin$x$ < a (|a| < 1 tengsizlikning yechilishi. 140, 141-rasmlardan ko‘rinib turibdiki, tengsizlikning [–π; π] kesmadagi yechimi x ning (–π – arcsin$a$; arcsin$a$) oraliqdagi qiymatlaridan, butun sonlar o‘qida esa (2kπ – π – arcsin$a$; 2kπ + arcsin$a$), $k\in Z$ oraliqdagi qiymatlar to‘plamidan iborat.



140-rasm



141-rasm



Shunday qilib, $\sin x < a$ tengsizlikning yechimlar to'plami

$$2k\pi - \pi - \arcsin a < x < 2k\pi + \arcsin a, k \in Z \qquad (2)$$

formula bilan ifodalanadi.

3-misol. $\sin 2x < -\frac{\sqrt{2}}{2}$ tengsizlikni yeching.

Yechilishi. 142-rasmdagi chizmadan ko'rinib turibdiki,

$2k\pi - \frac{3\pi}{4} < 2x < 2k\pi - \frac{\pi}{4}$. Bundan $k\pi - \frac{3\pi}{8} < x < k\pi - \frac{\pi}{8}$, $k \in Z$.

Javob: $\left(k\pi - \frac{3\pi}{8}; -\frac{\pi}{8} + k\pi\right)$, $k \in Z$.

4-misol. $2\cos^2 x + \sin x > 2$ tengsizlikni yeching.

Yechilishi. $2\cos^2 x + \sin x > 2 \Leftrightarrow 2(1 - \sin^2 x) + \sin x > 2 \Leftrightarrow 2\sin^2 x - - \sin x < 0 \Leftrightarrow \sin x\,(2\sin x - 1) < 0$. Berilgan tengsizlikka teng kuchli bo'lgan bu tengsizlikda $\sin x = y$ belgilash orqali yangi o'zgaruvchi kiritamiz va

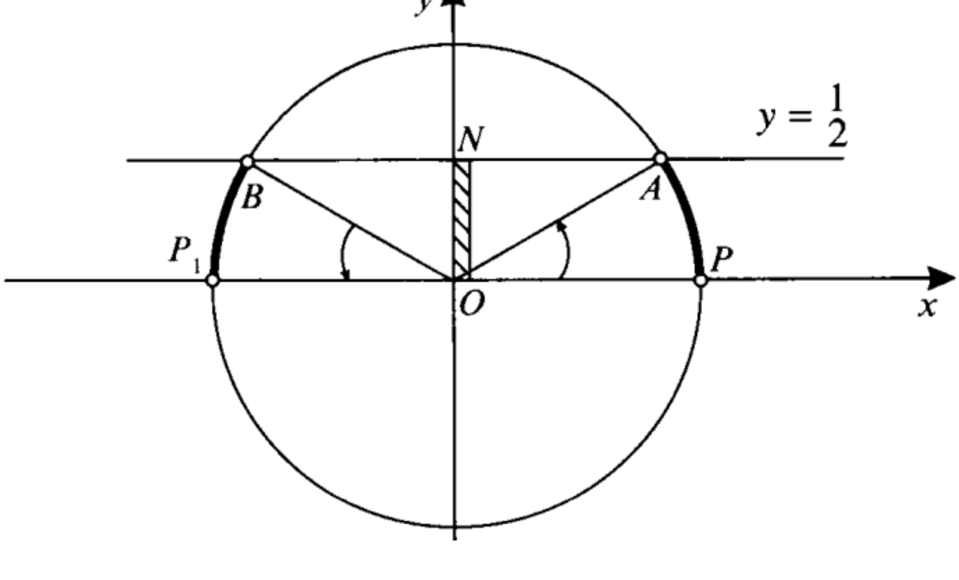
$$y\,(2y - 1) < 0$$

algebraik tengsizlikni hosil qilamiz. Bu tengsizlikning yechimi

$0 < y < \frac{1}{2}$.

Shunday qilib, $0 < \sin x < \frac{1}{2}$. Bu tengsizlikning yechimlar to'plami $2k\pi < x < \frac{\pi}{6} + 2k\pi$, $k \in Z$ yoki $\frac{5\pi}{6} + 2k\pi < x < \pi + 2k\pi$, $k \in Z$ oraliqlardan iborat (143-rasm).

Javob: $\left(2k\pi; \frac{\pi}{6} + 2k\pi\right) \cup \left(\frac{5\pi}{6} + 2k\pi; \pi + 2k\pi\right)$, $k \in Z$.



4.2. $\cos x > a$, $\cos x < a$ tengsizliklarni yechish. $|\cos x| \leq 1$ bo'lganligi sababli quyidagi tasdiqlar o'rinli.

Agar:

$a \leq -1$ bo'lsa, $\cos x < a$; $a > 1$ bo'lsa, $\cos x \geq a$;

$a < -1$ bo'lsa, $\cos x \leq a$; $a \geq 1$ bo'lsa, $\cos x > a$

tengsizliklar yechimga ega emas.

Agar:

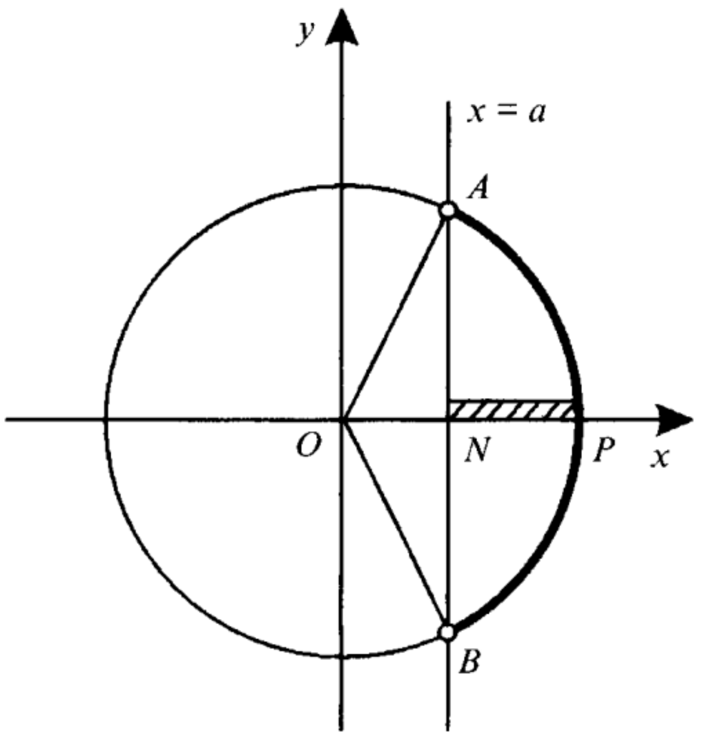
$a > 1$ bo'lsa, $\cos x < a$; $a \leq -1$ bo'lsa, $\cos x \geq a$;

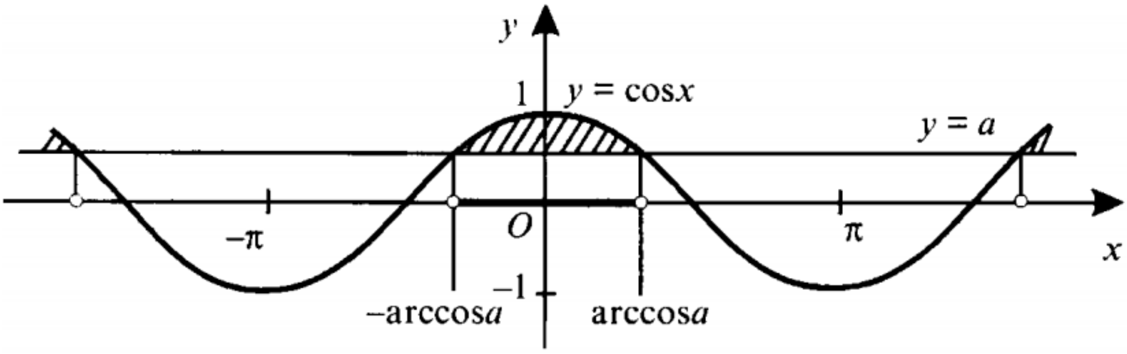
$a \geq 1$ bo'lsa, $\cos x \leq a$; $a < -1$ bo'lsa, $\cos x > a$

tengsizliklar x ning har qanday qiymatlarida bajariladi.

1. $\cos x > a$ ($|a| < 1$) tengsizlikning yechilishi

Birlik aylanada ordinatalar o'qiga parallel $x = a$ to'g'ri chiziqn chizamiz. Bu to'g'ri chiziq birlik aylanani A va B nuqtalarda kesil o'tadi (144-rasm). A va B nuqtalarning absissalari a ga teng bo'lib NP oraliqda x ning barcha qiymatlari a dan katta; birlik aylana BPA yoyining barcha nuqtalari a dan katta abssissaga ega. Shuning uchun $\cos x > a$ tengsizlikning $[-\pi; \pi]$ kesmadagi yechimlar $(-\arccos a; \arccos a)$ oraliqqa tegishli barcha x sonlar bo'lad (145-rasm). $\cos x$ ning davriyligini hisobga olib tengsizlikning butun sonlar o'qidagi yechimlar to'plamini





145-rasm

$$2k\pi - \arccos a < x < 2k\pi + \arccos a,\ k \in Z \qquad (3)$$

formula bilan berilishi mumkin.

$\cos x > 0$ tengsizlikning yechimlar to‘plami

$2k\pi - \frac{\pi}{2} < x < \frac{\pi}{2} + 2k\pi,\ k \in Z.$

$\cos x < 0$ tengsizlikning yechimlar to‘plami

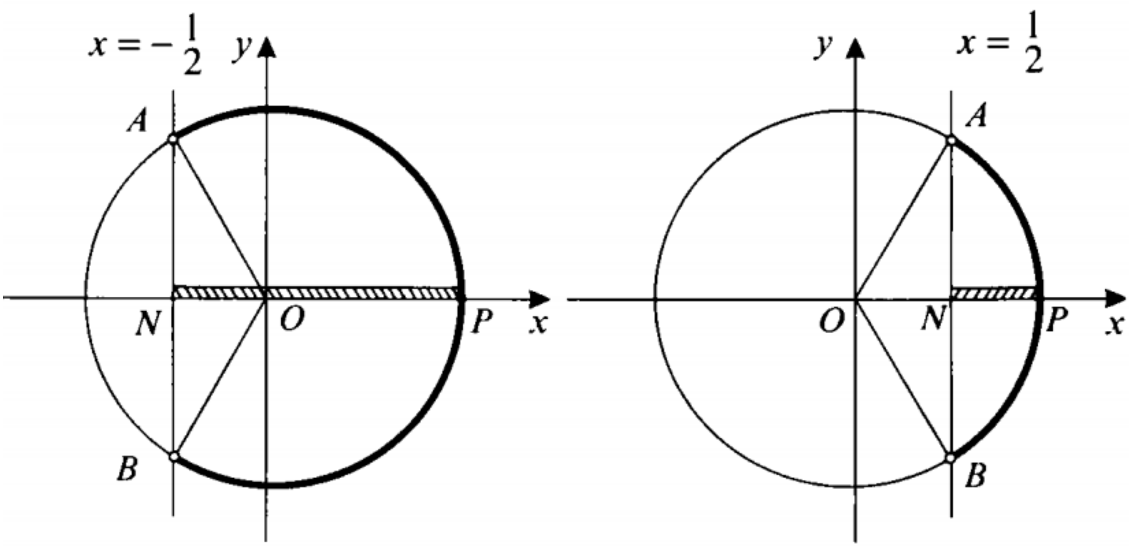
$2k\pi + \frac{\pi}{2} < x < \frac{3\pi}{2} + 2k\pi,\ k \in Z$ ekanligini yodda tuting.

5-misol. $\cos x > -\frac{1}{2}$ tengsizlikni yeching.

Yechilishi. Berilgan tengsizlikni (3) formuladan foydalanib yechamiz:

$$2k\pi - \arccos\left(-\frac{1}{2}\right) < x < 2k\pi + \arccos\left(-\frac{1}{2}\right) \Leftrightarrow 2k\pi - \left(\pi - \frac{\pi}{3}\right) <$$

$$< 2k\pi + \left(\pi - \frac{\pi}{3}\right) \Leftrightarrow 2k\pi - \frac{2\pi}{3} < x < 2k\pi + \frac{2\pi}{3},\ k \in Z.$$



Tengsizlikning yechimi 146-rasmda tasvirlangan.

Javob: $\left(2k\pi - \frac{2\pi}{3};\ 2k\pi + \frac{2\pi}{3}\right),\ k \in Z.$

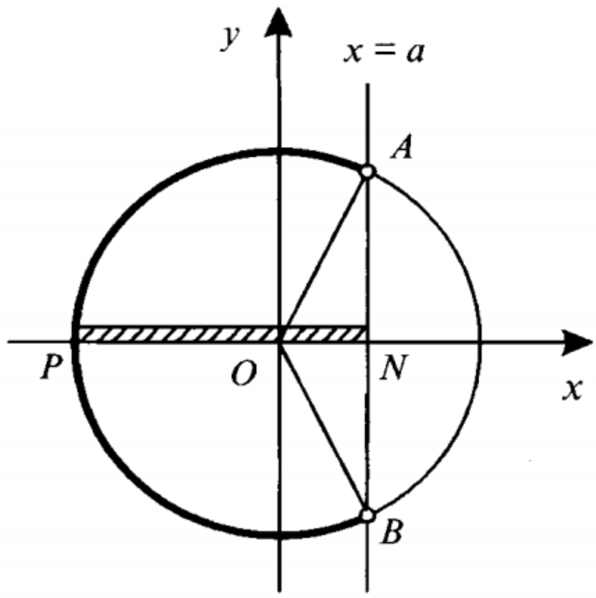
6-misol. $2\cos^2 x - 9\cos x + 4 < 0$ tengsizlikni yeching.

Yechilishi. $\cos x = y$ belgilash kiritamiz. U holda

$$2y^2 - 9y + 4 < 0 \Leftrightarrow 2\left(y - \frac{1}{2}\right)(y-4) < 0 \Rightarrow \frac{1}{2} < y < 4.$$

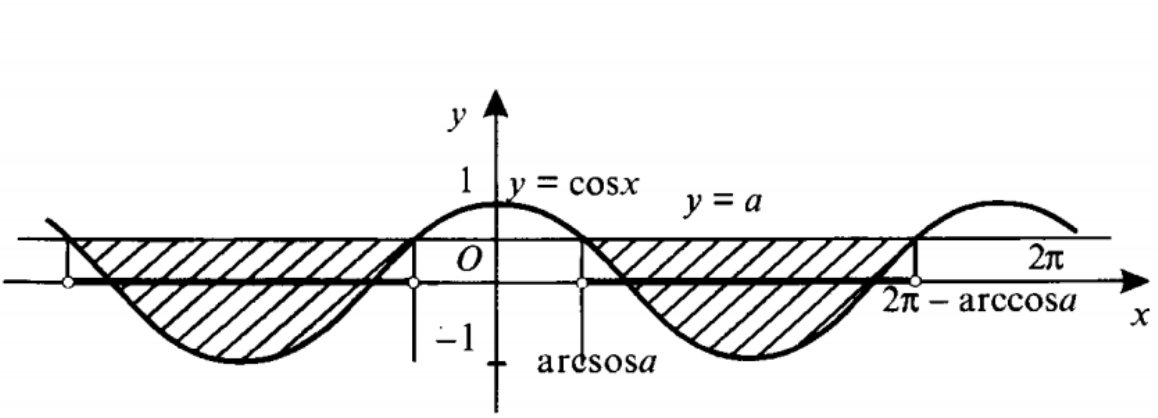
$|\cos x| \le 1$ bo‘lganligi uchun $\frac{1}{2} < \cos x \le 1$ tengsizlikka ega bo‘lamiz. Uning yechimlar to‘plami $2k\pi - \frac{\pi}{3} < x < \frac{\pi}{3} + 2k\pi,\ k \in Z$ (147-rasm).

Javob: $\left(2k\pi - \frac{\pi}{3};\ 2k\pi + \frac{\pi}{3}\right), k \in Z.$



148-rasm

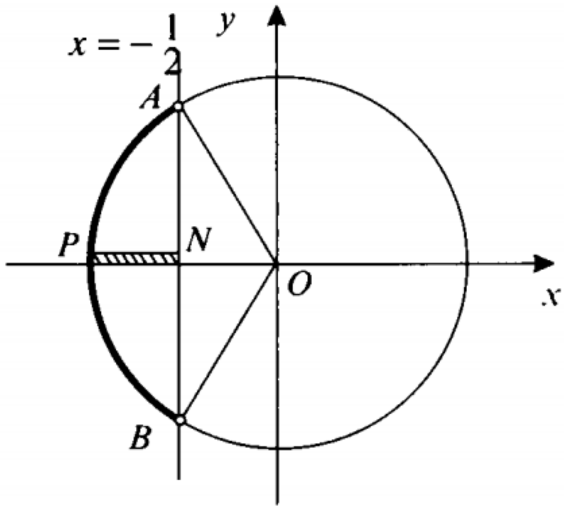
2. $\cos x < a$ ($|a|<1$) tengsizlikning yechilishi. 148, 149-rasmlardan ko‘rinib turibdiki, tengsizlikning [0; 2π] kesmadagi yechimi x ning ($\arccos a$; $2\pi - \arccos a$) oraliqdagi qiymatlaridan iborat. $\cos x$ ning davriyligini hisobga olib tengsizlikning butun sonlar o‘qidagi yechimlari to‘plamini yozamiz:



$$2k\pi + \arccos a < x < 2k\pi + 2\pi - \arccos a, k \in Z \qquad (4)$$

7-misol. $\cos 2x < -\frac{1}{2}$ tengsizlikni yeching.

Yechilishi. $2x$ ni α deb belgilasak, berilgan tengsizlik $\cos\alpha < -\frac{1}{2}$ ko'rinishni oladi. Bu tengsizlikni birlik aylananing abssissasi $-\frac{1}{2}$ dan kichik bo'lgan barcha nuqtalari qanoatlantiradi (150-rasm), shuning uchun $\cos\alpha < -\frac{1}{2}$ tengsizlikning $[0;\ 2\pi]$ kesmadagi yechimi



150-rasm

$$\arccos\left(-\frac{1}{2}\right) < \alpha < 2\pi - \arccos\left(-\frac{1}{2}\right) \Rightarrow \pi - \arccos\frac{1}{2} < \alpha <$$

$$< 2\pi - \left(\pi - \arccos\frac{1}{2}\right) \Rightarrow \pi - \frac{\pi}{3} < \alpha < 2\pi - \left(\pi - \frac{\pi}{3}\right) \Rightarrow \frac{2\pi}{3} < \alpha < \frac{4\pi}{3}$$

kabi topiladi. $\cos\alpha$ ni davriyligini hisobga olsak, $2k\pi + \frac{2\pi}{3} <$ $< \alpha < 2k\pi + \frac{4\pi}{3}$, $k \in Z$.

Endi x o'zgaruvchiga o'tib, berilgan tengsizlik yechimini yozamiz:

$$2k\pi + \frac{2\pi}{3} < 2x < 2k\pi + \frac{4\pi}{3} \Leftrightarrow k\pi + \frac{\pi}{3} < x < k\pi + \frac{2\pi}{3},\ k \in Z.$$

Javob: $\left(k\pi + \frac{\pi}{3};\ k\pi + \frac{2\pi}{3}\right)$, $k \in Z$.

8-misol. $7\cos^2 x - 5\cos x + \sin^2 x \le 0$ tengsizlikni yeching.

Yechilishi.

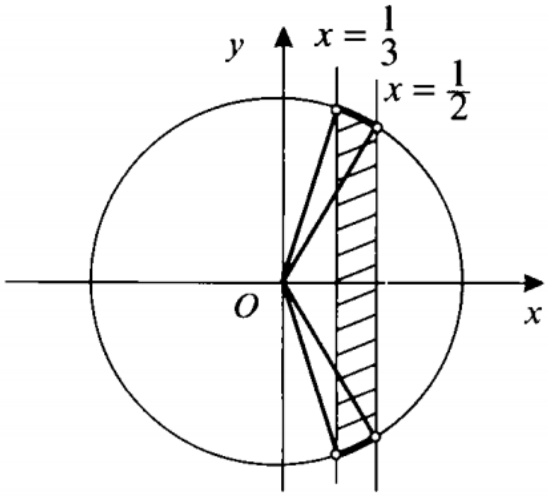
$$7\cos^2 x - 5\cos x + \sin^2 x \le 0 \Leftrightarrow 7\cos^2 x - 5\cos x + 1 - \cos^2 x \le 0 \Leftrightarrow$$

$$\Leftrightarrow 6\cos^2 x - 5\cos x + 1 \le 0$$

Berilgan tengsizlikka teng kuchli bo'lgan bu tengsizlikni yechish uchun $\cos x = y$ belgilash orqali yangi o'zgaruvchi kiritamiz. U holda

$$6y^2 - 5y + 1 \le 0 \Leftrightarrow 6\left(y - \frac{1}{3}\right)\left(y - \frac{1}{2}\right) \le 0 \Rightarrow \frac{1}{3} \le y \le \frac{1}{2}.$$

x o'zgaruvchiga o'tib, $\frac{1}{3} \le \cos x \le \frac{1}{2}$ tengsizlikka ega bo'lamiz.

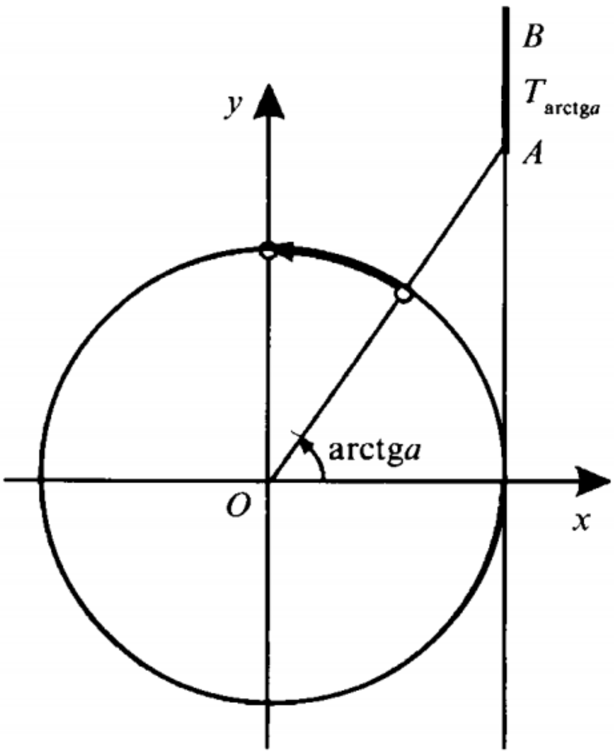


151-rasm

Bu tengsizlikni qanoatlantiruvchi nuqtalar esa $x = \frac{1}{3}$ to‘g‘ri chiziqdan o‘ngda, $x = \frac{1}{2}$ to‘g‘ri chiziqdan chapda yotadi (151-rasm). Birlik aylananing bu qismlariga mos keluvchi burchaklar oraliqlari berilgan tengsizlikning yechimlar to‘plamidan iborat. Shunday qilib,

$2k\pi + \frac{\pi}{3} \leq x \leq 2k\pi + \arccos\frac{1}{3}$, $k \in Z$ va $2k\pi - \arccos\frac{1}{3} \leq x \leq 2k\pi - \frac{\pi}{3}$, $k \in Z$ oraliqlar birlashmasi tengsizlikning yechimi bo‘ladi.

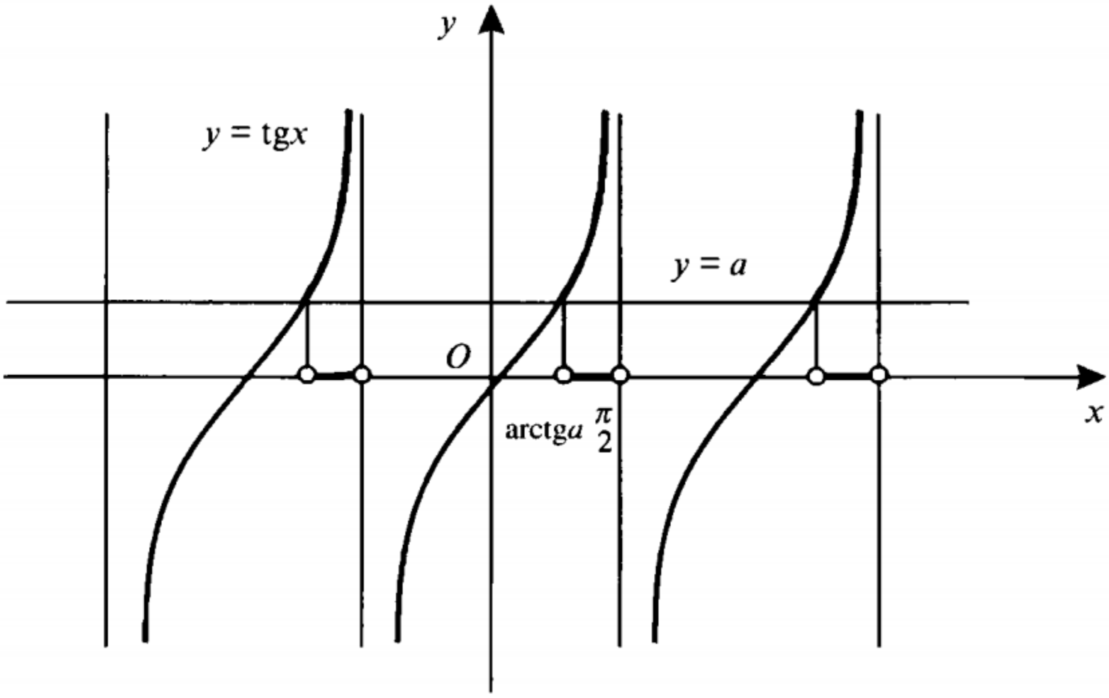
Javob: $\left[2k\pi + \frac{\pi}{3}; 2k\pi + \arccos\frac{1}{3}\right] \cup \left[2k\pi - \arccos\frac{1}{3}; 2k\pi - \frac{\pi}{3}\right]$, $k \in Z$.



152-rasm

4.3. tg$x > a$, tg$x < a$ **tengsizliklarni yechish.** Bu tengsizliklar a ning har qanday qiymatlarida yechimga ega bo‘lib, ularni yechishda ham birlik aylanadan yoki $y = \text{tg}x$, $y = \text{ctg}x$ funksiyalarning grafiklaridan foydalaniladi.

1. tg$x > a$ $(a \in R)$ tengsizlikning yechilishi. Birlik aylanani chizib, aylanaga (1; 0) nuqtada urinma bo‘lgan $T_{\text{arctg}a}$ — tangenslar chizig‘ini chizamiz (152-rasm). Tan-



153-rasm

genslar chizig'ida ordinatalari a dan katta bo'lgan barcha nuqtalar AB nurda yotibdi. Birlik aylananing bu nurga mos qismi 152-rasmda ajratib ko'rsatilgan. Birlik aylananing bu qismidagi har qanday nuqtada

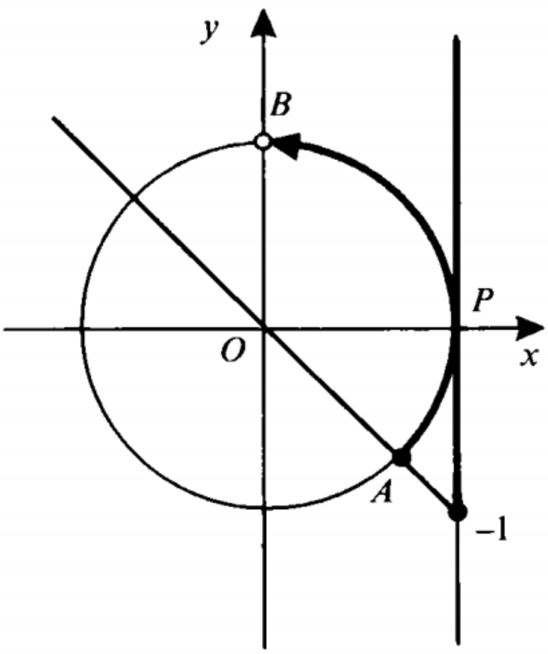
$$\text{arctg}a < x < \frac{\pi}{2}$$

tengsizlik bajariladi. 153-rasmda bu yechim $y = \text{tg}x$ funksiyaning grafigi orqali tasvirlangan, tengsizlikning butun sonlar o'qidagi yechimlari to'plamini yozamiz:

$$k\pi + \text{arctg}a < x < \frac{\pi}{2} + k\pi,\ k \in Z \qquad (5)$$

9-misol. $\text{tg}x \geq -1$ tengsizlikni yeching.

Yechilishi. Tengsizlikni yechishda birlik aylanadan foydalanamiz. 154-rasmdan ko'rinib turibdiki, tengsizlik $-\frac{\pi}{4} \leq x < \frac{\pi}{2}$ oraliqda bajarila-

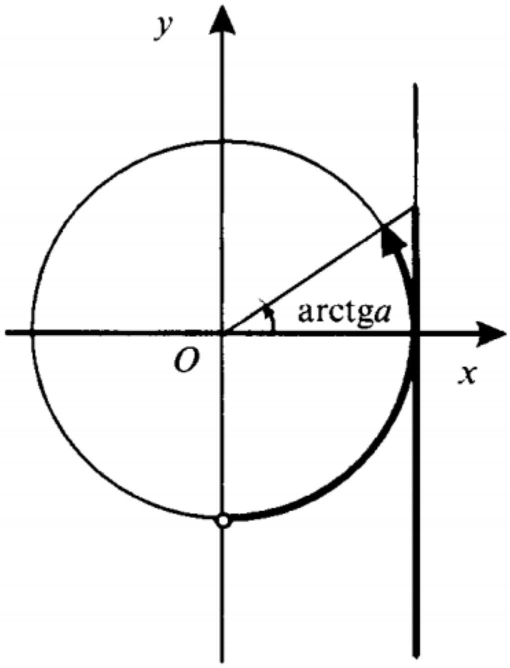


154-rasm

di. (tg$x$ funksiya $x = \frac{\pi}{2}$ da aniqlanmagan). Tengsizlikning davriyligidan foydalanib, butun sonlar o'qi uchun yechimlar to'plamini yozamiz:

$$k\pi - \frac{\pi}{4} \le x < \frac{\pi}{2} + k\pi, \ k \in Z.$$

Javob: $\left[k\pi - \frac{\pi}{4}; \frac{\pi}{2} + k\pi\right), \ k \in Z.$



155-rasm

2. tg$x < a$ ($a \in R$) **tengsizlikning yechilishi.** Bu tengsizlik 155-rasmda tasvirlangan birlik aylana va tangenslar chizig'ida ajratib ko'rsatilgan oraliqlarda bajariladi. Shu sababli tangensning davriyligini hisobga olib, tengsizlikning butun sonlar o'qidagi yechimlar to'plami

$$k\pi - \frac{\pi}{2} < x < \text{arctg}\,a + k\pi, \ k \in Z \quad (6)$$

formula bilan ifodalanadi.

tg$x > 0$ tengsizlikning yechimlar to'plami $k\pi < x < \frac{\pi}{2} + k\pi, \ k \in Z,$ tg$x < 0$ tengsizlikning yechimlar to'plami $k\pi - \frac{\pi}{2} < x < k\pi, \ k \in Z$ ekanligini yodda tuting.

10-misol. $\text{tg}\left(2x - \frac{\pi}{3}\right) \le \frac{1}{\sqrt{3}}$ tengsizlikni yeching.

Yechilishi. (6) formuladan foydalanamiz:

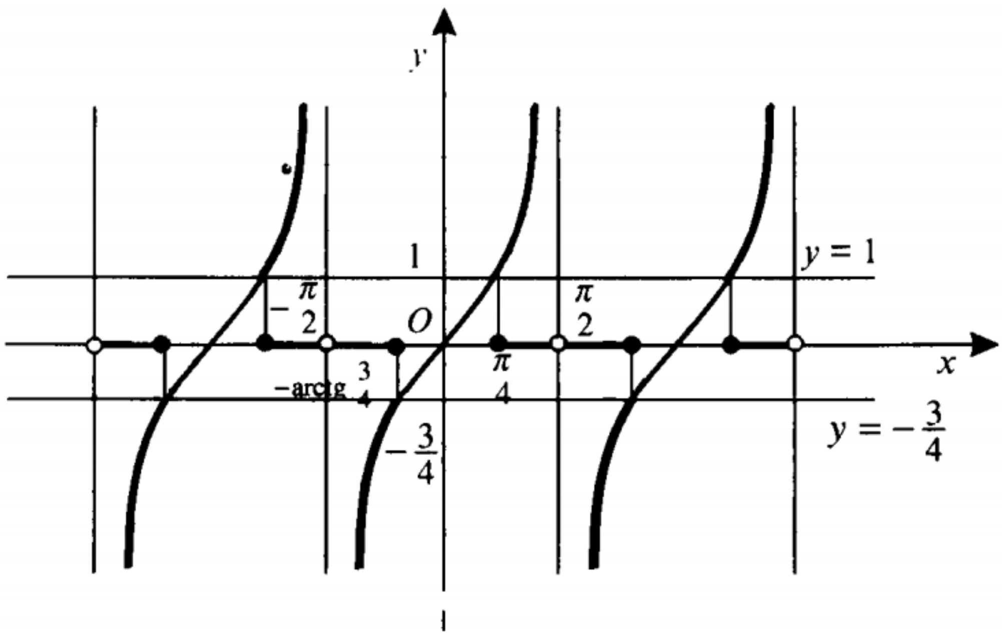
$$k\pi - \frac{\pi}{2} < 2x - \frac{\pi}{3} \le \text{arctg}\,\frac{1}{\sqrt{3}} + k\pi \Leftrightarrow k\pi - \frac{\pi}{2} + \frac{\pi}{3} < 2x \le \frac{\pi}{6} + \frac{\pi}{3} + k\pi \Leftrightarrow$$

$$\Leftrightarrow k\pi - \frac{\pi}{6} < 2x \le \frac{\pi}{2} + k\pi \Leftrightarrow \frac{k\pi}{2} - \frac{\pi}{12} < x \le \frac{\pi}{4} + \frac{k\pi}{2}, \ k \in Z.$$

Javob: $\left(\frac{k\pi}{2} - \frac{\pi}{12}; \frac{\pi}{4} + \frac{k\pi}{2}\right), \ k \in Z.$

11-misol. $4\text{tg}^2 x - \text{tg}x - 3 \ge 0 \ \left(x \ne \frac{\pi}{2} + k\pi\right)$ tengsizlikni yeching.

Yechilishi. tg $x = y$ belgilash orqali yangi o'zgaruvchi kiritamiz. U holda:



156-rasm

$$4y^2 - y - 3 > 0 \Leftrightarrow 4\left(y+\frac{3}{4}\right)(y-1) \geq 0.$$

Bu tengsizlik yechimlari to‘plamlarining birlashmasi $\left(-\infty; -\frac{3}{4}\right] \cup [1; +\infty)$ dan iborat. x o‘zgaruvchiga qaytib,

$$\left[\begin{array}{l} \text{tg}x \leq -\frac{3}{4}; \\ \text{tg}x \geq 1 \end{array}\right.$$

tengsizliklar sistemasiga ega bo‘lamiz. Bu sistemaning $\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ oraliq uchun yechimi $\left(-\frac{\pi}{2}; -\text{arctg}\,\frac{3}{4}\right]$ va $\left[\frac{\pi}{4}; \frac{\pi}{2}\right)$ oraliqlar birlashmasidan iborat (156-rasm). Tangensning davriyligini hisobga olib berilgan tengsizlikning butun sonlar o‘qidagi yechimlari to‘plamini yozamiz:

$$\left(k\pi - \frac{\pi}{2}; k\pi - \text{arctg}\,\frac{3}{4}\right] \cup \left[k\pi + \frac{\pi}{4}; \frac{\pi}{2} + k\pi\right),\ k \in Z.$$

Javob: $\left(k\pi - \frac{\pi}{2}; k\pi - \text{arctg}\,\frac{3}{4}\right] \cup \left[k\pi + \frac{\pi}{4}; \frac{\pi}{2} + k\pi\right),\ k \in Z.$

MUSTAQIL ISHNING DAVOMI…

25*. $\sin^3 x + \cos^3 x = 1$ tenglamani yeching.

A) $k\pi, k \in Z$; B) $\frac{\pi}{2} + 2k\pi, k \in Z$; C) 2π;

D) $2k\pi; \frac{\pi}{2} + 2k\pi, k \in Z$; E) $\pi + k\pi, k \in Z$.

26*. $\sin^4 x + \cos^4 x = \sin x \cos x$ tenglamani yeching.

A) $\frac{\pi}{4} + k\pi, k \in Z$; B) $\frac{\pi}{4} + 2k\pi, k \in Z$; C) $\frac{3\pi}{4} + k\pi, k \in Z$;

D) $\frac{3\pi}{2} + k\pi, k \in Z$; E) $\frac{\pi}{2} + 2k\pi, k \in Z$.

27*. $\sin^6 x + \cos^6 x = \frac{13}{14}(\sin^4 x + \cos^4 x)$ tenglamani yeching.

A) $\pm\frac{\pi}{6} + k\pi, k \in Z$; B) $\pm\frac{\pi}{12} + \frac{k\pi}{2}, k \in Z$;

C) $(-1)^k \frac{\pi}{6} + k\pi, k \in Z$; D) $\frac{\pi}{12} + k\pi, k \in Z$;

E) $\pm\frac{\pi}{8} + \frac{k\pi}{2}, k \in Z$.

28*. $81^{\sin^2 x} + 81^{\cos^2 x} = 30$ tenglamani yeching.

A) $\pm\frac{\pi}{3} + k\pi, k \in Z$; B) $\frac{\pi}{6} + k\pi, k \in Z$; C) $\frac{\pi}{6} + 2k\pi, k \in Z$;

D) $\pm\frac{\pi}{6} + \frac{k\pi}{2}, k \in Z$; E) $\pm\frac{\pi}{12} + \frac{k\pi}{2}, k \in Z$.

29*. $\frac{\cos 4x + 1}{\text{ctg}x - \text{tg}x} = \frac{1}{2}\cos^4 2x - 8\sin^4 x \cos^4 x$ tenglamani yeching.

A) $\frac{\pi}{16} + \frac{k\pi}{4}, k \in Z$; B) $\frac{3\pi}{4} + k\pi, k \in Z$; C) $\frac{\pi}{12} + \frac{k\pi}{4}, k \in Z$;

D) $\pm\frac{\pi}{12} + \frac{k\pi}{4}, k \in Z$; E) $\pm\frac{\pi}{6} + 2k\pi, k \in Z$.

30*. $\frac{1}{1-\mathrm{tg}2x}+\frac{2\cos^2 x-1}{\cos 2x+\sin 2x}=\frac{2\sqrt{3}\mathrm{tg}2x}{1-\mathrm{tg}^2 2x}$ tenglamani yeching.

A) $\frac{\pi}{6}+k\pi, k\in Z$; B) $\frac{\pi}{6}+\frac{k\pi}{2}, k\in Z$; C) $\frac{\pi}{12}+k\pi, k\in Z$;

D) $\frac{\pi}{3}+k\pi, k\in Z$; E) $\frac{\pi}{12}+\frac{k\pi}{2}, k\in Z$.

31*. $\begin{cases}\cos x+\cos y=\sqrt{3},\\ x+y=\frac{\pi}{3}\end{cases}$ sistemani yeching.

A) $\left(\frac{\pi}{6}+2k\pi; \frac{\pi}{6}-2k\pi\right), k\in Z$; B) $\left(\frac{\pi}{3}+2k\pi; \frac{\pi}{3}-2k\pi\right), k\in Z$;

C) $\left(\frac{\pi}{6}+k\pi; \frac{\pi}{6}-k\pi\right), k\in Z$; D) $\left(\frac{\pi}{3}+k\pi; \frac{\pi}{3}+k\pi\right), k\in Z$;

E) $\left(\pm\frac{\pi}{6}+2k\pi; \pm\frac{\pi}{6}+2k\pi\right), k\in Z$.

32*. $\cos x\cos 2x\cos 4x\cos 8x=\frac{1}{8}\cos 15x$ tenglamani yeching.

A) $\frac{\pi}{12}+k\pi, k\in Z$; B) $\frac{k\pi}{14}, k\in Z$; C) $\frac{k\pi}{30}, k\in Z$;

D) $\frac{k\pi}{16}, k\in Z$; E) $\frac{k\pi}{12}, k\in Z$.

33*. $\mathrm{tg}x+\mathrm{tg}2x-\mathrm{tg}3x=0$ tenglamaning $\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ oraliqdag chimlari yig‘indisini toping.

A) 0; B) $\frac{\pi}{6}$; C) $\frac{\pi}{4}$; D) $\frac{5\pi}{12}$; E) $-\frac{\pi}{6}$.

34. $9^{\cos x}+2\cdot 3^{\cos x}=15$ tenglamani yeching.

A) $2k\pi, k\in Z$; B) 0; C) 2π; D) $\pi+2k\pi, k\in Z$;

E) $\frac{\pi}{2}+2k\pi, k\in Z$.

35. $\sqrt{1-\cos x}=\sin x$ tenglamani yeching.

A) $\frac{\pi}{2}+k\pi; 2k\pi,\ k\in Z$; B) $\frac{\pi}{2}+2k\pi,\ k\in Z$;

C) $\frac{\pi}{2}+2k\pi; 2k\pi,\ k\in Z$; D) $2k\pi, k\in Z$; E) $-\pi+2k\pi, k\in Z$.

36*. $\cos^{10}x+\sin^{15}x=1$ tenglamani yeching.

A) $k\pi; \frac{\pi}{2}+k\pi, k\in Z$; B) $2k\pi, k\in Z$; C) $2k\pi; \frac{\pi}{2}+2k\pi, k\in Z$;

D) $\pi+2k\pi; \frac{\pi}{2}+k\pi,\ k\in Z$; E) $k\pi; \frac{\pi}{2}+2k\pi,\ k\in Z$.

37*. $\frac{\cos 3x}{\sin 3x - 2\sin x} = \operatorname{tg} x$ tenglamani yeching.

A) $\frac{\pi}{4} + \frac{k\pi}{2},\ k \in Z$; B) $\frac{\pi}{4} + 2k\pi,\ k \in Z$; C) $\frac{\pi}{4} + k\pi,\ k \in Z$;

D) $\frac{\pi}{3} + \frac{k\pi}{2},\ k \in Z$; E) $\frac{\pi}{3} + k\pi,\ k \in Z$.

38*. $\sqrt{1 - \sin 2x} = \sin 3x + \cos 3x$ tenglama $\left[\frac{3\pi}{2}; 2\pi\right]$ oraliqda nechta ildizga ega?

A) 4; B) 3; C) 2; D) 1; E) ko'rsatilgan oraliqda ildizi yo'q.

39*. $\log_3(-\cos x) - \log_9 \sin x + \frac{1}{4} = -\log_9 2$ tenglamani yeching.

A) $(-1)^k \frac{\pi}{6} + k\pi,\ k \in Z$; B) $(-1)^k \frac{\pi}{3} + k\pi,\ k \in Z$;

C) $\frac{5\pi}{6} + 2k\pi,\ k \in Z$; D) $\frac{\pi}{3} + 2k\pi,\ k \in Z$; E) $\frac{2\pi}{3} + 2k\pi,\ k \in Z$.

40*. $5\sin 3x - 6\cos 3x = a$ tenglama a ning qanday qiymatlarida yechimga ega?

A) $-1 \le a \le 1$; B) $-\frac{5}{6} \le a \le \frac{5}{6}$; C) $-\sqrt{11} \le a \le \sqrt{11}$;

D) $-\sqrt{61} \le a \le \sqrt{61}$; E) $-\sqrt{14} \le a \le \sqrt{14}$.

41. $3\arccos(2x + 3) = \frac{5\pi}{2}$ tenglamani yeching.

A) $\frac{\sqrt{3}}{2}$; B) $-\frac{\sqrt{3}}{2}$; C) $-\frac{6+\sqrt{3}}{4}$; D) $-\frac{6-\sqrt{3}}{4}$; E) $-1{,}25$.

42*. $\arcsin x \cdot \arccos x = \frac{\pi^2}{18}$ tenglamani yeching.

A) $\frac{\sqrt{3}}{2}; \frac{\sqrt{2}}{2}$; B) $-\frac{\sqrt{3}}{2}; \frac{1}{2}$; C) $\frac{\sqrt{3}}{2}; \frac{1}{2}$; D) $\frac{1}{2}; -\frac{1}{2}$; E) $\frac{\sqrt{2}}{2}; \frac{1}{2}$.

43. $\operatorname{arctg}(1 - x) + \operatorname{arctg}(1 + x) = \frac{\pi}{4}$ tenglamani yeching.

A) ± 2; B) $\pm\sqrt{2}$; C) $\pm\sqrt{3}$; D) $\sqrt{2}$; $\sqrt{3}$; E) 1; $\sqrt{2}$.

44. $2(\arcsin x)^2 + \pi^2 = 3\pi \arcsin x$ tenglamani yeching.

A) 1; B) 0; C) $\frac{\sqrt{3}}{2}$; D) $\frac{1}{2}$; E) $\frac{\sqrt{2}}{2}$.

45*. $2\arcsin 2x = \arcsin 7x$ tenglama nechta ildizga ega?

A) 1; B) 2; C) 3; D) 4; E) $\varnothing$.

46. $\sqrt{2} \sin\left(\frac{\pi}{2} - 2x\right) > 1$ tengsizlikni yeching.

A) $\left(2k\pi-\frac{\pi}{4};2k\pi+\frac{\pi}{4}\right); k\in Z;$ B) $\left(k\pi-\frac{\pi}{4};k\pi+\frac{\pi}{4}\right); k\in Z;$

C) $\left(2k\pi-\frac{\pi}{8};2k\pi+\frac{\pi}{8}\right); k\in Z;$ D) $\left(k\pi-\frac{\pi}{8};k\pi+\frac{\pi}{8}\right); k\in Z;$

E) $\left(k\pi+\frac{\pi}{8};k\pi+\frac{3\pi}{8}\right); k\in Z.$

47. $2\cos\left(\frac{3\pi}{2}+3x\right)\le-\sqrt{2}$ tengsizlikni yeching.

A) $\left[\frac{2k\pi}{3}-\frac{5\pi}{12};\frac{\pi}{12}+\frac{2k\pi}{3}\right], k\in Z;$ B) $\left[\frac{2k\pi}{3}-\frac{\pi}{4};\frac{2k\pi}{3}-\frac{\pi}{12}\right], k\in Z;$

C) $\left[\frac{k\pi}{3}-\frac{5\pi}{12};\frac{\pi}{12}+\frac{k\pi}{3}\right], k\in Z;$ D) $\left[\frac{k\pi}{3}-\frac{\pi}{4};\frac{k\pi}{3}-\frac{\pi}{12}\right], k\in Z;$

E) $\left[\frac{2k\pi}{3}+\frac{\pi}{12};\frac{3\pi}{12}+\frac{2k\pi}{3}\right], k\in Z.$

48. $\operatorname{tg}\left(\pi+\frac{x}{3}\right)+1\ge0$ tengsizlikni yeching.

A) $\left[k\pi-\frac{\pi}{4};\frac{\pi}{2}+k\pi\right), k\in Z;$ B) $\left[k\pi+\frac{\pi}{4};\frac{\pi}{2}+k\pi\right), k\in Z;$

C) $\left[3k\pi-\frac{3\pi}{4};\frac{3\pi}{2}+3k\pi\right), k\in Z;$ D) $\left[k\pi-\frac{3\pi}{4};\frac{3\pi}{2}+k\pi\right), k\in Z;$

E) $\left(3k\pi-\frac{3\pi}{2};\frac{3\pi}{2}+3k\pi\right), k\in Z.$

49. $\operatorname{ctg}\left(\frac{3\pi}{2}-\frac{x}{2}\right)\le\sqrt{3}$ tengsizlikni yeching.

A) $\left(2k\pi-\pi;\frac{2\pi}{3}+2k\pi\right], k\in Z;$ B) $\left(k\pi-\frac{\pi}{2};\frac{\pi}{3}+k\pi\right], k\in Z;$

C) $\left(2k\pi-\pi;\frac{\pi}{3}+2k\pi\right], k\in Z;$ D) $\left(k\pi-\frac{\pi}{2};\frac{\pi}{6}+k\pi\right], k\in Z;$

E) $\left(-\pi;\frac{2\pi}{3}\right].$

50. $\frac{1}{2}<\sin x\le\frac{\sqrt{2}}{2}$ tengsizlikning $[0; 2\pi]$ oraliqdagi yechimlarini toping.

A) $\left(\frac{\pi}{6};\frac{\pi}{4}\right]\cup\left[\frac{3\pi}{4};\frac{5\pi}{6}\right);$ B) $\left(0;\frac{\pi}{4}\right]\cup\left[\frac{3\pi}{4};2\pi\right);$ C) $\left(\frac{\pi}{6};\frac{5\pi}{6}\right);$

D) $\left(\frac{\pi}{6}+2k\pi;\frac{\pi}{4}+2k\pi\right], k\in Z;$

E) $\left(\frac{\pi}{6}+2k\pi;\frac{\pi}{4}+2k\pi\right]\cup\left[\frac{3\pi}{4}+2k\pi;\frac{5\pi}{6}+2k\pi\right),\ k\in Z.$

51. $-\frac{\sqrt{3}}{2} \le \cos x < \frac{2}{3}$ tengsizlikni yeching.

A) $\left[2k\pi - \frac{5\pi}{6}; 2k\pi - \arccos\frac{2}{3}\right), k \in Z;$

B) $\left[2k\pi - \frac{5\pi}{6}; 2k\pi - \arccos\frac{2}{3}\right) \cup \left(\arccos\frac{2}{3} + 2k\pi; \frac{5\pi}{6} + 2k\pi\right], k \in Z;$

C) $\left(2k\pi - \frac{5\pi}{6}; 2k\pi - \arccos\frac{2}{3}\right], k \in Z;$

D) $\left(2k\pi - \frac{5\pi}{6}; 2k\pi - \arccos\frac{2}{3}\right) \cup \left[\arccos\frac{2}{3} + 2k\pi; \frac{5\pi}{6} + 2k\pi\right), k \in Z;$

E) $\left(\arccos\frac{2}{3} + 2k\pi; \frac{5\pi}{6} + 2k\pi\right), k \in Z.$

52. $2\text{tg}^2 2x - 1 > 0$ tengsizlikning $\left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ oraliqdagi yechimla-ni toping.

A) $\left(k\pi - \frac{\pi}{2};\ k\pi - \frac{1}{2}\text{arctg}\ \frac{1}{\sqrt{2}}\right), k \in Z;$

B) $\left(k\pi + \frac{\pi}{2};\ k\pi + \text{arctg}\ \frac{1}{\sqrt{2}}\right), k \in Z;$

C) $\left(\frac{k\pi}{2} - \frac{\pi}{4}; \frac{k\pi}{2} - \frac{1}{2}\text{arctg}\frac{1}{\sqrt{2}}\right) \cup \left(\frac{k\pi}{2} + \frac{1}{2}\text{arctg}\frac{1}{\sqrt{2}}; \frac{k\pi}{2} + \frac{\pi}{4}\right), k \in Z;$

D) $\left(-\frac{\pi}{4}; -\frac{1}{2}\text{arctg}\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{2}\text{arctg}\frac{1}{\sqrt{2}}; \frac{\pi}{4}\right), k \in Z;$

E) $\left(-\frac{1}{2}\text{arctg}\ \frac{1}{\sqrt{2}}; \frac{1}{2}\text{arctg}\ \frac{1}{\sqrt{2}}\right).$

53. $2\sin^2 x - 5\sin x + 2 < 0$ tengsizlikning $[0; 2\pi]$ oraliqdagi yechim-ri to‘plamini toping.

A) $\left(\frac{\pi}{6}; \frac{5\pi}{6}\right)$; B) $\left[0; \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}; 2\pi\right]$; C) $\left[0; \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}; 2\pi\right]$;

D) $\left[0; \frac{\pi}{3}\right) \cup \left(\frac{2\pi}{3}; 2\pi\right]$; E) $\varnothing$.

54*. $y = \sqrt{1 - 2\sin^2 x} + \sqrt{\sin 2x}$ funksiyaning aniqlanish sohasi-i toping.

A) $\left[k\pi - \frac{\pi}{4}; k\pi + \frac{\pi}{4}\right], k \in Z$; B) $\left[k\pi; \frac{\pi}{2}\right], k \in Z$;

C) $\left[k\pi; \frac{\pi}{4} + k\pi\right], k \in Z$; D) $\left[0; \frac{\pi}{2}\right]$; E) $\left[2k\pi; \frac{\pi}{2} + 2k\pi\right], k \in Z.$

55. $f(x)=\sqrt{\cos\left(x-\frac{\pi}{4}\right)}$ funksiyaning aniqlanish sohasini toping.

A) $\left[-\frac{\pi}{4};\frac{\pi}{4}\right]$; B) $[0;\pi]$; C) $\left[0;\frac{\pi}{2}\right]$;

D) $\left[-\frac{\pi}{4}+2k\pi;\frac{3\pi}{4}+2k\pi\right]\ k\in Z$; E) $\left[-\frac{\pi}{4}+2k\pi;\frac{\pi}{4}+2k\pi\right]\ k\in Z$.

56. $y=3\sqrt{\sin 2x}-2\sqrt{\mathrm{ctg}2x}$ funksiyaning aniqlanish sohasini toping.

A) $\left(k\pi;\frac{\pi}{4}+k\pi\right],k\in Z$; B) $\left(k\pi;\frac{\pi}{2}+k\pi\right),k\in Z$;

C) $\left(\frac{\pi}{2}+k\pi;\pi+k\pi\right),k\in Z$; D) $(0;\pi)$; E) $\left(\frac{3\pi}{2}+k\pi;2k\pi\right),k\in Z$.

57*. $y=\sqrt{1+\log_{\frac{1}{2}}\cos x}$ funksiya x ning qanday qiymatlarida aniqlangan ($x\in[0;2\pi]$)?

A) $\left[0;\frac{\pi}{3}\right]\cup\left[\frac{5\pi}{3};2\pi\right]$; B) $\left[0;\frac{\pi}{2}\right)\cup\left(\frac{3\pi}{2};2\pi\right]$; D) $[0;\pi]$;

E) $\left[0;\frac{\pi}{4}\right]\cup\left[\frac{7\pi}{4};2\pi\right]$.

58*. $\cos x<\sin x$ tengsizlikni yeching.

A) $\left(\frac{\pi}{4}+2k\pi;\frac{5\pi}{4}+2k\pi\right),k\in Z$; B) $\left(\frac{\pi}{4}+k\pi;\frac{3\pi}{4}+k\pi\right),k\in Z$;

C) $\left(\frac{\pi}{4}+2k\pi;\frac{3\pi}{4}+2k\pi\right),k\in Z$; D) $\left(\frac{\pi}{4}+k\pi;\frac{5\pi}{4}+k\pi\right),k\in Z$;

E) $(2k\pi;\pi+2k\pi),k\in Z$.

59. $\left(\frac{\pi}{6}-\frac{e}{6}\right)^{\ln(2\cos x)}\geq 1$ tengsizlikni yeching ($x\in[0;2\pi]$).

A) $\left[\frac{\pi}{3};\frac{\pi}{2}\right)\cup\left(\frac{3\pi}{2};\frac{5\pi}{3}\right]$; B) $\left[\frac{\pi}{3};\frac{\pi}{2}\right]\cup\left[\frac{3\pi}{2};\frac{5\pi}{3}\right]$; C) $\left[\frac{3\pi}{2};\frac{5\pi}{3}\right]$;

D) $\left[\frac{\pi}{3};\frac{\pi}{2}\right)$; E) $\left[\frac{\pi}{6};\frac{\pi}{2}\right)\cup\left(\frac{3\pi}{2};\frac{5\pi}{6}\right]$.

60*. $y=\arccos(2\sin x)$ funksiyaning aniqlanish sohasiga tegishli bo‘lgan x ning $[-\pi;\pi]$ kesmadagi barcha qiymatlarini aniqlang.

A) $\left[-\pi; -\frac{2\pi}{3}\right]\cup\left[-\frac{\pi}{3}; \frac{\pi}{3}\right]\cup\left[\frac{2\pi}{3}; \pi\right]$; B) $\left[-\frac{\pi}{3}; \frac{\pi}{3}\right]$;

C) $\left[-\frac{\pi}{4}; \frac{\pi}{4}\right]$; D) $\left[-\frac{\pi}{6}; \frac{\pi}{6}\right]$; E) $\left[-\pi; -\frac{5\pi}{6}\right]\cup\left[-\frac{\pi}{6}; \frac{\pi}{6}\right]\cup\left[\frac{5\pi}{6}; \pi\right]$.

61*. $\frac{2\sin 3x-4}{2x-3x^2+1} < 0$ tengsizlik nechta butun yechimga ega?

A) cheksiz ko‘p; B) 4; C) 3; D) 2; E) 1.

62*. $\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x > 0$ tengsizlikni yeching.

A) $(2k\pi; \pi + 2k\pi)$, $k \in Z$; B) $\left(k\pi; \frac{\pi}{2} + k\pi\right)$, $k \in Z$;

C) $\left(\frac{2k\pi}{3}; \frac{\pi}{3} + \frac{2k\pi}{3}\right)$, $k \in Z$; D) $\left(2k\pi; \frac{\pi}{2} + 2k\pi\right)$, $k \in Z$;

E) $\left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right)$, $k \in Z$.

63*. $\arcsin x < \arcsin(1 - x)$ tengsizlikni yeching.

A) $\left(-\infty; \frac{1}{2}\right)$; B) $(0; 2)$; C) $\varnothing$; D) $[-1; 1]$; E) $\left[0; \frac{1}{2}\right)$.

64*. $\arcsin x > \arccos x$ tengsizlikni yeching.

A) $\left[0; \frac{\sqrt{2}}{2}\right)$; B) $[0; 1]$; C) $\left[\frac{\sqrt{2}}{2}; 1\right]$; D) $\left(\frac{\sqrt{2}}{2}; 1\right)$; E) $\varnothing$.

65*. $x^2 - 4x + \arccos(x^2 - 4x + 5) < 0$ tengsizlikni yeching.

A) $\{2\}$; B) $[-1; 1]$; C) $[-2; 2]$; D) $(-2; 0)$; E) $(0; 2)$.

11-Mavzu: Trigonometriyani planimetrik masalalarni yechishga tatbiqlari.

2.1.1-ta'rif. Bir to'g'ri chiziqda yotmagan uchta A, B, C nuqtadan va bu nuqtalarni ikkitalab tutashtiruvchi uchta kesmadan iborat figuraga uchburchak deyiladi va ΔABC kabi belgilanadi. A, B, C nuqtalar uchburchakning uchlari, AB, BC, CA kesmalar uning tomonlari deyiladi (2.1.1-chizma). $\angle CAB$, $\angle CBA$, $\angle ACB$ burchak lar ABC uchburchakning ichki burchaklari deyiladi, ular ba'zan bitta harf orqali ham belgilanadi: $\angle A$, $\angle B$, $\angle C$. Uchburchakning AC tomonini C nuqtadan o'ngga davom ettiramiz. Natijada hosil qilingan $\angle BCD$ burchak ABC uchburchakning tashqi burchagi deyiladi. Tomonlariga ko'ra uchburchaklar uch turga: teng yonli, teng tomonli yoki muntazam, turli tomonli uchburchaklarga bo'linadi.

2.1.2-ta'rif. Ikki tomoni bir-biriga teng bo'lgan uchburchak teng yonli uchburchak deyiladi.

2.1.3-ta'rif. Uchta tomoni o'zaro teng bo'lgan uchburchak teng tomonli yoki muntazam uchburchak deyiladi.

2.1.4-ta'rif. Tomonlari har xil uzunliklarga ega bo'lgan uchburchak turli tomonli uchburchak deyiladi.

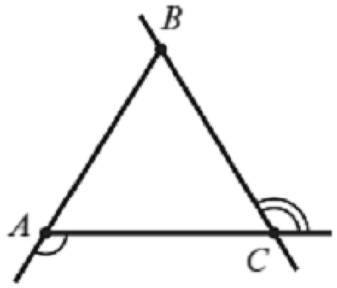
Burchaklariga ko'ra uchburchaklar uch xil bo'ladi.

2.1.5-ta'rif. Barcha ichki burchaklari o'tkir bo'lgan uchburchak o'tkir burchakli uchburchak deyiladi.

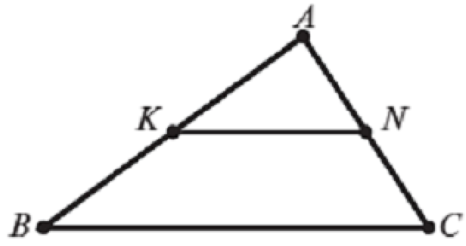
2.1.6-ta'rif. Bitta ichki burchagi o'tmas bo'lgan uchburchak o'tmas burchakli uchburchak deyiladi.

2.1.7-ta'rif. Bitta ichki burchagi $90°$ ga teng bo'lgan uchburchak to'g'ri burchakli uchburchak deyiladi.

2.1.8-ta'rif. ABC uchburchakning AB va AC tomonlari o‘rtalari Kva N nuqtalarni tutashtiruvchi kesma uchburchakning o‘rta chizig‘i deyiladi (2.1.2-chizma).



2.1.1-chizma.Uchburchak



2.1.2-chizma.Uchburchak o’rta chizig’i

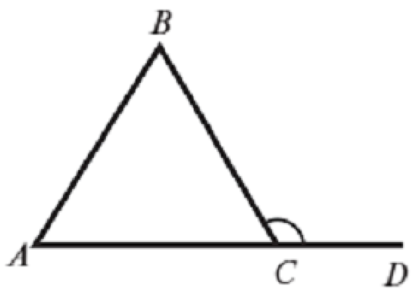
2.1.1-teorema. Uchburchakning o‘rta chizig‘i uning asosiga parallel va asosi uzunligining yarmiga teng: $BC \parallel KN$ va $BC = 2 \cdot KN$.

2.1.2-teorema. Uchburchak ichki burchaklarining yig’indisi $180°$ ga teng.

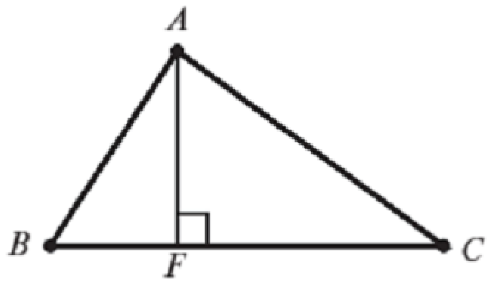
2.1.3-teorema. Uchburchakning tashqi burchagi unga qo‘shni bo‘lmagan ichki burchaklar yig‘indisiga teng (2.1.3-chizma):

$$\angle BCD = \angle BAC + \angle ABC.$$

ABC uchburchakning A uchidan BC to‘g‘ri chiziqqa perpendikulyar tushiramiz va F ularning kesishish nuqtasi bo’lsin. U vaqtda AF kesma uchburchakning balandligi deyiladi (2.1.4-chizma). Uchburchakda uchta balandlik o’tkazish mumkin.



2.1.3-chizma.Tashqi burchak.



2.1.4-chizma.Uchburchak balandligi

Berilgan ΔABC ning tomonlari $AB = c$, $BC = a$, $AC = b$ bo‘lsin (2.1.5-chizma). Unda $AK \perp BC$ balandlik o‘tkazamiz. Agar $\angle B = 90°$ bo‘lsa, to‘g‘ri burchakli ABK va ACK uchburchaklardan $b^2 = AK^2 + KC^2$, $AK^2 = c^2 - BK^2$ ifodalarni topamiz. Ulardan $b^2 = c^2 - BK^2 + (a - BK)^2 = c^2 - BK^2 + a^2 - 2a \cdot BK + BK^2$,

$b^2 = a^2 + c^2 - 2a \cdot BK$ bo’ladi. Bundan, $BK = \frac{a^2+c^2-b^2}{2a}$ kelib chiqadi. Olingan ifodani AK uchun yuqorida olingan ifodaga keltirib qo’yamiz:

$$AK^2 = c^2 - BK^2 = c^2 - \left(\frac{a^2+c^2-b^2}{2a}\right)^2 = \left(c - \frac{a^2+c^2-b^2}{2a}\right) \cdot \left(c + \frac{a^2+c^2-b^2}{2a}\right) = \left(\frac{2ac-a^2-c^2+b^2}{2a}\right)\left(\frac{2ac+a^2+c^2-b^2}{2a}\right) = \frac{(b^2-(a-c)^2)((a+c)^2-b^2)}{4a^2}$$

. Bundan,

$AK^2 = \frac{(b-a+c)(b+a-c)(a+c-b)(a+c+b)}{4a^2}$ bo’ladi. $a + b + c = p$ deb belgilab, qolgan ko’paytuvchilarni p orqali ifodalaymiz: $b + c - a = a + b + c - 2a = 2p - 2a$, $a + c - b = 2(p - b), a + b - c = 2(p - c)$. Natijada AK balandlik uchun

$AK^2 = \frac{16p(p-a)(p-b)(p-c)}{4a^2}$ va $AK = h_a = \frac{2}{a}\sqrt{p(p-a)(p-b)(p-c)}$ ifodani olamiz. Qolgan h_b va h_c balandliklar uchun ham, yuqoridagiga o’xshash,

$$h_b = \frac{2}{b}\sqrt{p(p-a)(p-b)(p-c)}$$

$$h_c = \frac{2}{c}\sqrt{p(p-a)(p-b)(p-c)}$$

formulalarni hosil qilamiz.

$$h_a : h_b : h_c = \frac{1}{a} : \frac{1}{b} : \frac{1}{c} = bc : ac : ab$$

Misol: $\frac{1}{r} = \frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c}$ ni isbot qiling.

$$\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} = \frac{1}{2R}\left(\frac{1}{sinBsinC} + \frac{1}{sinCsinA} + \frac{1}{sinAsinB}\right) = \frac{sinA + sinB + sinC}{2RsinAsinBsinC}$$

$$= \frac{4cos\frac{A}{2}cos\frac{B}{2}cos\frac{C}{2}}{16Rsin\frac{A}{2}sin\frac{B}{2}sin\frac{C}{2}cos\frac{A}{2}cos\frac{B}{2}cos\frac{C}{2}} = \frac{1}{4Rsin\frac{A}{2}sin\frac{B}{2}sin\frac{C}{2}}$$

2.1.4-teorema. Uchburchakning yuzi uning ikki tomoni bilan shu tomonlar orasidagi burchak sinusi ko'paytmasining yarmiga teng.

$$S = \frac{1}{2}absinC = \frac{1}{2}bcsinA = \frac{1}{2}acsinB$$

Isbot. h_b b tomoniga tushirilgan balandlik bo'lsin, unda $h_b = asinC$ bo'lganidan

$$S = \frac{1}{2}bh_b = \frac{1}{2}absinC$$

kelib chiqadi. Shu yo'l bilan S uchun boshqa ikki xil ifodani ham hosil qilamiz. shu ikki o'lchovli elementga ega bo'lganidan $\sqrt{S}$ chiziqli (bir o'lchovli) elementga ega bo'ladi. Bu chiziqli elementga tegishli burchak elementli formula

$$S = \sqrt{\frac{1}{2}sinAsinBsinC}$$

dan iboratdir.

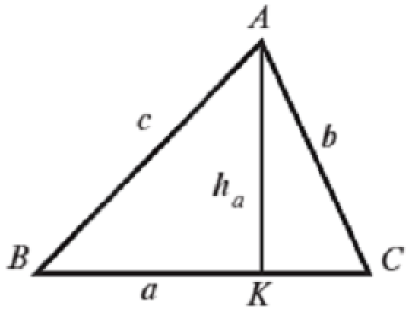
Uchburchakning berilgan uchidan chiqib qarama-qarshi tomonining o'rtasini tutashtiruvchi kesmaga mediana deyiladi. ΔABC da AD mediana va AK balandlik o'tkazilgan bo'lsin (2.1.6-chizma). Kesmalar uzunliklari uchun quyidagi belgilashlarni kiritamiz: $AB = c$, $AC = b$, $BC = a$, $AD = m_a$ AD mediana bo'lganligidan. Endi ΔABC uchun Stuart teoremasini yozamiz:

$$AC^2 \cdot BD + AB^2 \cdot DC = AD^2 \cdot BC + DC \cdot BD \cdot BC$$

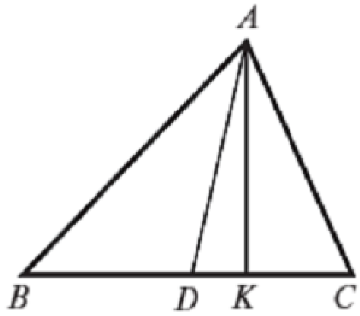
yoki $b^2 \cdot \frac{a}{2} + c^2 \cdot \frac{a}{2} = m_a^2 \cdot a + \frac{a^2}{4} \cdot a$. Bu tenglikning ikkala tomonini a ga qisqartiramiz: $\frac{b^2+c^2}{2} = m_a^2 + \frac{a^2}{4}$. Bundan, $m_a^2 = \frac{b^2+c^2}{2} - \frac{a^2}{4}$. $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$ ifodani olamiz. Yuqoridagiga o'xshash m_b, m_c medianalar uchun ushbu ifodalarni olamiz:

$$m_b = \frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2};\ m_c = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2}.$$

$m_a^2 + m_b^2 + m_c^2 = \frac{3}{4}(a^2 + b^2 + c^2)$, $m_a = \frac{1}{2}\sqrt{2sin^2B + 2sin^2C - sin^2A}$.



2.1.5-chizma.Uchburchak



2.1.6-chizma.Uchburchak medianasi.

Uchburchak burchagi bissektrisasining shu uchni uning qarshi tomondagi nuqta bilan tutashtiruvchi kesmasiga uchburchakning bissektrisasi deyiladi. Uchburchak bissektrisalarini hisoblash formulalarini keltirib chiqaramiz. a)Tomonlari $AB = c$, $AC = b$, $BC = a$ bo'lgan ΔABC da AD bissektrisani o'tkazamiz (2.1.7-chizma) va uning l_a uzunligini a, b, c orqali ifodalaymiz. Uchburchak ichki burchagi bissektrisasi xossasiga ko'ra: $\frac{BD}{DC} = \frac{AB}{AC}$ yoki $BD = \frac{ac}{b+c}$ va $DC = \frac{ab}{b+c}$ munosabatlarni olamiz. Bu qiymatlarni Stuart teoremasidagi $AC^2 \cdot BD + AB^2 \cdot DC = BD \cdot DC \cdot BC + AD^2 \cdot BC$ ifodaga keltirib qo'yamiz.

$$b^2 \cdot \frac{ac}{b+c} + c^2 \cdot \frac{ab}{b+c} = \frac{ac}{b+c} \cdot \frac{ab}{b+c} \cdot a + l_a^2 \cdot a.$$

Oxirgi ifodani a ga qisqartirib, $l_a^2 = \frac{bc(b+c)}{b+c} - \frac{a^2 \cdot b \cdot c}{(b+c)^2} = \frac{bc((b+c)^2 - a^2)}{(b+c)^2}$. Yoki

$$l_a = \frac{\sqrt{bc((b+c)^2 - a^2)}}{b+c}$$

ifodaga ega bo'lamiz. Agar yuqoridagi kabi $a + b + c = 2p$ deb belgilasak, $b + c - a = a + b + c - 2a = 2p - 2a = 2(p - a)$ bo'ladi. u holda oxirgi formula $l_a = \frac{2\sqrt{bc}}{b+c}\sqrt{p(p-a)}$ ko'rinishni oladi. Xuddi shunday

$$l_b = \frac{2\sqrt{ac}}{a+c}\sqrt{p(p-b)}\,; \; l_c = \frac{2\sqrt{ba}}{b+a}\sqrt{p(p-c)}$$

Endi uchburchak bissektrisasini uchburchakning burchaklari orqali ifoda qilamiz. b) Agar $AD = l_a$ ABC uchburchakning A burchagining bissektrisasi bo'lsa, u holda sinuslar teoremasi bo'yicha ΔABD da $\frac{l_a}{sinB} = \frac{c}{\sin(\angle ADB)}$. Bu yerda

$$\angle ADB = \pi - \left(B + \frac{A}{2}\right) = \pi - B - \frac{\pi-(B+C)}{2} = \frac{\pi}{2} - \frac{B-C}{2}$$

bo'lganidan

$$\sin(\angle ADB) = \sin\left(\frac{\pi}{2} - \frac{B-C}{2}\right) = cos\,\frac{B-C}{2}$$

kelib chiqadi. Bundan:

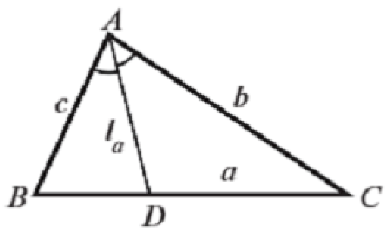
$$\frac{l_a}{sinB} = \frac{c}{cos\frac{B-C}{2}} \text{ va } l_a = \frac{csinB}{cos\frac{B-C}{2}}.$$

Bu ifodani uchburchakning burchak elementlari orqali ifoda qilsak, burchak bissektrisasi

$$l_a = \frac{sinBsinC}{cos\frac{B-C}{2}}$$

bo'ladi. Xuddi shunday

$$l_b = \frac{sinAsinC}{cos\frac{A-C}{2}}, \quad l_c = \frac{sinAsinB}{cos\frac{A-B}{2}}$$



12-Mavzu: Trigonometriyani planimetrik masalalarni yechishga tatbiqlari.

Sinuslar va kosinuslar teoremalari.

2.2.1-teorema (sinuslar teoremasi). Har qanday uchburchakning tomonlari ular qarshisidagi burchaklarning sinuslariga proporsionaldir:

$$\frac{a}{sinA}=\frac{b}{sinB}=\frac{c}{sinC}$$

Isbot. a) ΔABC ning bitta tomoni unga tashqi chizilgan aylananing markazidan o'tadi, ya'ni $AB=2R$ deb olamiz (2.2.1-chizma), bunda R –tashqi chizilgan aylananing radiusi. $\angle ACB$ diametrga tiralganligidan $\angle ACB=90°$. Sinusning ta'rifiga ko'ra $\frac{BC}{AB}=sinA$, $\frac{a}{c}=sinA$, $\frac{a}{2R}=sinA$, $\frac{a}{sinA}=2R$ bo'ladi.

Xuddi shunga o'xshash, $\frac{b}{sinB} = 2R$ ekanligini ham ko'rsatish mumkin. $sin90° = 1$ ekanligini hisobga olib, $AB = c$ gipotenuza uchun $\frac{c}{sinC} = 2R$ deb yozish mumkin. Hosil qilingan ifodalarni taqqoslab, $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} = 2R$ munosabatlarga ega bo'lamiz.

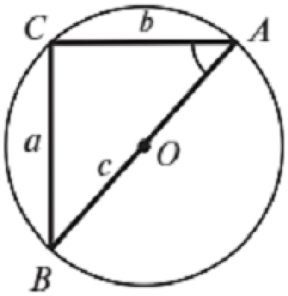
b) ΔABC ning barcha tomonlari unga tashqi chizilgan aylananing O markazidan bir tomonda yotgan bo'lsin. Uchburchakning B uchidan aylananing BA_1 diametrni o'tkazamiz (2.2.2-chizma). U vaqtda ΔA_1BC to'g'ri burchakli bo'ladi va $A_1B = 2R$. ΔA_1BC dan $BC = 2Rsin\angle BA_1C$ bo'lishini topamiz. Lekin $\angle BA_1C$ va $\angle BAC$ lar aylanaga ichki chizilgan va BC tomonga tiralgan bo'lganligi uchun o'zaro teng,

ya'ni $\angle BA_1C = \angle BAC$. Demak, $BC = 2RsinA$ va $\frac{a}{sinA} = 2R$. Qolgan tengliklar ham shunga o'xshash isbotlanadi.

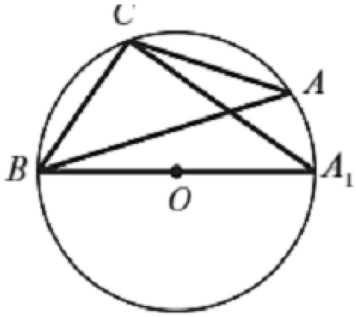
d) ΔABC ga tashqi chizilgan aylananing O markazi ΔABC ning ichida yotgan holni qaraymiz (2.2.3-chizma). Uchburchakning B uchidan aylananing BA_1 diametrini o'tkazamiz hamda A_1 va C nuqtalarni tutashtiramiz. $A_1B = 2R$ bo'lganligidan ΔA_1BC –to'g'ri burchakli bo'ladi va aylanaga ichki chizilgan burchaklar sifatida $\angle BA_1C = \angle BAC$ bo'ladi. ΔA_1BC dan $BC = A_1BsinA$ yoki $BC = 2RsinA$ bo'ladi. Demak, $\frac{a}{sinA} = 2R$ .Qolgan $\frac{b}{sinB} = 2R$, $\frac{c}{sinC} = 2R$ tengliklar ham shunga o'xshash isbotlanadi.

2.2.1-natija. Ixtiyoriy uchburchak tomonining shu tomon qarshisidagi burchak sinusiga nisbati bu uchburchakka tashqi aylana diametriga teng.

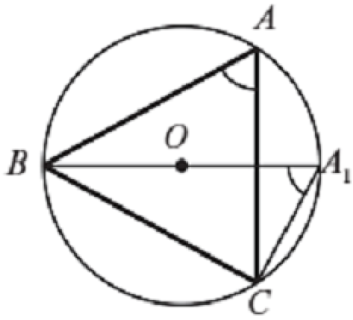
$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} = 2R$$



2.2.1-chizma.



2.2.2-chizma.



2.2.3-chizma.

(2.2.1-chizma), (2.2.2-chizma) va (2.2.3-chizma) lar sinuslar teoremasidan

3.2.2-teorema (kosinuslar teoremasi). Uchburchak istalgan tomonining kvadrati qolgan ikki tomon kvadratlari yig'indisidan, shu ikki tomon bilan ular orasidagi burchak kosinusining ikkilangan ko'paytmasini ayirish natijasiga teng:

$a^2 = b^2 + c^2 - 2bc \cdot cosA$
$b^2 = a^2 + c^2 - 2ac \cdot cosB$
$c^2 = a^2 + b^2 - 2ab \cdot cosC$

Isboti. Uchta holni qarab chiqamiz. 1- hol. a) $\angle A$ o'tkir, ya'ni $\angle A < 90°$ va $\angle B$ o'tkir burchak bo'lsin. Uchburchakning C uchidan $CD \perp AB$ o'tkazamiz (2.2.4 a-chizma). U vaqtda $AD = b_c$ va $DB = a_c$ lar $AC = b$ va $BC = a$ tomonlarining $AB = c$ tomonga proyeksiyasidan iborat bo'ladi. $CD = h_c$ deb belgilaymiz.

To'g'ri burchakli ΔBCD va ΔACD lardan, pifagor teoremasiga ko'ra,

$$\begin{cases} a^2 = h_c^2 + a_c^2 \\ h_c^2 = b^2 - b_c^2 \end{cases}$$

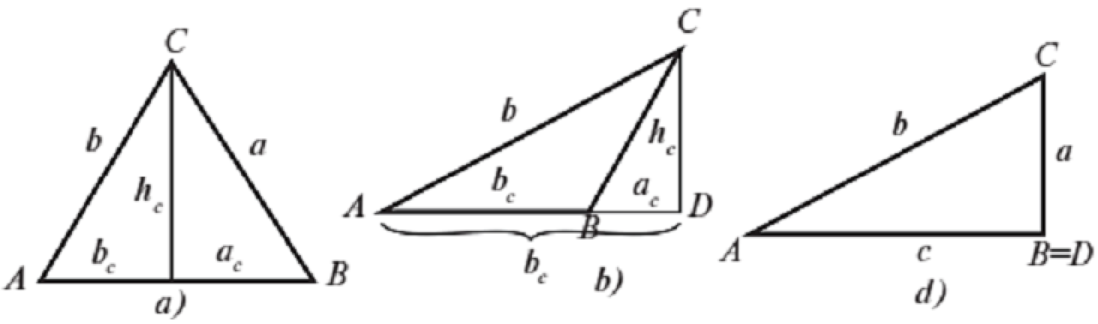
munosabatlarni olamiz. h_c ning qiymatini birinchi ifodaga qo'yib, $a_c = c - b_c$ munosabatdan foydalangan holda, $a^2 = b^2 - b_c^2 + (c - b_c)^2$ ifodani olamiz yoki $a^2 = b^2 - b_c^2 + c^2 - 2 \cdot c \cdot b_c + b_c^2 = b^2 + c^2 - 2 \cdot \mathrm{c} \cdot b_c$ bo'ladi. ΔACD da b_c kesma $\angle A$ ga yopishgan katet bo'lganligidan $b_c = b \cdot cosA$. Olingan qiymatni a^2 ning ifodasiga keltirib qo'ysak, talab qilingan $a^2 = b^2 + c^2 - 2 \cdot a \cdot c \cdot cosA$ tenglikni olamiz.

b) $\angle A -$ o'tkir, $\angle B -$o'tmas burchak bo'lgan holni qaraymiz. Bu holda $b_c = AD = c + a_c$, $a_c = BD$ bo'ladi (2.2.4 b-chizma). To'g'ri burchakli ΔBCD va ΔACD lardan, pifagor teoremasiga ko'ra

$$\begin{cases} a^2 = h_c^2 + a_c^2 \\ h_c^2 = b^2 - AD^2 = b^2 - (c + a_c)^2 \end{cases}$$

munosabatlarni olamiz. Bundan, $a^2 = b^2 - (c + a_c)^2 + a_c^2 = b^2 - c^2 - 2 \cdot c \cdot a_c$ bo'ladi. to'g'ri burchakli ΔBCD dan $AD = b \cdot cosA$ yoki $c + a_c = b \cdot cosA$ bo'lishi kelib chiqadi. U vaqtda $a_c = b \cdot cosA - c$ bo'ladi va a^2 uchun olingan ifodaga keltirib qo'ysak, $a^2 = b^2 - c^2 - 2c(b \cdot cos\angle A - c) = b^2 - c^2 - 2b \cdot c \cdot cosA + 2c^2$ va $a^2 = b^2 + c^2 - 2b \cdot c \cdot cosA$ bo'ladi.

d) $\angle A$ – o'tkir, $\angle B$ –to'g'ri burchak bo'lgan holni qaraymiz. Bu holda CB va CD kesmalar bir-biriga teng bo'ladi va $a_c = 0$ (2.2.4 d-chizma). To'g'ri burchakli ΔABC dan Pifagor teoremasiga ko'ra $a^2 = b^2 - c^2$ bo'ladi. bu ifodani quyidagicha yozamiz: $a^2 = b^2 + c^2 - 2c^2 \cdot c$ katet $\angle A$ ga yopishganligini hisobga olsak, $c = b \cdot cosA$ bo'ladi va natijada talab qilingan, $a^2 = b^2 + c^2 - 2b \cdot c \cdot cosA$ tenglikka ega bo'lamiz. Shunday qilib, bu holda ham kosinuslar teoremasi o'rinli bo'lar ekan.



2.2.4-chizma.Kosinuslar teoremasidan.

2-hol. $\angle A$ –o'tmas burchak, ya'ni $\angle A > 90°$ bo'lgan holni qaraymiz. $CD \perp AB$ to'g'ri chiziqni o'tkazamiz va $BD = a_c$, $AD = b_c$ deb belgilaymiz. To'g'ri burchakli ΔACD va ΔBCDifagor teoremasiga ko'ra $h_c^2 = b^2 - b_c^2$, $h_c^2 = a^2 - (c + b_c)^2$ munosabatlarni olamiz. Olingan tengliklarning o'ng tomonlarini

tenglashtirib, $b^2 - b_c^2 = a^2 - (c + b_c)^2$, $b^2 - b_c^2 = a^2 - c^2 - 2cb_c - b_c^2$ ifodalarga ega bo'lamiz. Ulardan $a^2 = b^2 + c^2 + 2cb_c$ ifoda kelib chiqadi. To'g'ri burchakli ΔACD ni qaraymiz. $\angle CAD = 180° - \angle A$ $\cos(180° - \angle A) = -b\cos\angle A$ bo'ladi. Shunday $a^2 = b^2 + c^2 - 2b \cdot c \cdot \cos\angle A$.

3-hol. $\angle A$ –to'g'ri burchak, ya'ni $\angle A = 90°$ bo'lgan holni qaraymiz. $\angle A$ = bo'lganligidan, $BC = a$ tomon ΔABC ning gipotenuzasidir va Pifagor teorem ko'ra, $a^2 = b^2 + c^2 - 2bc\cos\ \angle A$ ko'rinishda yozish mumkin. Teorema isbotlandi.

Kosinuslar teoremasidan uchburchak burchagining kosinusini tomonlari orqali ifoda etishimiz mumkin: $cosA = \frac{b^2+c^2-a^2}{2bc}$. Buni hisoblashda, $\frac{C}{2}$ burchaklarning trigonometrik funksiyalar qiymatlarini logarifm jadvalidan t mumkin. Yarim burchak kosinusi formulasidan

$$\cos\frac{A}{2} = \sqrt{\frac{1+cosA}{2}} = \sqrt{\frac{2bc+b^2+c^2-a^2}{4bc}} = \sqrt{\frac{(a+b+c)(b+c-a)}{4bc}},$$ bunda $a + b + c$ =

shuning uchun

$$\cos\frac{A}{2} = \sqrt{\frac{p(p-a)}{bc}},\quad \cos\frac{B}{2} = \sqrt{\frac{p(p-b)}{ac}},\quad \cos\frac{C}{2} = \sqrt{\frac{p(p-c)}{ac}}.$$

Yarim burchak sinusi formulasidan

$$\sin\frac{A}{2} = \sqrt{\frac{1-cosA}{2}} = \sqrt{\frac{1-\frac{b^2+c^2-a^2}{2bc}}{2}} = \sqrt{\frac{2bc-b^2-c^2+a^2}{4bc}} =$$

$$= \sqrt{\frac{a^2-(b^2-2bc+c^2)}{4bc}} = \sqrt{\frac{(p-b)(p-c)}{bc}}.$$

Shu xilda

$$\sin\frac{B}{2} = \sqrt{\frac{(p-a)(p-c)}{ac}},\quad \sin\frac{C}{2} = \sqrt{\frac{(p-a)(p-b)}{ab}}.$$

Shularga asoslanib,

$$tg\frac{A}{2} = \sqrt{\frac{(p-b)(p-c)}{p(p-a)}},\ tg\frac{B}{2} = \sqrt{\frac{(p-a)(p-c)}{p(p-b)}} \text{ va } tg\frac{C}{2} = \sqrt{\frac{(p-a)(p-b)}{p(p-c)}}$$

kelib chiqadi.

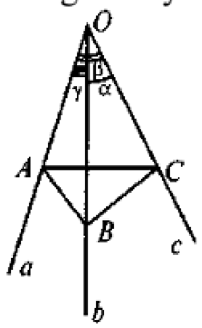
13-Mavzu: Trigonometriyani streometrik masalalarni yechishga tatbiqlari.

Uch yoqli burchak

Fazodagi ixtiyoriy *O* nuqtadan bitta tekislikda yotmaydigan uchta *α, b, c* yarim to'g'ri chiziq o'tkazilgan bo'lsin. Bu yarim to'g'ri chiziqlar juft-juft ravishda uchta (αb), (be), (αc) yassi burchak tashkil qiladi (15.15- chizma).

8-1 a' r i f. *Uchta yassi burchakdan va har bir yarim to'g'ri chiziqlar juftlari orasidagi yarim tekisliklarning qismlaridan tashkil topgan shakl* ***uch yoqli burchak*** *deyiladi.*

S — uch yoqli burchakning *uchi, a, b, c* yarim to'g'ri chiziqlar uning *qirralari,* tekis burchaklar va qirralar bilan chegaralangan tekisliklar qismlari uch yoqli burchakning *yoqlari (tomonlari)* deyiladi. Uch yoqli burchaklar tomonlarining (yoqlarining) har bir jufti ikki yoqli burchak hosil qiladi. Ular *a* qirradagi, b qirradagi va *c* qirradagi ikki yoqli burchaklardir.



15.15- chizma.

10- t e o r e m a (kosinuslar formulasi). ***Agar α, β, γ — uch yoqli burchakning yassi burchaklari, A, B, C — ular qarshisidagi ikki yoqli burchaklar bo'lsa,***

$$\cos\gamma = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta\cdot\cos C$$

munosabat bajariladi.

I s b o t i. Uch yoqli burchakning *c* qirrasida ixtiyoriy *C* nuqtani olamiz va $CB\perp c$, $CA\perp c$ to'g'ri chiziqlarni o'tkazamiz

(15.15-chizma), bunda *A* va B nuqtalar *CA* va CB perpendikularlarning *a* va *b* qirralar bilan kesishgan nuqtalaridir. *A* va B nuqtalarni tutashtirib, ΔABC ni hosil qilamiz. Kosinuslar teoremasiga ko'ra, ΔABC dan $AB^2 = AC^2 + BC^2 - 2AC\cdot BC\cdot\cos C$ va ΔABO dan

$$AB^2 = AO^2 + BO^2 - 2AO\cdot BO\cdot\cos\gamma$$

munosabatlarga ega bo'lamiz. Bu tengliklarning ikkinchisidan birinchisini ayiramiz:

$$AO^2 + BO^2 - AC^2 - BC^2 + 2AC\cdot BC\cdot\cos C - 2AO\cdot BO\cdot\cos\gamma = 0. \quad (1)$$

ΔABC va ΔABO to'g'ri burchakli bo'lganligidan,

$$AO^2 - AC^2 = OC^2 \text{ va } BO^2 - BC^2 = OC^2 \qquad (2)$$

bo'ladi. U holda (1) va (2) tengliklardan $AO\cdot BO\cdot\cos\gamma = OC^2 + AC\cdot BC\cdot\cos C$ ifodani hosil qilamiz. Lekin

$$\frac{OC}{AO} = \cos\beta,\quad \frac{OC}{BO} = \cos\alpha,\quad \frac{AC}{AO} = \sin\beta,\quad \frac{BC}{BO} = \sin\alpha$$

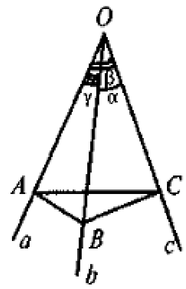
ekanligini hisobga olsak, talab qilingan

$$\cos\gamma = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta\cdot\cos C \qquad (3)$$

formulani olamiz. (3) tenglik uch yoqli burchak uchun *kosinuslar formulasi* deyiladi.
11-teorema (sinuslar formulasi). ***Agar α, β, γ — uch yoqli burchakning yassi burchaklari, A, B, C — ular qarshisidagi ikki yoqli burchaklar bo'lsa*** (15.16- chizma),

$$\frac{\sin\alpha}{\sin A}=\frac{\sin\beta}{\sin B}=\frac{\sin\gamma}{\sin C} \quad (4)$$

tenglik bajariladi.



15.16- chızma.

I s b o t i. (3) kosinuslar formulasidan cosC ni topamiz:

$$\cos C=\frac{\cos\gamma-\cos\alpha\cdot\cos\beta}{\sin\alpha\cdot\sin\beta}.$$

Endi bizga ma'lum formuladan

$$\sin^2 C=1-\cos^2 C=1-\frac{(\cos\gamma-\cos\alpha\cdot\cos\beta)^2}{\sin^2\alpha\cdot\sin^2\beta}=$$

$$=\frac{\sin^2\alpha\cdot\sin^2\beta-(\cos\gamma-\cos\alpha\cdot\cos\beta)^2}{\sin^2\alpha\cdot\sin^2\beta}=$$

$$=\frac{(1-\cos^2\alpha)(1-\cos^2\beta)-(\cos\gamma-\cos\alpha\cdot\cos\beta)^2}{\sin^2\alpha\cdot\sin^2\beta}=$$

$$=\frac{1-\cos^2\alpha-\cos^2\beta-\cos^2\gamma+2\cos\alpha\cdot\cos\beta\cdot\cos\gamma}{\sin^2\alpha\cdot\sin^2\beta}$$

bo'lishi kelib chiqadi. Oxirgi tenglikning ikki tomonini sin^ ga bo'lamiz:

$$\frac{\sin^2 C}{\sin^2\gamma}=\frac{1-\cos^2\alpha-\cos^2\beta-\cos^2\gamma+2\cos\alpha\cdot\cos\beta\cdot\cos\gamma}{\sin^2\alpha\cdot\sin^2\beta\cdot\sin^2\gamma}. \quad (5)$$

(5) tenglikning o'ng tomoni α, β, γ miqdorlarga nisbatan simmetrikdir. Agar nisbatlarni ham hisoblasak,

$$\frac{\sin^2 A}{\sin^2\alpha} \text{ va } \frac{\sin^2 B}{\sin^2\beta}$$

o'ng tomonda (5) ningo'ng tomonidagi ifodani hosil qilamiz. Shu sababli bu nisbatlar o'zaro teng:

$$\frac{\sin^2 A}{\sin^2\alpha}=\frac{\sin^2 B}{\sin^2\beta}=\frac{\sin^2 C}{\sin^2\gamma}.$$

(4) formula *sinuslar formulasi* deyiladi.
Natijalar: 1. *Uchyoqliburchakningharbiryassiburchagi uning qolgan ikkita yassi burchagi yig'indisidan kichik.*
2. Uch yoqli burchak yassi burchaklarining yig'indisi 360° dan kichik.

14-Mavzu: Trigonometriyani streometrik masalalarni yechishga tatbiqlari.

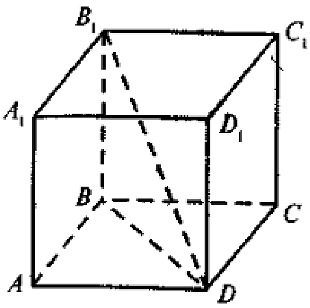
Stereometriyaning eng muhim obyektlari hech qanday tekislikda yotmaydigan fazoviy jismlar, masalan, shar, sfera, kub, parallelepiped, prizma, piramida, konus, silindr kabilar hisoblanadi. Geometrik jismlarning katta gunihini ko'pyoqlar tashkil qiladi.

1. Ko'pyoqlar. Sirti chekli sondagi ko'pburchaklardan iborat jism *ko'pyoq* deyiladi. Ko'pyoqni chegaralovchi ko'pburchaklar uning *yoqlari* deyiladi. Ko'pyoq qo'shni yoqlarining umumiy tomonlari uning *qirralari* deyiladi. Ko'pyoqning bitta nuqtada uchrashadigan yoqlari ko'p yoqli burchak tashkil qiladi va bunday ko'p yoqli burchaklarning uchlari ko'pyoqning *uchlari* deyiladi. Ko'pyoqning bitta yog'ida yotmagan ixtiyoriy ikkita uchini tutashtiruvchi to'g'ri chiziqlar uning *diagonallari* deyiladi. O'zining har bir yog'i tekisligining bir tomonida joylashgan ko'pyoq *qavariq ko'pyoq* deyiladi. Masala prizma, kub, parallelepiped, piramida qavariq ko'pyoqlardir.

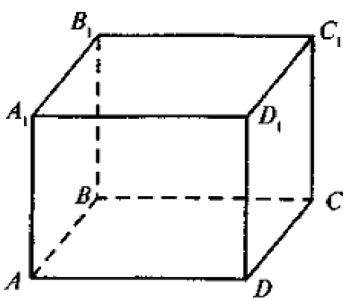
Endi ko'pyoqlarning ba'zilarini qarab chiqamiz.

Kub — barcha yoqlari kvadratlardan iborat ko'pyoqdir. Kubning yon yoqlari kesishadigan AA_1, BB_1, CC_1, DD_1 kesmalar kubning *yon qirralari,* $AB, BC, CD, DA, A_1B_1, B_1C_1, C_1D_1, D_1A_1$ lar esa kub *asoslarining qirralari* deyiladi (13.2-chizma). Kubning uchta yog'i kesishadigan $A, B, C, D, A_1, B_1, C_1, D_1$ nuqtalar uning *uchlari* deyiladi.

Parallelepiped — barcha yoqlari parallelogrammlardan iborat ko'pyoqdir (13.3- chizma). Yon yoqlari to'g'ri to'rtburchaklardan iborat parallelepiped to'g'ri parallelepiped, hamma yoqlari to'g'ri to'rtburchaklardan iborat parallelepiped *to'g'ri burchakli parallelepiped* deyiladi. Parallelepipedning qirralari va uchlari tushunchalari kubniki kabidir.



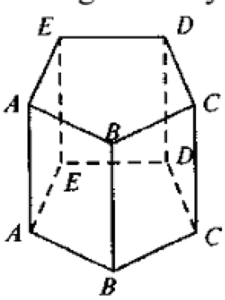
13.2- chizma.

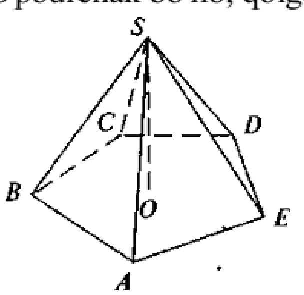


13.3- chizma.

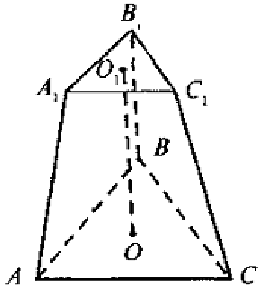
Prizma — asoslar deb ataladigan ikki yog'i parallel tekisiiklarda yotuvchi, qolgan yoqlari parallelogrammlardan iborat ko'pyoqdir (13.4- chizma). Yon yoqlari asosga perpendikular bo'lsa, prizma *to'g'ri prizma* deyiladi. Asoslari muntazam ko'pburchaklar bo'lib, yon yoqlari to'g'ri to'rtburchaklardan iborat bo'lgan prizma *muntazam prizma* deyiladi.

Piramida — asos deb ataladigan bitta yog'i ixtiyoriy ko'pburchak bo'lib, qolgan





Agar piramidaning asosi muntazam ko'pburchak bo'lib, piramidaning *SO* balandligi asosining markazi orqali o'tsa, piramida *muntazam piramida* deyiladi. Muntazam piramida yon yog'ining balandligi uning *apofemasi* deyiladi. Agar piramida asosiga parallel tekislik bilan kesilsa va uning $ABCA_1B_1C_1$ qismi qaralsa (13.6- chizma), bu qism *kesik piramida*deyiladi.



13.6- chizma.

Ko'pyoqning yoqlari soni *Y,* uchlari soni *U va* qirralari soni *Q* lar orasidagi bog'liqlik quyidagi teorema orqali beriladi.
1-t eorema (Eyler). ***Ixtiyoriy n yoq uchun Y+ U-Q=2 munosabat bajariladi.***
Quyidagi jadvaldan buni yaqqol ko'rish mumkin:

Ko'pyoq	Y	U	Q
Tetraedr	4	4	6
Parallelepiped	6	8	12
Olti burchakli prizma	8	12	18
O'n bir yoq	11	11	20
O'n ikki yoq	12	18	28

2- t eorema. ***Ko'pyoq tekis burchaklarining soni uning qirralari sonidan ikki marta ko'p.***
Nat i j alar.
1. *Ko 'pyoq tekis burchaklarining soni har doim juftdir.*
2. *Agar ko 'pyoqning har bir uchida bir xil k sondagi qirralar tutashsa,*

$$U\cdot k = 2Q$$

munosabat o 'rinli.
3. *Agar ko'pyoqning barcha yoqlari bir xil n tomonli ko 'pburchaklardan tashkil topgan bo 'Isa,*

$$Y\cdot n = 2Q$$

munosabat o 'rinli.
3-1 teorema. ***Yoqlari soni Y va qirrralari soni Q bo'lgan ko'pyoq tekis burchaklarining yig'indisi uchun***

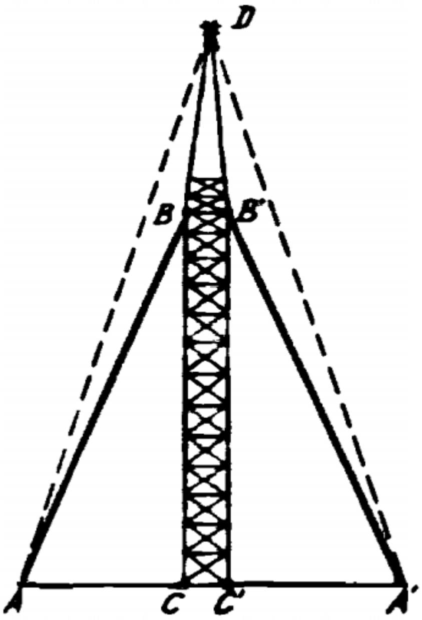
$$360°\ (Y - Q)$$

munosabat bajariladi.
Agar ko'pyoq modelini tayyorlash talab qilinsa, u tekis ko'pburchaklarni — ko'pyoqning yoqlarini bir-biriga yopishtirish natijasida hosil qilinadi. Bunda faqat ko'pburchaklar majmuyiga ega bo'libgina qolmasdan, qaysi ko'pburchaklarni o'zaro yopishtirish zarurligini ham bilish lozim bo'ladi.Biror ko'pyoq yoqlariga teng ko'pburchaklar majmuyi, qaysi tarafini, mos ravishda, yopishtirish kerakligi ko'rsatilgan holda, ko'pyoqning yoyilmasi deyiladi.Ko'pyoq berilganda uning yoyilmasini yasash mumkin. Teskari masala esa, ya'ni berilgan yoyilma bo'yicha ko'pyoqni yasash, quyidagi shartlar bajarilganda yechimga ega bo'ladi:
1) yoyilmaning har bir tomoniga qolgan tomonlarning faqat bittasi mos kelishi;
2) agar α va β yoqlari umumiy *A* uchga ega bo'lsa, qolgan yoqlardan faqat o'sha A uchga ega bo'lganlarini tanlab olish zarur;

15-Mavzu: Trigonometriyani tatbiqiga doir aralash geometrik masalalar.

QO'SHIMCHA MASALALAR

166. Radiomachta AB, AD troslar bilan mahkamlangan. A nuqta machta asosidan 75 m uzoqlikda joylashgan va $\angle ABD=150°$, $BD=24$ m bo'lsa, AD trosning uzunligini toping (61- rasm).



61- rasm.

167. ABC to'g'ri burchakli uchburchakning AC gipotenuzasi 16 sm va katetlaridan biri 11 sm. Uning o'tkir burchaklari sinus va kosinuslari qiymatlarini toping.

168. To'g'ri burchakli uchburchakning katetlari 8 va 9,5 m ga teng bo'lsa, shu katetlarga yopishgan burchaklarning kosinuslarini toping.

169. Teng tomonli uchburchakning burchaklari kosinuslari tengligini isbotlang.

170. Agar to'g'ri burchakli uchburchak burchagining sinusi ma'lum bo'lsa, uning kosinusini topish mumkinmi? Misollar keltiring.

171. Quyidagi ma'lumotlarga ko'ra to'g'ri burchakli uchburchakning noma'lum tomonini, o'tkir burchaklar sinuslar va kosinuslarini toping:

1) ikki kateti bo'yicha:

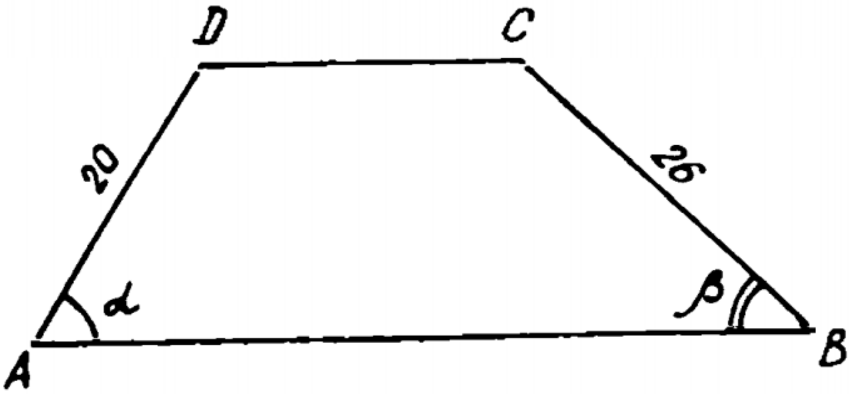
a) $a=3$, $b=4$; e) $a=7$, $b=8$;
b) $a=15$, $b=20$; f) $a=6$, $b=8$;
d) $a=9$, $b=11$; g) $a=5$, $b=8$;

2) gipotenuzasi va bir kateti bo'yicha:

a) $c=13$, $a=5$; e) $c=9$, $b=4$;
b) $c=10$, $a=5$; f) $c=60$, $b=25$;
d) $c=13$, $a=10$; g) $c=12$, $b=5$.

172. *ABCD* trapetsiya $DC \parallel AB$, $AD=20$ sm va $BC=26$ sm. Agar $\cos\alpha = 0{,}5$ bo'lsa, trapetsiyaning balandligini va $\cos\beta$ ni toping (62- rasm).

173. 1) $\cos\alpha = \frac{4}{7}$; 2) $\sin\alpha = \frac{4}{7}$; 3) $\cos\alpha = 0{,}5$ bo'lgan burchakni yasang.



62- rasm.

174. l uzunlikdagi kesma a to'g'ri chiziqqa l_1 uzunlikda proyeksiyalanadi va a to'g'ri chiziq bilan α ($\alpha \leq 90°$) burchak hosil qiladi. l, l_1 va α n i bog'lovchi ifodalarni yozing.

175. 174- masalada α ning qanday qiymatida kesma proyeksiyasi eng katta qiymatiga erishadi?

176. Burchak kosinusidan foydalanib, quyidagi masalani yeching: teng yonli uchburchakda a – asos, b – yon tomon, h_a – asosga tushirilgan balandlik, α – asosidagi burchaklar. Agar:

a) h_a va b; e) b va α;

b) h_a va a; f) b va β;

d) a va b;

ma'lum bo'lsa, qolgan noma'lum kattaliklarni toping (63- rasm)

177. To'g'ri burchakli uchburchak katetlarining nisbati ma'lum bo'lsa, uning burchaklari kosinuslarini toping.

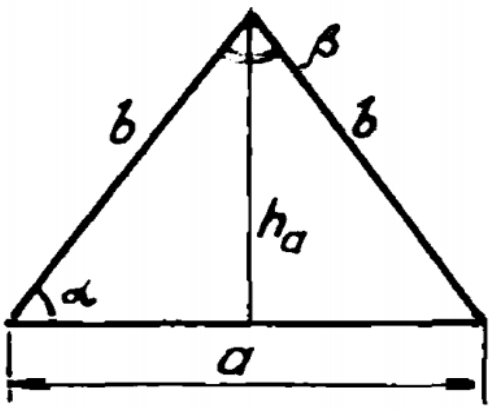
178. Vertikal turgan to'sinning balandligi 21 m, soyasi esa 7 m ga teng. Quyoshning gorizontdan balandligini graduslarda ifodalang.

179. Agar teng yonli trapetsiyaning tomonlari ma'lum bo'lsa, uning burchaklarini toping.

180. Teng yonli trapetsiyaning diagonali va uning asos bilan tashkil qilgan burchagi ma'lum bo'lsa, yuzini hisoblang.

181. Ikki medianasi teng bo'lgan uchburchak teng yonli uchburchak ekanini isbotlang.

182. Uchburchakning a, b, c tomonlari berilgan. Shu tomonlarga o'tkazilgan m_a, m_b, m_c medianalarni toping.



63-rasm.

183. Uchburchakning katta medianasi uning kichik tomoniga va kichik medianasi katta tomoniga o'tkazilishini isbotlang.

184. α va β burchaklari hamda a masofa bo'yicha binoning x balandligini qanday topish mumkinligini tushuntiring (64-rasm).

185. Parallelogrammning c va d diagonallari hamda ular orasida α burchak berilgan. Parallelogrammning tomonlarini toping.